CHAPTER 7

REPLENISHMENT POLICY FOR TWO LEVEL SUPPLY CHAIN STRUCTURE EMPLOYING POSTPONEMENT STRATEGY WITH PERMISSIBLE DELAY IN PAYMENTS

7.1 INTRODUCTION

Postponement, also known as late customization or delayed product differentiation, refers to delaying some product differentiation processes in a supply chain as late as possible until the supply chain is cost effective. Postponement strategy is highly successful in a wide range of industries that require high differentiation like high-tech industry, food industry and fashion industry. Postponement is not an omnipotent strategy. It has both advantages and disadvantages. The advantages include, following the JIT principles of production, reducing end-product inventory, making forecasting easier and pooling risks. Thus the concept is to delay the point of product differentiation at a closer point to the customer. This involves designing and developing generic products that can be customized once the actual demand is known. It also involves the implementation of precise inventory approach to position inventory farther away from the customer while satisfying the service levels and reducing the inventory costs. Postponement lessens the forecasting horizon and thereby solves the uncertainty of end product demand. The high cost of redesigning and manufacturing generic components is the main drawback of postponement.
Thus evaluation of postponement structure is an important issue. One practical example is Hewlett-Packard Development Company. HP produces generic printers in its factories and distributes them to local distribution centers, where power plugs with appropriate voltage and user manuals in the right language are packed. They have saved a lot of money every year by adopting the postponement strategy.

To the best of our knowledge, there exists no work that study the interaction between inventory and postponement in a supply chain for non-instantaneous deteriorating items under the conditions of permissible delay in payments. In this chapter, we frame an EOQ-based model for non-instantaneous deteriorating items to analyze postponement when the supplier provides a permissible delay in payments.

Here it is assumed that the supplier permits the buyer certain fixed period of time (permissible delay in time) for settling the account and does not charge any interest from the buyer on the amount owed during this period. In a supply chain this concept will play a more vital role. When the vendor offers credit period, both the buyer and the vendor will be mutually benefitted. We have incorporated this concept in the proposed model.

When the supplier offers trade credit period and when it is utilized the total inventory cost is reduced to a greater extent. The objective of this chapter is to show that the postponement system can outperform the independent system with non-instantaneous deteriorating items when the supplier offers permissible delay in payments. The impact of permissible delay period ($M$) and the deterioration rate on the inventory replenishment
policy are discussed using numerical examples. Further, the impact of deterioration rate on the difference in cost between the two systems is also analyzed.

In general, we show that when the permissible delay period is utilized to a greater extent in a postponement system considerable amount of cost is saved. Specifically, we establish a mathematical model, which is a general framework of number of previous existing models.

The rest of the chapter is organized as follows. In the next section problem description of this chapter is given followed by the notations and assumptions used throughout this chapter. In section 7.3, we have formulated a mathematical model to evaluate the impact of deterioration on optimal replenishment policy. Some useful theorems are developed in section 7.4 to characterize the optimal solution. An algorithm is also given to find the optimal solutions. In section 7.5, we compare and discuss about the postponement and independent systems. In section 7.6, number of numerical examples is given to illustrate the model and sensitivity analysis with respect to major parameters of the system is carried out and the results are discussed in detail. This is followed by conclusion in section 7.7.
7.2 PROBLEM DESCRIPTION

In this chapter, we develop a supply chain model involving a retailer and \( n \) customers. The retailer orders \( n \) different products in response to the demands of the customers. It is assumed that the \( n \) end-products are manufactured from the same raw material or semi-manufactured products. The end-products belong to the same product category, but they have slight differences and the customization process can be delayed after ordering. The retailer can order the \( n \) end-products independently in an independent system. The retailer can order the raw material (or the semi-manufactured product) and finish the customization process. The customization is postponed after ordering. The ordering decisions of the raw materials of all the products can be combined. This can be viewed as a postponement system. Whereas, in an independent system \( n \) different decisions have to be taken for ordering \( n \) different products. (For example, retailers of a soft drink supplier orders concentrated syrup and mix it with carbonated water to make different soda products for sale at their retail stores. In this situation the retailer makes only one decision (ordering policy) to acquire the concentrated syrup rather than making many different decisions to acquire different products marketed by the soft drink supplier). Further, it is assumed that the supplier offers trade credit to the buyer and that the deterioration of raw materials and end-product is non-instantaneous. Some useful theorems are developed in this chapter to characterize the optimal solutions. We investigate the impact of product deterioration rate on the inventory replenishment policy. The difference between the postponement and independent systems are discussed.
7.2.1 NOTATIONS

Despite the common notations given in section 1.8, the following notations are used only in this chapter.

\( i \)  
end-product, \( i=1,2,\ldots, n \)

\( D_i \)  
demand rate of an end-product \( i \), \( D_i > 0 \)

\( \theta \)  
deterioration rate of end-products and raw materials \( \theta \geq 0 \)

\( p \)  
common purchase cost per unit \( p > 0 \)

\( A \)  
common fixed ordering cost per order \( A > 0 \)

\( h \)  
common unit holding cost per unit per year excluding interest charges \( h > 0 \)

\( T_i \)  
total cycle time for the end-product \( i \), \( T_i > 0 \)

\( M \)  
trade credit period

\( TC (T_i) \)  
total average cost per unit time for ordering and keeping end-product \( i \)

TCI  
total average cost per unit time for ordering and keeping \( n \) end-products in an independent system

TCP  
total average cost per unit time for ordering and keeping \( n \) end-products in a postponement system (excluding the customization cost)

7.2.2 ASSUMPTIONS

1. The replenishment rate is infinite and the lead time is zero.
2. The end-product demand rates \( D_i \) are deterministic and constant.
3. Shortages are not allowed.
4. All the end-products are produced from the same type of raw materials and the ratio of raw material to end-product is 1:1.

