APPENDIX A - UNCERTAINTY ANALYSIS

This section demonstrates the estimation of Type-B uncertainty for minichannel heat sink experiments utilizing the information provided by the manufacturers of measuring instruments and from the calibration data. The Type-B uncertainty for the measurement of volume, time, temperature and pressure drop are provided in Table A.1.

Table A.1. Uncertainties of important variables measured during the experiments on assessment of heat transfer performance of minichannel heat sink.

<table>
<thead>
<tr>
<th>Quantity measured</th>
<th>Instrument used</th>
<th>Uncertainty involved</th>
<th>Maximum uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>K type thermocouple with digital indicator</td>
<td>± 0.1 °C</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Volume of the test fluid</td>
<td>Measuring jar</td>
<td>± 0.25 mL</td>
<td>± 0.5</td>
</tr>
<tr>
<td>Time</td>
<td>Digital stopwatch</td>
<td>± 0.5 s</td>
<td>± 3.33</td>
</tr>
<tr>
<td>Pressure drop</td>
<td>Digital manometer</td>
<td>±0.003 psi</td>
<td>±0.3</td>
</tr>
</tbody>
</table>

The Type-B uncertainty \( (U_p) \) for any calculated parameter \( (p) \) was determined from the uncertainty of measured parameters \( (X_j)'s \) as follows [123]:

\[
U_p = \pm \left[ \sum_{j=1}^{N} \left( U_{X_j} \frac{\partial p}{\partial X_j} \right)^2 \right]^{0.5}
\]  

(A1)
In the above equation, $U_{Xj}$ is the Type-B uncertainty of measured parameter ‘$X_j$’.

The percentage uncertainty ($\%U_p$) was obtained from $U_p$ and the value of parameter (p) as follows:

$$\%U_p = \frac{U_p \times 100}{\text{Value of } p}$$  \hspace{1cm} (A2)

### A1.1 Volumetric flow rate

Volumetric flow rate ($\dot{V}$) was determined as the ratio of volume of fluid collected (vol) per unit time ($t$).

$$\dot{V} = \frac{\text{Volume of fluid collected}}{\text{time}} = \frac{\text{vol}}{t}$$  \hspace{1cm} (A3)

The application of Eq. (A.1) for the estimation of Type-B uncertainty for volumetric flow rate gives

$$U_v = \pm \left[ \left( U_{vol} \frac{\partial \dot{V}}{\partial \text{vol}} \right)^2 + \left( U_t \frac{\partial \dot{V}}{\partial t} \right)^2 \right]^{0.5}$$  \hspace{1cm} (A.4)

$$U_v = \pm \left[ \left( U_{vol} \frac{1}{t} \right)^2 + \left( -U_t \frac{\text{vol}}{t^2} \right)^2 \right]^{0.5}$$  \hspace{1cm} (A.5)
For a typical experimental run, vol = 45 mL and t = 15 s. Accordingly, the Type-B uncertainty in the measurement of volumetric flow rate for this experimental run was calculated to be 0.103 mL/s. In terms of percentage uncertainty this was calculated to be 3.34 %. The percentage uncertainty for volumetric flow rate ranged between 3.34 % and 3.38 % for the experimental runs.

A1.2 Pumping power

Pumping power was determined as the product of volumetric flow rate and pressure drop (Δp) as

\[ P = \dot{V} \Delta p \]  \hspace{1cm} (A.6)

\[ U_p = \pm \left( \frac{U}{\dot{V}} \frac{\partial P}{\partial \dot{V}} \right)^2 + \left( \frac{U_{\dot{V}}}{\Delta p} \right)^2 \right]^{0.5} \hspace{1cm} (A.7) \]

\[ U_p = \pm \left( \left( \frac{U}{\dot{V}} \Delta p \right)^2 + \left( \frac{U_{\dot{V}}}{\dot{V}} \right)^2 \right)^{0.5} \hspace{1cm} (A.8) \]

For a typical experimental run, \( \dot{V} = 3\times10^{-6} \text{ m}^3/\text{s} \) and \( \Delta p = 372.317 \text{ Pa} \). Accordingly, the Type-B uncertainty in the measurement of pumping power for this experimental run was calculated to be 3.78937\times10^{-5} \text{ W}. In terms of percentage uncertainty this was calculated to be 3.35 %. The percentage uncertainty for pumping power ranged between 3.35 % and 3.39 % for the experimental runs.
A1.3 Heat transfer coefficient determined

Heat transfer coefficient was determined as the ratio of heat transfer rate to the temperature difference between the average wall temperature and average fluid.

\[ h = \frac{q A_h}{(T_{\text{wall,avg}} - T_{f,avg}) A_{\text{wall}}} \]  \hspace{1cm} (A.9)

\[ U_h = \left[ \left( U_q \frac{\partial h}{\partial q} \right)^2 + \left( U_{T_{\text{wall,avg}}} \frac{\partial h}{\partial T_{\text{wall,avg}}} \right)^2 + \left( U_{T_{f,avg}} \frac{\partial h}{\partial T_{f,avg}} \right)^2 \right]^{0.5} \]  \hspace{1cm} (A.10)

\[ U_h = \left[ \left( U_q A_b (T_{\text{wall,avg}} - T_{f,avg}) \right)^2 + \left( U_{T_{\text{wall,avg}}} A_{\text{wall}} (T_{\text{wall,avg}} - T_{f,avg}) \right)^2 + \left( U_{T_{f,avg}} A_{\text{wall}} (T_{\text{wall,avg}} - T_{f,avg}) \right)^2 \right]^{0.5} \]  \hspace{1cm} (A.11)

For a typical experimental run, \( q = 51487 \text{ W/m}^2 \), \( T_{\text{wall,avg}} = 307.22 \text{ K} \), \( T_{f,avg} = 301.3 \text{ K} \), \( A_b = 0.01116 \text{ m}^2 \) and \( A_{\text{wall}} = 0.018936 \text{ m}^2 \). Accordingly, the Type-B uncertainty in the measurement of heat transfer coefficient for this experimental run was calculated to be 40.74 W/m²K. In terms of percentage uncertainty this was calculated to be 1.38%. The percentage uncertainty for heat transfer coefficient ranged between 1.38% and 2.49% for the experimental runs.

