Chapter 3

Using Genetic Algorithm for Solving Quadratic Bilevel Programming Problems via Fuzzy Goal Programming

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3.1 Introduction

Bilevel programming problem (BLPP) is a special case of multiobjective decision making (MODM) problem in a hierarchical decision system. It is actually the most widely used and primitive version of a multilevel programming problem (MLPP) having multiple decision makers (DMs) with multiplicity of objectives in a large hierarchical decision making organization.

In a BLPP, two DMs are located at two different hierarchical decision levels and each one independently controls a vector of decision variables for optimizing individual objective functions, where objectives often conflict each other in the decision making horizon. Although, execution of the decision power is sequential from the upper-level DM (leader) to lower-level DM (follower), the decision of the leader is often affected by the reaction of the follower owing to his/her dissatisfaction with the decision of the leader. As a consequence, decision deadlock often arises and the problem of distribution of proper decision powers to the DMs is encountered in most of the hierarchical decision making context.

From the historical perspective, the concept of hierarchical decision problem as special structured mathematical programming (MP) problem was first introduced by Burton and Obel [95] for solving decentralized planning problems of large hierarchical decision organizations. Thereafter, various versions of MLPPs and BLPPs were studied by the active researchers in the field.

Most of the approaches for BLPPs developed so far in the past are based on vertex enumeration method [100] or transformation approach [70], which are actually the extensions of stackelberg strategy for solving two-person non-zero and non-cooperative games. But, in practical decision situations, they are computationally not efficient, especially for large and complex hierarchical decision problems. Again, most of the classical approaches developed for BLPPs often lead to the paradox that the leader’s decision power is dominated by the follower.

To overcome the above difficulties, an efficient multiobjective solution technique with post optimality analysis on the objective values based on the three compromise solutions: ideal point, threat point and ideal threat point, was suggested by Wen and Hsu [677]. But, their approach does not always lead to the satisfactory decision in a highly conflicting hierarchical decision situation [73, 372].
Now, in a hierarchical decision making context, it has been realized that each of the DMs should have a motivation to cooperate each other and relaxation of decision for a minimum level of satisfaction of the lower-level DM must be considered by the upper-level DM for survival and sustainable growth of the organization. Further, in actual practice, ambiguity often arises in making decision owing to imprecision in human judgments as well as inherent inexactness of model parameters in most of the real-world problems. To cope with the practical situations and to overcome the shortcomings of the classical approaches, the use of the concept of membership functions in fuzzy sets, introduced by Zadeh [698], has been suggested by Lai [372] to solve BLPPs as well as MLPPs. Thereafter, the supervised search procedure with the use of max-min operator introduced by Bellman and Zadeh [61] has been studied by Shih et al. [616]. The conventional fuzzy programming (FP) approaches discussed by Zimmerman [712] have been further extended by Shih and Lee [617], Sakawa and Nishizaki [581], and others to solve hierarchical decision problems from the view point of making a balance of decision powers of the DMs in the decision making environment.

But, the main difficulty of using the conventional FP approach is that there is a possibility of rejecting the solution again and again by the leader/follower due to his/her dissatisfaction with the solution, and computational load increases for involvement of re-evaluation of the problem repeatedly with the elicited membership values and so the decision is frequently found not closer to the highest membership value (unity) to reach the optimal solution.

To overcome the above difficulty in solving FP problems with multiplicity of objectives, fuzzy goal programming (FGP) as an extension of conventional goal programming (GP) [294, 553] based on the goal satisficing philosophy exposed by Simon [621], has been introduced by Pal and Moitra [512] to the field of conventional FP for making decision with regard to achieving multiple fuzzy goals efficiently in the decision making environment. The concept has been further extended by Pal et al. [515], Toksari [659], and others and implemented to different real-world problems [74, 481, 496, 497, 502, 517] in the past.

In FGP approach, achievement of fuzzy goals to their aspired levels in terms of achieving the highest membership value (unity) of each of them is considered. In
FGP model formulation, the membership functions of the defined fuzzy goals are transformed into flexible membership goals by assigning the highest membership value (unity) as the aspiration levels and introducing under- and over-deviational variables to each of them in an analogous to conventional GP approach.

Here, in the goal achievement function, only the under-deviational variables of the membership goals are minimized, which can easily be realized from the characteristics of membership functions defined for the fuzzy goals.

Now, in the area of MODM, the methodological aspects of nonlinear programming with multiplicity of objectives have been studied [287, 633] extensively in the past. The potential use of different versions of conventional GP approaches [28,296, 552] to several nonlinear MODM problems have been surveyed by Saber and Ravindran [568] in the last century and widely circulated in the literature.

The methodological aspects of solving FP problems having nonlinear objectives have been discussed by Yang and Ignizio [689]. Although, the study on fuzzily described MODM problems having fractional objectives has been well documented in the literature, methodological development for solving general nonlinear programming problems including quadratic programming problems from the viewpoint of potential use to real-world problems [14] is yet to be widely circulated in the literature.

But, in most of the previous studies, the conventional linear approximation approaches [355, 633] are used, which involve huge computational load and the approximation errors [258] inherently occur there in the decision making context.

To overcome the computational difficulties arising out of using traditional (single-point based) solution search approaches, genetic algorithms (GAs) based on the natural selection and population genetics, initially introduced by Holland [276, 277], have appeared as volume-oriented global solution search tools to solve complex real-world problems. The deep study on GA based solution methods made in the past century has been well documented by Goldberg [238] and Michalewicz [437, 438] among others.