5. During the trade credit period \( M \), the account is not settled; generated sales revenue is deposited in an interest bearing account. At the end of the period, the retailer pays of all units bought and starts to pay the capital opportunity cost for the items in stock.

6. \( p_i \geq p_i^l; I_p \geq I_e; pI_p \geq p_i^lI_e \).

7. Deterioration of raw materials and end-product is considered only after they have been received into inventory and the items start to deteriorate after some time \( "t_d" \). That is deterioration is non-instantaneous. There is no replacement of deteriorated inventory.

8. The product life (time to deterioration) \( t \) has a probability density function \( f(t) = \theta e^{-\theta (t-t_d)} \) for \( t > t_d \). The cumulative distribution function of \( t \) is given by \( F(t) = \int_{t_d}^{t} f(x)dx = 1 - e^{-\theta (t-t_d)} \) for \( t > t_d \); so that the deterioration rate is \( r(t) = \frac{f(t)}{1 - F(t)} = \theta \) for \( t > t_d \).

9. The length of time in which the product has no deterioration, \( t_d \) can be estimated by utilizing the random sample data of the product during past time and statistical maximum likelihood method. For simplicity, we assume \( t_d \) is a given constant and \( t_d \leq T \).

10. The time horizon is infinite.

11. \( T^* \) is the decision variable. \( T^* \) is the optimal length of replenishment cycle.
7.3 MODEL FORMULATION

![Graphical representation of inventory level](image)

**Figure 6. Graphical representation of inventory level**

We assume that the demand rates of end products are independent and constant. We formulate two EOQ based models to describe the supply chain. In the first model, the retailer orders the \( n \) end-products independently with different schedules, so there are \( n \) EOQ decisions. However, in the second model customization is postponed after ordering and the ordering decisions can be combined so that a single EOQ decision is made. This practice can be viewed as a form of postponement strategy.

The inventory system evolves as follows: \( I_m \) units of item arrive at the inventory system at the beginning of each cycle. During the time interval \([0,t_d]\), the inventory level is decreasing only owing to demand and the inventory level is dropping to zero due to demand and deterioration of the item during the time interval \([t_d,T]\) (illustrated in Figure 6). As
described above the inventory level \( I(t) \) of an end-product at time \( t \) is governed by the following differential equation.

\[
\frac{dI(t)}{dt} = \begin{cases} 
-D & \text{if } 0 \leq t \leq t_d \\
-D - \theta I(t) & \text{if } t_d \leq t \leq T 
\end{cases}
\]  

(83)

with boundary conditions \( I(0) = I_m, I(T') = 0 \).

Solution of equation (83) is

\[
I(t) = \begin{cases} 
I_1(t) & \text{if } 0 \leq t \leq t_d \\
I_2(t) & \text{if } t_d \leq t \leq T 
\end{cases}
\]  

(84)

where \( I_1(t) = D(t_d - t) + \frac{D}{\theta} \left[ e^{\theta(t-T_d)} - 1 \right] \) and \( I_2(t) = \frac{D}{\theta} \left[ e^{\theta(T-t)} - 1 \right] \).

Based on equation (84), we can obtain the total average cost per unit time for ordering and keeping the end-product. In order to establish the total average inventory cost function we consider the following cases:

Case (i) \( 0 < M \leq t_d \),

Case (ii) \( t_d < M \leq T \) and

Case (iii) \( M > T \).

Now, we have the total average cost per unit time as a function of \( T \) as follows: (refer to Appendix E)

\[
TC(T) = \begin{cases} 
TC_1(T) & \text{if } 0 < M \leq t_d \\
TC_2(T) & \text{if } t_d < M \leq T \\
TC_3(T) & \text{if } M > T 
\end{cases}
\]  

(85)

where ,
\[ TC_1(T) = \frac{A}{T} + pD + D\left[ht_d + pI_p(t_d - M)\right]\left[\frac{e^{\theta(T - t_d)}}{\theta T} - 1\right] \]
\[ + \frac{D(h + p\theta + pI_p)}{\theta^2 T}\left[e^{\theta(T - t_d)} - \theta(T - t_d) - 1\right] + \frac{pI_pD(t_d - M)^2 + hDt_d^2}{2T} \]
\[ - \frac{p_t I_v DM^2}{2T}, \] (86)

\[ TC_2(T) = \frac{A}{T} + pD + \frac{Dht_d}{\theta T}\left[e^{\theta(T - t_d)} - 1\right] + \frac{D(h + p\theta)}{\theta^2 T}\left[e^{\theta(T - t_d)} - \theta(T - t_d) - 1\right] \]
\[ + \frac{pI_pD}{\theta^2 T}\left[e^{\theta(T - M)} - \theta(T - M) - 1\right] + \frac{hDt_d^2}{2T} - \frac{p_t I_v DM^2}{2T} \] (87)

\[ \text{and } TC_3(T) = \frac{A}{T} + pD + \frac{hDt_d}{\theta T}\left[e^{\theta(T - t_d)} - 1\right] + \frac{D(h + p\theta)}{\theta^2 T}\left[e^{\theta(T - t_d)} - \theta(T - t_d) - 1\right] \]
\[ + \frac{hDt_d^2}{2T} - p_t I_v D\left(M - \frac{T}{2}\right). \] (88)

