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A GENETIC ALGORITHM BASED GOAL PROGRAMMING APPROACH FOR SOLVING INTERVAL VALUED BILEVEL PROGRAMMING PROBLEMS

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Abstract: In this article, the efficient use of genetic algorithm (GA) for solving goal programming (GP) formulations of interval valued bilevel programming problems (IVBLPPs) in a hierarchical decision making organization is presented.

In the proposed approach, first the individual best and least solutions of the objectives of both the decision makers (DMs) located at the two hierarchical decision levels are determined by using an GA method. Then, the target interval for achievement of each of the objectives as well as the target interval of the decision vector controlled by the upper-level DM are defined in the inexact decision making environment.

In the executable goal programming (GP) formulation of the problem, the defined interval valued goals are transferred into the standard goals in GP by using the interval arithmetic technique in interval programming (IP) approach.

In the solution process, both the aspects of minsum and minmax GP formulation are adopted to minimize the lower bounds of the regret intervals for goal achievement within the specified interval from the
optimistic point of view and thereby distribution of proper decision powers to the DMs of the hierarchical levels.

In the GA based solution search process, the conventional Roulette wheel selection scheme, arithmetic crossover and random mutation are taken into consideration to reach a satisfactory decision.

In the decision process, the GA scheme is directly used to the achievement function for minimizing the deviational variables of the goals in the decision making situation.

A numerical example is solved to illustrate the proposed approach, and the obtained solution is compared with the conventional approach studied previously.

Keywords: Bilevel Programming, Genetic Algorithm, Goal Programming, Interval Programming, Interval Valued Bilevel Programming.

1. INTRODUCTION

Bilevel programming problem (BLPP) is a special case of Multilevel programming problem (MLPP) of a large hierarchical decision system. In a BLPP, two DMs are located at two different hierarchical levels, each independently control one set of decision variables for optimizing the individual objective function which often conflict each other in the decision making situation.

In a hierarchical decision process, the lowerlevel DM (the follower) executes his/her decision powers, after the execution of decision of the upper-level DM (the leader). Although the leader independently optimizes its own benefits, the decision may be affected by the reactions of the follower. As a consequence, decision deadlock arises frequently and the problem of distribution of proper decision powers is encountered in most of the practical hierarchical decision situations.

The concept of BLPP was first introduced by Candler and Townsley in (6) in 1982. Thereafter, various versions of bilevel programming (BLP) have been presented in (2, 3). During 1980s, several solution approaches have been developed in (2, 3, 6, 10, 20, 21, 25) for solving BLPPs as well as MLPPs in general from the view point of their potential applications to hierarchical decentralized decision systems as economic systems, warfare, network designs and is specially applicable to conflict resolution.
Multiobjective solution techniques for BLPPs with post optimality analysis on the objective values to obtain efficient solutions have been suggested by Wen and Hsu in (25). In their approach, three efficient compromise solutions, threat point, ideal point and ideal-threat point dependent solutions depending on the DM’s preference have been provided. The use of the concept of membership function of fuzzy set theory to BLPPs as well as MLPPs for satisfactory decision was first introduced by Lai in (10). Thereafter, Lai’s satisfactory solution concept was extended by Shih et. al in (20, 21). The basic concept of the fuzzy programming (FP) approaches in (20) is that there is a possibility of rejecting the solution again and again by the leader and re-evaluation of the problem with the elicited membership values of the membership functions is repeatedly introduced in the solution search process due to the conflict in nature of the objectives. Again, for the use of such approaches, decision deadlock frequently arises due to the paradox that the leader’s decision power is dominated by that of the follower.

To overcome the above difficulties, the BLPPs in the framework of fuzzy goal programming (FGP) in (5) have been also studied by Pal et. al in (4, 15), and others in the past.

Now, it may be noted that although FP as well as FGP have been successfully implemented to the BLPPs as well as MLPPs, the main difficulty of using such approaches is that it may not always be possible for the DM to assigning the fuzzy aspiration levels to the objectives and thereby defining the tolerance ranges of goal achievement in the actual decision situations.

To overcome the above difficulty, IP approaches in (23, 24) have appeared as the prominent tools for solving multiobjective decision problems in imprecise environment. The IP problems have also been studied by Pal et. al. in (16, 17) in the recent past.

However, the methodological extension of IP problems is still at an early stage. Further, IP approach to hierarchical decision problems is yet to appear in the literature.

In this article, interval valued objectives of both the DMs located at two hierarchical decision levels are considered. In the objectives of the DMs, both the coefficients and the target values are taken as intervals. Then the interval valued objectives are transformed into
objective goals in the standard GP formulation using interval arithmetic technique in (12) by introducing under- and over-deviational variables to each of them. In the model formulation of the problem, both the aspects of GP, minsum GP in (4) and minmax GP in (19) are taken into consideration to construct the achievement function for goal achievement of the interval goals defined in the decision situation.

In the solution process, an GA method in (7, 8) is introduced to the GP formulation of IVBLPP. The GA approach in solving multiobjective decision making problems has been investigated by various researchers in (1, 11, 20) in the past. In using the GA process, the goal satisficing philosophy in GP, expound by Simon in (22) in 1957, for linear goals defined for the interval objectives is taken into consideration to reach a satisfactory decision.

The proposed approach is illustrated by a numerical example and the model solution is compared with the solution approach studied previously.

2. PROBLEM FORMULATION

Let $X_1, X_2$ be the vectors of decision variables controlled by the leader and the follower, respectively, in the hierarchical decision system, and $F_1, F_2$ be the objectives to be optimized at the two decision levels.

Then, the IVBLPP can be constructed as:

Find $X(X_1, X_2)$ so as to:

$$\max_{x_1} F_1 (X_1, X_2) = (c_{11L}, c_{11U}) X_1 + (c_{12L}, c_{12U}) X_2$$

(leader's problem) ...(2.1)

$$\max_{x_2} F_2 (X_1, X_2) = (c_{21L}, c_{21U}) X_1 + (c_{22L}, c_{22U}) X_2$$

(follower's problem) ...(2.2)

subject to

$$X \in S = [(X_1, X_2) | A_1 X_1 + A_2 X_2 = b, X \geq 0],$$

$$X_1, X_2 \in R^n, b \in R^m.$$

...(2.3)

where, $(c_{ijL}, c_{ijU}) i, j = 1, 2$, are the vector of interval coefficients, $i$, and $U$ denote the lower and upper bounds, respectively, of the defined
interval. \( A_1, A_2 \) are the constant matrices and \( h \) is a constant vector. It is assumed that the feasible region \( S (\neq \Phi) \) is bounded.

Now, using the rules of interval arithmetic operations in (9), the interval valued objectives in (2.1) and (2.2) can be successively expressed as:

\[
\max_{x_1} F_1(X_1, X_2) = (c_{11L}X_1 + c_{11U}X_2, c_{12L}X_1 + c_{12U}X_2) \\
\text{ (leader's problem) } \quad \text{... (2.4)}
\]

and

\[
\max_{x_2} F_2(X_1, X_2) = (c_{21L}X_1 + c_{21U}X_2, c_{22L}X_1 + c_{22U}X_2) \\
\text{ (follower's problem) } \quad \text{... (2.5)}
\]

Now, due to inexactness of the decision environment, it becomes generally difficult for a DM to assign the exact target levels for achievement of the objective. In such a case, interval valued target called the target interval for goal achievement is to be taken into account by the DM.

In the present decision situation, the individual best and least objective values can be reasonably taken into account as the upper and lower bounds of the target intervals for goal achievement.

Now, to determine the target intervals and thereby to make a satisfactory solution, an GA scheme is introduced here in the solution search process.

3. DESIGN OF THE GA SCHEME

In the literature of the GAs, there is a number of schemes in (7, 8) for generation of new populations with the use of the different operators: selection, crossover and mutation. Here, the binary coded representation of a candidate solution called chromosome is considered to perform genetic operations in the solution search process. The conventional Roulette wheel selection scheme in (7), singlepoint crossover (11) and bit-by-bit mutation operations are adopted to generate offspring in new population in search domain defined in the decision making environment. The fitness score of a chromosome \( v \) (say) in evaluating a function, say, \( \text{eval}(E_v) \), based on maximization or minimization of an objective function defined on the basis of DMs' needs and desires in the decision making context.
Now, the model formulation of the problem is described in the Section 4.

4. INTERVAL VALUED MODEL FORMULATION

To introduce the target intervals to the objectives \( F_1 \) and \( F_2 \) as well as the decision vector \( X \), of the leader, the individual best and least solutions of the objectives are determined by employing the proposed GA scheme. Let the individual best and least solutions of the leader be \((X_1^b, X_2^b; T_{1l})\) and \((X_1^l, X_2^l; T_{1u})\) respectively, where

\[
\begin{align*}
\text{Max} & \quad T_{1l} = (X_1, X_2) \in S^{F_1}(X_1, X_2) \\
\text{Min} & \quad T_{1u} = (X_1, X_2) \in S^{F_1}(X_1, X_2)
\end{align*}
\]

Similarly, let \((X_1^f, X_2^f; T_{2l})\) and \((X_1^f, X_2^f; T_{2u})\) be respectively the individual best and least solutions of the follower, where

\[
\begin{align*}
\text{Max} & \quad T_{2l} = (X_1, X_2) \in S^{F_2}(X_1, X_2) \\
\text{Min} & \quad T_{2u} = (X_1, X_2) \in S^{F_2}(X_1, X_2)
\end{align*}
\]

Then, the objectives in (2.4) and (2.5) with target intervals can be successively presented as

\[
\begin{align*}
(c_{11}, X_1 + c_{12}, X_2, c_{11l}, X_1 + c_{12l}, X_2) = (T_{1l}, T_{1u}), \\
\text{(leader's problem)} & \quad \text{... (4.1)} \\
(c_{21}, X_1 + c_{22}, X_2, c_{21l}, X_1 + c_{22l}, X_2) = (T_{2l}, T_{2u}), \\
\text{(follower's problem)} & \quad \text{... (4.2)}
\end{align*}
\]

Again, since the leader has a higher power of making decision, a certain relaxation of \( X_1^b \) as a lower tolerance limit should be given for searching of a better decision by the follower.

Let \( X_1^f (X_1^l < X_1^f < X_1^b) \) be the lower tolerance limit of the decision vector \( X_1 \) controlled by the leader.

Using the concept of mid-point interval arithmetic in (12), the control vector \( X_1 \) with target intervals can be obtained as

\[
X_1 = (X_1^f, X_1^b) \quad \text{... (4.3)}
\]
Now, the GP formulation of the problem is discussed in the Section 4.1.

### 4.1 GP Formulation

To formulate the GP model of the problem, the interval valued goals in (4.1), (4.2) and (4.3) are to be transformed into the conventional form of goals of GP methodology by using the rules of interval arithmetic in IP in (9), and under- and over-deviational variables are introduced to each of them.

Now, from the expression in (4.1), the leader’s objective goals appear as:

\[
\begin{align*}
    c_{11L}X_1 + c_{12L}X_2 + d_{1L}^- - d_{1L}^+ &= T_{1L}, \\
    c_{11U}X_1 + c_{12U}X_2 + d_{1U}^- - d_{1U}^+ &= T_{1U},
\end{align*}
\]  

... (4.4)

Similarly, from the expression in (4.2), the follower’s objective goals take the form:

\[
\begin{align*}
    c_{21L}X_1 + c_{22L}X_2 + d_{2L}^- - d_{2L}^+ &= T_{2L}, \\
    c_{21U}X_1 + c_{22U}X_2 + d_{2U}^- - d_{2U}^+ &= T_{2U},
\end{align*}
\]  

... (4.5)

where \(d_{1L}, d_{1U}^-\) and \(d_{1U}^+\) (\(\geq 0\)), \(k = 1, 2\) represent the under- and over-deviational variables, respectively, associated with the respective goals. Again, the expression in (4.3) can be transformed to the conventional goals as

\[
\begin{align*}
    X_1 + d_- = X_1^L \\
    X_1 + d_+ = X_1^U
\end{align*}
\]  

... (4.6)

where, \(d_- (\geq 0)\) and \(d_+ (\geq 0)\) represents the vectors of under- and over-deviational variables, and dimension of them depends on the dimension of \(X_1\).

Now, in the possibility approach in IP (9) method, the aim of the DMs is to minimize the possible regrets for under-deviations from \(T_{1L}\) and over-deviations from \(T_{1U}\) \((k = 1, 2)\) and also the regrets for vectors of under- and over-deviations from \(X_1^L\) and \(X_1^U\) respectively, defined for the leader’s control vector \(X_1\) to reach a solution for achieving the aspired goal levels within their respective target intervals specified in the decision situation.
As a matter of fact, minimization of the deviational variables $d_i$ and $d_{ik}$, $k = 1, 2$, and the vectors of deviational variables $d_i$ and $d_{ik}$ in the context of minimizing the total possible regrets for goal achievement is taken into account, and then the designed objective function of the proposed GP model is termed as the 'regret function'.

4.2 Construction of Regret Function

The development of regret function, for minimization of the deviational variables can be made in different ways (9) depending on the needs and desires of the DMs in the decision making context.

Here, from the optimistic point of view of the DMs, both the aspects of GP, \( \min_{\text{sum}} \) GP in (4) for minimizing the sum of the weighted unwanted deviational variables as well as \( \min_{\text{max}} \) GP in (19) for minimizing the maximum of the deviations, are simultaneously taken into account as a convex combination of them to reach a satisfactory solution for goal achievement within the specified target intervals of the defined goals of the problem.

Now, for model simplification let it be assumed that \( n_1 (n_1 < n) \) be the number of decision variables involved with the control vector \( x_i \).

The regret function appears as:

\[
\text{Minimize } Z = \lambda \left[ \sum_{i=1}^{n_1+2} (w_{iL} d_{iL}^+ + w_{iU} d_{iU}^+) \right] + (1 - \lambda) \left[ \max_{i \in n_1+2} (d_{iL}^+ + d_{iU}^+) \right] \quad ... (4.7)
\]

where \( Z \) represents the regret function for goal achievement; \( d_{iL}^+, d_{iU}^+ (\geq 0) \) \( (i = 1, 2, ..., n_1 + 2) \), are successively renamed for \( d_{iL}^-, d_{iU}^- \) \( (k = 1, 2) \) and \( n_1 \) components of each of the \( d_{iL}^-, d_{iU}^- \), and where \( w_{iL}, w_{iU} (\geq 0) \) with \( \sum_{i=1}^{n_1+2} (w_{iL} + w_{iU}) = 1 \) are the numerical weights of importance of achieving the goals within their respective target intervals, and \( 0 < \lambda < 1 \).