The efficient use of GA solution search methods for solving conventional GP models of nonlinear MODM problems have also been studied by Zheng et al. [707]
and Gen et al. [225] with a view to avoid the computational load for linearizing objectives in the process of solving the problems.

The extensive study on the use of GAs as goal satisficers rather than objective optimizers to multiobjective decision problems in crisp decision environment has been discussed by Deb [157] in 2002. The GA based solution methods to fuzzy multiobjective optimization problems have been well documented by Sakawa [573].

The potential use of GAs to FGP models of real-life problems have also been studied by Pal et al. [490, 494] in the past.

The GA based solution approach to BLPPs in crisp decision environment was first studied by Mathieu et al. [430]. Thereafter, the computational aspects of using GAs to fuzzily described hierarchal decision problems have been investigated by Sakawa et al. [579], and further extended by Sakawa and Nishizaki [580], Hejazi et al. [271], Nishizaki and Sakawa [472], and others in the past.

But, most of the GA based FP approaches studied previously are the extended versions of the conventional crisp interactive approaches, where elicited membership values of objectives of DMs are considered as achievement of fuzzy goals and repeated evaluation of a problem for conflict resolution is inherently involved there in the decision search process. However, the extensive study in this area is yet to be widely circulated in the literature.

Now, it is worthy to mention here that there are several real-life socio-economic MODM problems in which the objectives are found incommensurable in nature and inherently quadratic in form in the decision making horizon. For example, in the case of thermal power plant operations and planning for electric power generation, the objectives of economic power generation by controlling the generators and optimal reduction of emissions following Pollution Control Act are incommensurable and their characteristics are quadratic in nature [11, 240]. In such a case, the use of conventional MODM methods (crisp or fuzzy) with the incorporation of traditional approximation technique creates decision trouble for optimizing both the objectives simultaneously. To overcome the situation, it seems that quadratic bilevel programming problem (QBLPP) formulation of the given problem in the framework of FGP and the use of the GA scheme efficiently for selection of generators as
control variables at the two levels in a hierarchical decision system become prominent in searching an appropriate decision.

In this Chapter, in order to show how an GA method can be potentially used for modelling and solving real-life decentralized hierarchical decision making problems, a priority based FGP formulation of a general BLPP with quadratic objective functions is presented. In a hierarchical decision system, since the objectives of both the levels are incommensurable and decision conflict often arises for simultaneous optimization of the objectives, fuzzy description of them by assigning imprecise aspiration level to each of them for goal achievement within certain tolerance ranges is taken into consideration. Further, since the upper-level DM has a higher power of making decision, the decision vector controlled by him / her is fuzzily described to achieve the objective value of the lower-level DM to a certain level of satisfaction in the decision making horizon. In the decision making context, achievement of aspired levels of the defined fuzzy goals in terms of membership values to the extent possible on the basis of priorities is taken into account.

In the proposed approach, the GA method is implemented at two different stages. At the first stage, individual optimal decisions of the leader and follower are determined for fuzzy representation of the problem. At the second stage, evaluation of goal achievement function on the basis of priorities of achieving the highest membership value (unity) of the defined fuzzy goals to the extent possible by minimizing the under-deviational variables of the associated membership goals is taken into account.

In the decision process, the sensitivity analysis with the variations of priority structure of model goals is performed to present how the solution is sensitive to the changes in priority structure of goals. The Euclidean distance function is then used to identify the appropriate priority structure under which the most satisfactory decision can be reached in the decision making environment.

The proposed approach is illustrated by a numerical example and the model solution is compared with the solutions obtained by using the approaches studied by Tiwari et al. [657] and Pal and Moitra [511] in the past.


3.2 Literature Review

In the field of hierarchical decision making, the concept of MP formulation of BLPP was introduced separately by Fortuny-Amat and McCarl [207] and Candler and Townsley [102]. Thereafter, during the early 1980s, various versions of BLPPs were studied by Bard [43, 44, 45], Bialas and Karwan [70], Candler [100], and other pioneer researchers in the field from the point of view of their potential use to different real-world problems, viz network design [421, 422], firm management [47], transportation system [588], pollution control [15], economic policy system [191], supply chain model in inventory management [550], and especially for conflict resolutions [17] in hierarchical decision making situations.

The different methodological aspects studied for solving various hierarchical decision problems have been well documented by Shimizu et al. [619], Sakawa and Nishizaki [582].

The FP approaches to hierarchical decision making problems with fractional criteria [79, 450], as a special field of study in nonlinear programming have also been investigated by Sakawa et al. [587], Sakawa and Nishizaki [579] in the past from the view point of their potential use to real-life problems [470]. Tiryaki [658] has made an extensive study on FP approach to decentralized multi-level linear programming problems.

The FGP approach to linear BLPP studied by Moitra and Pal [456] has been extended to interactive FGP approach to BLPPs and MLPPs by Arora and Gupta [23], Biswas and Pal [73], respectively, in the past. The FGP approach to multiobjective fractional programming problems discussed by Pal et al. [515] has also been extended to BLPPs by Baky [37]. The concept of using FGP approach to QBLPPs has also been discussed by Pal and Moitra [511], where linear approximation technique has been used in the process of solving the problem. The methodological aspects of solving general nonlinear MLPPs in fuzzy environment have been discussed by Osman et al. [480].

The computational aspects of using GAs to fuzzily described hierarchical decision making problems have been investigated by Sakawa et al. [584], and further extended by Sakawa and Nishizaki [580], Hejazi et al. [271], Nishizaki and Sakawa [472], and others in the past.
The efficient use of GAs to minsum FGP [515] formulation of hierarchical decision problems has been studied by Pal et al. [495] in the recent past. But, the deep study on GA based FGP approaches to hierarchical decentralized planning problems is at an early stage. Again, the use of GAs as global solution search approaches to FGP models of QBLPPs as well as other general nonlinear hierarchal decision problems is yet to be documented in the literature.