7.4 THEORETICAL RESULTS

Case (i) \(0 < M \leq t_d\)

The first order necessary condition for the total average inventory cost per unit time in (86) to be minimum is \(dTC_1(T)/dT = 0\), which leads to

\[ -A + \frac{D[ht_d + pI_p(t_d - M)]}{\theta}\left[\theta Te^{\theta(T - t_d)} - e^{\theta(T - t_d)} + 1\right] \]
\[ + \frac{D(h + p\theta + pI_p)}{\theta^2}\left[\theta Te^{\theta(T - t_d)} - e^{\theta(T - t_d)} + 1\right] - \frac{D(h + p\theta + pI_p)}{\theta} \]
\[ - \frac{D[pI_p(t_d - M)^2 + ht_d^2]}{2} + \frac{p_t I_v DM^2}{2} = 0. \] (89)
Now we will show that the value of $T$ which satisfies (89) not only exists but also unique under certain condition. We have the following lemma asserted by Ouyang et al. [102]. Let $\Delta_1 = Dl_0^2(h + pI_p) + DM^2(p_1I_e - pI_p)$.

**Lemma 7.1.** For $0 < M \leq t_d$,

a) If $2A \geq \Delta_1$, then the point $\left(T^* = T_i > 0\right)$ where $T_i \in [t_d, \infty)$ and satisfies equation (89) exists and is unique. The point $\left(T^* = T_i > 0\right)$ is also the unique global optimum for the problem $\min \{TC_1(T/\theta): 0 < T < \infty\}$.

b) If $2A < \Delta_1$, then the total cost per unit time $TC_1(T/\theta)$, $(0 < T < \infty)$ has a minimum value at the boundary point $T = t_d$.

Thus $T^*$ can be uniquely determined as a function of $\theta$, say $T^* = T(\theta)$.

This also implies that $TC_1(T^*|\theta) = TC_1(T(\theta)|\theta)$.

**Theorem 7.1.**

$T\hat{C}_1(\theta) = TC_1(T^*|\theta)$ is an increasing and continuous function of $\theta$ in $[0, +\infty)$ and

$$\lim_{\theta \to 0} T\hat{C}_1(\theta) = pD + \sqrt{2A_iDg_2 - pI_pMD},$$

where $A_i = A + \frac{DM^2[pI_p - p_1I_e]}{2}$, $g_2 = h + pI_p$.

**Proof.** Recalling that the power series for $e^x$ as $\sum_{n=0}^{\infty} \frac{x^n}{n!}$, we have,

$\text{TC}_1(T/\theta) = \frac{A}{T} + pD + \frac{hT_d + pI_p(t_d - M)}{T} \left(\frac{T - t_d}{T}\right) \sum_{n=1}^{\infty} \frac{\theta (T - t_d)}{n!}^{n-1}$

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\[
\frac{D(h + p\theta + pI_p)}{T}(T - t_d) + \sum_{n=2}^{\infty} \left[\theta(T - t_d)\right]^{n-2} + \frac{pI_p D(t_d - M)^2 + hD_t^2}{2T} - \frac{pI_e DM^2}{2T}
\]

For \(\theta \geq 0\), it is obvious that \(TC_1(T|\theta)\) is an increasing function of \(\theta\) for each fixed value of \(T > 0\). If \(\theta_1 < \theta_2\) we have,

\[TC_1(T(\theta_1)|\theta_2) > TC_1(T(\theta_2)|\theta_1) \geq TC_1(T(\theta_1)|\theta_1) = TC_1(T(\theta_1)).\]

Thus \(TC_1(T|\theta)\) is an increasing function of \(\theta\) in \([0, +\infty)\). Let

\[f_1(T, \theta) = -A + D\left[ht_d + pI_p(t_d - M)\right] - \frac{\theta T e^{\theta(T - t_d)} - e^{\theta(T - t_d)} + 1}{\theta} + \frac{D(h + p\theta + pI_p)(T - t_d)}{\theta} - \frac{D\left[pI_p(t_d - M)^2 + hD_t^2\right] + pI_e DM^2}{2T}.
\]

We have,

\[\frac{\partial f_1}{\partial T} = DT\theta e^{\theta(T - t_d)}\left\{ht_d + pI_p(t_d - M) + \frac{(h + p\theta + pI_p)}{\theta}\right\} > 0.
\]

Now \(T(\theta)\) is a continuous function of \(\theta\) in \([0, +\infty)\). Moreover \(TC_1(T|\theta)\) is a continuously differentiable real function for \(T > 0\) and \(\theta \geq 0\). Thus \(TC_1(T|\theta)\) is also a continuous function of \(\theta\) in \([0, +\infty)\).

