CHAPTER 2

FINITE ELEMENT ANALYSIS AND ANALYTICAL METHOD BASED PERFORMANCE PREDICTION OF SWITCHED RELUCTANCE MACHINE

2.1 Introduction

The design of SR motors must suit certain specific requirements with regard to rating, torque, peak current, temperature rise, etc. The power supply requirements and the intended use of the motor as a fixed or variable speed drive are to be considered. These requirements coupled with the need for design optimization necessitate accurate performance prediction at the design stage. The requirement for predicting the performance characteristics of the SRM is to generate the relationships between the flux linkages vs. rotor position as a function of the machine phase currents (Krishnan 2001). The area enclosed between the curves flux linkage vs. current for unaligned and aligned positions of the rotor poles with a set of stator poles gives the maximum work done for one stroke of the motor. The electromagnetic torque can then be calculated if the angular movement is known from the mechanical work. For a fixed supply voltage, phase current is dependent on the flux-linkage or inductance at given rotor position. Furthermore, SRMs are commonly operated during magnetic saturation and their excitation waveforms are non-sinusoidal. Hence, precise measurement of the magnetic flux-linkage characteristics is important to verify machine design and for accurately predicting the performance of SRM drives.
In this study two methods have been used to predict the performance of SRM. One is based on FEA (Arumugam et al 1985, Lindsay et al 1986, Kôibuchi et al 1997) and the other is based on analytical method (Praveen 2001). This chapter discusses FEA and analytical design procedures for predicting the performance of SR machine.

2.2 FINITE ELEMENT ANALYSIS OF SRM

2.2.1 Outline of FEA

With the advent of modern digital computers, numerical methods have become practical in engineering design and are nowadays, extensively being used to improve the accuracy of magnetic field computations. FEA has emerged in the past decade as a useful numerical method for magnetic field analysis of electrical machines.

The method is based on formulating the magnetic field equations in terms of magnetic vector potential. The solution is obtained by reformulating the resulting partial differential equation using variation terms and extremizing the associated energy functional by a set of trial functions. To implement the variational formulation, the entire problem region is divided into numerous sub regions called finite elements. In this thesis triangular elements are used. Inside each elemental triangle the magnetic vector potential is linearly interpolated by a first order polynomial from the three vertex values. The complex motor configuration and the nonlinear material characteristic are taken into account in FEA. The following assumptions are made to determine the magnetic field distribution inside the motor:

(a) The magnetic field outside the motor periphery is negligible and hence the outer periphery of the motor can be treated as a zero vector potential line.
(b) The magnetic material of the stator and rotor cores is isotropic and the magnetization curve is single valued, i.e., hysteresis effects are neglected.

(c) The magnetic vector potential, $A$ and the current density vector, $J$ have only axially directed components and are invariant in that direction.

(d) The magnetic field distribution is constant along the axial direction of the motor.

(e) The electromagnetic field is quasistationary, i.e., displacement currents are neglected.

(f) Time harmonic effects are absent.

(g) In the two dimensional analysis the end effects are neglected.

The fundamental laws governing all electromagnetic fields can be expressed by the well known Maxwell’s equations. With reference to the above assumptions the Maxwell’s equation become,

\[ \text{Curl } H = J \]  \hspace{1cm} (2.1)

\[ \text{Div } B = 0 \]  \hspace{1cm} (2.2)

The magnetic flux density $B$ in a magnetic material can be given as

\[ B = \mu H = \frac{H}{\gamma} \]  \hspace{1cm} (2.3)

Where $H$ is the magnetic field intensity, $\mu$ is the permeability of the magnetic material and $\gamma$ is the reluctivity of the magnetic material.
From Equations (2.1) and (2.3)

\[ \text{Curl } (B) = \mu J \]  \hspace{1cm} (2.4)

By defining the magnetic vector potential, A as

\[ B = \text{Curl } A \] \hspace{1cm} (2.5)

\[ \text{Curl } (\text{Curl } A) = \mu J \] \hspace{1cm} (2.6)

Setting \( \nabla \cdot A = 0 \), and from the vector identity for the curl of the curl of the vector

\[ \nabla (\nabla \cdot A) - \nabla^2 A = \mu J \] \hspace{1cm} (2.7)

This implies that

\[ \nabla^2 A = -\mu J \] \hspace{1cm} (2.8)

Since A has only z-directed components the above expression can be written as

\[ \frac{\partial}{\partial x} \left( \frac{\partial A}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial A}{\partial y} \right) - \mu J \] \hspace{1cm} (2.9)

The solution of (2.9) yields the magnetic vector potential A inside the motor using appropriate boundary conditions. The nonlinearity of the magnetization curve of the iron and the complex geometry of the stator and rotor do not permit an analytical solution for the magnetic vector potential and hence numerical methods are chosen. The solution is obtained using an interpolation technique by minimizing the nonlinear energy functional,
\[ F = \int_{\Omega} \int_{R} (H \cdot dB - J \cdot A) \, dR \]  \hspace{1cm} (2.10)

where \( R \) is the problem region of integration.