Now, let

\[
\max_{i \in n_1 + 2} \left[ (d_{iL}^+ + d_{iU}^+) \right] = d \quad ... (4.8)
\]
Then, the executable GP model of the problem can be presented as:

Find $X (X_1, X_2)$ so as to:

$$\text{Minimize} \ Z = \lambda \left[ \sum_{i=1}^{n+2} (w_{ii} d_{ii}^- + w_{ii}^+ d_{ii}^+) \right] + (1 - \lambda) d$$

and satisfy the goal expressions in (4.4), (4.5) and (4.6) subject to

$$\left( d_{ii}^- + d_{ii}^+ \right) \leq d, \quad i = 1, 2, ..., n_i + 2 \quad \text{(4.9)}$$

and the given system constraints in (2.3).

Now, for the developed GP model of the proposed problem, the task of both the DMs is to search the solution to satisfy the goal levels to the extent possible by evaluating the defined regret function for overall benefit of the organization in the decision making context.

Now, since GA is a goal satisficer in (7) rather than optimizer, the proposed GA scheme can be employed here to minimize the regret function $Z$ and thereby to reach a satisfactory decision for distribution of proper decision powers to the DMs.

Here, the fitness function appears as:

$$\text{eval} \ (E_v) = (Z)_v$$

$$\left\{ \lambda \left[ \sum_{i=1}^{n+2} (w_{ii} d_{ii}^- + w_{ii}^+ d_{ii}^+) \right] + (1 - \lambda) d \right\}_v \quad \text{(4.10)}$$

where, $v$ represents a chromosome.

Here, the best chromosome $E^*$ with highest fitness score at a generation is determined by

$$E^* = \min_v \left[ \text{eval} \ (E_v) \right]$$

To illustrate the approach, a numerical example is solved in the Section 5.

5. AN ILLUSTRATIVE EXAMPLE

The BLPP with interval coefficients can be considered as:
Find $X(x_1, x_2)$ so as to:

$$\max_{x_1} F_1(x_1, x_2) = (1, 3)x_1 - (1, 2)x_2 \quad \text{ (leader's problem)}$$

where, for given $x_1, x_2$ solves

$$\max_{x_2} F_2(x_1, x_2) = (1, 2)x_1 + (1, 3)x_2 \quad \text{ (follower's problem)}$$

subject to

$$3x_1 - 5x_2 \leq 15, \ 3x_1 - x_2 \leq 21,$$

$$3x_1 + x_2 \geq 27, \ 3x_1 + 4x_2 \leq 45,$$

$$x_1 + 3x_2 \leq 30, \ x_1, x_2 \geq 0. \quad ... (5.1)$$

Now, the use of different genetic parameters and their values adopted in the solution search process are defined as follows.

The input parameters:

- $\text{Max}_\text{gen}$; // number of generations
- $\text{Pop}_\text{size}$; // population size
- $\text{pc}$; // probability of crossover
- $\text{pm}$; // probability of mutation

The programming language $C$ is used in the process of coding the evaluation program. The environment of execution is Intel Pentium IV with 2.66 GHz. Clock-pulse and 1 GB RAM.

The, chromosome length = 30 is considered with a view to searching solution in the domain of feasible solution set ($S$) defined in the decision situation. The population size as in the standard GA method in (7) is taken 100.

The number of generations = 300 is initially taken to conduct the experiment. The different experiments with the different values of $p_c (0 < p_c < 1)$ and $p_m (0 < p_m < 1)$, in the ranges ($0.7 \leq p_c \leq 0.9$) and ($0.03 \leq p_m \leq 0.8$) are made in the proposed GA scheme. It is found that $p_c = 0.8$ and $p_m = 0.08$ are successful in the decision search process.

Now, using the interval arithmetic rule and employing the GA scheme, the individual best and least solutions of the leader are obtained as
A Genetic Algorithm Based Goal Programming...

\[(x_1, x_2, T_{ul}) = (8, 3; 13)\] and \[(x_1, x_2, T_{ul}) = (7, 6; 8),\]

respectively.

Similarly, the best and least solutions of the follower are obtained as
\[(x_1, x_2, T_{2ul}) = (7, 6; 19)\] and \[(x_1, x_2, T_{2ul}) = (8, 3; 14),\]

respectively.

Then, following the procedure, the leader’s problem with the target interval can be obtained as:
\[(1, 3)x_1 - (1, 2)x_2 = (8, 13)\]

Similarly, the follower’s problem is obtained as:
\[(1, 2)x_1 + (1, 3)x_2 = (14, 19)\]

Now, the decision situation, the leader feels that the achieved value 8 of \(x_1\) can be relaxed up to 7.5 but not beyond of it. So
\[x_1^L = 7.5 (x_1^L < 7.5 < x_1^H)\]
is considered as the lower bound of \(x_1\) of its target interval.

Then, the control variable \(x_1\) of the leader with interval coefficients and target interval can be presented as:
\[(1, 1)x_1 = (7.5, 8)\]

Now, by following the procedure, the linear objective goals are obtained as:
\[x_1 - x_2 + d^d_L - d^d_u = 8\]
\[3x_1 - 2x_2 + d^d_L - d^d_u = 13\]
\[x_1 + x_2 + d^d_L - d^d_u = 14\]
\[2x_1 + 3x_2 + d^d_L - d^d_u = 19\]
\[x_1 + d^d_L - d^d_u = 7.5\]
\[x_1 + d^d_L - d^d_u = 8\] ... (5.2)

where, \(d^d_L, d^d_u, d^*_L, d^*_u \geq 0, \ i = 1, 2, 3.\)

Now, in the decision making context, the deviational variables to be minimized in the regret function depend on the needs and desires of the DMs.

In the present decision situation, the executable GP model with the regret function is constructed as:

Find \(X(x_1, x_2)\) so as to
Minimize \( Z = \lambda \sum_{i=1}^{3} \left( w_{it}^+ d_{it} + w_{it}^- d_{it}^- \right) + (1 - \lambda) d \)

subject to
\[
\sum_{i=1}^{3} d_{it}^- \leq d, \\
\sum_{i=1}^{3} d_{it}^+ \leq d, \ldots (5.3)
\]

and the goal constraints in (5.2) and the system constraints in (5.1).

Now, assigning equal weights, each value of which = \( \frac{1}{6} \), to all the goals for their achievement and introducing \( \lambda = 0.5 \), the problem is solved by employing the GA scheme with fitness function \( Z \) defined in (5.3) and the same parameter values considered previously.

The resultant decision is obtained as
\[(x_1, x_2) = (8, 3), \text{ with } (F_1, F_2) = (13, 14).\]

The result shows that a satisfactory decision is reached in the decision making environment.

Note: The BLPP with crisp coefficients in the fuzzy decision environment is considered by Pal and Moitra (14) is

\[
\max_{x_1} F_1 (x_1, x_2) = 2x_1 - x_2 \quad \text{(leader's problem)}
\]

where, for given \( x_1, x_2 \) solves

\[
\max_{x_2} F_2 (x_1, x_2) = x_1 + 2x_2 \quad \text{(follower's problem)}
\]

subject to the given system constraints in (5.1).

In the FGP formulation, if the best solutions of leader and follower are considered as the fuzzy aspiration levels of the respective objectives and their least solutions as the lower tolerance limits, then the solution of the problem in the framework of FGP is obtained as

\[(x_1, x_2) = (8.6, 4.8) \text{ with } (F_1, F_2) = (12.4, 18.2).\]

A comparison shows that a better solution is achieved here in terms of balancing the decision powers to the DMs of both the hierarchical levels. Now, it is to be followed that if the IP technique is used to the problem, then the solution is found as \((x_1, x_2) = (8, 3) \text{ with } (F_1, F_2) = (13, 14).\)
A further comparison shows that the solution under the present approach is a superior one from the viewpoint of satisfaction of both the DMs in the hierarchical decision making environment.

6. CONCLUSION

In this paper, how the GA method can be efficiently employed to solve BLPPs is presented. In future study, the proposed method can be extended to solve MLPPs as well as the other decentralized planning problems from the viewpoint of their potential use to real-life problems.

The proposed approach can be extended to solve chance constrained hierarchical decision problems, which may be a problem for future research.

However, it is expected that the approach presented here may lead to further research on real-life hierarchical decision problems in the inexact decision making environment.

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REFERENCES

A genetic algorithm based stochastic simulation approach to chance constrained interval valued multiobjective decision making problems

ABSTRACT

This article presents how the stochastic simulation through genetic algorithm (GA) can be used to modeling and solving chance constrained interval valued multiobjective decision making (MODM) problems. In the proposed method, a stochastic simulation approach to the chance constraints is employed for interval valued goal representation of the objectives as well as decision identification through the use of a GA method in an inexact decision making context. In the executable goal programming (GP) model of the problem, both the aspects of the GP, min-sum GP and min-max GP, are addressed within goal achievement function for minimizing possible regrets associated with the deviational variables of the defined goals for goal achievement within the target intervals specified in the decision making environment. A numerical example is solved and a comparison is made with the conventional GP approach.

INDEX TERMS

Index Terms are available to subscribers and IEEE members.
A Genetic Algorithm Based Stochastic Simulation Approach to Chance Constrained Interval Valued Multiobjective Decision Making Problems

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Abstract - This article presents how the stochastic simulation through genetic algorithm (GA) can be used to modeling and solving chance constrained interval valued multiobjective decision making (MODM) problems.

In the proposed method, a stochastic simulation approach to the chance constraints is employed for interval valued goal representation of the objectives as well as decision identification through the use of an GA method in an inexact decision making context.

In the executable goal programming (GP) model of the problem, both the aspects of the GP, minsum GP and minmax GP [1], are addressed within goal achievement function for minimizing possible regrets associated with the deviational variables of the defined goals for goal achievement within the target intervals specified in the decision making environment.

A numerical example is solved and a comparison is made with the conventional GP approach.

Keywords- Chance constrained programming, Genetic algorithm, Goal programming, Interval programming, Stochastic simulation.

I. INTRODUCTION

In the real-world decision making situations, decision makers (DMs) are frequently faced with the three types of uncertainties, stochastic, fuzzy and interval valued ness, to setting off the parameter values of the decision problems due to imprecise in nature of human judgments.

The stochastic programming (SP) based on the probability theory, initially introduced by Charnes and Cooper [2], has been studied [3] extensively in the past and applied to various real-life problems [4,5].

On the other hand, fuzzy programming (FP) based on the theory of fuzzy sets, initially introduced by Zadeh [6] in 1965, has been studied [7,8,9], deeply during the last 38 years from the viewpoint of the potential use to real-life problems [10, 11] with imprecise data.

Now, in a certain decision situation, it has been realized that parameter values are found to be neither probabilistic nor fuzzy, but they are rather in the form of intervals with certain lower- and upper- bounds. To cope with the situation, interval programming (IP) approaches have been studied [12] in the past, and implemented to real life problems [13] in the recent past.

But, methodological extension of the field of IP and wide application of it to practical problems is at an early stage.

However, in the real-world decision situations, it has been recognized that the combination of any two or all three types of the defined data are increasingly involved in introducing the parameter values of the problems. Here, consideration of such an aspect creates a great challenge to the researchers for modelling and solving MODM problems.

The modelling aspect of MODM problems under randomness and fuzziness was first studied by Luhandjula [14] in 1983. The extensive study on fuzzy stochastic programming (FSP) made in the past has been surveyed by Luhandjula [15] in 2006.

But, in most of the previous studies [16], the chance constraints having independent normally distributed random parameters are converted into their deterministic equivalent to solve the problems by using conventional deterministic tools. But, in an uncertain decision environment, different types of discrete as well as continuous random parameters
are frequently involved, and computational difficulty often arises in converting them to deterministic equivalent. To overcome the difficulty, the chance constrained programming (CCP) [17] with the stochastic simulation [18] has been studied [19] deeply in the past. The use of stochastic simulation to fuzzy goal programming (FGP) [20] formulations of chance constrained MODM problems [21, 22] have been studied by Pal et al. [23] in the past. However, the extensive study on stochastic simulation approach to fuzzy MODM problems is yet to be well documented in the literature. Further, the CCP approach to chance constrained MODM problems is yet to be circulated in the literature.

Now, in the area of MODM, GAs [24, 25] based on natural selection and population genetics and as the decision satisfiers [26] rather than objective optimizers have appeared as prominent tools for solving MODM problems in deterministic as well as inexact decision making environments. The use of GAs to different real-life problems have also been studied by Pal et al. [27, 28], and others in the past.

The use of an GA approach to interval valued decision problems has also been presented by Pal and Gupta [29] in the past. However, the study on the use of GA method to chance constrained IP problems with multiplicity of objectives is yet to be documented in the literature.

In this article, the use of stochastic simulation to multiobjective chance constrained interval valued decision problems having random parameters with lognormal probability distribution is presented. In the proposed approach, first the objectives with interval coefficients are converted into the interval valued goals by employing an GA scheme through the use of stochastic simulation technique. Then, the defined goals are transformed into the standard goals in conventional GP formulation by using interval arithmetic rule [30] and introducing under- and over-deviational variables to each of them.

In the executable GP model of the problem, both the minsum and minmax GP approach [1] are adopted in the regret function defined for goal achievement by minimizing the under- and over-deviational variables of the goals on the basis of the weights of importance of achieving the goal values within their specified target intervals.

In the solution process, the proposed GA scheme is employed in the same decision environment to reach a most satisfactory decision in the decision making horizon.

Now, the chance constrained interval valued problem formulation is presented in the following Section II.

II. PROBLEM FORMULATION

The generic form of chance constrained MODM problem can be stated as

\[
\text{Find } X(x_1, x_2, ..., x_n) \text{ so as to:}
\] 

Maximize \( F_k(X) = [a^l_k, a^u_k] X, \ k = 1, 2, ..., K \) 

subject to 

\[ X \in S = \{X \in \mathbb{R}^n \mid \Pr[AX \leq b] \geq p, X \geq 0, b \in \mathbb{R}^m \}, \]

where \([a^l_k, a^u_k]\) are the vector of interval coefficients of the \(k\)-th objective and where \(L\) and \(U\) stand for lower- and upper-bounds, respectively. \(X\) is the vector of decision variables, \(A\) is the technological coefficient matrix, \(b\) is the resource vector, \(p\) (0 < p < 1) is the vector of satisficing probability levels defined for the random occurrence of the parameters involved with the constraints set, and \(Pr\) indicates the probabilistically defined constraints. It is assumed that the feasible region \(S\) is bounded.

Now, the stochastic simulation approach to the chance constraints for estimation of random parameters in the solution search process is described in the following Section A.

A. Stochastic Simulation for Parameter Estimation

The generation of random numbers during the simulation run for constraint satisfying has been well documented in [31] and widely used in [3, 18] for solving CCP problems. In the present decision situation, without loss of generality, it is assumed that random parameters follow lognormal probability distributions.