Let \(G_1(T)\),

\[G_1(T) = \frac{A}{T} + D\left[ht_d + pI_p(t_d - M)\right] - \frac{(h + p\theta + pI_p)}{T} + \frac{D(T - t_d)^2(h + pI_p)}{2T} + \frac{pI_p D(t_d - M)^2 + hD_t^2}{2T} - \frac{pI_e DM^2}{2T}.
\]

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Because $T\hat{C}_1(\theta)$ is continuous in $[0, +\infty)$ we have,

$$\lim_{\theta \to 0} T\hat{C}_1(\theta) = T\hat{C}_1(0)$$

$$= \min_{0 < T < \infty} \{ G_1(T) + pD \}$$

$$= pD + \min_{0 < T < \infty} \{ G_1(T) \}$$

$$\geq pD + \sqrt{2A_i Dg_2} - pl_{p} MD.$$ 

Therefore, $\lim_{\theta \to 0} T\hat{C}_1(\theta) \geq pD + \sqrt{2A_i Dg_2} - pl_{p} MD$;

where $A_i = A + \frac{DM^2 [pl_{p} - pl_{e}]}{2}$; $g_2 = h + pl_{p}$.

**Case 2.** $t_d < M \leq T$

The first-order necessary condition for $TC_2(T)$ to be minimum is $dTC_2(T)/dT = 0$, which leads to

$$- A + \frac{D(\theta h_d + h + p\theta)}{\theta^2} \left[ \theta Te^{\theta(T-t_d)} - e^{\theta(T-t_d)} + 1 \right] + \frac{Dpl_{p}}{\theta^2} \left[ \theta Te^{\theta(T-M)} - e^{\theta(T-M)} + 1 \right]$$

$$- \frac{D[(h + p\theta)I_d + pl_{p} M]}{\theta} - \frac{hDt_d^2}{2} + \frac{pl_{e} DM^2}{2} = 0. \quad (91)$$

Under certain conditions, we can show that the value of $T$ which satisfies equation (91) not only exists but also is unique. Therefore, we have the following lemma asserted by Ouyang et al. [102].

Let $\Delta_2 = Dl_d^2 (h + pl_{e})$; $\Delta_3 = \frac{2D(\theta h_d + h + p\theta)}{\theta^2} \left[ \theta Me^{\theta(M-t_d)} - e^{\theta(M-t_d)} + 1 \right]$ 

$$- \frac{2D(h + p\theta)I_d}{\theta} - hDt_d^2 + pl_{e} DM^2.$$
Lemma 7.2. For $t_d < M \leq T$,

(a) If $2A \geq \Delta_3$, then the point $(T^* = T_{H^*} > 0)$ where $T_{H^*} \in [M, \infty)$ and satisfies equation (91) exists and is unique. The point $(T^* = T_{H^*} > 0)$ is also the unique global optimum for the problem $\min \{ TC_2 (T/\theta) : 0 < T < \infty \}$.

(b) If $2A < \Delta_3$ then the total inventory cost per unit time $TC_2(T/\theta)$, $(0 < T < \infty)$ has a minimum value at the boundary point $T = M$.

Thus in this case also $T^*$ can be uniquely determined as a function of $\theta$, say $T^* = T(\theta)$. This also implies that $TC_2(T^*/\theta) = TC_2(T(\theta)/\theta)$.

Theorem 7.2. $T^*_C(\theta) \Delta TC_2(T^*/\theta)$ is an increasing and continuous function of $\theta$ in $[0, +\infty)$ and $\lim_{\theta \to 0} T^*_C(\theta) = pD + \sqrt{2A_1Dg_2 - pl_pMD}$, where $A_1 = A + \frac{DM^2}{2}[pD_p - pD_p], \quad g_2 = h + pl_p$.

Proof. Using a similar proof of Theorem 7.1, we can obtain easily the validity of Theorem 7.2.

Case 3. $M > T$

The first-order necessary condition for $TC_3(T)$ to attain its minimum value is $dTC_3(T)/dT = 0$, which leads to

$$- A + \frac{D(hD_d + h + p\theta)}{\theta^2} \left[ \theta T e^{\theta(t-d)} - e^{\theta(t-d)} + 1 \right] - \frac{D(h + p\theta)D_d}{\theta} - \frac{hD_d^2}{2} + \frac{pD_pDT^2}{2} = 0. \quad (92)$$

Again it is not easy to find the closed-form solution of $T$ from equation (92). The following lemma asserted by Ouyang et al. [102] shows
that the value of $T$ which satisfies equation (92) not only exists but also is unique under certain conditions.

**Lemma 7.3.** For $M > T$,

(a) If $\Delta_2 \leq 2A \leq \Delta_3$, then the point $(T^* = T_{\text{opt}} > 0)$ where $T_{\text{opt}} \in [t_d, M]$ and satisfies equation (92) exists and is unique. The point $(T^* = T_{\text{opt}} > 0)$ is also the unique global optimum for the problem $\min \{ T C_3(T|\theta): 0 < T < \infty \}$.

(b) If $2A > \Delta_3$, then the total inventory cost per unit time $T C_3(T|\theta)$, $0 < T < \infty$ has a minimum value at the boundary point $T = M$.

(c) If $2A < \Delta_2$, then the total inventory cost per unit time $T C_3(T|\theta)$, $0 < T < \infty$ has a minimum value at the boundary point $T = t_d$.

**Theorem 7.3**

$T \hat{C}_3(\theta) \Delta T C_3(T^*|\theta)$ is an increasing and continuous function of $\theta$ in $[0, +\infty)$ and $
lim_{\theta \to 0} T \hat{C}_3(\theta) = pD + \sqrt{2ADg_1} - p_1 MD$ where $g_1 = h + pL_e$.

