CHAPTER 1

INTRODUCTION

1.1 PROLOGUE

In today’s highly competitive market, the success of a venture depends on its ability to provide services to customers or users and also remain financially viable. For an organization which is supplying goods to its customers, the major activity is to have suitable products available at an acceptable price within a reasonable timescale. This has forced manufacturers, suppliers and business firms to focus their attention on their supply chains. A supply chain is a group of organizations (suppliers, manufacturers, distributors and retailers) that are working together to profitably provide the right product or service to the right customer at the right time. To minimize the total cost of the entire system and to maintain a high service level, an effective supply chain strategy, must take into account, the interrelation between financial and inventory decisions at different levels of the supply chain.

The term supply chain management can be defined as the task of integrating organizational units along a supply chain and coordinating material, information and financial flows in order to fulfill customer demands with the aim of improving the competitiveness of a supply chain as a whole.
Many companies have saved large sums of money by using scientific inventory management. Inventory control has a major role in developing the economy of a country. Inventory control is the activity which organizes the availability of items to the customers. It co-ordinates purchasing, manufacturing and distribution functions to meet the marketing needs.

The purpose of inventory control function in supporting the business activities is to optimize three targets namely customer service, inventory costs and operating costs.

Inventory control may be defined as ‘the function of directing the movement of goods through the entire manufacturing cycle from raw materials to the inventory of finished goods in an orderly manner to meet the objectives of maximum customer-service with minimum investment at low-cost plant operation’. Inventory is viewed as a necessary evil - too little of it causes costly interruptions and too much of it results in idle capital.

1.2 MOTIVATION

Supply chain management is a critically important approach toward producing and delivering goods and services in a cost-effective, timely manner. Supply Chain Management (SCM) is the term used to describe the management of the flow of materials, information, and funds across the entire supply chain, from suppliers to component producers to final assemblers to distribution (warehouses and retailers), and ultimately to the consumer. In fact, it often includes after-sales service and returns or
recycling. SCM typically involves coordination of information and materials among multiple firms. SCM has generated much interest in recent years for a number of reasons.

- Many managers now realize that actions taken by one member of the chain can influence the profitability of all others in the chain.
- Firms are increasingly thinking in terms of competing as part of a supply chain against other supply chains, rather than as a single firm against other individual firms. Also, as firms successfully streamline their own operations, the next opportunity for improvement is through better coordination with their suppliers and customers. The cost of poor coordination can be extremely high. Each and every member of a supply chain is mutually benefitted by proper coordination.

**Inventory** has been and continues to be the lifeblood of supply chains. When properly managed, it drives revenue and efficiency for companies. The total investment in inventories is enormous, and the control of capital is tied up in raw material, work-in-progress, and finished goods offer a very important potential for improvement. Scientific methods for inventory control can give a significant competitive advantage.
1.3 INVENTORY CONTROL

1.3.1 MEANING OF INVENTORY CONTROL

The word inventory refers to any kind of resource having economic value and is maintained to fulfill the present and future needs of an organization. It is the idle resource of any kind provided such a resource has economic value. Such resources may be

- Physical resources such as raw materials, semi-finished goods, finished goods, spare parts, lubricants, etc.
- Human resources such as unused labour and
- Financial resources such as working capital, etc.

The control and maintenance of inventories of physical goods is a problem common to all enterprises in any sector of a given economy. For example, inventories must be maintained in agriculture, industry, retail establishments and the military. The fundamental reason for the organizations to maintain inventories of goods is that it is either physically impossible or economically unsound to have goods arrive in a given system precisely when demands for them occur. Without inventories, customers would have to wait until their orders were filled from a source or were manufactured. In general, customers cannot wait for long periods of time. For this reason, carrying of inventories is necessary to almost all organizations that supply physical goods to “customers”.

The need for stocking any item arises because of the imbalance between the supply and demand of that commodity at a particular instant of
time. In a typical industry, inventories of the following items become necessary:

1. Raw materials
2. Finished goods
3. Intermediate goods which are
   a. Manufactured in house
   b. Purchased from outside
4. Spare parts (or) repaired parts

1.3.2 DIFFERENT TYPES OF INVENTORY PROBLEMS

Alternative specifications of the following characteristics give rise to various types of inventory problems:

1. The nature of demand
   • Demand can be Static, Dynamic or Stochastic.
   • Demand in one period may be dependent or independent of demand in other periods.
   • In case of shortages, demand can be backlogged, it can be treated as lost sales or partially backlogged.