In order to search efficiently, the entire problem region is subdivided into triangular finite elements that coincide with the boundary of each material. Smaller elements are used where the magnetic flux density is expected to change very much. Such regions are the pole tips and the air gap between the overlapping poles.

### 2.2.2 MagNet Package

The FEA is performed using Computer Aided Design (CAD) package MagNet. It involves preprocessing, problem editing, solving and post processing operations.

Preprocessing comprises the specification and subdivision of the motor geometry to be analyzed. With the knowledge of the main dimensions, the motor geometry that could be used for analysis is determined based on the geometric and magnetic symmetry that exists for a given rotor position. Preprocessing also includes, the material characteristic modeling and setting up boundary conditions. Cubic hermite interpolation polynomials are used to represent the B-H curve of the core material. The material library contains the database of hundreds of predefined materials that can be applied to the model.

The subdivision of the problem region is done using triangular elements of arbitrary shape and size with the restriction that every single element shall be entirely in one region of the same material and that the union of all elements shall create a problem region without overlapping.

Problem editing requires the specification of the excitation in the winding region. For the winding region, the excitation in ampere conductors
and its polarity are given as the input. The magnetization characteristic for the core material, the type of solver to be used and the tolerance during numerical iterations are also specified during problem editing.

Once the problem is edited, then the CAD package does not require user interaction during the solution process. The output of the solver will be a digital version of the magnetic field. Further manipulations are required to extract engineering information from the mathematical solutions.

During post processing, the mathematical solution which is in the form of the nodal potential values is processed to obtain specific results such as the flux linkages, inductance, torque, etc.

### 2.2.3 Analysis of 8/6 Motor

The schematic diagram of 8/6 SRM is shown in Figure 2.1 and its dimensions are given in Table 2.1. The finite element subdivisions of an 8/6 SRM at aligned position is shown in Figure 2.2. The field solution is obtained by solving Equation (2.9), satisfying the assumptions made earlier.

**Figure 2.1 Schematic of 8/6 SRM**
Table 2.1 Main dimensions of the sample Motor

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator pole arc $\beta_s$</td>
<td>21 degrees</td>
</tr>
<tr>
<td>Rotor pole arc $\beta_r$</td>
<td>24 degrees</td>
</tr>
<tr>
<td>Air gap length $g$</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Stator diameter $D_0$</td>
<td>90 mm</td>
</tr>
<tr>
<td>Bore diameter $D$</td>
<td>48 mm</td>
</tr>
<tr>
<td>Stack length $L$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Shaft diameter $D_{sh}$</td>
<td>8.5 mm</td>
</tr>
<tr>
<td>Back iron thickness $C$</td>
<td>11.25 mm</td>
</tr>
<tr>
<td>Height of stator pole $h_s$</td>
<td>9.25 mm</td>
</tr>
<tr>
<td>Height of rotor pole $h_r$</td>
<td>9 mm</td>
</tr>
<tr>
<td>Turns Per Phase</td>
<td>316</td>
</tr>
<tr>
<td>Rated current</td>
<td>4.5 A</td>
</tr>
</tbody>
</table>

The flux plots obtained for the motor in the aligned and unaligned positions of the rotor are shown in Figures 2.3 and 2.4 respectively. The plots shown are for a steady excitation of 3 Amperes in one phase winding. The flux linkages of one pole winding for varying excitation currents and rotor positions are shown in Figure 2.5. It is seen that flux linkages increase
monotonically with excitation but decrease as the rotor moves away from the aligned position. The area under each curve gives the coenergy for a given rotor position. As the torque developed depends on the change in coenergy during rotor movement from one position to the other, the areas between the curves are important for the motor design.