**Proof.** Recalling that the power series for $e^x$ as $\sum_{n=0}^{\infty} \left( x^n / n! \right)$, we have

$$T C_3(T|\theta) = \frac{A}{T} + pD + \frac{hD t_d}{T} (T - t_d) \sum_{n=1}^{\infty} \left[ \theta (T - t_d) \right]^{n-1}$$

$$+ \frac{D(h + p\theta)}{T} (T - t_d)^2 \sum_{n=2}^{\infty} \left[ \frac{\theta (T - t_d)}{n!} \right]^{n-2} + \frac{hD^2 t_d}{2T} - p_1 D \left( M - \frac{T}{2} \right) \tag{93}$$

For $\theta \geq 0$, it is obvious that $T C_3(T|\theta)$ is an increasing function of $\theta$ for each fixed value of $T > 0$. If $\theta_1 < \theta_2$ we have

$$T \hat{C}_3(\theta_2) = T C_3(T(\theta_2)|\theta_2) > T C_3(T(\theta_2)|\theta_1) \geq T C_3(T(\theta_1)|\theta_1) = T \hat{C}_3(\theta_1).$$

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Thus $T\hat{C}_3(\theta)$ is an increasing function of $\theta$ in $[0, +\infty)$. Let

$$f_3(T, \theta) = -A + \frac{D(\theta t_d + h + p\theta)}{\theta^2} \left[ \theta T e^{\theta(T-t_d)} - e^{\theta(T-t_d)} + 1 \right] - \frac{D(h + p\theta) t_d}{\theta} - \frac{h T^2}{2} + \frac{p_1 I_e DT^2}{2}.$$  

We have, $\frac{\partial f_3}{\partial T} = D T e^{\theta(T-t_d)} [\theta h t_d + h + p\theta] + p_1 I_e DT > 0$.

Now $T(\theta)$ is a continuous function of $\theta$ in $[0, +\infty)$. Moreover $TC_3(T|\theta)$ is a continuously differentiable real function for $T > 0$ and $\theta \geq 0$.

Thus $T\hat{C}_3(\theta)$ is also a continuous function of $\theta$ in $[0, +\infty)$.

Let,

$$G_3(T) = \frac{A}{T} + \frac{D h t_d (T-t_d)}{T} + \frac{D h (T-t_d)^2}{2T} + \frac{h T^2}{2T} - p_1 I_e D \left( M - \frac{T}{2} \right).$$

Because $T\hat{C}_3(\theta)$ is continuous in $[0, +\infty)$ we have,

$$\lim_{\theta \to 0} T\hat{C}_3(\theta) = T\hat{C}_3(0)$$

$$= \min_{0 < T < \infty} \{ G_3(T) + p D \}$$

$$= p D + \min_{0 < T < \infty} \{ G_3(T) \}$$

$$\equiv p D + \sqrt{2 A D g_1} - p_1 I_e M D.$$  

Therefore, $\lim_{\theta \to 0} T\hat{C}_3(\theta) \equiv p D + \sqrt{2 A D g_1} - p_1 I_e M D$, where $g_1 = h + p I_e$;

$$\text{Note: } T\hat{C}(\theta) = \begin{cases} T\hat{C}_1(\theta) & \text{if } 0 < M \leq t_d \\ T\hat{C}_2(\theta) & \text{if } t_d < M \leq T \\ T\hat{C}_3(\theta) & \text{if } M > T \end{cases}$$
Theorem 7.4

(i) When $M \leq t_d$, we have the following results.

(a) If $2A \geq \Delta_1$, then $TC^*(T^*) = TC_1(T_i)$ and $T^* = T_i$, where $T_i \in [t_d, \infty)$ and satisfies equation (89).

(b) If $2A < \Delta_1$, then $TC^*(T^*) = TC_1(t_d)$ and $T^* = t_d$.

(ii) When $M > t_d$, we have the following results.

(a) If $\Delta_2 \leq 2A \leq \Delta_3$, then $TC^*(T^*) = TC_3(T_{III})$ and $T^* = T_{III}$.

(b) If $2A > \Delta_3$, then $TC^*(T^*) = \min\{TC_2(T_{II}), TC_3(M)\}$ and $T^* = T_{II}$ or $T^* = M$ associated with lowest cost.

(c) If $2A < \Delta_2$, then $TC^*(T^*) = TC_3(t_d)$ and $T^* = t_d$.

Proof. It immediately follows from Lemmas 7.1, 7.2 and 7.3.

Summarizing the result in Theorem 7.4, we can develop a simple algorithm to illustrate step-by-step solution procedure for finding the optimal replenishment cycle time as follows

7.4.1 ALGORITHM

Step 1. Compare the values of $M$ and $t_d$. If $M \leq t_d$, then go to Step 2. Otherwise, if $M > t_d$, then go to Step 3.

Step 2. Calculate $\Delta_1 = Dr_d^2\left(\frac{h + pI_p}{p}\right) + DM^2\left(p_I I_e - pI_p\right)$

(a) If $2A \geq \Delta_1$, then $TC(T^*) = TC_1(T_i)$ and $T^* = T_i$, where $T_i \in [t_d, \infty)$ and satisfies equation (89). Go to Step 4.

(b) If $2A < \Delta_1$, then $TC(T^*) = TC_1(t_d)$ and $T^* = t_d$. Go to Step 4.