2. The nature of costs involved
   • Set-up cost may be crashed or it may be constant.
   • Holding cost may be variable or constant.
   • Shortage cost may either be expressed directly or indirectly by a decision of the acceptable shortage level. Shortages may be partially backlogged, completely lost or fully backlogged.
3. The decision structure
   • The extent of the planning horizon may be finite or infinite.
   • Decisions could be taken on the basis of periodic reviews or continuous review.
   • Constraint on the inventory system.

4. Discounts
   • Permissible delay period for settling the accounts may be offered to the customers. It may or may not depend on the order quantity.
   • Freight rate discounts may be offered which may or may not depend on order quantity.

5. Nature of the items
   • The nature of the items may be deteriorating or perishable. The deterioration may be instantaneous or non-instantaneous.
   • Deterioration rate may be constant or it may vary.
   • Sometimes, it may be non-deteriorating or non-perishable.

Apart from the above factors, number of other factors and costs are involved in framing an inventory model.

1.3.3 BASIC FUNCTIONS OF INVENTORY CONTROL

There are two basic functions.

i. Maintaining an accounting record to handle the inventory transactions concerning each inventory item.
ii. Deciding inventory replenishment decisions. There are two basic replenishment decisions:

- When is it necessary to place an order to replenish inventory?
- How much to be ordered in each replenishment?

1.3.4 REASONS FOR CARRYING INVENTORIES

It is essential for any firm to have inventory, because of the following reasons

1. It provides adequate service to customers.
2. It helps in smooth and efficient running of business.
3. It reduces the possibility of duplicating orders.
4. Timely shipment of customers’ orders will improve cash flow.
5. It takes care of economic fluctuations.
6. It helps in minimizing the loss due to deterioration, obsolescence, damage, and so on.
7. It acts as a buffer stock, when raw materials are received late and shop rejections are too many.
8. It takes advantages of price discounts by bulk purchasing.
9. It improves the manpower, equipment and facility utilization because of better planning and scheduling.
10. It helps in avoiding stock-outs.

Solutions to inventory problems can be obtained by mathematical modeling of the inventory systems and computing the solutions to the problems. Thus, the main objective of this thesis is to find the optimal
replenishment policies for various inventory systems under static and dynamic environment.

1.4 PRELIMINARIES

- **Optimization** is the act of obtaining the best result under given circumstances. Optimization can be defined as the process of finding the conditions which give the maximum or minimum value of a function.

- **Statement of optimization problem**

  An optimization or a mathematical programming problem can be stated as follows.

  \[
  \begin{align*}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
  \end{align*}
  \]

  Find \( X = \begin{align*} x_1 \\
  x_2 \\
  \vdots \\
  x_n \end{align*} \) which minimizes (maximizes) \( f(X) \)

  subject to the constraints

  \[
  g_j(X) \leq 0, \quad j = 1, 2, \ldots, m
  \]

  \[
  l_j(X) = 0, \quad j = 1, 2, \ldots, p
  \]

  where \( X \) is an \( n \)-dimensional vector called the design vector, \( f(X) \) is termed as the objective function, \( g_j(X) \) and \( l_j(X) \) are known as inequality and equality constraints respectively. The number of variables \( n \) and the number of constraints \( m \) and/or \( p \) need not be related in any way. The problem stated above is called a **constrained optimization problem**. Some optimization problems do not involve any constraints and can be stated as
Find \( \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \) which minimizes (maximizes) \( f(\mathbf{X}) \).

Such problems are called \textit{unconstrained optimization problems}.

- \textbf{Decision Variable}

Any engineering system or component is defined by a set of quantities, some of which are viewed as variables during the design process. In general, certain quantities are usually fixed at the outset and these are called pre-assigned parameters. All the other quantities are treated as variables in the design process and are called design or decision variables \( x_i, i=1,2, \ldots, n \). The design variables are collectively represented as a design vector \( \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \).

