CHAPTER 9

REPLENISHMENT POLICY FOR SUPPLY CHAIN WITH QUALITY IMPROVEMENT UNDER TRADE CREDIT AND FREIGHT RATE DISCOUNT

9.1 INTRODUCTION

In recent years, the successful Japanese experience of employing just-in-time (JIT) production has triggered considerable attention. The ultimate goal of JIT from the production/inventory management standpoint is to produce small-lot sizes with high-quality products. Investing capital in improving quality is regarded as the most effective means of achieving this goal. In the classical inventory model, it is implicitly assumed that the quality level is fixed at an optimal level. That is, all items are assumed to have perfect quality. However, in the real production environment, it can often be observed that there are defective items being produced due to imperfect production processes. Hence the inventory policy determined by the conventional model might be inappropriate. Therefore, in this chapter, we have assumed that the supplier invests capital in quality improvement and that capital investment in quality improvement is a logarithmic function of quality level.

In the given intense market competition, transportation costs are critical in many operating decisions. An appropriate transportation cost function should be incorporated into models with all other relevant costs. In day-to-day dealing, it is found that a supplier allows a certain fixed
period to settle the account. During this credit period no interest is charged by the supplier, but beyond this period interest is charged by the supplier under the terms and conditions agreed upon, since inventories are usually financed through debt or equity. In case of debt financing, it is often a short-term financing. Thus, interest paid here is nothing but the cost of capital or opportunity cost. Also short term loans can be thought of as a return on investment since the money generated through revenue can be ploughed back in to the business. Therefore it makes economic sense for the customer to delay the settlement of the replenishment account up to the last day of the credit period allowed by the supplier. On the other hand retailer is also benefitted by earning interest on revenues generated during the free period. Buyers may be financed by their suppliers rather than by financial institutions. Supplier may offer credit period depending on the order quantity.

In general, almost all the researchers assume that the production rate and the unit production cost are assumed to be constant. In reality, it has been observed that the unit production cost vary with changes in the production rate. In this chapter, the production cost is assumed to be a convex function of the production rate.

In the existing literature, trade credit and transportation cost were concerned only with decisions that minimized the cost for the buyer or maximized the profit for the supplier. However, the complicated interaction and cooperation opportunities were considered trivial and ignored in their consideration. In reality, independent and conflicting production plans were
often made by members in the supply chain to achieve individual goals. Therefore, an aggregate approach to planning with an emphasis on improving supply chain efficiency is needed to help business survive in this fiercely competitive world. Cost reduction from either a trade credit or a discount in freight rate gives the buyer incentive to lower the retail price in order to increase market share.

To the best of our knowledge there is no existing integrated vendor buyer model in the literature when both trade credit and freight rate are linked to order quantity with capital investment in quality improvement. The remainder of the chapter is organized as follows. Section 9.2 gives the detailed description of the problem under consideration followed by assumptions and notations used throughout the chapter. Section 9.3 describes the mathematical modeling and the formulation of the model. Section 9.4 gives the solution procedure followed by an algorithm to find the optimal solution. In section 9.5, numerical example which elucidates the theory and solution procedure is given and it is followed by sensitivity analysis. Finally, we draw the conclusion of this chapter in section 9.6.

9.2 PROBLEM DESCRIPTION

In this chapter, we consider a single supplier and a single buyer integrated model. The production cost is assumed to be a convex function of the production rate. Here both trade credit and freight rate are linked to order quantity. Further, the supplier invests capital in quality improvement. Capital investment in quality improvement is assumed to be a logarithmic function of quality level. The objective of the chapter is to explore the
process of arriving at the best policy minimizing the total cost of the system.

9.2.1 ASSUMPTIONS

1. There is a single supplier and a single buyer for a single product.
2. Shortages are not allowed.
3. The market demand for the product is assumed to be sensitive to the buyer’s selling price \( p_1 \) (that is demand of a product increases as its selling price decreases) and is defined as \( D(p_1) = a_0 e^{-\tau} \) where \( a \) is a scaling factor that is greater than 0; \( \tau \) is a price-elasticity coefficient that is greater than 1. For notational simplicity we used \( D \) instead of \( D(p_1) \). Further, the selling price is an exogenous variable and constant.
4. The product is manufactured with a finite production rate \( P, P > D \).
5. Inventory is continuously reviewed.
6. The relationship between lot size and quality is formulated as follows: while the vendor is producing a lot, the process can go out of control with a given probability \( \alpha \) each time another unit is produced. The process is assumed to be in control in the beginning of the production process. Once out of control, the process produces defective items and continues to do so until the entire lot is produced.
7. The relationship between process quality, \( \alpha \) and capital investment in process quality improvement, \( \phi_\alpha \) is described by \( \phi_\alpha(\alpha) = \sigma_1 \ln \left( \frac{\alpha_0}{\alpha} \right) \),
where \( \frac{1}{\sigma_i} \) is the fraction of the reduction in \( \alpha \) per dollar increase in investment.