A1.4 Nusselt number

Nusselt number was determined as the ratio of product of heat transfer coefficient and hydraulic diameter to thermal conductivity.

\[ Nu_t = \frac{h D_h}{k} \]  \hspace{1cm} (A.12)
\[ U_{Nu} = \pm \left[ \left( U_h \frac{\partial N_u}{\partial h} \right)^2 + \left( U_k \frac{\partial N_u}{\partial k} \right)^2 \right]^{0.5} \] (A.13)

\[ U_{Nu} = \pm \left[ \left( U_h \frac{D_h}{k} \right)^2 + \left( -\frac{U_k h D_h}{k^2} \right)^2 \right]^{0.5} \] (A.14)

For a typical experimental run, \( h = 5016.18 \text{ W/m}^2 \text{ k}, D_h = 0.002667 \text{ m} \) and \( k = 0.6 \text{ W/m K} \). Accordingly, the Type-B uncertainty in the measurement of Nusselt number for this experimental run was calculated to be 0.18. In terms of percentage uncertainty this was calculated to be 1.38 %. The percentage uncertainty for Nusselt number ranged between 1.38 % and 2.49 % for the experimental runs.

### A1.5 Total thermal resistance

Total thermal resistance was determined as the ratio of difference in temperature between average bottom plate temperature and average fluid temperature to the heat flux.

\[ R_{th} = \frac{T_{b,avg} - T_{f,in}}{q} \] (A.15)

\[ U_{Ra} = \pm \left[ \left( U_q \frac{\partial R_{th}}{\partial q} \right)^2 + \left( U_{T_{b,avg}} \frac{\partial R_{th}}{\partial T_{b,avg}} \right)^2 + \left( U_{T_{f,in}} \frac{\partial R_{th}}{\partial T_{f,in}} \right)^2 \right]^{0.5} \] (A.16)

\[ U_{Ra} = \pm \left[ \left( U_q \frac{T_{b,avg} - T_{f,in}}{q^2} \right)^2 + \left( U_{T_{b,avg}} q \right)^2 + \left( U_{T_{f,in}} q \right)^2 \right]^{0.5} \] (A.17)

For a typical experimental run, \( q = 51487 \text{ W/m}^2 \), \( T_{b,avg} = 301.82 \text{ K} \) and \( T_{f,avg} = 294.9 \text{ K} \). Accordingly, the Type-B uncertainty in the measurement of total thermal resistance for
this experimental run was calculated to be 2.33E-04. In terms of percentage uncertainty this was calculated to be 1.93%.

**A1.6 Substrate temperature gradient**

Substrate temperature gradient was determined as the ratio of difference in temperature between maximum bottom plate temperature and minimum bottom plate temperature to the heat flux.

\[
\theta = \frac{T_{b,\text{max}} - T_{b,\text{min}}}{q}
\]  

(A18)

\[
U_e = \pm \left[ \left( U_q \frac{\partial \theta}{\partial q} \right)^2 + \left( U_{T_{b,\text{max}}} \frac{\partial \theta}{\partial T_{b,\text{max}}} \right)^2 + \left( U_{T_{b,\text{min}}} \frac{\partial \theta}{\partial T_{b,\text{min}}} \right)^2 \right]^{0.5}
\]

(A19)

\[
U_e = \pm \left[ \left( U_q \frac{T_{b,\text{max}} - T_{b,\text{min}}}{q^2} \right)^2 + \left( U_{T_{b,\text{max}}} \frac{T_{b,\text{max}}}{q} \right)^2 + \left( U_{T_{b,\text{min}}} \frac{T_{b,\text{min}}}{q} \right)^2 \right]^{0.5}
\]

(A20)

For a typical experimental run, \( q = 51487 \text{ W/m}^2 \), \( T_{b,\text{max}} = 304.6 \text{ K} \) and \( T_{b,\text{min}} = 299.8 \text{ K} \). Accordingly, the Type-B uncertainty in the measurement of substrate temperature gradient for this experimental run was calculated to be 2.13E-04. In terms of percentage uncertainty this was calculated to be 2.64%.

**A2.1 Uncertainty analysis for experiments involving U-shaped minitube**

The uncertainty analysis was carried out using the methodology presented above for the experiments involving U-shaped minitube. Table A.2 provides the maximum % uncertainty for different parameters for experiments carried out using U-shaped minitube.
Table A 2. Uncertainties of calculated variables for U-shaped minitube experiment.

<table>
<thead>
<tr>
<th>Quantity calculated</th>
<th>Maximum uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volumetric flow rate</td>
<td>± 1.41</td>
</tr>
<tr>
<td>Heat transfer rate</td>
<td>± 1.58</td>
</tr>
<tr>
<td>Log mean temperature difference</td>
<td>± 1.23</td>
</tr>
<tr>
<td>Coolant-side heat transfer coefficient</td>
<td>± 2.71</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>± 3.46</td>
</tr>
<tr>
<td>Nusselt number</td>
<td>± 3.09</td>
</tr>
<tr>
<td>Nusselt number ratio</td>
<td>± 4.37</td>
</tr>
</tbody>
</table>