In the present work, the efficient use of GA method for modelling and solving QBLPPs in the framework of priority based FGP is presented.

3.3 QBLPP Formulation

In a hierarchical decision system, let the BLPP be such that the DM at each level takes overall satisfactory balance of decision power and tries to optimize his/her own benefit paying serious attention to the preference of the other.

Let \( X = (x_1, x_2, \ldots, x_n) \) be the vector of decision variables involved in the hierarchical decision system.

Then, let \( X_1 \) and \( X_2 \) be the vectors of decision variables controlled by the leader and follower, respectively, and \( Z_1 \) and \( Z_2 \) be the objectives to be optimized at the two successive decision levels.

In a hierarchical decision situation, such a QBLPP in a generic form can be presented as:

Find \( X(X_1, X_2) \) so as to:

\[
\text{Maximize } Z_1(X_1, X_2) = C_1 X + \frac{1}{2} X^T D_1 X
\]

(leader's problem)

and, for given \( X_1, X_2 \) solves

\[
\text{Maximize } Z_2(X_1, X_2) = C_2 X + \frac{1}{2} X^T D_2 X
\]

(follower's problem)

subject to,

\[
X \in S = \{(X_1, X_2) \mid A_1 X_1 + A_2 X_2 \leq b, \ X \geq 0\}
\]

\[(3.1)\]

where \( X_1 \cap X_2 = \emptyset \) and \( X_1 \cup X_2 = X \), \( \cap \) and \( \cup \) stand for intersection and union, respectively, and where \( C_1, C_2 \) and \( b \) are constant vectors, the superscript \( T \) stands for
transposition, $A_1$ and $A_2$ are constant matrices, $D_1$ and $D_2$ are constant symmetric matrices. The functions $Z_1$ and $Z_2$ are assumed to be concave, differentiable, and bounded. It is also assumed that $S(\neq \varphi)$ is a convex set.

Now, in the decision making context, the structure of the objective functions in (3.1) indicates that the first objective (the leader's objective) is optimized first without paying any interest of optimizing the other objective (the follower's objective) by achieving the associated decision vector. Then, optimization of the follower's objective is considered with regard to the decision ($X_1$) made by the leader. Here, in the crisp sense, the leader's decision $X_1$ can never be sacrificed for any benefit of the follower.

Consequently, in the BLPP formulation of a hierarchical decision system, the execution of decision powers of the DMs is sequential from the leader to follower. Again, since the leader is in the leading position, it is often found that his/her decision dominates the follower in most of the decision situations. As such, decision trouble frequently arises with dissatisfaction of the follower. To overcome such a situation, certain relaxation on the decision of the leader needs to be given for a minimum level of satisfaction of the follower. Here, it is worth mentioning that the DMs in most of the times are confused with that of defining the aspiration levels crisply for optimizing the objectives, because of impression in human judgments as well as inexact nature of the model parameters of the problems.

In the above situation, fuzzy description of the problem seems to be appropriate one to take reasonable decision in the decision making environment.

Now, in the present decision making context, the GA scheme considered for modelling and solving the problem is presented in Section 3.4.

3.4 GA Scheme for QBLPP

In the literature of GAs, there is a variety of schemes [238, 276] for generating new population with the use of different operators: selection, crossover and mutation.

In the present GA scheme, binary representation of each candidate solution is considered in the genetic search process. The initial population (the initial feasible solution individuals) is generated randomly in the domain of feasible set defined in
The fitness of each feasible solution individual is then evaluated with the view of optimizing an objective function in the decision making context.

The basic steps of the GA scheme with the core functions adopted in the solution search process are presented in the following algorithmic steps.

Steps of the GA algorithm:

**Step1. Representation and initialization**

Let $E$ denote the binary coded representation of a chromosome in a population as $E = \{x_1, x_2, ..., x_n\}$. The population size is defined by pop_size, and pop_size chromosomes are randomly initialized in its search domain.

**Step2. Fitness function**

The fitness value of each chromosome is determined by the value of an objective function to be evaluated. The fitness function is defined as

\[
eval(E_v) = (Z_v)_i, \quad i = 1, 2; \quad v = 1, 2, ..., \text{pop\_size},
\]

where $Z_i$ represents the objective function of the $i$-th level DM given in (3.1), and the subscript $v$ is used to indicate the fitness value of the $v$-th chromosome, $v = 1, 2, ..., \text{pop\_size}$. The best chromosome with largest fitness value at each generation is determined as:

\[
E^* = \max \{\eval(E_v) | v = 1, 2, ..., \text{pop\_size}\},
\]

or,

\[
E^* = \min \{\eval(E_v) | v = 1, 2, ..., \text{pop\_size}\},
\]

which depends on searching of the maximum or minimum value of an objective function.

**Step3. Selection**

The simple roulette-wheel scheme [238] is used for the selection of parents for mating purpose in the genetic search process.

In roulette-wheel scheme, the fitness-proportionate strategy is used for selecting fitter chromosomes in the population to be introduced into the mating pool for reproduction. Here, probability distribution rule is used and
the strategy that the 'chance of an individual being selected is proportional to its fitness' is adopted. Eventually, an individual which has a higher probability is selected to be copied into the mating pool.

Again, there are different mechanisms to simulate roulette-wheel to compute the expected number of a fitter one being copied as member of mating pool. The simplest one is the weighted roulette-wheel, where the integer part of the product of the probability of selection and pop_size is conventionally adopted to reduce noise that inherently occurs for randomness of selection strategy.