Now, the simulation process adopted in the decision system can be defined as follows:

The probabilistic constraints in (2) can be explicitly presented as:

\[ \Pr \left[ \sum_{j=1}^{n} a^l_j x_j \leq b_i \right] \geq p_i, i = 1, 2, ..., m \]

Let 

\[ g_i(X, v) = \left( \sum_{j=1}^{n+1} a^l_j x_j - b_i \right) \]

Then, 

\[ \Pr[g_i(X, v) \leq 0] \geq p_i, i = 1, 2, ..., m. \]

Where \( v = (v_1, v_2, ..., v_{n+1}) \) is the \((n+1)\) component vector of random elements, where the dimension of each of which is \((m+1)\), and where, \( v^T = (a_{ij}, a_{ij}, ..., a_{ij}) \), \(j=1, 2, ..., n\), and \( v^T_{n+1} = (b_1, b_2, ..., b_m) \), \(T\) means transpose. Then, for a given vector \(X\), let \( R \) independent random vectors be generated in such a way that
\( \mathbf{v}^{(r)} = (v_1^{(r)}, v_2^{(r)}, ..., v_R^{(r)}), r = 1, 2, ..., R \)

for the given probability distributions of the defined vectors of random variables.

Let, \( R' \) be the number of occasion of \( R \) trials for which the expression \( g_i(X, v) \leq 0, i = 1, 2, ..., m \), in (4) is satisfied. Then, probability of satisfying the constraints appears as

\[ P = \frac{R'}{R} \]

Here, if \( P \geq p_i, \forall i \) is satisfied, \( X \) is reported as the feasible solution for optimizing the defined objectives. The process of simulation run is summarized in the following steps:

**Simulation Algorithm:**

**Step 1.** Initialize \( R' = 0 \).

**Step 2.** Generate vectors of random numbers

\( (\mathbf{v}^{(r)}, r = 1, 2, ..., R) \) according to the given distribution of the random parameters.

**Step 3.** If the constraints \( g_i(X, v) \leq 0, i = 1, 2, ..., m \), is satisfied, then set \( R' = R' + 1 \).

**Step 4.** Repeat the Step 2 and Step 3 \( P \) times,

**Step 5.** Compute \( P = \frac{R'}{R} \).

**Step 6.** Report the solution \( X \) as a feasible solution, where the constraints set are satisfied with the prescribed probabilities \( P \geq p_i, \forall i \).

Now, in the solution search process, an GA scheme through the use of the defined simulation technique is presented in the following Section B.

**B. Design of the GA Scheme**

The GA scheme employed in the solution search process is presented in the following algorithmic steps.

**Step 1. Representation and Initialization**

Let \( C_L \) denote the binary coded representation of chromosome in a population as \( C_L = \{e_i, e_{i1}, ..., e_{in}\} \) where \( e_i = 0 \) or 1, \( i = 1, 2, ..., n \). The population size is defined by \( \text{pop}_\text{size} \), and \( \text{pop}_\text{size} \) chromosomes are randomly initialized in its search domain.

**Step 2. Feasibility Verification**

The proposed stochastic simulation approach is used to test the feasibility of the candidate solution.

**Step 3. Fitness Function**

The fitness value of each chromosome is determined by the value of an objective function. The fitness function is defined as

\[ \text{eval}(C_L) = (F_k)_{k=L=1}, k=1, 2, ..., \text{pop}_\text{size} \]

where \( F_k \) is given in (1).

The best chromosome with largest fitness value at each generation is determined as:

\[ C^* = \max \{\text{eval}(C_L) | L = 1, 2, ..., \text{pop}_\text{size}\}, \]

or \( C^* = \min \{\text{eval}(C_L) | L = 1, 2, ..., \text{pop}_\text{size}\} \),

depending on the needs and desires of the DMs in the decision making situation.

**Step 4. Selection**

The simple roulette-wheel scheme [25] is used for selecting two parents for mating purposes in the genetic search process.

**Step 5. Crossover**

The parameter \( P_c \) is defined as the probability of crossover. The single-point crossover is applied here in the sense that the resulting offspring always satisfy the linear constraints set \( S \). Here a chromosome is selected as a parent, if for a defined random number \( r \in [0, 1] \), \( r < P_c \) is satisfied.

For example, arithmetic crossover for two parents \( C_1, C_2 \in S \) yields two offspring

\[ E_1 = \alpha_1 C_1 + \alpha_2 C_2, \quad E_2 = \alpha_2 C_1 + \alpha_1 C_2, \]

where, \( \alpha_1, \alpha_2 \geq 0 \) with \( \alpha_1 + \alpha_2 = 1, E_1, E_2 \in S \).

**Step 6. Mutation**

As in the conventional scheme, a parameter \( P_m \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a Random Number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

**Step 7. Termination**

The execution of the whole process terminates when the best-generated chromosome is reported in the genetic search process.

Now, the interval valued GP model formulation of the problem is presented in the Section III.

**III. INTERVAL VALUED GP MODEL FORMULATION**

To formulate the GP model of the problem, the objectives in (1) are to be transformed into interval valued goals by introducing target interval for goal achievement of each of them. The lower- and upper- bounds for interval specification can be obtained by means of estimating the best and least value of each of the objectives using the mid-point arithmetic rule [30] in IP to the interval coefficients and employing the proposed GA scheme.

Let the best and least solutions of the \( k \)-th objectives be \( F_{bk}^U \) and \( F_{bk}^L \), respectively,

\[ F_{bk}^U = \max_{X \in S} F_k(X) \quad \text{and} \quad F_{bk}^L = \min_{X \in S} F_k(X) \]

Then, the objective with interval coefficients and target interval appears as:

Maximize \( F_k(X) = [a_k^L, a_k^U] X = [F_{bk}^L, F_{bk}^U] \),

\[ k=1, 2, ..., K \] (7)
Now, the GP formulation of the problem is presented in the following Section A.

A. GP Formulation of the Problem

To formulate the GP model of the problem, the objectives in (7) are to be transformed into conventional goals in GP by using interval arithmetic rule [32] in IP and introducing under- and over-deviational variables to each of them.

Then the equivalent goal expressions of the expression in (7) in explicit form are obtained as [12]:

\[
\begin{align*}
\sum_{j=1}^{n} a^U_{ik} x_j + d^+_{ik} &= F^U_k, \quad k = 1, 2, ..., K, \\
\sum_{j=1}^{n} a^L_{ik} x_j + d^-_{ik} &= F^L_k, \quad k = 1, 2, ..., K,
\end{align*}
\]

where, \((d^-, d^-) \geq 0\) with \(d^+ = 0\) and \(d^+ = 0\) represent the under- and over-deviational variables respectively, and they are associated with the respective k-th goal.

Now, from the optimistic point of view of the DM, minimization of the possible regrets associated with the deviations of the goals defined in (8) for goal achievement with the specified target intervals of the goals is taken into account here in the decision making context. In such a case, a regret function with the view to minimizing the unwanted deviational variables of the goals is constructed as the objective function of the GP model. Here, in the GP model formulation, both the \(\text{minsum GP}\) and \(\text{minmax GP}\) aspects of the conventional GP are taken into account as the convex combination of them to reach a satisfactory decision in the decision making environment.

The executable GP model of the problem appears as:

Find \(X(x_1, x_2, ..., x_n)\) so as to

Minimize \(F = \sum_{k=1}^{K} (w^L_k d^L_{ik} + w^U_k d^U_{ik}) + (1 - \lambda) M\)

and satisfy the goal expressions in (8), subject to

\(X \in S, (d^L_{ik} + d^U_{ik}) \leq M, \quad k = 1, 2, ..., K; 0 < \lambda < 1,\)

where \(M = \max_{k=1}^{K} \{d^L_{ik} + d^U_{ik}\}\) and \(w^L_k\) and \(w^U_k \geq 0\) associated with \(d^L_{ik}\) and \(d^U_{ik}\), respectively, designates the numerical weights of importance of achieving the goals, and where

\(\sum_{k=1}^{K} (w^L_k + w^U_k) = 1.\)

Now, the proposed GA scheme can be used to solve the problem in (9).

The fitness function appears as:

\[\text{eval} (C_1) = (F_k)^L = \left[ \lambda \sum_{k=1}^{K} (w^L_k d^L_{ik} + w^U_k d^U_{ik}) \right] + (1 - \lambda) M \]

\(L = 1, 2, ..., \text{pop} \_ \text{size}.\) (10)

Here, the chromosome \(C^*\) with the best fitness value at each generation is determined as

\(C^* = \min \{\text{eval} (C_l) | L = 1, 2, ..., \text{pop} \_ \text{size}\}.\)

To illustrate the potential use of the approach, a numerical example is solved in the Section IV.

IV. An Illustrative Example

A modified version of the stochastic MODM problem with the crisp coefficients of the objective studied in [33] previously is considered to illustrate the potential use of the proposed approach.

The problem with interval coefficients of the objectives and system of chance constraints with certain restriction on the decision variables can be presented as:

Find \(X(x_1, x_2, x_3, x_4)\) so as to

Maximize \(F_1 = [13, 15] x_1 + [10, 12] x_2 + [2, 4] x_3 + [6, 8] x_4\)

and


subject to

\(\Pr [a_{i1} x_1 + a_{i2} x_2 + a_{i3} x_3 + a_{i4} x_4 < b_i] > 0.90,\)

\(\Pr [a_{i1} x_1 + a_{i2} x_2 + a_{i3} x_3 + a_{i4} x_4 < b_i] > 0.95,\)

\(\Pr [a_{i1} x_1 + a_{i2} x_2 + a_{i3} x_3 + a_{i4} x_4 < b_i] > 0.99,\)

\(0 \leq x_j < 3, \quad j = 1, 2, 3, 4.\) (11)

In the decision making environment, let it be assumed that the parameters \(a_{ij}\) and \(b_i\) are independent random variables having the characteristics of lognormal distribution.

Now, it is to be followed that a lognormal random variable is actually the exponential of a normal random variable [34]. Here, following the notion of distribution of random variables, the characteristics of a lognormal random variable can be defined.

Let \(y\) be lognormal random variable with known mean and variance. Then, \(\ln(y)\) would be normally distributed, and
\[ \ln(y) - \mu \]
follows conventional standard normal distribution, where \( \mu \) and \( \sigma^2 \) represents the mean and variance of the normally distributed random variables \( \ln(y) \), and where 'ln' means natural logarithm.

The values of \( \mu \) and \( \sigma^2 \) in terms of the known mean, \( E(y) \) and variance \( \text{Var}(y) \), can be defined as [34]:

\[
\mu = \ln(E(y)) - \frac{1}{2} \sigma^2,
\]

and

\[
\sigma^2 = \ln \left( 1 + \frac{\text{Var}(y)}{E(y)^2} \right).
\]

Now, using the data in Table I and Table II, \( \mu \) and \( \sigma^2 \) can easily be computed by following the expressions in (12).

The equivalent GP formulation of the original problem is obtained as:

\[
\text{Find } X(x_1, x_2, x_3, x_4) \text{ so as to Minimize } Z = \sum_{k=1}^{K} (w_{ik}d_{ik}^* + w_{ik}d_{ik}^*) + (1 - \lambda)M \]

and satisfy the goal expressions in (14), subject to \( (d_{ik}^* + d_{ik}^*) \leq M, \text{ k=1,2} \) and the given constraints set in (11).

Now, for simplicity, introducing the equal weights (0.25) to the goals for achievement of their aspiration levels and taking \( \lambda = 0.5 \), the problem in (15) is solved by employing the proposed GA scheme, the resulting solution of the problem is obtained as:

\[ (x_1, x_2, x_3, x_4) = (3, 3, 1.48, 1.32) \]

with \( F_1 = [79.88, 97.48] \) and \( F_2 = [100.24, 117.84] \).

Note: If the coefficients of the objectives are taken single valued instead of interval-valued, by using the mid-point arithmetic rule in IP [30], then the objectives in (11) successively take the form:

\[
\text{Maximize } Z_1 = 14x_1 + 11x_2 + 3x_3 + 7x_4
\]

and

\[
\text{Maximize } Z_2 = 12x_1 + 16x_2 + 8x_3 + 10x_4
\]

Evaluating the best values of the objectives in the same decision environment, and incorporating them to the program is performed in Dell Power Edge R900 Server with 2 GB RAM.

Now, following the procedure and employing the proposed GA scheme, the individual best (Max) and least (Min) values of the objectives are successively obtained as:

\( (F_1^*, F_1^*) = (88.56, 35), \)

and

\( (F_2^*, F_2^*) = (107.67, 46), \)
respective objectives as the aspiration levels, and introducing under- and over-deviational variables to each of them as made previously, the objective goals are obtained as:

\[
\begin{align*}
14x_1 + 11x_2 + 3x_3 + 7x_4 + d_1^- - d_1^+ &= 88.56 \\
12x_1 + 16x_2 + 8x_3 + 10x_4 + d_2^- - d_2^+ &= 107.67,
\end{align*}
\]

where \(d_i^-, d_i^+ (\geq 0), i = 1, 2\), represent the under- and over-deviational variables, respectively.

Then, the minsum GP model of the problem (for minimization of the unwanted under-deviational variables here) in its conventional form [1] is obtained as:

\[
\text{Minimize } Z = w_1 d_1^- + w_2 d_2^- \\
\text{subject to the given constraints set in (11).}
\]

Taking equal weights \((w_1 = w_2 = 0.5)\) for goal achievement, the solution of the problem in the same decision environment is found as:

\[
(x_1, x_2, x_3, x_4) = (2.93, 2.7, 1.27, 2)
\]

with \(Z_1 = 88.53\) and \(Z_2 = 108.52\). It is to be noted here that the achieved objective values are nearly the mid-point of the values of the objectives obtained previously in the form of the intervals for consideration of interval coefficient there.

Again, in the sequel of making decision, it may be pointed out here that for consideration of different coefficients values within the specified intervals, different achievement of objective values within the defined target intervals can be obtained without involving extra computational load. As a matter of fact, it may be claimed that the proposed formulation of an MODM problem is an efficient one for making decision with inexactness of model parameters in an uncertain decision environment.

V. CONCLUSION

In this article, how the stochastic simulation based GA method can be efficiently used to solve the chance constrained interval valued MODM problems is presented. The main advantage of using the stochastic simulation technique to the chance constraints is that the computational complexity for transforming the constraints to the deterministic equivalent does not arise here.

Further, consideration of interval valued goals in the GP formulation of the problem makes the approach an efficient one from the viewpoint of arriving at a potential solution on the basis of the needs and desires of the DM in the decision making context.

The proposed approach can be extended to different chance constrained MODM problems for making proper decisions in the current complex decision making arena.