8. For each unit of product, the supplier spends $p$ in production and receives $\nu$ from the buyer. The buyer then sells the product at $p_1$ to the customers. Here $p_1 > \nu > p$.

9. The capacity utilization \( \rho \) is the ratio of the demand rate, \( D \), to the production rate \( P \); it is always less than 1. ( \( \rho = D/P \) and \( \rho < 1 \)).

10. The unit production cost \( p(p) \) is a convex function of the production rate \( P \). That is, \( p(p) = a_0 + \frac{a_1}{p} + a_2 p \), where \( a_0, a_1 \) and \( a_2 \) are non-negative real numbers to be set to best fit the estimated unit production cost function. The fixed cost \( a_0 \) can be regarded as the material cost.

The cost component \( \frac{a_1}{p} \) decreases as the production rate \( (P) \) increases, representing costs such as labor cost and energy cost; both are equally distributed over a large number of units. The third term \( a_2 P \) denotes a cost component that increases with the production rate. Such cost would include additional tool or die wear at high production rate.

11. The supplier offers a credit period \( N_n \), which is linked to order size in the schedule as follows:
\[
\begin{array}{lll}
\eta & Q & N_\eta \\
1 & v_1 \leq Q \leq v_2 & N_1 \\
2 & v_2 \leq Q \leq v_3 & N_2 \\
\vdots & \vdots & \vdots \\
\lambda & v_\lambda \leq Q \leq v_{\lambda+1} & N_\lambda \\
\end{array}
\]
where, \(0 = v_1 < v_2 < \ldots < v_\lambda < v_{\lambda+1} = \infty\), each represents a boundary quantity. \(N_\eta\) denotes the credit period applicable to the orders whose order quantity \(Q\) falls in the interval \(v_\eta\) to \(v_{\eta+1}\) with \(0 < N_1 < N_2 < \ldots < N_\lambda\).

12. The supplier charges freight for shipping according to a weight schedule that is defined below:

\[
\begin{array}{lll}
\varepsilon & W & \chi_\varepsilon \\
1 & \mu_1 \leq W \leq \mu_2 & \chi_1 \\
2 & \mu_2 \leq W \leq \mu_3 & \chi_2 \\
\vdots & \vdots & \vdots \\
\mathfrak{m} & \mu_\mathfrak{m} \leq W \leq \mu_{\mathfrak{m}+1} & \chi_\mathfrak{m} \\
\end{array}
\]
where \(0 = \mu_1 < \mu_2 < \ldots < \mu_\mathfrak{m} < \mu_{\mathfrak{m}+1} = \infty\) is the boundary values of the freight weights at which freight rate break occurs. \(\chi_\varepsilon\) denotes that the freight rate applicable to the shipping weight \(W\) falls in the interval \(\mu_\varepsilon\) to \(\mu_{\varepsilon+1}\) with \(\chi_1 > \chi_2 > \ldots > \chi_\mathfrak{m} > 0\). The shipping weight of the product is \(\omega\) lbs per unit. That is \(W = \omega Q\). The unit shipping cost is therefore the unit shipping weight times the freight rate, which is \(F_\varepsilon = \omega \chi_\varepsilon\).

13. During the production period, the supplier manufactures in batches of size \(nQ\), where \(n\) is an integer. Once the first \(Q\) units are produced, the
supplier delivers them to the buyer and then continues making the delivery (on average), every $T (=Q/D)$ units of time until the supplier’s inventory level falls to zero.

14. During the credit period, the buyer sells the items and uses the sales revenue to earn interest at a rate of $I_{be}$. At the end of the credit period, the buyer pays the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of $I_{bp}$ for the items remaining in stock. $p_t I_{be} > v I_{bp}$.

15. In offering trade credit to the buyer the supplier endures a capital opportunity cost at rate $I_{vp}$ during the period between product shipped and paid for, where $I_{vp} \leq I_{bp}$.

9.2.2 NOTATIONS

Despite the common notations given in section 1.8, the following notations are used only in this chapter

$h_v$ per unit holding cost for the supplier per unit time

$h_b$ per unit holding cost for the buyer per unit time (interest charge is not considered)

$A_v$ setup cost for the supplier for each production run.