Step 3. Calculate $\Delta_2 = Dr_d^2\left(\frac{h + pI_e}{p}\right)$ and
\[ \Delta_3 = \frac{2D(\theta h t_d + h + p \theta)}{\theta^2} \left[ \theta M e^{\theta (M-t_d)} - e^{\theta (M-t_d)+1} \right] \]

\[ - \frac{2D(h + p \theta) t_d}{\theta} - h D t_d^2 + p I_e D M^2 \]

(a) If \( \Delta_2 \leq 2 \Delta \leq \Delta_3 \), then \( TC^*(T^*) = TC_3(T_{III}) \) and \( T^* = T_{III} \). Go to Step 4.

(b) If \( 2 \Delta > \Delta_3 \), then \( TC^*(T^*) = \min \{ TC_2(T_{II}), TC_3(M) \} \) and \( T^* = T_{II} \) or \( T^* = M \) associated with lowest cost. Go to Step 4.

(c) If \( 2 \Delta < \Delta_2 \), then \( TC^*(T^*) = TC_3(t_d) \) and \( T^* = t_d \). Go to Step 4.

Step 4. Stop.

7.5 COMPARISON BETWEEN THE POSTPONEMENT AND THE INDEPENDENT SYSTEMS

In this section, we compare and discuss about the postponement system and the independent system. In the independent system the raw materials are ordered independently (that is without postponement). The total average cost for ordering and keeping \( n \) end-products is

\[ TCI(\theta) = \sum_{i=1}^{n} TC(T_i|\theta), \]

where, \( TC(T_i|\theta) = \begin{cases} 
TC_1(T_i|\theta) & \text{if } 0 < M \leq t_d \\
TC_2(T_i|\theta) & \text{if } t_d < M \leq T_i \\
TC_3(T_i|\theta) & \text{if } M > T_i 
\end{cases} \)

Now,

\[ TC_1(T_i|\theta) = \frac{A}{T_i} + pD_i + \frac{D_i \left[ h t_d + p I_p (t_d - M) \right]}{\theta T_i} \left[ e^{\theta (T_i - t_d)} - 1 \right] \]
\[ TC_2 (T_i | \theta) = \frac{A}{T_i} + pD_i + \frac{D_i h t_d}{\theta T_i} \left[ e^{\theta (T_i - t_d)} - 1 \right] + \frac{D_i (h + p \theta)}{\theta^2 T_i} \left[ e^{\theta (T_i - t_d)} - \theta (T_i - t_d) - 1 \right] + \frac{p I_\ell D_i}{\theta^2 T_i} \left[ e^{\theta (T_i - M)} - \theta (T_i - M) - 1 \right] + \frac{hD_i t_d^2}{2T_i} - \frac{pI_\ell D_i M^2}{2T_i} \] (95)

and

\[ TC_3 (T_i | \theta) = \frac{A}{T_i} + pD_i + \frac{hD_i t_d}{\theta T_i} \left[ e^{\theta (T_i - t_d)} - 1 \right] + \frac{D_i (h + p \theta)}{\theta^2 T_i} \left[ e^{\theta (T_i - t_d)} - \theta (T_i - t_d) - 1 \right] + \frac{hD_i t_d^2}{2T_i} - pI_\ell D_i \left( M - \frac{T_i}{2} \right). \] (96)

In the form of postponement system, all the raw materials are ordered together (i.e., postponing the customization process) and the demand rate is \( \hat{D} = D_1 + D_2 + \ldots + D_n \). The total average cost for ordering and keeping the \( n \) end-products is given by (excluding the customization cost),
\[
TCP(\hat{t}|\theta) = \begin{cases} 
TCP_1(\hat{t}|\theta) & \text{if } 0 < M \leq t_d \\
TCP_2(\hat{t}|\theta) & \text{if } t_d < M \leq \hat{T} \\
TCP_3(\hat{t}|\theta) & \text{if } M > \hat{T}
\end{cases}
\]

Now, \( TCP_1(\hat{t}|\theta) = \frac{A}{T} + p \hat{D} + \frac{\hat{D}[ht_d + pI_p(t_d - M)]}{\theta^2 T} \left[ e^{\theta(\hat{t} - t_d)} - 1 \right] \]
\[+ \frac{\hat{D}(h + p\theta + pI_p)}{\theta^2 \hat{T}} \left[ e^{\theta(\hat{t} - t_d)} - \theta(\hat{t} - t_d) - 1 \right] + \frac{pI_p \hat{D}(t_d - M)^2 + h\hat{D}t_d^2}{2\hat{T}} \]
\[- \frac{p_1 I_e \hat{D}M^2}{2\hat{T}}, \quad \text{(97)} \]
\[
TCP_2(\hat{t}|\theta) = \frac{A}{T} + p \hat{D} + \frac{\hat{D}ht_d}{\theta^2 T} \left[ e^{\theta(\hat{t} - t_d)} - 1 \right] + \frac{\hat{D}(h + p\theta)}{\theta^2 \hat{T}} \left[ e^{\theta(\hat{t} - t_d)} - \theta(\hat{t} - t_d) - 1 \right] \]
\[+ \frac{pI_p \hat{D}}{\theta^2 \hat{T}} \left[ e^{\theta(\hat{t} - M)} - \theta(\hat{t} - M) - 1 \right] + \frac{h\hat{D}t_d^2}{2\hat{T}} - \frac{p_1 I_e \hat{D}M^2}{2\hat{T}}, \quad \text{(98)} \]
\[
TCP_3(\hat{t}|\theta) = \frac{A}{T} + p \hat{D} + \frac{\hat{D}t_d}{\theta \hat{T}} \left[ e^{\theta(\hat{t} - t_d)} - 1 \right] + \frac{\hat{D}(h + p\theta)}{\theta^2 \hat{T}} \left[ e^{\theta(\hat{t} - t_d)} - \theta(\hat{t} - t_d) - 1 \right] \]
\[+ \frac{h\hat{D}t_d^2}{2\hat{T}} - p_1 I_e \hat{D} \left( M - \frac{\hat{T}}{2} \right). \quad \text{(99)} \]