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REFERENCES


A genetic algorithm based goal programming method for solving patrol manpower deployment planning problems with interval-valued resource goals in traffic management system: A case study

ABSTRACT
This article demonstrates how the genetic algorithm (GA) method can be used to solve interval-valued goal programming (GP) model of patrol manpower allocation problem to various road-segment areas in different shifting times of Metropolitan cities to deter traffic violations and accidents. In the model formulation of the problem, the goals with target intervals are first converted into the standard goals in GP approach by using interval arithmetic technique. Then, the defined goals are transformed into the conventional form of goals by introducing under- and over-deviational variables to each of them to make a reasonable balance of decision in the deployment planning context. In the achievement function of the executable GP model, both the minsum and minmax aspects of GP are addressed to construct the achievement function for minimizing the possible regret towards achieving the goal values from the optimistic point of view in the decision making environment. A demonstrative example of the city Kolkata, West Bengal, India is solved and the model solution is compared with the solution of conventional GP approach studied previously.

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Abstract—This article demonstrates how the genetic algorithm (GA) method can be used to solve interval-valued goal programming (GP) model of patrol manpower allocation problem to various road-segment areas in different shifting times of metropolitan cities to deter traffic violations and accidents.

In the model formulation of the problem, the goals with target intervals are first converted into the standard goals in GP approach by using interval arithmetic technique. Then, the defined goals are transformed into the conventional form of goals by introducing under- and over-deviational variables to each of them to make a reasonable balance of decision in the deployment planning context.

In the achievement function of the executable GP model, both the minimum and maximin aspects of GP are addressed to construct the achievement function for minimizing the possible regret towards achieving the goal values from the optimistic point of view in the decision making environment.

A demonstrative example of the city Kolkata, West Bengal, India is solved and the model solution is compared with the solution of conventional GP approach [1] studied previously.

Keywords—Genetic algorithm, Goal programming, Interval arithmetic, Interval programming, Patrol manpower deployment.

I. INTRODUCTION

The rapid rate of development of society along with the upgrade of the technological aspects of producing wide variety of road-vehicles for the use in daily life, traffic load is increasing in an alarming rate in the recent years. As a result, accident frequency mainly in the major road-segments in the urban areas is increasing day-by-day through out the world. It has now become a great challenge to the traffic control section of law enforcement department for reducing accidents by enforcing traffic laws. Here, it may be mentioned that although different effective measures like construction of new roads, expansion of existing roads etc. have been taken by most of the metropolitan cities with high traffic density, police department is facing a lot of problems due to proper implementation of traffic enforcement program.

From the mid-'60s to early '70s of the last century, a considerable amount of efforts was devoted to analytical models [2,3,4,5,6] for allocation and deployment of police patrol units in the context of smoothing the traffic flows by enforcing traffic laws. The methodological aspects of the problem studied then was surveyed by Chaiken et. al. [7] in 1972.

During the late 1970s, patrol manpower planning problems were studied in [8,9,10] extensively and widely circulated in the literature. But, most of models developed then are concerned with the geographical patrol manpower allocation decision, and their usefulness is limited.

The first general mathematical model within the framework of goal programming (GP) [11,12] for state patrol deployment problems was introduced by Lee et. al. [13] in 1979. In their approach, the method of enumeration was used, which involves huge initial allocation to reach an optimal allocation decision. Again, in their model, the hidden nonlinearity [14] inherently involved with the deployment of patrol manpower and performance measures were not taken into account.

To overcome the above situation, a gradient search technique within the framework of GP was studied by Taylor et. al. [15] in 1985, where the fractional in nature of deployment policy goals have been taken into consideration. But, the approach presented there was illustrated by a hypothetical case example. The use of GP to patrolmen deployment planning problems was studied by Pal and Basu [16] in the past.

Now, it may be mentioned that the patrol manpower deployment activities are made by the trial and error methods in most of the cases, and they are mostly regional based. But, in most of the real-life decision situations, it is to be observed that the decision parameters are often imprecisely known due to the expert’s ambiguous understanding of the nature of them. As such, crisp definition of them and use of the conventional approaches creates decision trouble in practical multiobjective decision making (MODM) problems.
To overcome the above difficulty, FGP [17] in the framework of conventional GP and as an extension of fuzzy programming (FP) [18] have been studied in [19] in the past, and implemented to different real-world decision making problems [20,21]. The FGP based allocation and deployment of police patrol units has been studied by Pal et al [22] in the recent past.

However, it is to be observed that setting of fuzzy goal values in an imprecise environment may not always be possible in practice in a highly sensitive decision situation in an inexact environment.

To overcome the above situation, interval programming (IP) approach [23] has appeared as a prominent tool for solving multi-objective decision problems with interval parameter values in an inexact environment. The IP approaches have also been studied by Pal et al [24, 25] and implemented to real-life problems [26] in the recent past. However, the methodological extension of IP and its application to practical problem is still at an early stage.

In a multiobjective decision making environment, GAs based on natural selection and population genetics [27], have also appeared as robust computational tools for solving optimization problems [28]. GAs to real-world nonlinear multiobjectiv decision problems have been studied in [29] in the recent past to avoid the computational load involved with the traditional linearization approaches.

The use of IP approach as well as GA based approach to patrol manpower planning problems is at an early stage and yet to be widely circulated in the literature.

In this article, a GP formulation of a patrol unit deployment problem with the various performance criteria in fractional form is considered. In the solution process, a GA scheme is adopted to reach a satisfactory solution for proper patrolmen deployment in the planning context.

An illustrative case example of the metropolitan city Kolkata of India is presented to expound the potential use of the proposed approach.

II. PROBLEM FORMULATION

The two types of interval-valued goals, linear and fractional goals, with interval resource vectors are involved with the problem. Using the mid-point arithmetic in IP [23], the goals can be explicitly presented as follows.

A. Definition of Linear Goals

The linear goals with crisp coefficients and interval-valued resource vector appear as

\[
\text{Optimize } Z_k(X) : a_k X + \alpha_k = [t^L_k, t^U_k], \\
k = 1, 2, ..., k_f
\]

where \( X \in \mathbb{R}^n \) is the vector of decision variables, \( a_k \) and \( \alpha_k \), \( k=1, 2, ..., K_f \) are the vector of crisp coefficients and constants, respectively. \( t^L_k \) and \( t^U_k \), \( k=1, 2, ..., k_f \) represent the lower- and upper-levels of the target intervals and where \( L \) and \( U \) stand for lower- and upper-levels, respectively.

B. Definition of Fractional Goals

The goals in fractional form with crisp coefficients and target intervals can be presented as

\[
\text{Optimize } Z_k(X) : \frac{c_k X + \beta_k}{d_k X + \gamma_k} = [t^L_k, t^U_k], \\
k = k_f + 1, k_f + 2, ..., K
\]

where \( c_k \) and \( d_k \) are the coefficient vectors, and \( \beta_k \) and \( \gamma_k \) are the constants.

It is assume that \( (d_k X + \gamma_k) > 0 \) to avoid any infeasible solution in the decision making environment.

Now, in the decision making context, the interval-valued goals are to be transformed into the conventional goals for GP formulation of the problem.

C. Construction of Flexible Goals

Using interval arithmetic technique [24] and introducing under- and over-deviation variables, the goal expressions in (1) can be explicitly presented as follows.

The goal expression in (1) takes the form:

\[
\sum_{j=1}^{n} a_{kj} x_j + \alpha_k + \eta_{kl} - \eta_{kl} = t^L_k, \\
k=1, 2, ..., k_f
\]

\[
\sum_{j=1}^{n} a_{kj} x_j + \alpha_k + \eta_{kl} - \eta_{kl} = t^U_k, \\
k=1, 2, ..., k_f
\]

Similarly, the fractional goal expression in (2) appears as

\[
\sum_{j=1}^{n} c_{kj} x_j + \beta_k + \eta_{kl} - \eta_{kl} = t^L_k, \\
k=1, 2, ..., k_f
\]

\[
\sum_{j=1}^{n} c_{kj} x_j + \beta_k + \eta_{kl} - \eta_{kl} = t^U_k, \\
k=1, 2, ..., k_f
\]

Now, construction of objective function called the regret function in the GP formulation of the problem for goal achievement is presented in the following Section III.

III. CONSTRUCTION OF REGRET FUNCTION FOR GOAL ACHIEVEMENT

In the decision situation, decision maker's (DM's) objective for achievement of goal values within the specified ranges means minimization of the associated deviation variables to the extent possible in the decision making environment. The development of the regret function for minimization of the deviation variables can be made in different ways [30], and that depends on the DM's needs and desires in the decision making context.

In the present GP formulation, both the aspects of GP, \textit{minsum} and \textit{minmax} GP [11] for minimizing
the maximum of the deviations, are simultaneously taken into account as a convex combination of them to reach a satisfactory solution within the specified target intervals of the objective goals.

Now, from the optimistic point of view of the DM, minimization of the possible regrets on the basis of the priorities for goal achievement is taken into consideration.

The GP model of the problem under a preemptive priority structure can be presented as:

Find \( X() \) so as to

\[
\text{Minimize } Z = \sum_{r=1}^{R} \left( \sum_{i=1}^{K} \left( \frac{1}{P_r} \sum_{j=1}^{m} x_{ij} \right) \right) \]

and satisfy the expression in (3) - (6) such that

\[
\eta_{i1}^k + \eta_{i2}^k \leq V, \quad k=1,2,...,K
\]

where \( V = \max(\eta_{i1}^k + \eta_{i2}^k), \quad k=1,2,...,K \).

where the expression \( P_r(\eta) \) in the priority achievement function \( Z \) is of the form:

\[
P_r(\eta) = \sum_{k=1}^{K} w_k (\eta_{i1}^k + \eta_{i2}^k) + (1-\lambda)V_r,
\]

and \( V_r = \max(\eta_{i1}^k + \eta_{i2}^k), \quad 0 < \lambda < 1, \)

and where \( w_k (> 0) \) represents the numerical weights of importance of achieving the goals at the \( r \)-th priority level.

It may be noted here that the priority factors have the following relationship:

\[ P_1 > P_2 > ... > P_r > ... > P_R, \]

which means that the goal achievement under the priority factor \( P_r \) is preferred most to the next priority factor \( P_{r+1} \), \( r = 1,2,...,R-1 \).

Now, the different types of interval parameters and variables involved with the problem are defined in the Section IV.

IV. PARAMETERS AND VARIABLES

A. Definition of Interval Parameters

\([E^L, E^U]\) = The specified interval in which the total number of patrol units is required for deployment to the various road-segment areas in all the shifts of the time period.

\([F^L, F^U]\) = The specified interval in which the total number of patrol units required in the road-segment area \( i \) in the period.

\([A^L, A^U]\) = The specified interval in which the total number of patrol units required during the shift \( j \) in the period.

\([n^L, n^U]\) = Target interval of desired accident reduction at the road-segment area \( i \) in the period.

\([PC^L, PC^U]\) = Target interval of total physical contacts in all the road segment areas.

\([sp^L, sp^U]\) = Target interval of physical contacts at the road segment area \( i \) in the period.

\([SC^L, SC^U]\) = Target interval of the total sight contacts in all the road segment areas.

\([s^L, s^U]\) = Target interval of sight contacts at the road segment area \( i \) in the period.

\([CE^L, CE^U]\) = Target interval of total cash expenditure for deployment of the patrol units in all the road segment areas during all the shifts in the period.

B. Crisp Coefficients

\( C_k \) = Estimated cost for deployment of a patrol unit to the road-segment area \( i \) during the shift \( j \).

C. Definition of Decision Variables

1) Independent variable: \( x_{ij} \) = Deployment of patrol units to the road-segment area \( i \) during the shift \( j \).

2) Dependent variables: \( PC_i \) = Number of physical contacts made by a patrolman in the road-segment area \( i \) during shift \( j \).

\( SC_i \) = Number of sight contacts made by a patrolman in the road-segment area \( i \) during shift \( j \).

\( AR_i \) = Number of accident rate contributed by the road-segment area \( i \) during the shift \( j \).

Now, for the defined interval parameters and variables, the algebraic structures of the interval-valued goals are described in the following Section V.

V. DESCRIPTION OF INTERVAL-VALUED GOALS

A. Patrol manpower requirement goal

1) Total patrolmen allocation: A certain level of patrol manpower should be provided to deploy them to the road-segment areas during different shifts in the period.

The goal expression appears as

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = [E^L, E^U]
\]

2) Segment-wise patrolmen allocation: Depending on the accident frequency and traffic density, a minimum number of patrol units to be deployed to each of the road-segment areas in all the shifts.

The goal expression takes the form

\[
\sum_{j=1}^{n} x_{ij} = [F^L, F^U], \quad i = 1,2,...,m
\]

3) Shift-wise patrolmen allocation: For smoothing the traffic flow in all the road segment areas, a certain number of patrol units need be provided in each shift to all the road-segment areas.

The goal expression appears as
B. Deployment performance goal

Here, it is to be observed that the reduction of accident rate as well as the other performance measures increases with the increase of patrol units to the road-segment areas. But, increasing of them beyond a certain level do not show any additional effect and that depends on the traffic density and accident frequency of a road-segment-area.

In context to the above, a general mathematical egression for relationship with x_i and each of the PC_i, SC_i and AR_i in fractional form appears in connection to measuring the performance of different activities.

Now, following the expression (8), the expression for PC_i takes the form:

\[ PC_i = P_i \cdot \frac{P_j}{x_j}, \quad i = 1,2,\ldots, m; \quad j = 1,2,\ldots, n \]

(9)

where \( P_i \) and \( P_j \) are the estimated parameters.

Here, it is to be followed that the number of physical contacts increases with the increase of x_i, but at a decreasing rate.

Then, the goal expressions can be defined as follows:

(i) Segment-wise physical-contacts

Since the physical-contact operations are made in terms of the number contacts made on the basis of traffic density, segment-wise physical-contact goal expression appears as:

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_i \cdot \frac{P_j}{x_j} \right) = \left[ P_i \cdot P_j \right], \quad i = 1,2,\ldots, m \]

(ii) Total physical-contacts

To measure the overall performance against deployment of patrol units, the goal expression for total physical-contact takes the form:

\[ \sum_{i=1}^{m} \sum_{j=1}^{n} \left( P_i \cdot \frac{P_j}{x_j} \right) = \left( PC_i \cdot PC_j \right) \]

2) Sight-contact goals: Beyond physical-contact, sight-contact should be an integral and vital part to smoothing the vehicle speed, preventing against violation of general traffic rules, etc., during crossing of a road-segment areas.

The potential use of the expression in (8) in different phases of traffic operation activities is presented as follows:

1) Physical-contact goals: Physical-contact indicates the direct contact to the vehicles for spot-check of driving license, car registration, road tax clearance etc.