**Step 4. Crossover**

The parameter $p_c$ is defined as the probability of crossover. The arithmetic crossover operation (single-point crossover) [437] of a genetic system is applied here from the view point that the resulting offspring always satisfy the system constraints set $S(\neq \varphi)$. Here, a chromosome is selected as a parent for a defined random number $r \in [0,1]$, if $r < p_c$ is satisfied.

For instance, the arithmetic crossover for the parents $E_1, E_2 \in S$ is defined as

$$E_1' = \alpha_1 E_1 + \alpha_2 E_2, \quad E_2' = \alpha_2 E_1 + \alpha_1 E_2,$$

for producing two offspring $E_1'$ and $E_2'$ ($E_1' \in S$ and $E_2' \in S$), where $\alpha_1, \alpha_2 \geq 0$, with $\alpha_1 + \alpha_2 = 1$.

**Step 5. Mutation**

Mutation mechanism is applied over the population after performing crossover operation. It alters one or more genes of a selected chromosome to re-introduce the genetic material to gain extra variability for fitness strength in the population. As in the conventional GA scheme, a parameter $p_m$ of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < p_m$ is satisfied.
Step6. Termination

The execution of the whole process terminates when the fittest chromosome is reported at a certain generation number in the solution search process.

Now, FGP formulation of the problem (3.1) by defining the fuzzy goals is presented in Section 3.5.

3.5 FGP Problem Formulation

In FGP formulation of the problem, both the objectives $Z_1$ and $Z_2$ and the control vector $X_1$ are to be transformed into fuzzy goals by means of assigning an imprecise aspiration level to each of them. Then, the defined fuzzy goals are characterized by the membership functions to measure the degree of goal achievement in terms of membership values.

In the present decision situation, the individual best decisions of the DMs are taken into consideration and they are evaluated by using the proposed GA scheme.

Let, $(X'_1, X'_2; Z'_1)$ and $(X''_1, X''_2, Z''_1)$ be the optimal solutions of the leader and follower, respectively, when calculated in isolation over the feasible solution space $S$,

where $Z'_1 = \max_{(X_1, X_2)} Z_1(X_1, X_2)$ and $Z''_1 = \max_{(X_1, X_2)} Z_1(X_1, X_2)$

Then, the fuzzy objective goals appear as:

$$Z_1 \geq Z'_1, \quad Z_2 \geq Z''_1$$

The fuzzy goal for the control vector $X_1$ is obtained as

$$X_1 \geq X'_1$$

where ‘≥’ refers to the fuzziness of an aspiration level and it is to be understood as ‘essentially greater than’ in the sense of Zimmermann [712].

Now, in the decision situation, it is assumed that both the DMs have a motivation to cooperate each other to make a balance of decision powers, and they agree to give a possible relaxation of their individual optimal decision for a benefit of the other.

Then, the lower-tolerance limits of the respective fuzzy objective goals for the leader and follower can be determined as:
Further, since the leader has a higher power of making decision, a certain relaxation on the decision $X_1^l$ as a lower-tolerance limit should be given by the leader for searching a better decision by the follower. Let, $X_1^p < X_1^l < X_1^f$ be the lower tolerance limit of $X_1^l$.

Now, in a fuzzy decision situation, the fuzzy goals are characterized by their respective membership functions. The characterization of membership functions of the defined fuzzy goals are presented in the following Section 3.5.1.

### 3.5.1 Characterization of Membership Function

The membership function for the fuzzy objective goal of the leader appears as [713]:

$$
\mu_{z_1}[Z_1(X_1, X_2)] = \begin{cases} 
1 & \text{if } Z_1(X_1, X_2) \geq Z_1^l \\
\frac{Z_1(X_1, X_2) - Z_1^f}{Z_1^l - Z_1^f} & \text{if } Z_1^l \leq Z_1(X_1, X_2) < Z_1^f \\
0 & \text{if } Z_1(X_1, X_2) < Z_1^l 
\end{cases}
$$

(3.3)

Similarly, the membership function for the fuzzy objective goal of the follower takes the form:

$$
\mu_{z_2}[Z_2(X_1, X_2)] = \begin{cases} 
1 & \text{if } Z_2(X_1, X_2) \geq Z_2^l \\
\frac{Z_2(X_1, X_2) - Z_2^f}{Z_2^l - Z_2^f} & \text{if } Z_2^l \leq Z_2(X_1, X_2) < Z_2^f \\
0 & \text{if } Z_2(X_1, X_2) < Z_2^l 
\end{cases}
$$

(3.4)

The membership function for the fuzzy decision of the leader appears as:

$$
\mu_{X_1}[X_1] = \begin{cases} 
1 & \text{if } X_1 \geq X_1^l \\
\frac{X_1 - X_1^p}{X_1^f - X_1^p} & \text{if } X_1^f \leq X_1 < X_1^l \\
0 & \text{if } X_1 < X_1^p 
\end{cases}
$$

(3.5)

Now, the FGP model formulation is presented in the following Section 3.5.2.
3.5.2 FGP Model Formulation

In a fuzzy decision situation, the aim of each of the DMs is to achieve the highest membership value of his/her objective to the extent possible in the decision making horizon. Here, since the follower is in the lower position for making decision, it is always desired by him/her to introduce a lower limit for achievement of his/her control vector $X_2$ to ensure a minimum level of goal achievement for his/her own benefit in the environment of making cooperation with the leader for the benefit of the organization as a whole.