- \textbf{Local Minimum and Local Maximum}

A function of one variable \( f(x) \) is said to have a relative or \textit{local minimum} at \( x = x^* \) if \( f(x^*) \leq f(x^* + h) \) for all sufficiently small positive and negative values of \( h \). Similarly, a point \( x^* \) is called a relative or \textit{local maximum} if \( f(x^*) \geq f(x^* + h) \) for all values of \( h \) sufficiently close to zero.
• **Global Minimum and Global Maximum**

A function $f(x)$ is said to have a **global or absolute minimum** at $x^*$, if $f(x^*) \leq f(x)$ for all $x$, and not just for all $x$ close to $x^*$, in the domain over which $f(x)$ is defined. Similarly, a point $x^*$ will be a **global maximum** of $f(x)$ if $f(x^*) \geq f(x)$ for all $x$ in the domain.

• **A single variable optimization problem** is one in which the value of $x = x^*$ is to be found in the interval $[a, b]$ such that, $x^*$ minimizes $f(x)$.

• **Necessary and Sufficient conditions for the relative minimum of a function of single variable**

  **Necessary condition:**
  If a function $f(x)$ is defined in the interval $a \leq x \leq b$ and has a relative minimum at $x = x^*$, where $a < x^* < b$, and if the derivative $df(x)/dx = f'(x)$ exists as a finite number at $x = x^*$, then $f'(x^*) = 0$.

  **Sufficient condition:**
  Let $f'(x^*) = f''(x^*) = \cdots = f^{(n-1)}(x^*) = 0$, but $f^{(n)}(x^*) \neq 0$. Then $f(x^*)$ is
  
    i. a minimum value of $f(x)$ if $f''(x^*) > 0$ and $n$ is even.
    
    ii. a maximum value of $f(x)$ if $f''(x^*) < 0$ and $n$ is even.
    
    iii. neither a maximum nor a minimum if $n$ is odd.

• **Positive definite, Negative definite and Positive Semi-definite**

A matrix $A$ will be positive definite, if all its eigenvalues are positive; that is all the values of $\lambda$ that satisfy the determinantal
equation \( |A - \lambda I| = 0 \) should be positive. Similarly, the matrix \( [A] \) will be negative definite if its eigenvalues are negative.

Another test that can be used to find the positive definiteness of a matrix \( A \) of order \( n \) involves evaluation of the determinants

\[
A = |a_{11}|, \\
A_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}, \\
A_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}, \ldots, \\
A_n = \begin{vmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}.
\]

The matrix \( A \) will be positive definite, if and only if all the values \( A_1, A_2, A_3, \ldots, A_n \) are positive. The matrix \( A \) will be negative definite if and only if the sign of \( A_j \) is \((-1)^j\) for \( j = 1, 2, \ldots, n \). If some of the \( A_j \) are positive and the remaining \( A_j \) are zero, the matrix \( A \) will be positive semi definite.
• Necessary and Sufficient Conditions for the minimum or maximum of an unconstrained function of several variables

**Necessary condition:** If \( f(X) \) has an extreme point (maximum or minimum) at \( X = X^* \) and if the partial derivatives of \( f(X) \) exist at \( X^* \),

then \( \frac{\partial f}{\partial x_1} (X^*) = \frac{\partial f}{\partial x_2} (X^*) = \ldots = \frac{\partial f}{\partial x_n} (X^*) = 0 \)

**Sufficient condition:** A sufficient condition for a stationary point \( X^* \) to be an extreme point is that the matrix of second partial derivatives (Hessian matrix) of \( f(X) \) evaluated at \( X^* \) is

i. Positive definite when \( X^* \) is a relative minimum point, and

ii. Negative definite when \( X^* \) is a relative maximum point.

• Convex function

A function \( f(X) \) is said to be **convex**, if for any pair of points

\[
X_1 = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_n^{(1)} \end{bmatrix} \quad \text{and} \quad X_2 = \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ \vdots \\ x_n^{(2)} \end{bmatrix}
\]

and all \( \lambda, 0 \leq \lambda \leq 1 \),

\[
f[\lambda X_2 + (1-\lambda)X_1] \leq \lambda f(X_2) + (1-\lambda)f(X_1),
\]

that is, if the segment joining the two points lies entirely above or on the graph of \( f(X) \).
• **Concave function**

A function \( f(X) \) is called a **concave** function, if for any pair of points

\[
X_1 = \begin{bmatrix} x_1^{(1)} \\ x_2^{(1)} \\ \vdots \\ x_n^{(1)} \end{bmatrix} \quad \text{and} \quad X_2 = \begin{bmatrix} x_1^{(2)} \\ x_2^{(2)} \\ \vdots \\ x_n^{(2)} \end{bmatrix}
\]

and all \( \lambda, \ 0 \leq \lambda \leq 1 \), \( f[\lambda X_2 + (1 - \lambda)X_1] \geq \lambda f(X_2) + (1 - \lambda) f(X_1) \), that is, if the segment joining the two points lies entirely below or on the graph of \( f(X) \).