The difference in the optimal total average cost per unit time of the two systems is defined as \( z^* = TCP'(\theta) - TCI'(\theta). \)
7.6 NUMERICAL EXAMPLES

In order to illustrate the above solution procedure, we consider the following examples:

**Example 7.1.** Given an inventory system with the following parameter: \( A = 250, \quad h = 1.75, \quad p = 5, \quad p_i = 6, \quad D = 600, \quad \theta = 0.08, \quad I_p = 0.15, \quad I_e = 0.11, \quad t_d = 0.0822, \quad M = 0.3425 \) in appropriate units. This is the case where \( M > t_d \).

Applying the algorithm in section 7.4.1, we get the optimal value of cycle length, \( T^* = 0.5519 \) and the optimal value of total cost \( TC^* = 37,690 \).

**Example 7.2.** In order to study how various deterioration rates affect the optimal cost of the EOQ model, sensitivity analysis with respect to \( \theta \) is performed. The value of the deterioration rate varies as follows: \( (0, 0.02, 0.04, 0.06, 0.08, 0.10, 0.20, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80) \). The other related data are the same as the data of Example 7.1. Again applying the algorithm given in section 7.4.1 we derive the results shown in Table 10 and Figure 7. Theorems 7.1, 7.2 and 7.3 show that \( T\hat{C}(\theta) \) is an increasing and continuous function of \( \theta \). Further from Table 10 and Figure 7, the following observations can be made:

- \( T\hat{C}(\theta) \) is an increasing and concave function of \( \theta \) in \([0, +\infty)\).
- \( T(\theta) \) is a decreasing function of \( \theta \) in \([0, +\infty)\).
Table 10. Impact of deterioration rate on inventory replenishment policy

<table>
<thead>
<tr>
<th>θ</th>
<th>0</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^*$</td>
<td>0.5880</td>
<td>0.5781</td>
<td>0.5688</td>
<td>0.5601</td>
<td>0.5519</td>
<td>0.5441</td>
<td>0.4847</td>
<td>0.4494</td>
<td>0.4207</td>
<td>0.3968</td>
<td>0.3766</td>
</tr>
<tr>
<td>$TC^*$</td>
<td>3714</td>
<td>3728</td>
<td>3742</td>
<td>3755</td>
<td>3769</td>
<td>3781</td>
<td>3841</td>
<td>3895</td>
<td>3945</td>
<td>3991</td>
<td>4034</td>
</tr>
</tbody>
</table>

Table 11. Impact of permissible delay period ($M$) on inventory replenishment policy

<table>
<thead>
<tr>
<th>$M$</th>
<th>0</th>
<th>0.0411</th>
<th>0.0822</th>
<th>0.1233</th>
<th>0.1644</th>
<th>0.2055</th>
<th>0.2466</th>
<th>0.2877</th>
<th>0.3288</th>
<th>0.3699</th>
<th>0.4110</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TC^*$</td>
<td>3921</td>
<td>3902</td>
<td>3883</td>
<td>3856</td>
<td>3840</td>
<td>3823</td>
<td>3807</td>
<td>3791</td>
<td>3774</td>
<td>3758</td>
<td>3742</td>
</tr>
</tbody>
</table>
**Example 7.3.** In order to analyze how the change in permissible delay period \((M)\) affects the optimal cost of the model, sensitivity analysis with respect to \(M\) is performed. The value of \(M\) varies as given in Table 11. The other related data are the same as the data of Example 7.1. Again applying the algorithm given in section 7.4.1, we derive the results shown in Table 11 and Figure 8 from which the following observation is made.

- It can be found that the total cost decreases with an increase in the credit period (other parameters are kept unchanged). It implies that the longer the credit period supplier offer is, the lower the total inventory cost will be. From economic point of view, if the supplier provides a permissible delay in payments, the retailer should make use of it to the maximum extent to reduce his inventory cost.
Figure 8. The impact of permissible delay period $M$ on the total cost

Example 7.4. In order to study how various deterioration rates affect the difference in cost between the postponement system and the independent system, we assume that there are eleven end products. For the 11 products, we assume that $D_1=550$, $D_2=560$, $D_3=570$, $D_4=580$, $D_5=590$, $D_6=600$, $D_7=610$, $D_8=620$, $D_9=630$, $D_{10}=640$, $D_{11}=650$. The other related data are same as the data of Example 7.1. Applying the algorithm in section 7.4.1, we obtain the results of the sensitivity analysis with these parameters, which are shown in Table 12 and Figure 9 and the following analysis are made.