Figure 2.3 Flux plot at aligned position

Figure 2.4 Flux plot at unaligned position
2.2.4 Experimental Validation of FEA Model

To validate the FEA based model it is essential to compare the FE results with the static flux-linkage test results. Researchers have proposed wide variety of methods to experimentally determine the magnetization characteristics of SRM. The merits and demerits of each of the methods have been discussed in detail by Keunsoo Ha (2005). In this work the method proposed by (Ramanarayanan et al 1996) is applied to validate the FEA based SRM model. The dimension of the machine subjected to experimental study is given in Table 2.1.

The experimental method makes use of the voltage equation as the basis for determining the magnetization characteristics of the machine. When a voltage pulse is applied to one of the phases of SRM with all other phases open, its voltage equation is given by

\[ V = iR + \frac{d\psi}{dt} \]  

(2.11)
Where $V$ is the instantaneous voltage across the phase winding, $R$ is the resistance and $i$ is the current. The flux linkage is given by

$$\psi = \int (V - iR) \, dt$$

(2.12)

The flux-linkage can be determined for different values of current by using the Equation (2.12). The experimental setup for determining the magnetization characteristics is shown in Figure 2.6. The general practice is to apply a voltage pulse to the stator winding by clamping the rotor to a known position. The current rises up to a steady state level, and then the voltage is turned off, de-energizing the stator winding. Throughout this time, integration takes place to determine the instantaneous flux-linkage as a function of current and position. The current and voltage waveforms recorded while the rotor was locked at aligned position are shown in Figure 2.7.

Figure 2.6  Experimental setup for determining flux linkage vs. current characteristics
Figure 2.7 Waveforms recorded at aligned position

Table 2.2 shows a comparison of the flux-linkage by measurement and FEA methods at discrete points to evaluate the correlation between the results. The flux linkages obtained by FEA show higher values than those of experimental values. However the difference is small. Inspite of the same physical dimensions of the motor, differences are inevitable due to measurement errors in the experiment and tolerances in the numerical computation. The closeness of the results have confirmed and validated the FEA model.

Table 2.2 Comparison of measured flux-linkage

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Position (Degree)</th>
<th>Experimental (Wb-turns)</th>
<th>FEA (Wb-turns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
<td>0</td>
<td>0.0054</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.033</td>
<td>0.036</td>
</tr>
<tr>
<td>1.1</td>
<td>0</td>
<td>0.012</td>
<td>0.0107</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0487</td>
<td>0.055</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.0152</td>
<td>0.0184</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.0868</td>
<td>0.094</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.02567</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.135</td>
<td>0.138</td>
</tr>
</tbody>
</table>
2.3 ANALYTICAL METHOD

The performance of SRM can be computed using FEA techniques. There are some disadvantages in resorting to them. FEM calculations require much more set-up and execution time on a computer. Any change in one or many of the motor and control parameters requires an entire new FEM computation, either in two or three dimensions with a considerable amount of time (Krishnan 2001). To optimize a performance index, it is advantageous for a machine designer to know the set of implicit and explicit analytical expressions involving the motor dimensions and the inputs to performance variables. Therefore, researchers have proposed various improved analytical calculation method (Miller et al. 1990, Moallem et al. 1998, Radun 1999) capable of predicting the machine inductance at every rotor position and excitation level. The procedure outlined in this thesis is based on the method proposed by Praveen (2001). The following assumptions are valid with respect to the calculation of inductance.

1. The air gap flux lines consist of concentric arcs and straight line segments.

2. The flux lines enter and leave the iron normally.

3. The flux lines in the stator and rotor poles are parallel to the pole axes.

4. The flux lines in the stator back iron and rotor body are concentric.

5. The windings are rectangular blocks and the stator inter polar space is only partially filled with windings.

6. The shaft is purely non magnetic.
2.3.1 Calculation of Aligned Inductance

For calculating the value of inductance, a certain flux density is assumed in the stator pole and flux densities in other parts of the machine such as rotor pole, rotor back iron, stator yoke, and air gap are derived from the machine geometry and the assumed stator pole flux density. From the flux densities in various parts of the machine and the flux density vs. magnetic field intensity (B-H) characteristics of the lamination material, corresponding magnetic field intensities are obtained. Given the magnetic field intensities and the length of the flux path in each part, their product gives the magneto motive force (mmf). The mmfs for various parts are likewise obtained, and for the magnetic equivalent circuit and stator excitation ampere’s circuital law is applied. The error between the applied stator mmf and that given equivalently by various machine parts is used to adjust the assumed flux density in the stator pole. The entire iteration continuous until the error is reduced to a fixed tolerance value.