• **Test for convexity and concavity**

A function \( f(X) \) is convex, if the Hessian matrix \( H(X) = \left[ \frac{\partial^2 f(X)}{\partial x_i \partial x_j} \right] \) is positive semidefinite.

*Note:* Any local minimum of a convex function \( f(X) \) is a global minimum.

1.4.1 **BASICS IN INVENTORY THEORY**

• A model is a representation of a system under study.

• **Demand for Inventory systems**

The understanding of the nature of demand, that is its rate, size and pattern for the given inventory item is essential to determine an optimal inventory policy for that item.

- The quantity required to satisfy the demand for inventory will be called the **demand size**. The size of the demand may be either deterministic or probabilistic. In the deterministic case, the demand over a period of time is known with certainty. This can
be fixed (static) or can vary (dynamic) from period to period. But in the probabilistic case, the demand over a period of time is not known with certainty but the nature of such demand can be described by a known probability distribution. The known probability distribution may be either stationary or non-stationary.

- Demand rate is the demand size per unit time. If a demand size \( x \) quantity units occur over a period of \( t \) time units, the demand rate is equal to \( \frac{x}{t} \).

- The different ways by which demand occurs during a period is referred to as **demand pattern**. It is the manner in which inventory items are required by the customers. The demand for a given period of time may be satisfied instantaneously at the beginning of the period, or uniformly during that period.

- Inventory system in which the demand size is known will be referred to a **deterministic inventory system**. In this case, the demand size may be fixed (static) or can vary (dynamic) from time to time.

- Inventory system in which demand size is not known, but it is possible to ascertain its probability distribution is called **probabilistic inventory system**. In this case, the known probability distribution may be either stationary or non-stationary from time to time.

- **The scheduling period** is the length of time between consecutive decisions with respect to replenishments.
• The lead time is the length of time between scheduling replenishment and its actual addition to stock.

• Replenishment size, Replenishment period, Replenishment rate and Replenishment patterns:
  o The replenishment size is the quantity scheduled for replenishment. Let it be denoted by $q$.
  o The replenishment period is the length of time during which the replenishment size $q$ is being added to inventory. Let this period by denoted by ‘$t$’.
  o The average replenishment rate ‘$p$’ is the ratio of the replenishment size and the replenishment period. That is $p = q/t$.
  o When the replenishment period is insignificant $t = 0$ and the corresponding rate is infinite that is, $p = \infty$. In this case replenishment is said to be instantaneous. The manner in which replenishment occurs when replenishment is not instantaneous is known as replenishment pattern.

• The time period between placements of two successive orders is referred to as an order cycle.

• The time horizon is the period over which the inventory level will be controlled. This horizon may be finite or infinite depending on the time period over which demand can be forecast reliably.
• **Obsolescence** refers to items that lose their value through time because of rapid changes of technology or the introduction of a new product by the competitor. Style goods must be sharply reduced in price or otherwise disposed off after the season is over. For example, spare parts for military aircraft are style goods and they become obsolete when a replacement model is introduced.

• Inflation is a rise in the general level of prices of goods and services in an economy over a period of time. Inflation is also erosion in the purchasing power of money.

• Chief measure of price inflation is the **inflation rate**, the annualized percentage change in a general price index (normally the Consumer Price Index) over time.

• **Deterioration** refers to damage, breakage, spoilage, dryness, vaporization, etc. of the products.

• The products like foodstuff, human blood, photographic film, etc., which are having a maximum usable lifetime are known as **perishable products**.

• The products like alcohol, gasoline, radioactive substances, etc. which are having no shelf-life at all are known as **decaying products**.

• The shelf-life of some products can be indefinite and they fall under the category **no obsolescence/deterioration** category.
• Some of the goods would have a span of maintaining quality or the original condition, namely, during that period, there was no deterioration occurring. This phenomenon is known as non-instantaneous deterioration. This type of phenomenon exists commonly in first hand vegetables and fruits. They have a short span of maintaining fresh quality, in which there is almost no spoilage. Afterwards, some of the items will start to decay.