$A_b$ setup cost for the buyer

$I_{vp}$ vendor’s fractional opportunity cost of capital per dollar per unit time

$I_{bp}$ buyer’s fractional opportunity cost of capital per dollar per unit time

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\( I_{be} \) interest earned by the buyer per dollar per unit time

\( \alpha \) the percentage of defective items produced when the process is in the "out-of-control" state

\( \alpha_0 \) the original percentage of defective items produced when the process is in the out-of-control state prior to investment

\( \phi_a \) the capital investment in process quality improvement

\( T \) buyer's replenishment cycle length

\( Q \) buyer's order quantity \((Q = DT)\) per order

### 9.3 MODEL FORMULATION

Here, we assume that the supplier offers the buyer an order-size based credit period and a freight rate discount schedule. The two discount schedules given in the assumptions are reorganized into a new discount schedule which is given below. The reorganized process is accomplished by identifying all possible lot-size intervals with no breaks within each interval.

\[
\begin{array}{cccc}
J & W & M_j & F_j \\
1 & q_1 \leq Q \leq q_2 & M_1 & F_1 \\
2 & q_2 \leq Q \leq q_3 & M_2 & F_2 \\
: & : & : & : \\
: & : & : & : \\
k & q_k \leq Q \leq q_{k+1} & M_k & F_k \\
\end{array}
\]

Here \( k \leq \lambda + m \). For \( q_j \leq Q \leq q_{j+1}, \ j=1,2,\ldots,k, \ M_j \) is the length of credit period offered by the supplier and \( F_j \) dollars is the freight rate charged by
the supplier per unit. \(0 \leq q_1 < q_2 < \ldots < q_k < q_{k+1} = \infty, 0 < M_1 < M_2 < \ldots < M_k\)
and \(F_1 > F_2 > \ldots > F_k > 0\).

Now we consider the total cost function for the vendor and the buyer.

**Vendor’s total cost per unit time consists of the following components.**

- Set up cost per unit time is \(\frac{A_v}{nT}\).
- The inventory carrying cost includes storage and handling expenses, insurance and taxes as well as the time value of capital tied up in the inventories. Therefore, the carrying cost per unit time is given by \((h_v + pI_{vp})\frac{DT}{2} [(n-1)(1-\rho) + \rho]\).
- Defective items rework cost per unit time is \(\frac{nD^2 \alpha Tr_w}{2}\).
- Opportunity cost of quality improvement investment is given by \(I_{vp}\sigma_1 \ln\left(\frac{\alpha_0}{\alpha}\right)\).
- Opportunity cost per unit time for offering trade credit to the buyer is \(\nu I_{vp} DM_j\).

Therefore, the total cost of the vendor is given by

\[
TCV_V(\alpha, T, n) = \frac{A_v}{nT} + (h_v + pI_{vp})\frac{DT}{2} [(n-1)(1-\rho) + \rho] + \frac{nD^2 \alpha Tr_w}{2} + I_{vp}\sigma_1 \ln\left(\frac{\alpha_0}{\alpha}\right) + \nu I_{vp} DM_j
\]

(117)
Buyer’s total cost per unit time consists of the following elements.

- Buyer’s ordering cost per unit time is \( \frac{A_b}{T} \).
- Transportation cost per unit time is \( DF_j \).
- Inventory holding cost [excluding interest charges] per unit time is given by \( \frac{h_sDT}{2} \).
- During the credit period, the buyer sells the item and uses the sales revenue to earn interest at a rate of \( I_{be} \). At the end of the permissible delay period, the buyer pays the purchasing cost to the supplier and incurs a capital opportunity cost at a rate of \( I_{bp} \) for the items left unsold.

For \( M_j \) and \( T \), we have the following two cases a) \( T < M_j \)  b) \( T \geq M_j \).

Case 1. \( T < M_j \)

In this case, the inventory is completely depleted before the delayed payment due date. So the buyer need not pay an opportunity cost for the items kept in stock. But the buyer uses sales revenue and earns interest at a rate of \( I_{be} \). Therefore, the interest earned per unit time is

\[
\frac{p_1I_{be}}{T} \left[ \int_0^T dt \cdot DT(M_j - T) \right] = DP_1I_{be} \left[ M_j - T/2 \right].
\]

Case 2. \( T \geq M_j \)

In this case, the inventory is depleted after the delayed payment due date or on that date. Therefore the buyer pays an opportunity cost for the
items still kept in stock. The capital opportunity cost for the items still kept in stock. The capital opportunity cost per unit time is given by,

\[
\frac{v I_{bp}}{T} \int_{M_j}^T D(T-t)dt = \frac{v I_{bp} D}{2T} (T - M_j)^2.
\]

As the buyer does not pay the supplier until the end of the credit period, the buyer can use the sales revenue during the credit period to earn interest at a rate of \( I_{be} \). The interest earned by the buyer is given by,

\[
\frac{P_{I_{be}}}{T} \int_{0}^{M_j} D(t) dt = \frac{P_{I_{be}} DM_j^2}{2T}.
\]