As a matter of fact, bounds on the control vectors in the solution search process can be introduced as:

$$X_f^l \leq X \leq X^r_f \quad \text{and} \quad X^l_2 \leq X_2 \leq X^r_2,$$

(3.6)

where $X^l_2 (X_2^l < X_2^r < X_2^l)$ is the lower bound of $X_2$.

Now, in FGP formulation, the defined membership functions are transformed into membership goals by assigning the highest membership value (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. Then, minimization of the under-deviational variables of the stated membership goals on the basis of their weights of importance of achieving the aspired goal levels is taken into account.

In the proposed problem, since the objectives often conflict each other and they are not commensurable owing to their two different locations in the hierarchical decision system, achievement of goals on the basis of priorities is taken into consideration.

In priority based FGP, the goals are rank ordered on the basis of the priorities of achieving the target levels of them. The goals which seem to be equally important from the view point of assigning a priority are included at the same priority level and numerical weights are given to them on the basis of their weights of importance of achieving their aspired levels at the same priority level.
Then, a priority based FGP model of the problem can be presented as:

Find $X(X_1, X_2)$ so as to:

Minimize $Z = [P_1(d^-), P_2(d^-), P_k(d^-), \ldots, P_K(d^-)]$

and satisfy:

$$\mu_{z_1} : \frac{Z_1(X_1, X_2) - Z_1^f}{Z_1^f - Z_1^i} + d_1^- - d_1^+ = 1,$$

$$\mu_{z_2} : \frac{Z_2(X_1, X_2) - Z_2^f}{Z_2^f - Z_2^i} + d_2^- - d_2^+ = 1,$$

$$\mu_{z_i} : \frac{X_i - X_i^p}{X_i^p - X_i^f} + d_i^- - d_i^+ = 1,$$

subject to the system constraints in (3.1) and the decision restrictions in (3.6),

(3.7)

where $Z$ represents the vectors of the $K$ priority achievement functions, and $d_i^-, d_i^+ (\geq 0)$, with $d_i^- . d_i^+ = 0$ (i = 1, 2) represent the under- and over-deviational variables, respectively, associated with the $i$-th membership goals and $d_i^-, d_i^+ (\geq 0)$ with $d_i^-, d_i^+ = 0$ represent the vector of under- and over-deviational variables associated with the membership goals defined for the decision vector $X_i$, and $I_1$ is a column vector with all elements equal to 1 and the dimension of it depends on the dimension of the decision vector $X_i$.

$P_k(d^-)$ is a linear function of the weighted under-deviational variables at the $k$-th priority level and where $P_k(d^-)$ is of the form:

$$P_k(d^-) = w_{ik}^--d_{ik}^- + w_{2k}^-d_{2k}^- + w_{3k}^-d_{3k}^-,$$

$k = 1, 2, \ldots, K; \quad K \leq (n + 1)$

where $d_{ik}^-$ is renamed for $d_i^-$, (i = 1, 2, 3), to represent it at the $k$-th priority level, and $w_{ik}^-$, (i = 1, 2), and $w_{3k}^-$ are the numerical weights and vector of numerical weights, respectively, which represents the relative importance of achieving the aspired levels of goals that are grouped together at the same priority level $P_k$. 
The values of $w_{ik}(i = 1,2)$ and $w_{3k}$ are determined as [511]:

$$w_{ik} = \frac{1}{(Z_i^l - Z_i^r_k)}, \quad w_{2k} = \frac{1}{(Z_2^l - Z_2^r_k)}, \quad w_{3k} = \frac{1}{(X_i^l - X_i^r_k)},$$

where the suffix ‘ $k$ ’ is used to represent the values of the weights of achieving the aspired goal values at the $k$-th priority level.

It is worthy to mention here that the notion of pre-emptive priorities of the goals actually holds on the concept that the goals which are included at the $k$-th priority level $P_k$ are preferred most for achievement of their aspired levels before considering the achievement problem of the goals at the next priority $P_{k+1}$, regardless of any multiplier associated with $P_{k+1}$.

Also, the relationship among the priorities is

$$P_1 >>> P_2 >>> \ldots >>> P_k >>> \ldots >>> P_K,$$

where $>>>$ means ‘much greater than’ and implies that the goals at the first priority level ($P_1$) are achieved to the extent possible before considering the achievement of goals at the second priority level ($P_2$), and so forth.

Now, in a decision making situation, achievement of the highest membership value of each of the fuzzy goals is a trivial one. Again, the DM is frequently confused with that of assigning the proper priorities to the goals, because they often conflict each other for achieving their individual aspired levels in the decision making environment.

To overcome the above situation, the notion of Euclidean distance function for group decision analysis, introduced by Yu [697], can be used to achieve an ideal point dependent solution and thereby selecting the appropriate priority structure under which the most satisfactory decision can be reached.

The selection of appropriate priority structure for goal achievement is presented in the following Section 3.5.3.

3.5.3 Use of Euclidean distance Function for Priority Structure Selection

In the present decision situation, since the highest membership value of each fuzzy goal is unity, the ideal point would be a vector with each element equal to 1.
The Euclidean distance function can be presented as:

\[ D^k = [(1 - \mu_{Z_1}(\cdot))^2 + (1 - \mu_{Z_2}(\cdot))^2 + (1 - \mu_{Z_3}(\cdot))^2]^\frac{1}{2}, k = 1, 2, \ldots, K, \]

where \((\mu_{Z_1}(\cdot), \mu_{Z_2}(\cdot), \mu_{Z_3}(\cdot))\) are the actual utilities resulting from the decision \(X\) under the \(k\)-th priority structure of the goals, and \(I_2\) is the row vector with all elements equal to 1 and the dimension of it depends on \(X_i\). \(D^k\) indicates the distance associated between the achieved membership values of the goals and the ideal point when the problem is solved under the \(k\)-th priority structure.