![Figure 2.8 Magnetic equivalent circuit at aligned position](image)
The flux path at aligned position is shown in Figure 2.3. About 90% of the flux lines pass through the air gap between the stator and rotor. The magnetic equivalent circuit at the aligned position is shown in Figure 2.8. The length and area of cross section of the flux path is given in Table 2.3.

**Table 2.3 Length and area of flux path at aligned position**

<table>
<thead>
<tr>
<th>Segment</th>
<th>Path Length</th>
<th>Area of cross section of flux path</th>
</tr>
</thead>
<tbody>
<tr>
<td>Airgap</td>
<td>( l_g = g )</td>
<td>( A_g = \frac{L}{2} \left( \beta_s \frac{D}{2} + \beta_s \left( \frac{D}{2} - g \right) \right) )</td>
</tr>
<tr>
<td>Stator Pole</td>
<td>( l_s = h_s + \frac{C}{2} )</td>
<td>( A_s = \beta_s L \frac{D}{2} )</td>
</tr>
<tr>
<td>Rotor Pole</td>
<td>( l_r = \frac{D}{4} - \frac{g}{2} + \frac{h_r}{2} - \frac{D_{sh}}{4} )</td>
<td>( A_r = L \beta_r \left( \frac{D}{2} - g \right) )</td>
</tr>
<tr>
<td>Rotor Core</td>
<td>( l_{rc} = \pi \left( \frac{D}{4} - \frac{g}{2} + \frac{h_r}{2} - \frac{D_{sh}}{4} \right) )</td>
<td>( A_{rc} = L \left( \frac{D}{2} - g - h_r - \frac{D_{sh}}{2} \right) )</td>
</tr>
<tr>
<td>Yoke</td>
<td>( l_y = \pi \left( \frac{D_y}{2} - \frac{C}{2} \right) )</td>
<td>( A_y = \beta_s L \frac{D}{2} )</td>
</tr>
</tbody>
</table>

Applying amperes circuital law, the mmf calculated for each path are summed and given as

\[
F_C = 2 \left[ H_s l_s + H_g l_g + H_r l_r \right] + H_{rc} l_{rc} + H_y l_y
\]  
(2.13)

The error mmf is found as

\[
\Delta F = F_F - F_C = F_{ph} i - F_C
\]  
(2.14)
The error mmf is reduced by iteration and flux and inductance are determined using the following equations

\[
\phi_a = \frac{T_{ph} i}{2 \left( R_s + R_g + R_r \right) + \frac{R_e}{2} + \frac{R_y}{2}} \quad (2.15)
\]

\[
L_a = \frac{T_{ph} \phi_a}{i} \quad (2.16)
\]

Figure 2.9 Identification of flux paths at unaligned position

2.3.2 Unaligned Inductance Calculation

For calculation of unaligned inductance seven flux paths are identified as shown in Figure 2.9. For each flux path the magnetic equivalent circuit is derived and the inductance contributed by it is calculated. Figure 2.10 describes the flow chart for calculation of unaligned inductance.
Figure 2.10 Flowchart for calculating unaligned inductance

1. **Start**
2. Assume an initial value for stator pole flux density, \( B_{sp} \).
3. Find the stator pole flux, \( \Phi_{sp} \).
4. Calculate the flux in path 1 for various machine segments and area of cross-section of the various segments encountered by flux tube 1.
5. Evaluate for flux densities in the various segments.
6. From B-H curve of the lamination material, find \( H \) for corresponding \( B \) of segments.
7. Compute the length of the flux path in each segment, \( l \).
8. Compute mmf for each segment by taking the product of its \( H \) and \( l \).
9. Derive the magnetic equivalent circuit and write Ampere’s circuital equation.
10. Compute error mmf as \( \Delta F = F \cdot \Sigma H \cdot l \) and test whether it is within Error Tolerance (ET).
11. From the final \( B_{sp} \), compute reluctances and hence inductance contributed by flux tube 1.
12. Likewise, compute inductance contributed by various flux tubes.
13. Sum the inductances contributed by all flux tubes to obtain the inductance of the winding.
14. **Stop**

\[ B_{sp} = B_{sp} \pm \Delta B_{sp} \]
2.3.3 Calculation of Average Torque

For a constant current $I_p$, the average torque is calculated from the flux linkages vs. current characteristics at aligned and unaligned position. The area between these two curves is calculated and is denoted as $\delta W_m$, as shown in Figure 2.11.

$$\delta W_m = W_m^{aligned} - W_m^{unaligned} \Delta i \left( \frac{\lambda_1 + \lambda_2 + ... + \lambda_n}{2} - \frac{1}{2} \lambda_u I_p \right)$$  \hspace{1cm} (2.17)

$$\Delta i = \frac{I_p}{n}$$  \hspace{1cm} (2.18)