Therefore, total cost for the buyer is given by

\[
TCB_j = \begin{cases} 
TCB_{i,j} & \text{if } T < M_j \\
TCB_{2,j} & \text{if } T \geq M_j 
\end{cases}
\]

(118)

where

\[
TCB_{i,j} = \frac{A_b}{T} + DF_j + \frac{h_b DT}{2} - Dp_{I_{be}} [M_j - T/2] 
\]

(119)

\[
TCB_{2,j} = \frac{A_b}{T} + DF_j + \frac{h_b DT}{2} + \frac{u I_{bp} D}{2T} (T - M_j)^2 - \frac{P_{I_{be}} DM_j^2}{2T}
\]

(120)

Total cost for both buyer and vendor is given by

\[
TC_j = \begin{cases} 
TC_{1,j} & \text{if } T < M_j \\
TC_{2,j} & \text{if } T \geq M_j 
\end{cases}
\]

(121)

where,

\[
TC_{1,j}(\alpha, T, n) = \frac{1}{T} \left( \frac{A_v}{n} + A_b \right) + I_{vp} \sigma_1 \ln \left( \frac{\alpha_0}{\alpha} \right) + D \left[ F_j - M_j \left( P_{I_{be}} - u I_{vp} \right) 
\right.
\]

\[
+ \frac{T}{2} \left\{ \left( h_v + p I_{vp} \right) \left[ (n-1)(1-\rho) + \rho \right] + h_b + P_{I_{be}} + nD \sigma w \right\}
\]

(122)
\[ \begin{align*}
TC_{2j}(\alpha, T, n) &= \frac{1}{T}\left(\frac{A_v}{n} + A_p\right) + I_{vp}\sigma_1 \ln \left(\frac{\alpha_0}{\alpha}\right) \\
&+ D \left\{ F_j - M_j v(I_{bp} - I_{vp}) - \frac{M_j^2}{2T} \left( p_{1}I_{be} - vI_{bp} \right) \\
&+ \frac{T}{2} \left( h_v + pI_{vp}\rho \right) (n-1)(1-\rho) + \rho \right] + h_v + vI_{bp} + nD\alpha r_w \right\} \right. \\
&\left. \right. \quad \left. \right. \quad \left. \right. \quad \left. \right. \quad \left. \right. \quad \left. \right. \quad (123) \right\}
\end{align*} \]

9.4. SOLUTION PROCEDURE

To examine the effect of \( n \) on the total cost per unit time, we first take the second-order partial derivative of (121) with respect to \( n \), for \( j=1,2,\ldots,k \), to obtain

\[ \frac{\partial^2 TC_j(\alpha, T, n)}{\partial n^2} \left. = \frac{\partial^2 TC_i(\alpha, T, n)}{\partial n^2} = \frac{2}{Tn^3} A_v > 0 \quad \text{for} \quad i=1,2. \right. \]

This shows that \( TC_j(\alpha, T, n) \) for \( j=1,2,\ldots,k \) is a convex function in \( n \) for fixed \( \alpha \) and \( T \). Thus the search for finding the optimal shipment number \( n^* \) is reduced to finding a local optimal solution.

Now, for fixed \( n \), \( TC_i(\alpha, T, n) \) (for \( i=1,2 \)) is convex in \( \alpha \) and \( T \) (refer to Appendix G). Therefore there exists a unique value of \( \alpha \) which minimizes \( TC_i(\alpha, T, n) \). \( \alpha \) can be obtained by solving the equation

\[ \frac{\partial TC_i}{\partial \alpha} = 0 \quad \text{for} \quad i=1,2. \]

The value of \( \alpha \) is given by

\[ \alpha = \frac{2I_{vp}\sigma_1}{nD^2Tr_w}. \quad (124) \]
Also, there exists unique values of \( T \) namely \( T_{1j} \) and \( T_{2j} \) which minimizes \( TC_{1j}(\alpha, T, n) \) and \( TC_{2j}(\alpha, T, n) \) respectively. \( T_{1j} \) can be obtained by solving the equation \( \frac{\partial TC_{1j}}{\partial T} = 0 \) and \( T_{2j} \) can be obtained by solving the equation \( \frac{\partial TC_{2j}}{\partial T} = 0 \) and it is given by

\[
T_{1j} = \frac{2\bar{A}}{D\sigma + h_b + p_1I_{be} + nD \alpha r_w},
\]

(125)

\[
T_{2j} = \frac{2\bar{A} - DM_j^2[p_1I_{be} - \nu I_{bp}]}{D\sigma + h_b + \nu I_{bp} + nD \alpha r_w},
\]

(126)

where \( \bar{A} = \frac{A_1}{n} + A_b \), and \( \sigma = (h_v + p I_{bp}) \left[ (n-1)(1 - \rho) + \rho \right] \).