• Concept of **EOQ and EPQ model**

  Order quantity decisions affect the amount of inventory to be maintained at various stocking points. The ordering decision is stated in terms of Economic Order (or lot size) Quantity (EOQ). EOQ is the optimal quantity that should be ordered to replenish inventory so as to achieve the optimum total (or variable) inventory cost during the given period of time.

  EPQ is the Economic Production Quantity model. This model is similar to that of EOQ model. The only difference is, the time to replenish inventory. In this model, it is assumed that the replenishment is gradual. This is because, in many situations, the amount ordered is not delivered all at once, but ordered quantity is sent or received gradually over a length of time at a finite rate per unit of time.
• Postponement Strategy

Postponement, also known as late customization or delayed product differentiation, refers to delaying some product differentiation processes in a supply chain as late as possible, until the supply chain is cost effective.

A postponement strategy aims at delaying some supply chain activities until customer demand is revealed in order to maintain both low system wide cost and fast response. When postponement strategy is employed, the purchasing of some expensive and fragile materials is delayed. Products in semi-finished forms are customized quickly in production facilities close to customers. Further, finished products are kept in central location and are distributed quickly to customers.

Postponement is one of central features of mass customization. Postponement strategy is highly successful in a wide range of industries that require high differentiation, e.g., high-tech industry, food industry, and fashion industry, etc.

One practical example is Hewlett-Packard Development Company. HP produces generic printers in its factories and distributes them to local distribution centers, where power plugs with appropriate voltage and user manuals in the right language are packed. They have saved a lot of money every year by adopting the postponement strategy.
Reasons for using postponement strategy

- Reduces inventory cost
- Reduces transportation cost
- Reduces risk of obsolescence
- Reduces demand variability
- Improves competitiveness by offering customized products quickly.

• **Permissible delay in payment or Trade credit period**

For encouraging the retailer to buy more, the supplier allows a certain fixed period for settling the account and does not charge any interest from the retailer for the amount owed during this period. This period is known as trade credit period.

• **Relevant Inventory Costs**

The costs that are affected by the firm’s decision to maintain the optimal level of inventory are called relevant costs and they play an important role in the study of an inventory system. The costs which influence the operating doctrine are those costs which vary as the operating doctrine is changed. Costs that are independent of the operating doctrine used need not be included in any analysis, where costs are used as an aid in determining an operating doctrine. Various types of costs are as follows.

- The **cost of replenishing inventories**: Two types of costs must be considered in the replenishing cost namely setup cost and ordering cost
- **Set-up cost** – This is the cost associated with the setting up of machinery before starting production. Set-up cost is generally assumed to be independent of the quantity ordered for or produced.

- **Ordering cost** – This is a cost associated with ordering of raw material for production purposes, advertisements, consumption of stationery and postage, telephone charges, telegrams, rent for space used by the purchasing department, travelling expenditures incurred, etc., constitute the ordering cost.

- **Purchase or Production cost** – The cost of purchasing (or producing a unit of item is known as purchase (or production) cost. Purchase cost per unit item is affected by the quantity purchased due to quantity discounts or price breaks

  \[
  \text{Purchase cost} = \text{price per unit item} \times \text{Demand per unit time}
  \]

  When price-break or quantity discounts are available for bulk purchase of a specified quantity, the unit price becomes smaller as size of order (say \(Q\)) exceeds a specified quantity level. In those cases, purchase cost become variable and depends on the size of the order. In this case, Purchase cost = Price per unit when order size is \(Q \times \text{Demand per unit time}\).

- **Carrying or Holding cost** – The cost associated with carrying or holding goods in stock is called holding cost or carrying cost per unit of item for a unit of time. This cost generally includes invested capital cost, record-keeping and administrative cost,
handling costs, storage costs, taxes and insurance costs and others. Holding cost may be assumed to vary with time in some inventory models.

- **Shortage or Stock out cost** - The shortage of items occurs when actual demand cannot be fulfilled from the existing stock. These costs arise due to shortage of goods, and may result in loss of sales. There may be a loss in good will. The shortages may be completely backordered, completely lost or partially backlogged.

- **Deterioration cost** - The cost incurred due to the deterioration of items in inventory is referred to as the deterioration cost.

- **Opportunity Cost** - It is the cost of passing up the next best choice when making a decision. For example, if an asset such as capital is used for one purpose, the opportunity cost is the value of the next best purpose the asset could have been used for.