An instance of physical-contact is presented in the Fig 2.
Now, similar to the physical-contact, the expression of $SC_i$ takes the form:

$$SC_y = S_y - \frac{x_y}{x_y}, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n$$

(10)

where $S_y$ and $x_y (S_y > x_y)$ are estimated parameters.

The goal expressions can be defined as follows:

(i) Segment-wise sight-contact goals

For smoothing the traffic operations, the segment-wise sight-contact goal appears as:

$$\sum_{j=1}^{n} \left( S_y - \frac{x_y}{x_y} \right) = \left[ s_i^L, s_i^U \right], \quad i = 1, 2, ..., m$$

(ii) Total sight-contact goal

Similar to the total physical-contact goal, the goal expression here also appears as:

$$\sum_{i=1}^{m} \left( S_y - \frac{x_y}{x_y} \right) = \left[ SC_L, SC_U \right]$$

3) Accident reduction goals: The primary job of the deployed patrolmen is to reduce the rate of accident.

An instance a patrolman on-duty is presented in the Fig 4.

Fig 4: A patrolman on-duty

The expression of accident rate reduction, which increases with the increase of patrol units, appears as:

$$AR_y = R_y - \frac{r_y}{x_y}, \quad i = 1, 2, ..., m; \quad j = 1, 2, ..., n$$

(11)

where $R_y$ and $x_y (R_y > r_y)$ are the estimated parameters associated with $AR_y$ and $x_y$.

(i) Segment-wise accident reduction goals

The reduction of accident rate at a particular road-segment in all the shifts appears as:

$$\sum_{j=1}^{n} \left( R_y - \frac{x_y}{x_y} \right) = \left[ r_i^L, r_i^U \right], \quad i = 1, 2, ..., m$$

(ii) Total accident reduction goal

The goal expression for reduction of overall accident rate for all the road-segments areas during all the shifts of the period takes the form:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \left( R_y - \frac{x_y}{x_y} \right) = \left[ AR^L, AR^U \right]$$

C. Cash expenditure goal

An estimated amount of money should have to be provided for incurring different costs for patrolling to the road-segment areas in different shifts of the period.

The goal expression appears as:

$$\sum_{i=1}^{m} C_y x_y = \left[ CE_L, CE_U \right]$$

Now, it is to be observed that the goal expressions which are involved directly to the activities of traffic operations are fractional in nature. In the conventional single-objective optimization, the traditional linear approximation approach is used to solve the problems. But, in case of multiobjective optimization, the use of such an approach leads to a dissatisfactory solution due to the inherent approximation error as well as incommensurable and often conflict in nature of the objective goals.

To overcome the difficulty associated with linear approximation approach, the GA approach as a goal satisfier [28] for multiobjective decision making is introduced here in the solution search process.

The proposed GA method is presented in the following Section VI.

VI. GA FOR GP MODEL

The basic steps of the GA procedure with the core functions adopted in the solution process are presented via the following steps.

Step 1. Representation and initialization

Let $E$ denote the binary coded representation of chromosome in a population as $E = \{x_1, x_2, ..., x_n \}$. The population size is defined by $pop\_size$, and chromosomes are randomly initialized in the search domain.

Step 2. Fitness function

The fitness value of each chromosome is estimated by the value of an objective function. The fitness function is defined as:

$$eval(E_v) = (Z_v) = \frac{1}{Z_v}$$

(12)

where $Z_v$ is renamed for the goal achieving function $Z$ in (7) to represent it at the $r$-th priority level, and where the subscript $v$ refers to the fitness value of the selected $v$-th chromosome, $v = 1, 2, ..., pop\_size$.

The best chromosome with largest fitness value at each generation is determined as $E^* = \{\min\{eval(E_v) \mid v = 1, 2, ..., pop\_size\}$, which depends on searching of the best value of an objective.
Step 3. Selection

The simple roulette-wheel scheme [28] is used for selecting two parents for mating purposes in the genetic search process.

Step 4. Crossover

The parameter Pc is defined as the probability of crossover. The arithmetic crossover operation (single-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the linear constraints set to S (and S). Here a chromosome is selected as a parent, if for a defined random number r ∈ [0, 1], r < Pc is satisfied.

Here, arithmetic crossover for two parents E1, E2 ∈ S is defined as X1 = a1E1 + a2E2 and X2 = a2E1 + a1E2, for producing two offspring X1 and X2, where a1, a2 ≥ 0 with a1 + a2 = 1 always belong to S, and where S is a convex set.

Step 5. Mutation

As in the conventional GA scheme, a parameter Pm of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a defined random number r ∈ [0, 1], a chromosome is selected as a parent, if for a defined random number r ∈ [0, 1], r < Pm is satisfied.

Now, regarding traffic operations, it is to be followed that there is no additional effect for deployment of more than 5 patrolmen in any road-segment area during any shift.

Now, following the records, the different target intervals as well as the crisp coefficients are presented in the Tables I-V.

Now, following the records, the different target intervals as well as the crisp coefficients are presented in the Tables I-V.

<table>
<thead>
<tr>
<th>TABLE I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource Utilization levels</td>
</tr>
<tr>
<td>[50, 200]</td>
</tr>
<tr>
<td>[2150, 2300]</td>
</tr>
<tr>
<td>[670, 790]</td>
</tr>
<tr>
<td>[2130, 2230]</td>
</tr>
<tr>
<td>[2, 9, 3.04]</td>
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</tbody>
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<table>
<thead>
<tr>
<th>TABLE II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Description of Segment-Wise Resource Utilization</td>
</tr>
<tr>
<td>Segment area (i)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>[9, 12]</td>
</tr>
<tr>
<td>[8, 10]</td>
</tr>
<tr>
<td>[7, 10]</td>
</tr>
<tr>
<td>[1, 1.15]</td>
</tr>
<tr>
<td>[1, 1.02]</td>
</tr>
<tr>
<td>[0, 0.87]</td>
</tr>
<tr>
<td>[250, 280]</td>
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<tr>
<td>[200, 240]</td>
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<tr>
<td>[750, 790]</td>
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<tr>
<td>[710, 770]</td>
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<tr>
<td>[640, 670]</td>
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</tbody>
</table>

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<tr>
<th>TABLE III</th>
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<tbody>
<tr>
<td>Data Description of Shift-Wise Resource Utilization</td>
</tr>
<tr>
<td>Shift (j)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
</tr>
<tr>
<td>[8, 10]</td>
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<td>[10, 12]</td>
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<tr>
<td>[6, 8]</td>
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<tr>
<td>[7, 9]</td>
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<table>
<thead>
<tr>
<th>TABLE IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data Description of Cg (Cost in Rs.) for Deployment of Patrol Unit</td>
</tr>
<tr>
<td>Road segment area (i)</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>75</td>
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<tr>
<td>80</td>
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<tr>
<td>70</td>
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<td>75</td>
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<tr>
<td>80</td>
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<tr>
<td>70</td>
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<tr>
<td>75</td>
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</tbody>
</table>

TABLE V

<table>
<thead>
<tr>
<th>Parameters Value for ARi, PCi, SCi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Road segment area (i)</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
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<tr>
<td>4</td>
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</table>
| Shif...
Now, using the data presented in the Table I - V, and following the procedure the goals of model for the defined interval-valued goals are constructed in the following Section A.

A. Construction of model goals

1) Patrolmen allocation goals:
(i) Total patrolmen goals
\[ \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{il} - \eta_{il} = 35, \quad \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 50 \]

(ii) Segment-wise patrolmen allocation goals
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 9, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 12 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 8, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 10 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 7, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 8 \]

(iii) Shift-wise patrolmen allocation goals
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 8, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 10 \]
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 10, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 12 \]
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 6, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 8 \]
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 7, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 9 \]

2) Accident reduction goals:
(i) Segment-wise accident reduction goals
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 1.1, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 1.15 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 1, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 1.02 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 2.9, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 3.04 \]

(ii) Total accident reduction goals
\[ \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{il} - \eta_{il} = 250, \quad \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 280 \]
\[ \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{il} - \eta_{il} = 220, \quad \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 270 \]
\[ \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{il} - \eta_{il} = 200, \quad \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 240 \]

3) Physical-contact goals:
(i) Segment-wise physical-contact goals
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 670, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 790 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 750, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 790 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 730, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 770 \]
\[ \sum_{j=1}^{4} \xi_{ij} + \eta_{il} - \eta_{il} = 650, \quad \sum_{j=1}^{4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 670 \]

(ii) Total physical-contact goals
\[ \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{il} - \eta_{il} = 2130, \quad \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 2230 \]

4) Sight-contact goals:
(i) Segment-wise sight-contact goals
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 750, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 790 \]
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 730, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 770 \]
\[ \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{il} - \eta_{il} = 650, \quad \sum_{j=1}^{4} \sum_{i=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 670 \]

(ii) Total sight-contact goals
\[ \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{il} - \eta_{il} = 2130, \quad \sum_{i=1}^{4} \sum_{j=4} \xi_{ij} + \eta_{iu} - \eta_{iu} = 2230 \]

5) Cash expenditure goals:
\[ \sum_{k=1}^{4} \sum_{j=4} (75x_{ij} + 80x_{ij} + 70x_{ij}) + \eta_{il} - \eta_{il} = 2130, \]
\[ \sum_{k=1}^{4} \sum_{j=4} (75x_{ij} + 80x_{ij} + 70x_{ij}) + \eta_{il} - \eta_{il} = 2300 \]

Now, for the stated goals of the problems and the decision restriction given in (13), and following the expression in (7), the executable GP model under the three assigned priorities is obtained as:

Find \( \{x_{ij} \} \) \( i=1,2,3; j=1,2,3,4 \) so as to

Minimize
\[ Z = P_{1} [\lambda \sum_{k=1}^{4} w_{k}(\eta_{il} + \eta_{il}) + (1-\lambda) V_{l}] + P_{2} [\lambda \sum_{k=1}^{4} w_{k}(\eta_{il} + \eta_{il}) + (1-\lambda) V_{l}] + P_{3} [\lambda \sum_{k=1}^{4} w_{k}(\eta_{il} + \eta_{il}) + (1-\lambda) V_{l}] \]
and satisfy the defined goal expressions and the variable restriction in (13), subject to $\eta_{kl}^r + \eta_{kl}^L \leq V_i$, $k = 9, 10, 11, 12$ and $k = 13, 14, \ldots, 20$, $\eta_{kl}^r + \eta_{kl}^L \leq V_i$, for $k = 21, 22, \ldots, 30$, with $\gamma_{kl} \geq 0$, $\forall k = 1, 2, \ldots, 21$.

Now, for simplicity, equal weights are introduced to the goals for achievement of their aspired levels at the different priority levels. Actually, the weight structure $(w_{w_1}, w_{w_2}, w_{w_3}) = (1/4, 1/16, 1)$ to the goals at the different priority levels and $? = 0.5$ is taken into account in the decision making situation.

Then, the GA method presented in the Section VI is employed to solve the problem. The achievement function in (14) here appears as the fitness function in the solution search process.

In the genetic search process, the following parameter values are introduced:
- Probability of crossover $P_c = 0.8$
- Probability of mutation $P_m = 0.08$
- Population size = 100
- Chromosome length = 30

The GA based program is designed in Programming Language C. The execution is done in a Intel Pentium IV with 2.66 GHz. Clock-pulse and 1GB RAM.

The model solution of the problem under the three priority factors are presented in the Table VI.

<table>
<thead>
<tr>
<th>TABLE VI</th>
<th>Patrolmen Allocation Under the Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift (j)</td>
<td>6 a.m. to 10 a.m.</td>
</tr>
<tr>
<td>B. T Road</td>
<td>2</td>
</tr>
<tr>
<td>A. J. C Road</td>
<td>2</td>
</tr>
<tr>
<td>EM By-pass</td>
<td>2</td>
</tr>
</tbody>
</table>

Note: If the conventional minsum GP approach in [1] is used to the problem under the same priority structure, where minimization of the sum of the weighted deviational variables are considered, then without linearizing the fractional goals and using the LINGO (var.6.0) the obtained solution is presented in the Table VII.

<table>
<thead>
<tr>
<th>TABLE VII</th>
<th>Patrolmen Allocation Under the minsum GP Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shift (j)</td>
<td>6 a.m. to 10 a.m.</td>
</tr>
</tbody>
</table>

The graphical representations of the model solutions under the GA based interval-valued GP and conventional minsum GP approach for patrolmen deployment to the three road segment areas in four different shifts are displayed in the Fig 5.

**REFERENCES**


A genetic algorithm based hybrid goal programming approach to land allocation problem for optimal cropping plan in agricultural system

ABSTRACT
This paper presents how the fuzzy goal programming (FGP) and interval valued goal programming (IVGP) can be efficiently used for modelling and solving land allocation problems for optimal production of seasonal crops in agricultural system. In the proposed approach, utilization of cultivable land, production of crops and target level of profit are fuzzily described. The supply and utilization of productive resources are considered interval valued to reach a satisfactory decision in the decision making environment. In the solution process, a genetic algorithm (GA) scheme is employed for achievement of the different goals on the basis of assigned weights of importance in the decision making situation. The potential use of the approach is demonstrated by a case example of the Nadia District, West Bengal (W. B.), INDIA.

INDEX TERMS
Index Terms are available to subscribers and IEEE members.
A Genetic Algorithm Based Hybrid Goal Programming Approach to Land Allocation Problem for Optimal Cropping Plan in Agricultural System

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Abstract- This paper presents how the fuzzy goal programming (FGP) and interval valued goal programming (IVGP) can be efficiently used for modelling and solving land allocation problems for optimal production of seasonal crops in agricultural system.

In the proposed approach, utilization of cultivable land, production of crops and target level of profit are fuzzily described. The supply and utilization of productive resources are considered interval valued to reach a satisfactory decision in the decision making environment.

In the solution process, a genetic algorithm (GA) scheme is employed for achievement of the different goals on the basis of assigned weights of importance in the decision making situation.

The potential use of the approach is demonstrated by a case example of the Nadia District, West Bengal (W. B.), INDIA.

Keywords- Cropping Plan, Fuzzy Goal Programming, Goal Programming, Genetic Algorithm, Hybrid Goal Programming.

I. INTRODUCTION

The increase of health awareness in the technological affluent society along with the alarming rate of increase of human population in the past few years, worldwide alertness for sustainable growth of agricultural products to meet the primary need food has taken place in the recent years.

In an agricultural planning situation, optimal production of crops highly depends on the proper allocation of land in different seasons for cultivating the crops and adequate supply of productive resources to the planning unit.

From the mid-'60s to '70s of the last century, the extensive study on the mathematical programming (MP) models to agricultural planning problems was surveyed by Nix [12] in 1979.

Since, the agricultural planning problems involve multiplicity of objectives, goal programming (GP) [18] as a prominent tool for multiobjective decision analysis has been widely used to farm management problems. The deep study in this area has been surveyed by Glen [4] in the past.