To ensure \( T_{1j} < M_j \) (i.e., Case 1), we substitute (125) into the inequality \( T_{1j} < M_j \). We obtain that \( T_{1j} < M_j \) if and only if \( 2\bar{A} < \Delta_j \), where

\[
\Delta_j = DM_j^2\left[ \sigma + h_b + p_1I_{be} + nD \alpha r_w \right].
\]

(127)

Similarly to ensure \( T_{2j} \geq M_j \) (i.e., Case 2), we substitute (126) into the inequality \( T_{2j} \geq M_j \) and we obtain that \( T_{2j} \geq M_j \) if and only if \( 2\bar{A} \geq \Delta_j \), where

\[
\Delta_j = DM_j^2\left[ \sigma + h_b + p_1I_{be} + nD \alpha r_w \right].
\]

(128)

Now consider the constraint \( 0 < \alpha \leq \alpha_0 \). From equation (124), we note that \( \alpha^* \) is positive. Also if \( \alpha^* < \alpha_0 \), then \( (T^*, \alpha^*) \) is an interior optimal solution for given \( n \). However, if \( \alpha^* \geq \alpha_0 \), then it is unrealistic to invest in
changing the current process quality level. For this special case, take \( \alpha^* = \alpha_0 \).

**Theorem 9.1**

For any given \( n \) and \( j = 1, 2, \ldots, k \) we have

(a) If \( 2\overline{a} < \Delta_j \), then the buyer's optimal replenishment cycle length is \( T_j^* = T_{1j} \).

(b) If \( 2\overline{a} \geq \Delta_j \), then the buyer's optimal replenishment cycle length is \( T_j^* = T_{2j} \).

**Proof.**

It immediately follows from equations (127) and (128).

**Theorem 9.2**

For any given \( n \) and \( j = 1, 2, \ldots, k \), let \( Q_j = DT_j^* \), we have

(a) If \( q_j \leq Q_j < q_{j+1} \), then \( TC_j(\alpha, T, n) \) at the point \( T = T_j^* \) has a minimum value.

(b) If \( Q_j \geq q_{j+1} \), the lot size ordering by the buyer is over the upper-bound under the credit period \( M_j \), then \( T_j^* \) is not a feasible solution.

(c) If \( Q_j < q_j \), the lot size ordering by the buyer is less than the lower-bound under the credit period \( M_j \), then \( T_j^* \) is not a feasible solution. However,

\[
TC_j(\alpha, T, n) \text{ is a strictly increasing function in } T \in \left[ \frac{q_j}{D}, \frac{q_{j+1}}{D} \right].
\]

Hence,

\[
TC_j(\alpha, T, n) \text{ at point } T = \frac{q_j}{D} \text{ has a minimum value.}
\]
Proof.

The proof immediately follows from Theorem 9.1 and the fact that \( TC_j(\alpha, T, n) \) is a convex function of \( T \) and \( \alpha \) and the discount schedule restructured from assumptions 11 and 12. From Theorem 9.2 and discussions in section 9.4 we frame the following algorithm.

9.4.1 ALGORITHM

Step 1. Set \( n = 1 \).

Step 2. Start with \( \alpha = \alpha_0 \).

Step 3. For \( j = 1, ..., k \).

(a) If \( 2 \Delta_j \leq \Delta_j \) then \( T_j = T_{1j} \). Find \( T_j \) from (125).

(b) If \( 2 \Delta_j \geq \Delta_j \) then \( T_j = T_{2j} \). Find \( T_j \) from (126).

Step 4. Calculate \( Q_j = DT_j^* \) and check each \( Q_j \) under \( M_j, \ j = 1, 2, ..., k \).

Step 4.1 for \( j = 1, 2, ..., k - 1 \).

(a) If \( q_j \leq Q_j < q_{j+1} \), then \( T_j^* \) is a feasible solution. Determine \( TC_j \) using equation (122) or (123). That is

\[
TC_j = \begin{cases} 
TC_{1j} & \text{if } T_j < M_j \\
TC_{2j} & \text{if } T_j \geq M_j 
\end{cases}
\]

(b) If \( Q_j \geq q_{j+1}, T_j^* \) is not a feasible solution. Set \( TC_j = 0 \).

(c) If \( Q_j < q_j \), \( T_j^* \) is not a feasible solution and hence

\[ TC_j(\alpha, T, n) \] has a minimum value at the point \( T_j^* = \frac{q_j}{D} \).
Obtain $TC_j$ using equation (122) or (123). That is

$$TC_j = \begin{cases} 
TC_{1j} & \text{if } T_j < M_j \\
TC_{2j} & \text{if } T_j \geq M_j
\end{cases}. $$

**Step 4.2 for $j = k$**

(a) If $Q_k \geq q_k$, then $T_k^*$ is a feasible solution.