In the priority selection process, it can easily be realized that the solution which is closest to the ideal solution point would be the most satisfactory one. As such, priority structure that corresponds to the minimum of the distances obtained for arrangement of different priority structures might be considered as the appropriate priority structure for achievement of goals in the decision making environment.

Here, it can easily be realized that the solution which is closest to the ideal point must correspond to:

\[ \min_{k \in K} \{D^k\} = D^r \text{ (say), } 1 \leq r \leq K. \]

Then, the \(r\)-th priority structure would be considered as an appropriate one to reach the most satisfactory decision in the decision making situation.

Now, it is worthy to note that computational complexity often arises to solve problems with nonlinear objectives/goals by using traditional approximation approaches, and use of such an approach in most of the times leads to a local optimal solution rather than global one. Further, computational load owing to linearization and decision trouble due to approximation error are frequently involved in using conventional methods.

To overcome the above situation, an GA method as a volume-oriented (global one) search method and a promising tool as goal satisficer rather than objective optimizer can be effectively employed to the proposed FGP model to arrive at a reasonable solution for proper distribution of decision powers to the leader and follower in the decision making environment.

Now, in the genetic search process, the fitness function for executing the problem in (3.7) is defined in Section 3.6.
3.6 Use of GA to FGP Model

The goal achievement functions $Z$ appears as the evaluation function in the GA search process of solving the problem.

Here, the evaluation function to determine the fitness of a chromosome appears as:

$$\text{Eval} (E_v)_k = (Z_k)_v = (\sum_{i=1}^{2} w_{ik}^+ d_{ik}^+ + w_{ik}^- d_{ik}^-)_v, \quad v = 1, 2, ..., \text{pop}_\text{size}$$

where $(Z_k)_v$ is used to represent the achievement function $Z$ in (3.7) for measuring the fitness value of the $v$-th chromosome, when the problem of achieving the goals at the $k$-th priority level $P_k$ is taken into account.

The best objective value $(Z_k^*)$ for the fittest chromosome at a generation in the solution search process is determined as:

$$Z_k^* = \min \{ \text{Eval} (E_v)_k \mid v = 1, 2, ..., \text{pop}_\text{size} \}, \quad k = 1, 2, ..., K.$$

In the solution process, the step by step execution of the problem by employing the GA scheme for achievement of model goals on the basis of priorities is briefly discussed as follows.

At the first step, the execution is only performed for searching the minimum of $Z_1$ in order to achieve the aspired levels of goals at the first priority level ($P_1$) in the domain of solution search space. When minimum of $Z_1$ is reached, i.e., the value $Z_1^*$ is achieved for a chromosome at a certain generation, the functional expression of the achievement function $Z_1$ is crisply introduced into the system by incorporating $Z_1^*$ as its upper-bound, which acts as insurance against deterioration of the achieved values of goals at the priority level $P_1$ for any further execution to be made for evaluation of the problem. Then, execution counter is shifted to the next step to evaluate $Z_2$ for achievement of aspired levels of goals included at the second priority level ($P_2$). The continuation of execution for searching solution with sequential selection of priorities (step by step) is made until the evaluation of $Z_K$ is completed and thereby the fittest chromosome as a candidate solution for final decision is reached in the decision making environment.

The potential use of the proposed approach is illustrated by a numerical example in Section 3.7.
3.7 An Illustrative Example

The example studied previously by Pal and Moitra [511] is solved. The QBLPP with two decision variables is of the form:

Find \( X(x_1, x_2) \) so as to:

Maximize \( Z_1(x_1, x_2) = x_1 + 2x_1^2 - (x_2 - 2)^2 \) \hspace{1cm} \text{(leader's problem)}

and for given \( x_1, x_2 \) solves

Maximize \( Z_2(x_1, x_2) = (x_1 - 2)^2 + x_2^2 \) \hspace{1cm} \text{(follower's problem)}

subject to

\[
\begin{align*}
    x_1 + x_2 &\leq 6, \\
    x_1 + x_2 &\geq 2, \\
    -x_1 + x_2 &\leq 2, \\
    -x_1 + x_2 &\geq -2, \\
    x_1, x_2 &\geq 0.
\end{align*}
\]

(3.9)

Now, to solve the problem by employing the proposed GA scheme, the following genetic parameters are adopted in the solution search process.

- probability of crossover \( p_c = 0.8 \)
- probability of mutation \( p_m = 0.08 \)
- population size = 100
- Chromosome length = 30.

The GA is implemented using the Programming Language C. The execution is made in an Intel Pentium IV with 2.66 GHz clock-pulses and 1 GB RAM.

Then, following the procedure, the individual best solutions of the DMs are obtained as:

\[
\begin{align*}
    (x_1^l, x_2^l; Z_1^l) &= (4, 2; 36) \hspace{1cm} \text{(leader's solution)} \\
    (x_1^f, x_2^f; Z_2^f) &= (2, 4; 16) \hspace{1cm} \text{(follower's solution)}
\end{align*}
\]

Then, the fuzzy goals of the problem can be successively presented as:

\[ Z_1 \geq 36, \ Z_2 \geq 16 \ \text{and} \ \ x_1 \geq 4. \]

The lower tolerance limits of \( Z_1 \) and \( Z_2 \) are found as

\[ Z_1^f = 6 \ \text{and} \ Z_2^f = 8, \ \text{respectively.} \]
Now, let the leader feel that the value of his/her control variable $x_1$ can be relaxed up to 2.35, and not beyond it for a benefit of the follower. As such, the lower tolerance limit of $x_1$ is obtained as

$$x^l_1 = 2.35, \text{ where } x^l_1 < 2.35 < x^u_1$$

Again, let the follower feel that a certain lower limit on the decision $x_2$ would have to be maintained to obtain a satisfactory level of his/her objective beyond the achieved least value. Let $x^l_2 = 2.5$ be the lower bound of $x_2$.