- **Set-up crashing cost** - The cost incurred in allocating capital to reduce, setup cost is called the set-up crashing cost.

### 1.5 PHENOMENA OF THE THESIS

The control and maintenance of inventory are vital as the effective control of inventories can provide better customer service and improve profitability. Demand plays a major role in control of inventories. Some inventory models are formulated in a static environment. That is demand is assumed to be steady and constant over a finite horizon. The demand
may be constant under certain circumstances for certain products. But, constant demand is not always applicable in all situations.

In supermarkets, the display of the consumer goods in large quantities attracts more customers and generates higher demand. That is demand rate may go up or down with the on hand stock-level. At times, the presence of inventory has a motivational effect on the people around it. It is common belief that large piles of goods displayed in a supermarket will lead the customers to buy more.

Sometimes, demand may be time-dependent. Many inventoried items such as electronic products, fashionable clothes, tasty food products and domestic goods generate increasing sales after gaining consumers acceptance. The sales for other products may decline drastically due to the introduction of more competitive products or due to the change in consumers’ preferences. The demand of the products during its growth and decline phases can be well approximated by continuous time-dependent functions such as linear or exponential.

The demand for certain other products is price-sensitive. That is demand is a function of selling price. Demand for seasonal products like television, refrigerator, cellular phones etc. depends on selling price. In order to generate sufficient demand, the prices of the product should be adjusted.

In economics, inflation is erosion in the purchasing power of money. Inflation can have both positive and negative effects over economy. Most
of the countries have suffered from large scale inflation and sharp decline in purchasing power of money last several years. Therefore inflation and time value of money cannot be ignored when framing an inventory model as they would influence the inventory policy to any significant degree.

When the quality of the product declines by damage, breakage, spoilage, dryness etc the product is said to deteriorate. Some product would retain their quality for certain period and start deteriorating afterwards. This phenomenon is termed as non-instantaneous deterioration. Both deterioration and non-instantaneous deterioration may occur depending upon the products and circumstances. Thus, it is very much important to control and maintain inventories for deteriorating/non-instantaneous deteriorating items.

Further, when shortages occur, some customers are willing to wait for backorders to be fulfilled and others may turn to buy from other sellers. That is when shortages occur; in reality they are partially backlogged. In some cases, the backlogging rate may be constant during the shortage period. In some inventory systems, such as fashionable commodities, the length of the waiting time for the next replenishment would determine whether the backlogging will be accepted or not. Therefore, the backlogging rate is variable and dependent on the waiting time for the next replenishment.

It is assumed in general that the payment must be made to the supplier for the items immediately after receiving the consignment. But this is not true in all cases. Practically, for encouraging the retailer to buy more,
the supplier seldom allows a certain fixed period for settling the account and does not charge any interest from the retailer on the amount owed during this period. Often the suppliers offer a fixed credit period (permissible delay in payment) to the customers. This credit period may be independent or dependent upon the order quantity. Some suppliers may offer a freight rate discount which depends upon the order quantity. This will attract new customers and increase the order quantity. Both the supplier and the retailer are mutually benefitted.

Other factors which influence the inventory models are setup cost, holding cost and quality of the product. Setup cost includes the cost associated with setting up of machinery parts before starting a production, administrative cost, record keeping cost etc. These costs can be reduced by employing skilled laborers and by using high-tech machineries and so on. Thus by investing some capital, the setup cost can be crashed. The quality level is not always fixed at an optimal level. In real production environment, some defective items are being produced due to imperfect production processes. The defective items must be rejected, repaired or reworked or if they have reached the customer refunded to increase good will among the customers. In all the cases, substantial costs are incurred. Thus for the system with an imperfect production process, it is advisable to consider investing capital on quality improvement. This would reduce the quality-related costs.

Further holding cost need not be constant always. It depends upon the time spent by the items in storage. The holding cost may be represented
as an increasing step function of the storage time. This structure is representative of many real-life situations in which the storage times can be classified into different stages each with its distinctive unit holding cost. This is particularly true in the storage of deteriorating and perishable items such as food products. The longer these food products are kept in storage, the more sophisticated the storage facilities and services needed, and therefore, the higher the holding cost.

Postponement is one of the central features of mass customization. It has been found that postponement is highly successful in a wide range of industries that require high differentiation like high-tech industry, food industry, fashion industry etc. Thus, it is very important to analyze the effect of employing postponement strategy in a supply chain.