The use of GP to farm planning has also been studied by Pal and Basu [13] in the past.

Now, in a practical decision situation, assigning of certain target values to the objectives become fuzzy due to imprecise nature of human judgments.

In such a case, fuzzy programming (FP) approach [20] as well as FGP [14] approach to crops production planning has been studied in [2, 15, 17] in the past.

Although, FGP has been successfully implemented to real-life problems, the difficulty of assigning imprecise aspiration levels to the goals sometime arises in a highly sensitive inexact decision environment. To overcome the situation, IVGP approach [10], where the parameters are considered in interval forms to decision problems, has been considered. But the use of such an approach to practical decision problems is at an early stage.

Now, in the real-world decision making environment, GAs [3, 5] based on the natural selection and population genetics have also appeared as robust computational tools for solving optimization problems.

The use of GAs to real-life problems in the framework of FGP has also been studied by Pal et al. [16] in the past. But, exploration of the potential use of GAs to multiobjective decision making (MODM) problems is yet to be widely circulated in the literature.

Again, the use of an GA method to GP formulation having both the fuzzy and interval valued goals is yet to circulate in the literature.

In this article, a GP formulation called hybrid GP (HGP) with incorporation of both the fuzzy and interval valued goals of a cropping plan system is introduced. In the solution process, an GA scheme as a goal satisficer [3] rather than optimizer is adopted to reach a satisfactory decision for optimal production of crops. Now, the general model formulation of the problem is presented in the Section II.
II. PROBLEM FORMULATION

The general format of an MODM problem having fuzzy and interval valued goals can be stated as:

Find \( X \) so as to

\[
\text{satisfy } Z_k(X) = \frac{g_k}{g_k} \leq g_k, \quad k \in K_1
\]

\[
Z_k(X) = [l_k^*, u_k^*], \quad k \in K_2
\]

subject to

\[
X \in S(X \in \mathbb{R}^n | f(X) \geq b, X \geq 0, b \in \mathbb{R}^m)
\]

where, \( X \) is the vector of decision variables, \( Z_k(X), k \in K_1 \) represents the k-th fuzzy goal with the aspiration level \( g_k \), \( a_k \) is the vector of crisp coefficients and \( c_k \) is a constant, respectively. Also it is assumed that \( l_k^* \) and \( u_k^* \) are the lower- and upper-bounds of the target interval of the k-th objective \( Z_k(X), k \in K_2 \), where \( L \) and \( U \) stand for lower- and upper-bounds, respectively, \( f(X) \) is a function (linear/nonlinear) representing the constraints set, \( b \) is the right-hand-side vector of resources of the system constraints. It is assumed that the feasible region \( S(X) \) is bounded, and \( K_1 \cup K_2 = \{1,2,...,K\} \) with \( k_1 \cap k_2 = \emptyset \).

Now, in the model formulation of the problem, both types of goals are to be transformed into the standard goals. Here, the fuzzy goals in (1) are first characterized by their membership functions in [20] to measure the degree of achievement of the goals. Then, they are transformed into fuzzy goals [16] for achievement of the highest membership value (unity) to the extent possible.

Again, the interval valued goals in (2) are to be transformed into the conventional goals in GP for achievement of the goal values within the specified ranges by minimizing the regrets associated with them.

A. Construction of Membership Goals of Fuzzy Goals

The membership goal expression of the membership function \( \mu_k(X) \) defined for the fuzzy goal \( Z_k(X) \geq g_k \) appears as:

\[
\mu_k(X) = \frac{Z_k(X) - g_k}{g_k} + d_k^* - d_k^* = l, \quad k \in K_1
\]

where, \( g_k \) and \( g_k - g_k \) represent the lower tolerance limit and tolerance range, respectively, for achievement of the associated k-th fuzzy goal. Also, \( d_k^* \geq 0 \) and \( d_k^* \geq 0 \) are the under- and over-deviational variables, respectively, of the k-th membership goal \( \mu_k(X) \).

Similarly, the membership goal expression for the fuzzy goal \( Z_k(X) \leq g_k \), takes the form:

\[
\mu_k(X) = \frac{g_k - Z_k(X)}{g_k - g_k} + d_k^* - d_k^* = l, \quad k \in K_1
\]

B. Construction of Conventional Goal of the Interval Valued Goal

Using the mid-point rule in interval programming (IP) approach [10] and introducing under- and over-deviational variables, the two goal expressions from the expressions in (2) can be explicitly obtained as

\[
\frac{\sum_{j=1}^{n} a_{kj} x_j + a_k + d_k^L - d_k^U}{\sum_{j=1}^{n} a_{kj} x_j + a_k + d_k^L - d_k^U} = l_k^*, \quad k \in K_2
\]

where, \( (d_k^L, d_k^U), (d_k^L, d_k^U) \geq 0 \) represent lower- and upper-deviational variables associated with the respective goals.

Now, construction of the HGP model for goal achievement on the basis of needs and desires of the decision maker (DM) in the decision making context is presented in the Section III.

III. HGP MODEL FORMULATION

In the decision situation, since the DM’s objective is to achieve the fuzzy goal values by achieving certain goal levels of interval valued goals within the specified ranges, consideration of both types of goals regarding their achievement in the achievement function of the GP model is called the hybridization of the GP model.

Now, from the optimistic point of view of the DM, \( \min\text{sum} \) GP approach [16] to fuzzy goals as well as \( \min\text{sum} \) and \( \min\text{max} \) GP approaches [18] as a convex combination of them to the interval valued goals are simultaneously taken into account to reach a satisfactory decision by minimizing the possible regrets on the basis of the weights of importance of achieving the goals in the decision situation.

The executable HGP model can be presented as:

Find \( X(x_1, x_2, ..., x_n) \) so as to

Minimize \( Z = \sum_{k=1}^{K} w_k d_k^{L - d_k^U} \lambda k \in K_1 + l - \lambda) V \)

and satisfy the goal expressions in (4)-(6), subject to \( d_k^{L - d_k^U} \leq V, k = K_1 + l, K_2 + 2, ..., K \)

where, \( V = \max_{k \in K_2} \) represents the goal achievement function, \( w_k^{L - d_k^U} \geq 0, \) \( k = 1,2,...,K_1 \), represent the numerical weights associated with the respective under-deviational variables, and where \( w_k^{L - d_k^U} (k=1,2,...,K_1) \) are determined as [16].

\[
w_k^{L - d_k^U} = \frac{1}{g_k - g_k}, \quad \text{for the } \mu_k \text{ defined in (4)
}
\]

\[
w_k^{L - d_k^U} = \frac{1}{g_k - g_k}, \quad \text{for the } \mu_k \text{ defined in (5)
}
\]
Again, \( w_{kj}, w_{kj}(>0) \) with \( \sum_{k=K_j+1}^{K} (w_{kj} + w_{kj}) = 1 \), represent the numerical weights associated with the respective deviational variables.

Now, in a MODM situation, it is to be observed that the goal objectives often conflict each other for achieving their goal levels in the decision environment. Further, when nonlinearity in an objective or in a system constraint is involved, the computational complexity in [9] arises in the decision process. Here, the use of conventional approximation approach [19] involves inherent error and increases computational load.

To overcome the above difficulty, an GA scheme is introduced here to make a proper cropping plan in the decision situation. The core functions adopted in the GA scheme is presented in the following Section A.

A. GA for HGP Model

In the literature of the GAs, there are a number of schemes [5] for generation of new populations with the use of the different operators: selection, crossover and mutation. Here, the binary coded representation of a candidate solution called chromosome is considered to perform genetic operations in the search process. The conventional Roulette wheel selection scheme in [5], single-point crossover [11] and bit-by-bit mutation operations are adopted to generate offspring in new population in the search domain defined in the decision making environment. The fitness score of a chromosome \( v \) (say) in evaluating a function, say, \( \text{eval}(v) \), is based on maximization or minimization of an objective function defined on the basis of DM’s needs and desires in the decision making context.

Now, the HGP model formulation of the proposed problem is presented in the following Section A.

IV. HGP MODEl OF THE PROBLEM

A. Definition of Decision Variables and Parameters

- Decision variables:

\[ x_{cs} = \text{Allocation of land for cultivating the crop } c \text{ during the season } s, \quad c = 1, 2, ..., C; \quad s = 1, 2, ..., S. \]

- Parameters:

1) Fuzzy goal levels:

\[ \text{T}_{Lcs} = \text{Total area of land (in hectares (ha)) currently in use for cultivating the crop } c \text{ during the season } s. \]

\[ \text{AP}_{c} = \text{Annual production level (in qtls) of the crop } c. \]

\[ \text{EM} = \text{Estimated total amount of money (in Rupees (Rs.)) required per annum for supply of the productive resources.} \]

\[ \text{EMV} = \text{Estimated total market value (in Rs.) of all the crops yield during the plan period.} \]

2) Crisp coefficients:

\[ \text{MH}_{cs} = \text{Average machine-hours (in hrs.) required for tillage per ha of land for cultivating the crop } c \text{ during the season } s. \]

\[ \text{MD}_{cs} = \text{Man-days (in days) required per ha of land for cultivating the crop } c \text{ during the season } s. \]

\[ \text{WS}_{cs} = \text{Water supply (in inches) per ha of land for cultivating the crop } c \text{ during the season } s. \]

\[ \text{EP}_{cs} = \text{Estimated production of the crop } c \text{ per ha of land cultivated during the season } s. \]

\[ CP = \text{Average cost for the purchase of fertilizers, seeds and other different farm related materials per ha of land cultivated for the crop } c \text{ during the season } s. \]

\[ \text{MP}_{cs} = \text{Market price (Rs. / qtl.) at the time of harvesting the crop } c \text{ cultivated during the season } s. \]

3) Crisp target levels:

\[ \text{PR}_{ij} = \text{The ratio of annual production of the } i \text{-th and } j \text{-th crop, } \quad (i, j = 1, 2, ..., C; \quad i \neq j). \]

\[ \text{PR}_{ij} = \text{The ratio of annual profits obtained from the } i \text{-th and the } j \text{-th crops, } \quad (i, j = 1, 2, ..., C; \quad i \neq j). \]

4) Interval valued production resources:

\[ \text{[MP}_{i}^{L}, \text{MP}_{i}^{U}] = \text{Target interval specified for total machine-hours (in hours (hrs)) required during the season } s. \]

\[ \text{[MP}_{i}^{L}, \text{MP}_{i}^{U}] = \text{Target interval specified for total man-days (in days) required during the season } s. \]

\[ \text{[WS}_{i}, \text{WS}_{i}^{U}] = \text{Target interval specified for total water supply (in inch) required during the season } s. \]

B. Description of Goals and Constraints

1) Fuzzy goals :

(i) Land utilization goals

The fuzzy goal expression for utilization of total cultivable land appears as:

\[ \sum_{c=1}^{C} x_{cs} \leq \text{T}_{Lcs}, \quad s = 1, 2, ..., S. \]

(ii) Production achievement goal

To meet the increasing demand of agricultural products in society, the fuzzy production achievement goal for each crop cultivated in different season appears as:

\[ \sum_{c=1}^{C} \text{EP}_{cs} x_{cs} \geq \text{AP}_{c}, \quad c = 1, 2, ..., C. \]

(iii) Cash requirement goal

An estimated amount of money (in Rs.) is essentially involved for acquiring the productive resources.

The fuzzy goal takes the form

\[ \sum_{c=1}^{C} \sum_{s=1}^{S} \text{CP}_{cs} x_{cs} \leq \text{EMV} \]

(iv) Profit achievement goal

A minimum level of profit from the farm is highly expected by the farm manager.

The fuzzy profit goal appears as

\[ \sum_{c=1}^{C} \sum_{s=1}^{S} (\text{MP}_{cs} - CP_{cs}) x_{cs} \geq \text{EMV} \]

2) Interval valued productive resource goals :

(i) Machine-hour goal

An estimate number of machine-hours (in hrs.) within an interval are required to till the land in the season.

The interval valued resource goal appear as
\[
\sum_{c=1}^{C} MH_{c},x_{c} = [MH_{c}^{1}, MH_{c}^{2}], \quad s = 1,2, ..., S.
\]

(ii) Manpower goal
To avoid the uncertainty of labourers and involvement of extra money for hiring them at the peak time, a certain number of labourers in an estimated interval should be employed during the period.

The interval valued goal appear as
\[
\sum_{c=1}^{C} MD_{c},x_{c} = [MD_{c}^{1}, MD_{c}^{2}], \quad s = 1,2, ..., S.
\]

(iii) Water supply goal
To meet the target levels of production of all the seasonal crops, water supply within a certain target interval must be provided during any season s.

The interval valued water supply goal appears as
\[
\sum_{c=1}^{C} WS_{c},x_{c} = [WS_{c}^{1}, WS_{c}^{2}], \quad s = 1,2, ..., S.
\]

3) Crisp constraints:
(i) Production ratio constraint
It is to be mentioned that the few major crops serve almost the same purpose in terms of their consumption. So certain ratios between the total productions of two major crops sowed have to be maintained.

The ratio constraints appear as
\[
\sum_{c=1}^{C} E_{c},x_{c} = [E_{c}^{1}, E_{c}^{2}], \quad i, j = 1,2, ..., C \text{ and } i \neq j.
\]

(ii) Profit ratio constraint
From the socio-economic point of view, beyond the meeting of demand of food grains in society, the attention for cultivation of the profitable crops need be paid in the planning horizon. Here, certain ratios of crops production are to be maintained from the view point of making profit from the farm. The profit ratio constraint appear as
\[
\sum_{c=1}^{C} (MP_{c},EP_{c} - CP_{c}) x_{c}/\sum_{c=1}^{C} (MP_{c},EP_{c} - CP_{c}) x_{c} = PR_{ij};
\]
\[i, j = 1,2, ..., C \text{ and } i \neq j\]

Now, the executable HGP model formulation of the problem and use of the proposed GA scheme is demonstrated via the following case example presented in the Section V.

V. A CASE STUDY OF THE NADIA DISTRICT, W.B. (INDIA)

The data of the planning year 2005-2006 were collected from different agricultural planning units. The sources are: District Statistical Hand Book, Nadia, 2005-2006 [8]; Action Plan Records (2005-2006 and 2004-2005) [6]; Soil Testing and Fertilizer Recommendation [1]; The Nadia Gramin Bank; Department of Agri-Irrigation [7].

Now, the three seasonal crop-cycles: Pre-kharif, Kharif and Rabi successively appear in W.B. during a planning year, and they are used to designate the time periods for crops production during summer, rainy and winter seasons, respectively, in a year.

The decision variables and different types of data involved with the problem are summarized in the following TABLES I-IV.
Now, using the data of Tables I-IV, the membership goals of the defined fuzzy goals and the conventional goals of the defined interval valued goals can be constructed by using the expressions in (4), (5) and (6), respectively.