Then, the variables with their crisp bounds can be obtained as:

$$2.35 \leq x_1 \leq 4$$

and

$$2.5 \leq x_2 \leq 4.$$  

(3.10)

Now, the membership functions of the defined fuzzy goals can be obtained by using (3.3), (3.4) and (3.5). Following the procedure, the membership goals are then obtained as:

$$\mu_{z_1} : \frac{x_1 + 2x_1^2 - (x_2 - 2)^2 - 6}{30} + d_1^- - d_1^+ = 1,$$

$$\mu_{z_2} : \frac{(x_1 - 2)^2 + x_2^2 - 8}{8} + d_2^- - d_2^+ = 1,$$

$$\mu_{x_1} : \frac{x_1 - 2.35}{1.65} + d_1^- - d_1^+ = 1.$$  

(3.11)

The executable FGP model of the problem can be obtained by using (3.7).

Following the proposed approach, two priority factors $P_1$ and $P_2$ are assigned to the model goals in (3.11) for achievement of the associated fuzzy goals, and three priority structures are considered to perform sensitivity analysis in the process of solving the problem in the environment of the given system constraints in (3.9) and structural constraints in (3.10).

The priority achievement functions under the three Runs and the results obtained by employing the GA scheme with the consideration of the evaluation function defined in (3.8) are displayed in the Table 3.1.
Table 3.1
Priority Structure and Solution Achievement

<table>
<thead>
<tr>
<th>Run</th>
<th>Priority structure for goal achievement</th>
<th>Decision $(x_1, x_2)$</th>
<th>Membership value $(\mu_{z_1}, \mu_{z_2}, \mu_{z_3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$[P_1(\frac{1}{30}d_1^1 + \frac{1}{1.65}d_1^2), \ P_2(\frac{1}{8}d_1^2)]$</td>
<td>$(3.2753, 2.5714)$</td>
<td>$(0.6135, 0.0298, 0.5608)$</td>
</tr>
<tr>
<td>2</td>
<td>$[P_1(\frac{1}{30}d_1^1), \ P_2(\frac{1}{8}d_2^1 + \frac{1}{1.65}d_2^2)]$</td>
<td>$(3.5000, 2.5000)$</td>
<td>$(0.7250, 0.0625, 0.6970)$</td>
</tr>
<tr>
<td>3</td>
<td>$[P_1(\frac{1}{8}d_1^2 + \frac{1}{1.65}d_1^2), \ P_2(\frac{1}{30}d_2^1)]$</td>
<td>$(3.0874, 2.7633)$</td>
<td>$(0.5190, 0.1023, 0.4469)$</td>
</tr>
</tbody>
</table>

In the context of achieving the solutions in Table 3.1, it is worthy to mention that, in the process of executing the problem, the achieved values of membership goals of the first priority level $(P_1)$, $(0.6135, 0.0298, 0.5608)$, $(0.7250, 0.0625, 0.6970)$ and $(0.5190, 0.1023, 0.4469)$, under the three successive runs are not sacrificed for achievement of aspired level of any goals included at the second priority level $(P_2)$ and thereby arriving at the final decision against each of the runs.

Now, the Euclidean distances for the achieved membership values under the three successive Runs are obtained as:

$D^1 = 1.3401$, $D^2 = 1.2310$ and $D^3 = 1.3899$

The results reflect that the minimum distance corresponds to $D^2 = 1.2310$

Thus, the priority structure under the Run 2 is an appropriate one to reach the appropriate decision for satisfaction of both the leader and follower in the decision making environment. The resultant decision is

$(x_1, x_2) = (3.5000, 2.5000)$ with $(Z_1, Z_2) = (27.7500, 8.5000)$.

The achieved membership values of the objectives are

$(\mu_{z_1}, \mu_{z_2}) = (0.7250, 0.0625)$. 
Note 3.1: In the context of setting the GA parameter values to solve the problem, it may be mentioned that they are actually adopted from the test results. Here, the parameter setting is particularly adopted to avoid any early convergence with suboptimal decision and substantial increase in generation numbers in the decision making horizon.

Considering the genetic viability of the members in a gene pool of the search domain, it may be noted that the ranges \(0.6 \leq p_c \leq 0.9\) and \(0.06 \leq p_m \leq 0.09\) are also valid for making appropriate decision; otherwise inferior decisions are achieved there under each of the three given priority structures of the test model in the decision making situation. Again, inferior result occurs for selection of \(\text{pop}_-\) size below 50, and number of generations is considerably increased for any value higher than the adopted one to reach the decision. Moreover, Chromosome length below 20 is found not valid to reach the decision, and for any selection below and above the adopted one, variations of generation numbers are found there to arrive at the final decision.

However, it is worthy to mention here that the selection of GA parameter values highly depends on the characteristics as well as size of a problem in the decision making environment.

Remark 1: Regarding the use of the proposed priority based FGP approach, a question about involvement of computational burden for different arrangements of priorities in the decision making process may generally arise. Since, if ‘K’ be the total number of priorities, then consideration of \(K!\) priority structures may arise. But, as limited number of goals is involved with the proposed problem, such a computational burden does not arise here. Further, it is worthy to mention that not more than ‘two’ to ‘five’ priority levels typically arise in a real-world decision problem [296] and the conflict of assigning priorities arises for at most ‘two’ to ‘three’ priority levels.