1.6- METHODOLOGY

The approach in this thesis is based upon mathematical modeling and optimization. The inventory systems in the supply chain under various assumptions are studied and the problem is analyzed in detail. The next step is to formulate a mathematical model which depicts the system and its properties. Mathematical model is one in which the system is represented by the symbols that can be manipulated by using mathematical rules. Many inventory systems can be solved by forming a mathematical model and then optimal decision rules are derived from the model. It is generally more effective method of solving an inventory system and this model is called an **inventory model**. This is followed by identifying the decision variables and objective functions.
The main aim of the thesis is to find the optimal replenishment policy for various inventory systems in the supply chain under various circumstances. The objective is to optimize the total inventory cost or total profit of the inventory systems. Various solution procedures and methods are employed to find the optimal replenishment policy. Different algorithms are designed to match with the problem. Using those algorithmic approaches, the solutions are obtained. This is followed by evaluation of the model by numerical illustrations. These illustrations are used to demonstrate the theoretical approach. Further, analysis of the numerical illustrations are carried out and the implications are discussed in detail. Finally, we conclude by discussing the various managerial implications and insights obtained from the research work.

Thus, the methodology followed to find the optimal replenishment policies are listed as below.

- Description of the problem and mathematical modeling
- Solution procedure/designing algorithm to optimize the total cost/ total profit of the inventory system
- Numerical illustrations and analysis.
1.7 DESCRIPTION OF THE THESIS

1.7.1 A SHORT DESCRIPTION OF THE RESEARCH CONTRIBUTION IN THIS THESIS

- Inventory models for non-instantaneous deteriorating items under inflation with partial backlogging of shortages and stock dependent consumption rate over finite and infinite planning horizons are developed and their optimal replenishment policies are found.

- Various inventory models for non-instantaneous deteriorating items with time dependent and constant demand with permissible delay in payments are developed. The necessary and sufficient conditions for the existence and uniqueness of the optimal solution are derived.

- A two level supply chain structure employing postponement strategy with permissible delay in payments is developed.

- An inventory model is developed with storage time dependent holding cost where setup cost is reduced through capital investment.

- A Two level supply chain model for an integrated vendor buyer system, with variable production cost by considering quality improvement when the freight rate and trade credit are both linked to order quantity is developed.
1.7.2 BRIEF OUTLINE OF THE THESIS

Various inventory models involving different concepts are developed under static and dynamic environment and their optimal replenishment policies are found. The thesis is organized into ten different chapters. The chapter wise outline of the thesis is as follows:

CHAPTER 1

This chapter is introductory in nature. It explains the basic concepts involved in inventory theory. It also highlights the motivation behind this research work, phenomenon of the thesis and explains the methodology involved in the thesis. Further, a short description of various contributions in this thesis and the notations used throughout are given.

CHAPTER 2

Literature review is presented in this chapter. Various inventory models existing in the literature are discussed. It gives a brief report of the works done in the field of inventory control considering different aspects. Apart from this, the literature gap is also given in this chapter.

CHAPTER 3

In this chapter, an Economic Order Quantity (EOQ) model for non-instantaneous deteriorating items is developed. The demand is assumed to be stock-dependent with shortages partially backlogged. The effects of inflation and time value of money are considered. The model is developed by considering a fixed planning horizon. Optimization algorithm to find the optimal replenishment policy is given and numerical illustrations are given.
to elucidate the theory. Analysis with respect to various factors which affect the inventory system is carried out and the results are concluded.

CHAPTER 4

Finding out the replenishment policy is a hot topic of research in inventory management. In this chapter, we propose an optimal replenishment policy by considering stock dependent consumption rate for non-instantaneous deteriorating items with money inflation and time discounting. In this model, shortages are allowed and backlogging is partial. Here, we consider an infinite planning horizon. The optimal cycle time and the length of time in which there is no shortages are taken as decision variables. Solution procedure to determine the optimal values of the decision variables is given and explained with examples. The impact of various parameters of the system on the optimal solution is analyzed by carrying out sensitivity analysis with perturbations in various parameters.