Here, it is to be noted that the three consecutive seasons are required for yielding the crop sugarcane and all the other crops are single-season based.

Now, the goals are described as follows.

A. Membership goals of fuzzy goals:

1) Land utilization goals

\[
\begin{align*}
\mu_1: 8.32 - 0.027 (x_{11} + x_{12} + x_{13}) + d_i^1 - d_i^1 = 1 \quad \text{(Pre-kharif)} \\
\mu_2: 8.32 - 0.027 (x_{23} + x_{24}) + d_i^2 - d_i^2 = 1 \quad \text{(Kharif)} \\
\mu_3: 8.32 - 0.027 (x_{31} + x_{32} + x_{33} + x_{34}) + d_i^3 - d_i^3 = 1 \quad \text{(Rabi)}
\end{align*}
\]

(9)

2) Production achievement goals

\[
\begin{align*}
\mu_4: 0.03 x_{11} + 0.04 x_{12} + 0.05 x_{13} - 10.83 + d_i^* - d_i^* = 1 \quad \text{(Rice)} \\
\mu_5: 0.03 x_{21} - 18.74 + d_i^* - d_i^* = 0 \quad \text{(Jute)} \\
\mu_6: 0.03 x_{31} - 5.94 + d_i^* - d_i^* = 1 \quad \text{(Sugarcane)} \\
\mu_7: 0.03 x_{41} - 3.13 + d_i^* - d_i^* = 1 \quad \text{(Potato)} \\
\mu_8: 0.022 x_{53} - 9.99 + d_i^* - d_i^* = 0 \quad \text{(Mustard)}\
\mu_9: 0.03 x_{61} - 9.99 + d_i^* - d_i^* = 1 \quad \text{(Mustard)}
\end{align*}
\]

(10)

B. Standard goals of inter valued goals:

1) Machine-hour goals

\[
\begin{align*}
204x_{11} + 510x_{12} + 425x_{13} + d_{11} - d_{11} = 60704, \\
204x_{11} + 510x_{12} + 425x_{13} + d_{12} - d_{12} = 72845, \\
204x_{13} + d_{13} - d_{13} = 342622, \\
204x_{14} + d_{14} - d_{14} = 40469. \\
816x_{14} + 204x_{13} + 102x_{12} + 340x_{12} + 150x_{12} + d_{15} - d_{15} = 114238, \\
816x_{15} + 204x_{13} + 102x_{12} + 340x_{12} + 150x_{12} + d_{16} - d_{16} = 117227. \\
\end{align*}
\]

(13)

2) Power generation goals

\[
\begin{align*}
90x_{11} + 41x_{12} + 40x_{13} + 15x_{14} - d_{11} - d_{11} = 13774, \\
90x_{11} + 41x_{12} + 40x_{13} + 15x_{14} - d_{12} - d_{12} = 13973, \\
41x_{13} + 60x_{14} + d_{13} - d_{13} = 6702, \\
41x_{13} + 60x_{14} + d_{14} - d_{14} = 7176. \\
41x_{13} + 60x_{14} + 39x_{15} + 30x_{16} + 70x_{16} + 15x_{16} + d_{15} - d_{15} = 14962, \\
41x_{13} + 60x_{14} + 39x_{15} + 30x_{16} + 70x_{16} + 15x_{16} + d_{16} - d_{16} = 15352. \\
\end{align*}
\]

(14)

3) Water supply goals

\[
\begin{align*}
20x_{11} + 20x_{12} + 34x_{13} + d_{11} - d_{11} = 4037, \\
20x_{11} + 20x_{12} + 34x_{13} + d_{12} - d_{12} = 4064. \\
20x_{13} + 50x_{14} + d_{13} - d_{13} = 5062, \\
20x_{13} + 50x_{14} + d_{14} - d_{14} = 5979. \\
20x_{13} + 70x_{15} + 15x_{16} + 10x_{16} + d_{15} - d_{15} = 9586.
\end{align*}
\]

(15)

\[
20x_{11} + 70x_{15} + 15x_{16} + 10x_{16} + d_{11} - d_{11} = 9854. \\
\]

(Rabi) (15)

C. Crisp constraints:

1) Production-ratio constraint

The production ratio constraint appears as:

\[
(2.187x_{11} + 2.285x_{44} + 3.336x_{55})/1.882x_{63} = 7 \\
\]

(16)

2) Profit-ratio constraint

From the view point of making profit by exporting certain products the profit ratio of Jute and Aus-paddy in the pre-kharif season is taken into account here. The profit-ratio constraint takes the form

\[
259.84x_{11}/(99.06x_{21} + 969.45x_{31}) = 4 \\
\]

and satisfy the goal expressions in (9) - (15) subject to constraints (16) and (17), and

\[
(w_{n1}d_{11} + w_{n2}d_{12}) \leq V, k = 13,..., 21. \\
\]

Now, assigning the equal weights for minimizing the possible regrets to achieve the interval valued goals within their specified intervals, taking

\[
w_{n1} = w_{n2} = \frac{1}{18} \quad \text{and} \quad \lambda=0.5, \\
\]

the proposed GA scheme is employed to solve the problem in (18).

Now, the use of different genetic parameters and their values adopted in the solution search process are defined as follows.

The input parameters:

\[
\text{Max_gen; } \quad \text{number of generations} \\\n\text{Pop_size; } \quad \text{population size} \\\n\text{p_c; } \quad \text{probability of crossover} \\\n\text{p_m; } \quad \text{probability of mutation}
\]

The programming language C is used in the process of coding the evaluation program. The environment of execution is Intel Pentium IV with 2.66 GHz. Clock-pulse and 1 GB RAM.

The chromosome length 30 is considered from the view point of exploring the solution search domain defined by the constraints (16) and (17). The population size as in the standard GA method in [5] is taken 100.

The number of generations 300 is initially taken to conduct the experiment. The different experiments with the different values of \( p_c < 0.8 \) and \( p_m < 0.08 \), in the ranges \( 0.7 \leq p_c \leq 0.9 \) and \( 0.03 \leq p_m \leq 0.08 \) are made in the proposed GA scheme. Finally, \( p_c = 0.8 \) and \( p_m = 0.08 \) are found successful in the decision search process.

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The model solution is found at the generation number 200 of the genetic search process. The resulting decision of the cropping plan is presented in TABLE V.

<table>
<thead>
<tr>
<th>Crop (c)</th>
<th>Land allocation</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jute</td>
<td>123.37</td>
<td>332.27</td>
</tr>
<tr>
<td>Sugarcane</td>
<td>1.9</td>
<td>149.46</td>
</tr>
<tr>
<td>Rice</td>
<td>334.72</td>
<td>896.38</td>
</tr>
<tr>
<td>Wheat</td>
<td>55.05</td>
<td>117.34</td>
</tr>
<tr>
<td>Mustard</td>
<td>87.15</td>
<td>78.56</td>
</tr>
<tr>
<td>Potato</td>
<td>5.5</td>
<td>147.50</td>
</tr>
<tr>
<td>Pulses</td>
<td>49.95</td>
<td>41.55</td>
</tr>
</tbody>
</table>

The achieved annual profit = Rs. 110299.47 Lack.

The land allocation and production structure of the existing cropping plan (2005-2006) is presented in the TABLE VI.

<table>
<thead>
<tr>
<th>Crop (c)</th>
<th>Land allocation</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jute</td>
<td>120.20</td>
<td>325.15</td>
</tr>
<tr>
<td>Sugarcane</td>
<td>1.50</td>
<td>118.00</td>
</tr>
<tr>
<td>Rice</td>
<td>265.40</td>
<td>732.40</td>
</tr>
<tr>
<td>Wheat</td>
<td>47.10</td>
<td>100.40</td>
</tr>
<tr>
<td>Mustard</td>
<td>79.20</td>
<td>71.40</td>
</tr>
<tr>
<td>Potato</td>
<td>5.50</td>
<td>147.50</td>
</tr>
<tr>
<td>Pulses</td>
<td>46.40</td>
<td>38.60</td>
</tr>
</tbody>
</table>

Here, the achieved annual profit = Rs. 95803.01 Lack.

A comparison of the model solution with the result in the TABLE VI shows that a satisfactory decision for the optimal cropping plan is obtained here in the decision making environment.

VI. CONCLUSION

The land-use planning approach outlined in this article provides a good basis for analyzing the DM’s perception on the use of both the FGP and IVGP in the framework of the production planning model in farm management. Here, it is worth mentioning that achievement of all the aspired goals levels may not always be possible due to the limitation of productive resources, but the best possible decision can be obtained here under the proposed model.

Again, in different agricultural planning horizons, different regional based environmental constraints generally occur which can easily be incorporated under the framework of the proposed planning model.

However, the solution method presented in this article may open up many new vistas into the way of making proper decision in the current complex multiobjective planning problems in farm management.

ACKNOWLEDGMENT

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An Application of Genetic Algorithm for Solving Academic Personnel Planning Problems in University Management System via Fuzzy Goal Programming with Penalty Functions

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ABSTRACT
This article demonstrates how the genetic algorithm (GA) approach can efficiently be used to the penalty function based fuzzy goal programming (FGP) formulation for modelling and solving academic resource planning problems in University management system.

A demonstrative case example of the University of Kalyani, West Bengal (W.B), India is considered to expound the potential use of the approach. The proposed model is compared with the conventional FGP solution of the problem.

Keywords
Fuzzy programming, Goal programming, Fuzzy goal programming, Genetic algorithm, Penalty function.

1. INTRODUCTION
The increase of social awareness for betterment of social life as well as uplift in technological aspects has created a deep interest in higher education to the new study areas in the current technological affluent society in the recent years from the view points of widening the academic interest as well as opening the scope of employment to different emerging activity areas.

As a matter of fact, worldwide initiative for opening of new academic departments in university system has been taken in the past few years by most of the higher academic institutions. It is to be mentioned here that in most of the developed countries, tuition fees play the major roles to meet the economic constraints of running the academic organizations. But, in case of a developing country like India, universities are mainly run by the national income of a Government. Again, since the Universities are not the profit making organizations, opening of a new academic unit means increase of financial load to a University within the limited allocation of budget in a financial year.

In such a case, proper planning for academic staff allocation and setting of other assisting personnel within the existing infrastructure is inevitably needed for sustainable growth of the new units in the decision making horizon.

From a historical perspective, early works on education planning were studied by Gani [1] in 1963 and Platt [2] in 1962 as the applications of quantitative methods in the area of management science. During 1960s, studies on planning, programming and budgeting system in higher education were taken place rapidly and circulated widely in the literature. The survey of works in the field of higher education was conducted by Rath [3] and Hufner [4] in 1968. Different management science models to academic resource planning for enrichment of higher education were investigated [5, 6].

A comprehensive bibliography on the state-of-the-art of modelling aspects of academic planning for improvement of the quality of education studied from 1960s to early '70s was first prepared by Schroeder [7] in 1973. Thereafter, worldwide efforts for implementation of management science models to enrich the various activities of academic institutions were taken place from the view point of potential growth of socio-economic conditions of a nation.

Now, since most of the academic planning problems are multiobjective in nature, goal programming (GP) approach [8, 9] as one of the most promising and flexible tool for multiobjective decision analysis has been successfully implemented to university management system by Schroeder [10], Walters et al. [11], Franz et al. [12], and others. The extensive study on academic planning has been further surveyed by White [13] in 1987. The several other modelling aspects for different higher academic planning problems have also been studied by Pal and Basu [14] in 1997 and Kwak and Lee [15] in 1998.

Now, in a real-life decision situation, it is to be observed that the decision maker (DM) is often faced with the problem of setting precise parameter values to the decision problems due to inherent imprecise in nature of them as well as ambiguity in human judgments. In such a situation, crisp mathematical programming approaches fail to produce proper solutions in practical decision situations.

To overcome the above difficulty, FGP approach [16,17] in the field of fuzzy programming (FP) [18,19] and interval goal programming (IGP) [20] in the area of interval programming (IvP) [21] as the extension of conventional GP has been developed for solving multiobjective decision problems in imprecise (inexact) decision environment. The FGP as well as IGP approaches to university planning problems and other decision problems have been studied by Pal et al. [22, 23] in the past.

Now, in actual practice of using the above two approaches, it is found that the satisfactory solutions by satisfying the tolerance ranges in FGP as well as goal intervals in IGP specified by the DMs may not always be achieved due to different environmental restrictions imposed in the decision making situations.
The above difficulty may also arise due to inexactness of the expert's knowledge about the parameter values as well as setting of highly optimistic target levels of achieving the objectives. As a matter of fact, decision trouble is frequently encountered in solving practical decision problems.

Now, for managerial decision making, the problem of achieving goal values in different ranges instead of attaining the assigned target levels of the goals as in conventional GP formulation, the concept of penalty functions with the incorporation of marginal penalties for minimizing the deviations for goal achievement in different ranges has been well discussed by Romero [24] in 1991. The concept has been further extended by the active researchers [25, 26, 27, 28, 29, 30] in the past for solving IGP problems.

The penalty function approach to the GP model of an academic planning problem have also been studied by Pal et al. [14] in the past.

In the decision making environment, GAs based on natural selection and population genetics, initially introduced by Holland [31], have also appeared as robust computational tools in the field of optimization problems. The use of GAs on the framework of multiobjective decision making (MODEM) problems have been investigated by the pioneer researchers in the field and implemented to practical problems. But exploration of the potential use of GAs to real-life MODEM problems (crisp or fuzzy) is at an early stage.

The GA method [31, 32] as the direct random search approach to university management problems have been studied by Wang [33] and Mozos et al. [34] in the past. The use of GA to penalty function based FGP formulation of MODEM problems is yet to be documented in the literature.

In the proposed approach, requirement of total full-time teaching staff and allocation of pay-roll budget to each of academic departments are fuzzily described. The recruitment of minimum number of teaching and non-teaching staff and maintaining of certain ratios of part-time teaching staff and non-teaching staff individually with full-time teaching staff, and a ratio of total number of students with total teaching staff in each department for smooth functioning of the academic activities of the departments are considered as constraints in the academic planning horizon.

In the model formulation of the problem, the concept of penalty functions for measuring the degree of achievement of membership goals in different ranges for the defined fuzzy goal. Then, the membership function, say \( \mu_k(X) \), for the fuzzy goal \( Z_k(X) \) can be characterized as follows [17].

For \( \geq \) type of restriction, \( \mu_k(X) \) takes the form:

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } Z_k(X) \geq b_k, \\
\frac{Z_k(X) - (b_k - t_{uk})}{t_{uk}} & \text{if } b_k - t_{uk} \leq Z_k(X) < b_k, \\
0 & \text{if } Z_k(X) < b_k - t_{uk}, 
\end{cases}
\]

(3)

where \( (b_k - t_{uk}) \) represents the lower-tolerance limit for achievement of the stated fuzzy goal.