(b) If $Q_k < q_k$, set $T_k^* = \frac{q_k}{D}$.

(c) Obtain $TC_j$ using equation (122) or (123). That is

$$TC_k = \begin{cases} 
TC_{1k} & \text{if } T_k < M_k \\
TC_{2k} & \text{if } T_k \geq M_k
\end{cases}. $$

**Step 5.** Find $\min_{j=1,2,\ldots,k} TC_j(\alpha, T_j, n)$. Set $TC^* = \min_{j=1,2,\ldots,k} TC_j(\alpha, T_j, n)$. Then for given $n$ and $\alpha$ value of $T$ corresponding to $\min_{j=1,2,\ldots,k} TC_j(\alpha, T_j, n)$ is the optimal value. Therefore for given $n$ and $\alpha$, $(n, T^{(n)}, \alpha^{(n)})$ is the optimal value.

**Step 6.** Calculate $\alpha_i$ with the values of $T$ and $n$ obtained in Step 5. Set $\alpha = \alpha_1$. Repeat Steps 3 to 5 until no change occurs in the value of $T$ and $\alpha$.

**Step 7.** Let $n = n + 1$. Repeat Steps 2-6 to find $TC^*(n, T^{(n)}, \alpha^{(n)})$.

**Step 8.** If $TC^*(n, T^{(n)}, \alpha^{(n)}) \leq TC^*(n-1, T^{(n-1)}, \alpha^{(n-1)})$, go to Step 7. Otherwise the optimal solution is $(n^*, T^*, \alpha^*) = ((n-1), T^{(n-1)}, \alpha^{(n-1)})$. 

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9.5 NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS

9.5.1 NUMERICAL EXAMPLE

Example 9.1

In order to illustrate the above solution procedure, let us consider an inventory system with the following data in appropriate units:

\[ a = 10^5, \rho = 0.75; \ A_v = 5000, \ A_b = 3000, \ h_v = 5, \ h_b = 6, \ \sigma_1 = 100, \ \nu = 7, \ \tau = 3.5, \]

\[ p_1 = 12, \ I_{vp} = 0.04, \ I_{be} = 0.09, \ I_{bp} = 0.10, \ a_0 = 1, \ a_1 = 1.5, \ a_2 = 0.15, \ \alpha_0 = 0.02, \ r_w = 0.95 \]

and the product weighs \( \omega = 2 \) lbs per unit. In addition, we assume that the supplier offers credit terms as follows:

<table>
<thead>
<tr>
<th>( \eta )</th>
<th>( Q ) (units/order)</th>
<th>( N_\eta ) (days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 0 \leq Q &lt; 100 )</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>( 100 \leq Q &lt; 1000 )</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>( 1000 \leq Q )</td>
<td>35</td>
</tr>
</tbody>
</table>

A freight rate schedule is offered below:

<table>
<thead>
<tr>
<th>( \varepsilon )</th>
<th>( W ) (lbs/ship)</th>
<th>( \chi_\varepsilon ) $/lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 0 \leq W &lt; 100 )</td>
<td>0.80</td>
</tr>
<tr>
<td>2</td>
<td>( 100 \leq W &lt; 500 )</td>
<td>0.67</td>
</tr>
<tr>
<td>3</td>
<td>( 500 \leq W )</td>
<td>0.55</td>
</tr>
</tbody>
</table>

Combining these two tables, we obtain the following schedule.
<table>
<thead>
<tr>
<th>$J$</th>
<th>$Q$ (units/order)</th>
<th>$M$ (days)</th>
<th>$F($/unit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$0 \leq Q &lt; 50$</td>
<td>10</td>
<td>1.60</td>
</tr>
<tr>
<td>2</td>
<td>$50 \leq Q &lt; 100$</td>
<td>10</td>
<td>1.34</td>
</tr>
<tr>
<td>3</td>
<td>$100 \leq Q &lt; 250$</td>
<td>20</td>
<td>1.34</td>
</tr>
<tr>
<td>4</td>
<td>$250 \leq Q &lt; 1000$</td>
<td>20</td>
<td>1.10</td>
</tr>
<tr>
<td>5</td>
<td>$1000 \leq Q$</td>
<td>35</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Using the algorithm given above we have found the optimal solution which is given in Table 14 as system 1. In system 1 both freight rate and trade credit are linked to the order quantity. Optimal solutions obtained in other policies are presented as system 2, 3 and 4 in the same table. These policies illustrate the effects of the strategy represented by system 1 where both freight rate and trade credit are linked to the order quantity.