CHAPTER 5

In today’s time based competition, the demand for a product may grow or decline with time. In this chapter, we assume that the demand depends on time. Shortages are allowed. The unsatisfied demand is partially backlogged with a time-proportional backlogging rate. We formulate an inventory model for non-instantaneous deteriorating items. Further, the necessary and sufficient conditions for the existence and uniqueness of the optimal solutions are derived. Numerical examples are presented to demonstrate the developed model. The impact of various parameters of the system on the optimal solution is analyzed, by carrying out sensitivity
analysis and their effects are discussed in detail. According to the results of sensitivity analysis, we provide several ways for the retailer to effectively reduce the total inventory cost.

CHAPTER 6

In this chapter, Economic Order Quantity (EOQ) based model for non-instantaneous deteriorating items with permissible delay in payments is proposed. This model, aids in minimizing the total inventory cost by finding an optimal replenishment policy. In this model, shortages are allowed and are partially backlogged. The backlogging rate is variable and is dependent on the waiting time for the next replenishment. Some useful theorems are framed to characterize the optimal solutions. The necessary and sufficient conditions for the existence and uniqueness of the optimal solutions are also provided. An algorithm is designed to find the optimal replenishment cycle time and order quantity under various circumstances. Numerical examples are given to demonstrate the theoretical results. Sensitivity analysis of the optimal solution with respect to major parameters of the system are carried out and the implications are discussed in detail. In the discussions, suggestions are given to minimize the total cost of the inventory system.

CHAPTER 7

In the proposed model, we formulate an Economic Order Quantity-based model for non-instantaneous deteriorating items with permissible delay in payments. The purpose of the chapter is to evaluate the impact of postponement strategy on the retailer in a supply chain. Here, we have
developed some useful theorems to characterize the optimal solutions. An algorithm is provided to find the optimal replenishment cycle time under various circumstances. We formulate different models for a postponement system and an independent system to minimize the total average cost function per unit time for ordering and keeping $n$ end-products. An algorithm is given to derive the optimal solutions of the proposed models. Several numerical examples are given to demonstrate the theoretical approach. Sensitivity analysis with respect to various parameters of the system is carried out and the results are discussed in detail.

CHAPTER 8

In this chapter, we study an Economic Order Quantity problem for deteriorating items where demand depends on the inventory level. In traditional inventory models, it was assumed that the holding cost is constant and the set-up cost is not subject to control. But in practice, the holding cost depends upon the storage time. Thus, we assume that holding cost is an increasing function of the time spent in storage by considering two time-dependent holding cost step functions. Also, the set-up cost is reduced through capital investment. Further, in this chapter, shortages are allowed and partially backlogged. Backlogging rate is variable and it is dependent on the waiting time for the next replenishment. We have developed algorithms for finding the optimal values and explained the theory by numerical illustration.
CHAPTER 9

In this chapter, we formulate a supply chain model with single vendor and single buyer considering quality improvement. We assume that the buyer purchases under trade credit which is linked to the order quantity offered by the supplier. In addition, the buyer pays the freight charge according to the weight schedule. Further, the demand is assumed to be sensitive to the buyer’s selling price. Also the production cost of the supplier is assumed to be a convex function of the production rate. This model offers best possible solutions for both the buyer and the vendor to collaboratively agree on inventory control. An algorithm is furnished to determine the optimal solution. In addition, a numerical example is presented to illustrate the theoretical approach and results. Sensitivity analysis with respect to the major parameters of the system has been carried out. From the analysis, we offer some managerial insights to achieve significant decrease in total cost.

CHAPTER 10

This chapter sums up the thesis with the intention of further research.
1.8 NOTATIONS

Some of the common notations used throughout the thesis are given below:

- $A$: ordering cost per order ($ / order)
- $J(t)$: the inventory level at time $t$
- $0$: deterioration rate
- $p$: purchasing cost per unit ($ / unit)
- $Pi$: selling price per unit ($/unit)
- $s$: shortage cost for backlogged items per unit, per unit time ($/unit/unit time.)
- $n$: opportunity cost due to lost sales. ($/unit)
- $T$: the length of replenishment cycle
- $ti$: the time at which the inventory level drops to zero
- $td$: the length of period during which the product has no deterioration (fresh product time)
  denotes the optimum value
- $D(t)$: the demand rate at time $t$
- $H$: Planning horizon
- $Q$: Order quantity per cycle
- $P$: production rate
- $/*$: the rework cost for a defective item.
- $I_m$: maximum inventory level
- $h$: maximum amount of shortage demand to be backlogged
- $Ip$: the capital opportunity cost in stock per dollar per year
- $I_e$: the interest earned per dollar per year