Figure 2.11 Flux linkage vs. excitation current for aligned and unaligned rotor positions

The aligned co-energy is calculated using $n$ points on the $\lambda$ vs. $i$ curve with the trapezoidal integration algorithm. From the unaligned inductance value the coenergy at unaligned position is determined.
The average torque is given by

\[ T_{av} = \frac{\text{Total work done} / \text{revolution}}{2\pi} \]  \hspace{1cm} (2.19)

\[ \text{Total work done} / \text{revolution} = \frac{\delta W_{m,N_s,N_r}}{2} \]  \hspace{1cm} (2.20)

\[ T_{av} = \frac{\delta W_{m,N_s,N_r}}{4\pi} \]  \hspace{1cm} (2.21)

2.3.4 Validation of Analytical Model

The analytical computation technique is applied to predict the performance of 500 W and 5 HP 8/6 SRM with dimensions as given in Table 2.1 and Table 2.4 respectively.

Table 2.4 Main Dimensions of the 5 HP motor

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stator pole arc (\beta_s)</td>
<td>18 degrees</td>
</tr>
<tr>
<td>Rotor pole arc (\beta_r)</td>
<td>22 degrees</td>
</tr>
<tr>
<td>Air gap length (g)</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Stator diameter (D_0)</td>
<td>190 mm</td>
</tr>
<tr>
<td>Bore diameter (D)</td>
<td>100.6 mm</td>
</tr>
<tr>
<td>Stack length (L_{stk})</td>
<td>200mm</td>
</tr>
<tr>
<td>Shaft diameter (D_{sh})</td>
<td>28mm</td>
</tr>
<tr>
<td>Back iron thickness (C)</td>
<td>12 mm</td>
</tr>
<tr>
<td>Height of stator pole (h_s)</td>
<td>32.7 mm</td>
</tr>
<tr>
<td>Height of rotor pole (h_r)</td>
<td>19.8 mm</td>
</tr>
<tr>
<td>Turns per phase</td>
<td>154</td>
</tr>
<tr>
<td>Rated current</td>
<td>13 A</td>
</tr>
<tr>
<td>Lamination Material</td>
<td>M43</td>
</tr>
</tbody>
</table>
The results obtained by analytical model are compared and validated with FEA model. The flux linkage-current characteristics obtained by analytical model and FEA for both the machines are presented in Figure 2.12 and Figure 2.13 respectively. The flux linkages obtained by FEA are higher than that of the analytical values when the rotor is in the aligned position. The flux linkages in the unaligned position are almost the same. Table 2.5 presents the average torque value computed by two methods. The closeness of the results confirms and validates the analytical model.

Figure 2.12 Comparison of flux linkages for 500 W machine

Table 2.5 Comparison of average torque

<table>
<thead>
<tr>
<th>Average Torque</th>
<th>500 W motor</th>
<th>5 HP motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analytical model</td>
<td>1.31 Nm</td>
<td>23.14 Nm</td>
</tr>
<tr>
<td>FEA</td>
<td>1.38 Nm</td>
<td>23.61 Nm</td>
</tr>
</tbody>
</table>
2.4 CONCLUSION

In this chapter, the application of FEA to predict the performance of SRM is discussed. The basic equations used to get magnetic field solution, the assumptions based on which the formulation is done are given. A brief description of the use of MagNet CAD package and the various steps to be followed during SR machine analysis are given. The static modeling results of an 8/6 500 W SRM obtained by FEA are compared with experimental results. The flux linkage vs. current characteristics obtained by FEA agrees satisfactorily with the experimental values. The differences between the predicted and experimental values are very small.

Further an analytical approach to predict the performance of SR machine is described. The procedure for calculating the inductance at aligned and unaligned position, average torque and the assumptions made are discussed. To confirm the accuracy of the analytical method the results obtained by analytical approach for two 8/6 machine with different ratings are compared with FEA. From the results it is seen that the flux linkages obtained
by FEA are higher than that of the experimental and analytical values when the rotor is in the aligned position. However the difference is very small. At unaligned position the values obtained by FEA are almost the same as obtained by analytical values. Further the average torque produced by analytical method is closer to that obtained by FEA method. The results obtained confirm the application of analytical method for performance prediction of SRM. In this work both FEA and analytical approaches are applied to predict the performance of SRM during the optimization process.