In system 1 both the freight rate and credit period offered are dependent on the order quantity. The optimal joint total cost for the entire system is $1214.10 and the optimal order quantity is 98 units / order. Therefore the buyer pays the freight charge of $1.34 /unit and settles the account within 10 days after delivery. The optimal replenishment cycle time is 5.85 days. The supplier’s production lot size is 392 units per set up.

From Table 14, we see that the joint total cost is less in system 1 compared to other systems. The results in the table suggests that managers should consider a trade credit policy and freight rate policy as a marketing strategy and it should be linked with the order quantities.
Figure 11. Effect of capacity utilization $\rho$ on the total cost of the system
Table 14. Optimal solution for Example 9.1

<table>
<thead>
<tr>
<th>System</th>
<th>Trade Credit</th>
<th>Freight Rate Discounts</th>
<th>Optimal Paying Time</th>
<th>Optimal Freight Rate</th>
<th>$n^*$</th>
<th>$Q^*$</th>
<th>$n^<em>Q^</em>$</th>
<th>$D$</th>
<th>$T^*$</th>
<th>$\alpha$</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yes</td>
<td>10 days after delivery</td>
<td>1.34</td>
<td>4</td>
<td>98</td>
<td>392</td>
<td>17</td>
<td>5.85</td>
<td>0.0013</td>
<td>516.20</td>
<td>697.90</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
<td>Yes</td>
<td>Cash on delivery</td>
<td>1.34</td>
<td>3</td>
<td>107</td>
<td>322</td>
<td>17</td>
<td>6.42</td>
<td>0.0016</td>
<td>844.98</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>No</td>
<td>Order quantity related</td>
<td>Fixed 1.60</td>
<td>4</td>
<td>98</td>
<td>392</td>
<td>17</td>
<td>5.85</td>
<td>0.0013</td>
<td>524.55</td>
</tr>
<tr>
<td>4</td>
<td>No</td>
<td>No</td>
<td>Cash on delivery</td>
<td>Fixed 1.60</td>
<td>3</td>
<td>107</td>
<td>321</td>
<td>17</td>
<td>6.42</td>
<td>0.0016</td>
<td>853.32</td>
</tr>
</tbody>
</table>

Table 15. Sensitivity analysis with respect to $\rho$

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>Optimal paying time</th>
<th>Optimal Freight rate</th>
<th>$n$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$\alpha$</th>
<th>$nQ$</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.55</td>
<td>20 days after delivery</td>
<td>1.34</td>
<td>2</td>
<td>7.32</td>
<td>122</td>
<td>0.0021</td>
<td>245</td>
<td>500.25</td>
</tr>
<tr>
<td>0.65</td>
<td>20 days after delivery</td>
<td>1.34</td>
<td>3</td>
<td>6.30</td>
<td>105</td>
<td>0.0016</td>
<td>316</td>
<td>506.34</td>
</tr>
<tr>
<td>0.75</td>
<td>10 days after delivery</td>
<td>1.34</td>
<td>4</td>
<td>5.85</td>
<td>98</td>
<td>0.0013</td>
<td>391</td>
<td>516.21</td>
</tr>
<tr>
<td>0.85</td>
<td>10 days after delivery</td>
<td>1.34</td>
<td>5</td>
<td>5.73</td>
<td>96</td>
<td>0.0011</td>
<td>479</td>
<td>519.93</td>
</tr>
<tr>
<td>0.95</td>
<td>10 days after delivery</td>
<td>1.34</td>
<td>9</td>
<td>5.50</td>
<td>92</td>
<td>0.0006</td>
<td>827</td>
<td>528.20</td>
</tr>
</tbody>
</table>
9.5.2 SENSITIVITY ANALYSIS

In this section, sensitivity analysis with respect to major parameters are carried out.

- To see the effects of quality improvement, we find out the optimal values of the fixed quality level model with $\alpha=0.02$ (with other data as in Example 9.1) in Table 16. Comparing Tables 14 and 16, we see that savings on the total cost of the entire supply chain can be achieved from the efforts of quality improvement. The percentage saving is calculated and shown in Table 16.

- We study the effects of variable capacity utilization $\rho$ considering different values of $\rho$ and keeping the values of other parameters the same as in Example 9.1. The optimal values for various values of $\rho$ are given in Table 15 and graphically represented in Figure 11. From Table 15 and Figure 11 we note that the total cost for the entire supply chain decreases with increase in $\rho$. That is, the closer the production rate is to the demand rate, lesser is the cost incurred in the integrated mode. Thus, this implies that if the supplier and the buyer would work in a cooperative manner to coordinate supply with actual customer demand, significant cost savings can be achieved.