Note 3.2: If the conventional linear approximation approach studied by Pal and Moitra [511] is used to solve the problem, then under the same GA environment, the solution is found as:

\[(x_1, x_2) = (2.7854, 3.2146) \text{ with } (Z_1, Z_2) = (16.8270, 10.9505).\]
The obtained membership values are $(\mu_{Z_1}, \mu_{Z_2}) = (0.3609, 0.3688)$.

The result shows that the leader's decision power is dominated here by the follower. As a matter of fact, the solution is quite unacceptable in the decision situation owing to violation of hierarchical order of execution of decision powers of the DMs.

**Note 3.3:** If the additive-FGP approach studied by Tiwari et al. [657], where no priority structure is taken into account and maximization of $\sum_{k=1}^{K} \mu_k$ subject to $\mu_k \leq 1$ is taken and the defined system constraints set is considered for goal achievement, is used to solve the problem, then the solution obtained by employing the GA scheme is $(x_1, x_2) = (3.0000, 2.5080)$ with $(Z_1, Z_2) = (20.7419, 7.2900)$. The achieved membership values of the objectives are $(\mu_{Z_1}, \mu_{Z_2}) = (0.4914, 0)$. The result indicates that, although the hierarchical order of execution of decision powers of the DMs is preserved here, the solution is inferior in comparison to the solution obtained by using the proposed GA based FGP approach.

The graphical representation of the model solution and the solutions obtained by using the other two conventional approaches is displayed in the Figure 3.1.

![Comparison of achieved objective values](image)

*Figure 3.1: Achievement of objective values under the three approaches*
3.8 An Illustration for Performance Comparison

To illustrate more the efficient use of the proposed solution approach, a modified version of the problem presented in Section 3.7 is considered. In the present problem, the objectives of the leader and follower are interchanged, that is, the objectives of the leader and follower here are those of the follower and leader, respectively, in the previous example. Actually, the objectives of the problem appear as:

Maximize \[ Z_1(x_1, x_2) = (x_1 - 2)^2 + x_2^2 \] (leader's problem)

and, for given \( x_2, x_1 \) solves

Maximize \[ Z_2(x_1, x_2) = x_1 + 2x_1^2 - (x_2 - 2)^2 \] (follower's problem)

The system constraints and the other restrictions are identical to those given in (3.9) of the previous example.

It is to be followed that the roles of the leader and follower are simply interchanged here in the process of solving the problem. The best solutions of the leader and follower are obtained as:

(\( x_1^*, x_2^*; Z_1^* \)) = (2, 4; 16) and (\( x_1^*, x_2^*; Z_2^* \)) = (4, 2; 36), respectively.

Then, the fuzzy goals successively appear as:

\( Z_1 > 16, Z_2 > 36 \text{ and } x_2 > 4 \).

The lower tolerance limits of \( Z_1 \) and \( Z_2 \) are found as \( Z_1^l = 8 \) and \( Z_2^l = 6 \), respectively.

The lower tolerance limit of \( x_2 \) is defined as \( x_2^p = 2.5, \text{ where } x_2^l < 2.5 < x_2^l \).

The crisp bounds on the variables are considered the same as defined in (3.10) of the example presented in Section 3.7.

Now, in an analogous to the defined model goals in (3.11), the executable membership goals can be constructed and then a priority based FGP model of the problem can easily be obtained.

The resultant decision under the priority structure, which is similar to that in Run 2 of the previous example, is obtained as:

\( (x_1, x_2) = (2.3500, 3.6500) \text{ with } (Z_1, Z_2) = (13.4450, 10.6725) \).
The achieved membership values of the objectives are
\((\mu_{Z_1}, \mu_{Z_2}) = (0.6806, 0.1558)\).

The result reveals how the solution changes with the change of the hierarchical
structure of the problem in the decision making environment.

The results reflect that, the solution obtained by using the proposed GA based
FGP approach is better in comparison to that obtained by using the conventional
approaches studied previously from the viewpoint of more satisfaction of the DMs
with the decisions in the decision making environment.

**Remark 2:** From the above discussions and solution comparisons, it may be claimed
that the proposed approach is superior over the other approaches studied previously
with regard to proper distribution of decision powers of the DMs and thereby
arriving at the most satisfactory decision on the basis of the needs and desires of the
DMs in the decision making context.

Further, since GAs are population based global solution search methods, the
efficient use of an GA scheme to MODM problems always offers a satisfactory
decision in the decision making environment.

### 3.9 Conclusion

The main advantage of the proposed approach is that the computational load with
linearization of objectives as well as approximation error which is inherent to
conventional linearization approaches can be avoided here owing to the use of GA
method directly within the framework of the proposed model of the problem.

The proposed approach can easily be extended to BLPPs having more than one
objective (linear/non-linear) at any one of the hierarchical decision levels without
involving any computational difficulty.

Again, if fuzzy description of the system constraints is also needed in the decision
situation, then that can easily be accommodated within the framework of the
proposed FGP model of the problem.

Further, if nonlinearity in system constraints involves in a decision horizon, then
the proposed approach can easily be extended there and also the computational
complexity does not occur to take decision owing to the use of the population based global solution search scheme in the decision making environment.

The proposed approach can be extended to general nonlinear BLPPs as well as MLPPs with multiplicity of objectives in large hierarchical decentralized decision making organizations, which may be the problem of future study.

Finally, it is hoped that the approach presented here can lead to future research for solving practical problems of hierarchical decision organizations in the current complex uncertain decision making environment.