- The postponement system yields savings in the total average cost.
• The cost saving rate becomes larger when the deterioration rate increases. This implies that when the deterioration rate increases, the postponement strategy is more cost-effective.

![Graph showing the impact of deterioration rate on the difference in cost between two systems.](image)

**Figure 9. The impact of deterioration rate on the difference in cost between the two systems**

**Example 7.5.** Consider the inventory system with the following parameter:

- \( A = 1000 \), \( h = 1.75 \), \( p = 5 \), \( p_1 = 6 \), \( \theta = 0.08 \), \( I_p = 0.15 \), \( I_c = 0.11 \), \( t_d = 0.0822 \)
- \( M = 0.0548 \) in appropriate units. We assume that there are seven end products. For the 7 products, we assume that \( D_1 = 350 \), \( D_2 = 400 \), \( D_3 = 450 \), \( D_4 = 500 \), \( D_5 = 550 \), \( D_6 = 600 \), \( D_7 = 650 \). This is the case where \( M \leq t_d \). Applying the algorithm in section 7.4.1, we get the optimal value of total cost \( TCI^* = 29,277 \) and \( TCP^* = 21,780 \).
Table 12. The impact of deterioration rate on the difference in cost between the two systems

<table>
<thead>
<tr>
<th>$\theta$</th>
<th>0</th>
<th>0.02</th>
<th>0.04</th>
<th>0.06</th>
<th>0.08</th>
<th>0.10</th>
<th>0.20</th>
<th>0.30</th>
<th>0.40</th>
<th>0.50</th>
<th>0.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-z^*$</td>
<td>6530</td>
<td>6667</td>
<td>6802</td>
<td>6934</td>
<td>7063</td>
<td>7190</td>
<td>7780</td>
<td>8322</td>
<td>8822</td>
<td>9290</td>
<td>9725</td>
</tr>
<tr>
<td>$\frac{z^<em>/TCl}{</em>}$</td>
<td>0.1598</td>
<td>0.1626</td>
<td>0.1652</td>
<td>0.1678</td>
<td>0.1704</td>
<td>0.1729</td>
<td>0.1841</td>
<td>0.1942</td>
<td>0.2033</td>
<td>0.2116</td>
<td>0.2192</td>
</tr>
</tbody>
</table>

Table 13. The impact of permissible delay period ($M$) on inventory replenishment policy in a postponement system

<table>
<thead>
<tr>
<th>$M$</th>
<th>0</th>
<th>0.0411</th>
<th>0.0822</th>
<th>0.1233</th>
<th>0.1644</th>
<th>0.2055</th>
<th>0.2466</th>
<th>0.2877</th>
<th>0.3288</th>
<th>0.3699</th>
<th>0.4110</th>
</tr>
</thead>
<tbody>
<tr>
<td>TCP*</td>
<td>21924</td>
<td>21815</td>
<td>21708</td>
<td>21564</td>
<td>21469</td>
<td>21374</td>
<td>21279</td>
<td>21184</td>
<td>21089</td>
<td>20994</td>
<td>20899</td>
</tr>
<tr>
<td>TCI*</td>
<td>29427</td>
<td>29315</td>
<td>29203</td>
<td>28914</td>
<td>28819</td>
<td>28724</td>
<td>28629</td>
<td>28535</td>
<td>28440</td>
<td>28345</td>
<td>28250</td>
</tr>
</tbody>
</table>
Figure 10. Impact of permissible delay period $M$ on the total cost in a postponement system

**Example 7.6.** In order to analyze how the change in permissible delay period ($M$) affects the optimal cost in a postponement system, sensitivity analysis with respect to $M$ is performed. The value of $M$ varies as given in Table 13. The other related data are the same as the data of Example 7.5. Again applying the algorithm given in section 7.4.1, we derive the results shown in Table 13 and Figure 10 from which the following observation is made.

- It can be found that the total cost in a postponement system decreases with an increase in the credit period $M$ for fixed other parameters. It implies that the longer the credit period offered by the supplier is, the lower the total inventory cost in a postponement system will be. From
economic point of view, if the supplier provides a permissible delay in payments, the trader should make use of it to the maximum extent to reduce his/her inventory cost.

Thus employing postponement strategy will reduce cost to a significant degree. Further, when the permissible delay in payment is offered the trader should utilize it to a maximum extent, which will again reduce the total cost to a significant degree. From Table 13, we note that under the same data as in Example 7.5 the total cost in an independent system when no credit period is offered is 29,427. Whereas when the credit period is 0.4110, the total cost in a postponement system is 20,899. Thus significant cost savings can be achieved in the entire supply chain when both postponement strategy and permissible delay in payment are employed.
7.7 CONCLUSION

In this chapter, we have developed an EOQ model for non-instantaneous deteriorating items when permissible delay in payment is offered to evaluate the impact of postponement strategy on the retailer in a supply chain. Numerical examples shows that, the difference between the two strategies will become larger, when $\theta$ becomes larger. We have clearly shown that the postponement system can give a lower total average cost than the independent system. We have also analyzed the effects of major parameters like $M$, $\theta$ on the average total cost. In particular we have established a mathematical model, which is a general framework that comprises numerous previous models such as in Ghare and Schrader [49], Li et al. [82], and Ouyang, et.al [102] as special cases. According to the results obtained from the numerical examples, we obtain some managerial insights that guide the retailer to find a proper tradeoff between postponement and non-postponement.