- Sensitivity analysis with respect to the parameters $A_v$ and $A_b$ are carried out and the results are given in Table 17. When the setup cost $A_v$ is higher the supplier tends to produce more in one production run. Thus frequent deliveries are made to the buyer. The total cost of the system increases with increase in the setup cost. When the buyer’s ordering
cost increases, the total cost of the system also increases with increase in $A_b$.

- Table 14 implies that when the supplier offers both trade credit linked to the buyer’s order quantity and freight charge based on order size (system 1), we see that the total cost of the integrated supply chain and that of the buyer is minimum, when compared to all other systems. The cost of the entire system is minimized. The supplier is also benefitted as this will entice the buyer to enlarge his order size.
Table 16. Optimal solutions for fixed quality model

Fixed quality model $\alpha=0.02$

<table>
<thead>
<tr>
<th>System</th>
<th>Trade Credit</th>
<th>Freight Rate Discounts</th>
<th>Optimal Paying Time</th>
<th>Optimal Freight Rate</th>
<th>$n^*$</th>
<th>$Q^*$</th>
<th>$n^<em>Q^</em>$</th>
<th>$D$</th>
<th>$T^*$</th>
<th>$TC^0$</th>
<th>Percent Savings (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Order-quantity related</td>
<td>Yes</td>
<td>20 days after delivery</td>
<td>1.34</td>
<td>3</td>
<td>104</td>
<td>312</td>
<td>17</td>
<td>6.22</td>
<td>1251.50</td>
<td>2.99</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
<td>Yes</td>
<td>Cash on delivery</td>
<td>1.34</td>
<td>3</td>
<td>104</td>
<td>312</td>
<td>17</td>
<td>6.22</td>
<td>1499.00</td>
<td>2.45</td>
</tr>
<tr>
<td>3</td>
<td>Order quantity related</td>
<td>No</td>
<td>20 days after delivery</td>
<td>Fixed 1.60</td>
<td>3</td>
<td>104</td>
<td>312</td>
<td>17</td>
<td>6.22</td>
<td>1259.80</td>
<td>2.96</td>
</tr>
<tr>
<td>4</td>
<td>No</td>
<td>No</td>
<td>Cash on delivery</td>
<td>Fixed 1.60</td>
<td>3</td>
<td>104</td>
<td>312</td>
<td>17</td>
<td>6.22</td>
<td>1507.40</td>
<td>2.44</td>
</tr>
</tbody>
</table>

$TC^0$ – Total cost of the system with fixed quality; $TC$ – Total cost of the system when $\alpha$ is a variable.

Percent Savings = $\{(TC^0 - TC)/TC^0\} * 100$. 
Table 17. Sensitivity analysis with respect to $A_v$ and $A_b$

<table>
<thead>
<tr>
<th>Parameter change</th>
<th>$A_v$</th>
<th>$A_b$</th>
<th>Optimal paying time</th>
<th>Optimal Freight rate</th>
<th>$n$</th>
<th>$T$</th>
<th>$Q$</th>
<th>$\alpha$</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-50%</td>
<td>-50%</td>
<td>10 days after delivery</td>
<td>1.34</td>
<td>3</td>
<td>5.82</td>
<td>97</td>
<td>0.0017</td>
<td>1079.10</td>
</tr>
<tr>
<td></td>
<td>-25%</td>
<td>-25%</td>
<td>20 days after delivery</td>
<td>1.34</td>
<td>3</td>
<td>6.12</td>
<td>102</td>
<td>0.0016</td>
<td>1148.90</td>
</tr>
<tr>
<td></td>
<td>+25%</td>
<td>+25%</td>
<td>20 days after delivery</td>
<td>1.34</td>
<td>4</td>
<td>6.07</td>
<td>101</td>
<td>0.0012</td>
<td>1266.50</td>
</tr>
<tr>
<td></td>
<td>+50%</td>
<td>+50%</td>
<td>20 days after delivery</td>
<td>1.34</td>
<td>4</td>
<td>6.27</td>
<td>105</td>
<td>0.0012</td>
<td>1317.20</td>
</tr>
</tbody>
</table>


9.6 CONCLUSION

In this study, we formulated a supply chain considering quality improvement. Here the order size is linked to both trade credit and freight rate. The numerical example and analysis show that the managers should consider trade credit policy and freight rate policy as a marketing strategy and it should be linked with the order quantities. Further, closer the production rate of the supplier is to the demand rate of the buyer, lesser is the cost incurred in the integrated mode. Thus, the supplier and buyer should work cooperatively and coordinate supply with actual demand to achieve significant cost savings. Significant cost savings on the total cost of the entire supply chain can also be achieved by quality improvement. Thus our model efficiently determines the appropriate ordering policy when the quality of the product is improved and offers some managerial insights to reduce the total cost of the entire supply chain so that both the supplier and the buyer are mutually benefitted.