Again, for \( \leq \) type of restriction, \( \mu_k(X) \) becomes

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } Z_k(X) \leq b_k, \\
\frac{(b_k + t_{uk}) - Z_k(X)}{t_{uk}} & \text{if } b_k < Z_k(X) \leq b_k + t_{uk}, \\
0 & \text{if } Z_k(X) > b_k + t_{uk}, 
\end{cases}
\]

(4)

In the solution process, an GA scheme is iteratively used to solve the problem for achievement of the defined goals on the basis of the priorities assigned to them without linearizing the defined fractional constraints, unlike the classical approaches, in the decision making environment.

A case example of the University of Kalyani, W.B, India is considered to illustrate the proposed model. The model solution is compared with the existing staff allocation pattern as well as the conventional Minsum FGP solution approach in the decision making environment.

Now, the general FGP problem formulation is presented in the following Section 2.

2. PROBLEM FORMULATION

The generic form of a multiobjective FP problem can be presented as:

Find \( X(x_1, x_2, \ldots, x_n) \) so as to

satisfy \( Z_k(X) \geq b_k, \quad k = 1, 2, \ldots, K \) \hspace{1cm} (1)

subject to

\[
X \in S = \left\{ X \in \mathbb{R}^n \mid AX \geq \begin{bmatrix} b_1 & \cdots & b_m \end{bmatrix}, b_1 \geq 0, b \in \mathbb{R}^m \right\}
\]

(2)

where \( X \) is the vector of decision variables, \( \geq \) and \( \leq \) indicate the fuzziness of \( \geq \) and \( \leq \) restrictions, respectively, in the sense of Zimmermann [36], and where \( b_k \) is the imprecise aspiration level of the \( k \)-th objective \( Z_k(X) \), \( k = 1, 2, \ldots, K \), \( A \) is the technological coefficient matrix, \( b \) is the vector of right-hand side values. It is assumed that the feasible region \( S(\Phi) \) is bounded.

Now, in a fuzzy decision making situation, the fuzzy goals are to be characterized by their respective membership functions.

2.1 Characterization of Membership Function

Let \( t_{uk} \) and \( t_{uk} \) be the lower- and upper-tolerance ranges, respectively, for achievement of the aspirered level \( b_k \) of a \( k \)-th fuzzy goal. Then, the membership function, say \( \mu_k(X) \), for the fuzzy goal \( Z_k(X) \) can be characterized as follows [17].

For \( \geq \) type of restriction, \( \mu_k(X) \) takes the form:

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } Z_k(X) \geq b_k, \\
\frac{Z_k(X) - (b_k - t_{uk})}{t_{uk}} & \text{if } b_k - t_{uk} \leq Z_k(X) < b_k, \\
0 & \text{if } Z_k(X) < b_k - t_{uk}, 
\end{cases}
\]

(3)

where \( (b_k - t_{uk}) \) represents the lower-tolerance limit for achievement of the stated fuzzy goal.

Again, for \( \leq \) type of restriction, \( \mu_k(X) \) becomes

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } Z_k(X) \leq b_k, \\
\frac{(b_k + t_{uk}) - Z_k(X)}{t_{uk}} & \text{if } b_k < Z_k(X) \leq b_k + t_{uk}, \\
0 & \text{if } Z_k(X) > b_k + t_{uk}, 
\end{cases}
\]

(4)

2.2 FGP Model Formulation

In FGP model formulation, the membership functions are transformed into membership goals by assigning the highest degree (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. Then, in the goal achievement function, the under-deviational variables
are minimized on the basis of the importance of achieving the aspired goal levels in the decision making context.

Now, since multiple goals are involved with the problem and they often conflict with each other for achievement of their aspired goal levels, a priority based FGP model for goal achievement is considered in the decision making situation.

The FGP model of the problem for the defined membership functions under a pre-empptive priority structure appears as

\[
\text{Minimize } Z = \left[ P_1(d'), P_2(d'), \ldots, P_h(d'), \ldots, P_H(d') \right]
\]

and satisfy

\[
\begin{align*}
\frac{Z_k(X) - b_k - d_k^* - d_k'^*}{t_{a_k}} &= 1, \\
\frac{(b_k + t_{a_k}) - Z_k(X) + d_k^* - d_k'^*}{t_{a_k}} &= 1, \\
d_k^*, d_k'^* &\geq 0, \quad k = 1, 2, \ldots, H,
\end{align*}
\]

where, \( P_h(d') \) represents the vector of H priority achievement function, and \( d_k^*, d_k'^* \) are the under- and over-deviational variables of the k-th goal. \( P_h(d') \) is a linear function of the weighted under-deviational variables, where \( P_h(d') \) is of the form:

\[
P_h(d') = \sum_{k=1}^{K} w_{hk} d_{hk}, \quad h = 1, 2, \ldots, H; (H \leq K),
\]

where \( d_{hk} \) is renamed for \( d_k^* \) to represent it at the h-th priority level, \( w_{hk} \) is the numerical weight associated with \( d_{hk} \) and it designates the weight of importance of achieving the aspired level of the k-th goal relative to other which are grouped at the h-th priority level and where \( w_{hk} \) values are determined as [17]:

\[
w_{hk} = \begin{cases} 
\frac{1}{\mu_h(X)}, & \text{for the defined } \mu_h(X) \text{ in (3)} \\
\frac{1}{\mu_{hk}(X)}, & \text{for the defined } \mu_{hk}(X) \text{ in (4)}
\end{cases}
\]

where, \( \mu_h(X) \) and \( \mu_{hk}(X) \) are used to present \( t_{hk} \) and \( t_{hk} \) respectively, at the h-th priority level.

It is worthy to mention here that the notion of pre-empptive priorities of the goals actually hold on the concept that the h-th priority \( P_h \) is preferred to the next priority \( P_{h+1} \), regardless of any multiplier associated with \( P_{h+1} \), \( h = 1, 2, \ldots, H \). Also, the relationship among the priorities is:

\[ P_1 >> P_2 >> \ldots >> P_h >> \ldots >> P_N, \]

which implies that the goals at the highest priority level (\( P_1 \)) are achieved to the extent possible before the set of goals at the second priority level (\( P_2 \)) is considered, and so forth.

Now, an GA scheme for solving the problem (5) is presented in the following Section 3.

**3. DESCRIPTION OF GA SCHEME**

Now, in the literature of GAs, there is a large number of schemes [31, 32] for generating new populations with the use of different operators: selection, crossover and mutation. However, the basic steps of the GA procedure with the core functions adopted in the solution process are presented via the following steps.

**Step1. Representation and Initialization**

Let \( M \) denote the binary coded representation of a chromosome in a population as \( M = [g_1, g_2, \ldots, g_n] \). The population size is defined by \( \text{pop.size} \), and \( \text{pop.size} \) chromosomes are randomly initialized in its search domain.

**Step2. Fitness Function**

The fitness value of each chromosome is judged by the value of the goal achievement function defined with the priorities of the goals. The fitness function is defined as

\[ \text{eval} (M_v) = \left( \sum_{v=1}^{H} w_{hk} d_{hk} \right) \]

where \( M_v \) represents the h-th priority factor of the goal achievement of the function \( Z \) and the subscript 'v' refers to the fitness value of the selected \( v \)-th chromosome, \( v = 1, 2, \ldots, \text{pop.size} \). The best chromosome with largest fitness value at each generation is determined as

\[ M^* = \min \{ \text{eval}(M_v) \} \ v = 1, 2, \ldots, \text{pop.size}, \]

depending on searching of the best value of an objective.

**Step3. Selection**

The simple roulette-wheel scheme [32] is used for selecting two parents for mating purposes in the genetic search process.

**Step4. Crossover**

The parameter \( \text{pc} \) is defined as the probability of crossover. The arithmetic crossover (single-point crossover) operation of a genetic system is applied here in the sense that the resulting offspring appear with very close characteristics of parents and always satisfy the linear constraints set \( S (\neq \phi) \). Here, a chromosome is selected as a parent for a defined random number \( r \in [0, 1] \), if \( r < \text{pc} \) is satisfied. Again, in the reproduction process, the arithmetic crossover for two selected parents \( M_1, M_2 \) can be defined as

\[ G_1 = a_1 M_1 + a_2 M_2 \]

and

\[ G_2 = a_3 M_1 + a_4 M_2 \]

for producing two offspring \( G_1 \) and \( G_2 \) (\( G_1 \) and \( G_2 \) \( \in \bar{S} \)) where \( a_1, a_2 \geq 0 \) with \( a_1 + a_2 = 1 \).

**Step5. Mutation**

As in the conventional GA scheme, a parameter \( \text{pm} \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < \text{pm} \).

**Step6. Termination**

The execution of the whole process terminates when the fittest chromosome is reported at a certain generation number in the solution search process.

**Remark:** Since GAs are satisfiers rather than optimizers [31], in the process of using the GA method to the problem, the system constraints are also made flexible by introducing under- and over-deviational variables to each of them in the notion of searching the most satisfactory solution in the expanded flexible region \( S \) in goal satisfying philosophy of conventional GP [8,9]. The system constraints are then termed as the flexible goals.

The flexible goals in explicit form can be presented as:

\[ a_i x_j + d_i^- - d_i'^* = b_i, \quad i=1,2,\ldots,m; \quad j=1,2,\ldots,n \]

Now, in a MODM decision situation, it is to be observed that the goals often conflict each other for achieving the individual aspired goal levels, and happening of the situation
of achieving all the goal values does not generally occur in a practical decision situation. In such a case, goal achievement in different specified tolerance ranges with the defined marginal penalties can be taken into account in the decision making context. Here, incorporation of the penalty functions to FGP problem for analyzing the decision situation is presented in the Section 4.

4. FGP MODEL FORMULATION OF THE PROBLEM INCORPORATING PENALTY FUNCTION

The decision variables and different types of parameters involved with the academic planning problem are defined first to formulate the FGP model by incorporating penalty functions.

(i) Definition of Parameters:
The following parameters are involved in the proposed model.

- \( F_{ij} \): minimum number of full-time teaching staff (FTS) required in the department \( i \) (\( i = 1, 2, \ldots, I \)), rank \( j \) (\( j = 1, 2, \ldots, J \)) during the time period \( t \).
- \( [FTS]_i \): total FTS required in the department \( i \) at the period \( t \).
- \( N_i \): minimum number of non-teaching staff (NTS) required to run the department \( i \) at the time period \( t \).
- \( S_i \): total number of students (ST) in the department \( i \) at the time period \( t \).
- \( r_j \): ratio of part-time teaching staff (PTS) and FTS in the department \( i \).
- \( R_j \): ratio of NTS and total teaching staff (TTS) \( [FTS + PTS] \) in the department \( i \).
- \( s_j \): ratio of ST and TTS.
- \( TS_{ij} \): annual (average) salary of a FTS in the department \( i \), rank \( j \) at the time period \( t \).
- \( NS_{ij} \): annual (average) salary of a NTS at the time period \( t \).
- \( P_j \): annual remuneration of a PTS in the department \( i \).
- \( B_{it} \): pay-roll budget allocation to the department \( i \) at the time period \( t \).

(ii) Definition of decision variables:
The following decision variables are involved in the proposed model.

- \( f_{ij} \): number of FTS in the department \( i \), rank \( j \) at the time period \( t \).
- \( n_i \): number of NTS in the department \( i \) at the time period \( t \).
- \( P_j \): number of PTS employed in the department \( i \) at the time period \( t \).

Now, the fuzzy goals and constraints of the problem are described in the following Section 4.

4.1 Description of Fuzzy Goal and Constraints

4.1.1 Fuzzy goal description

Two types of fuzzy goals are involved with the problem. They are described as:

(i) FTS goals:

For potential academic performance of the departments, an estimated number of total FTS should always be employed to each of the departments by the University management. But, due to limitation of budget, requirement of FTS to an aspired level is fuzzy in nature at a planning period \( t \).

The fuzzy goal expressions appear as:

\[
\sum_{j=1}^{J} f_{ij} \geq [FTS]_i, \quad i = 1, 2, \ldots, I. \quad (8)
\]

Now, in conventional framework of GP with penalty functions, the goals are normalized to make the goals commensurable as well as to measure the goal achievement in terms of percentage.

Then, the goal expression in (8) takes the form:

\[
\sum_{j=1}^{J} \left( \frac{100}{[FTS]_i} \right) f_{ij} \geq 100, \quad i = 1, 2, \ldots, I \quad (9)
\]

(ii) Budget goal:

Due to limitation of total available budget to run the University, the pay-roll budget for each department is fuzzily described.

The budget goal expression appears as:

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} [FTS]_i f_{ij} + \sum_{i=1}^{I} [NS]_i n_i + \sum_{i=1}^{I} [P_j] P_j \leq [B]_i \quad (10)
\]

Then, in percentage scale, the expression in (10) takes the form:

\[
\sum_{i=1}^{I} \sum_{j=1}^{J} \left( \frac{100}{[B]_i} \right) [FTS]_i f_{ij} + \sum_{i=1}^{I} \left( \frac{100}{[B]_i} \right) [NS]_i n_i + \sum_{i=1}^{I} \left( \frac{100}{[B]_i} \right) [P_j] P_j \leq 100 \quad (11)
\]

Then, the membership function representations of the stated goals and measuring of the achievement grades of membership functions for incorporation of penalty functions are presented via a case example provided in the Section 5.

Now, the constraints of the problem are defined as follows:

4.1.2 Crisp constraint description

The two types of crisp constraints (linear and fractional) are involved with the staff allocation structure of the problem.

(i) FTS Constraints:

To run the academic curriculum, a minimum number of FTS at each rank need have to be provided in each of the departments.

The FTS constraints appear as:

\[
f_{ij} \geq [F]_i, \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J \quad (12)
\]

(ii) NTS constraints:

To perform the official and teaching related activities, a minimum number of NTS should be employed to each of the departments.

The NTS constraints appear as:

\[
n_i \geq [N]_i, \quad i = 1, 2, \ldots, I \quad (13)
\]

(iii) PTS-FTS ratio constraint:

When the FTS can not be employed at a time period \( t \), PTS at a certain ratio to FTS should be provided in each of the departments.
The ratio constraints in fractional form can be presented as:
\[
\frac{P_k}{\sum_{j=1}^{J} f_{ij}} = [r_k], \quad i=1,2,\ldots,J.
\]  
(14)

(iv) ST-TTS ratio constraint
For smoothing the academic activities, a certain ratio of ST and TTS should be maintained in each of the departments.
The constraints take the form:
\[
\frac{1}{\sum_{j=1}^{J} f_{ij} + p_{it}} = [s], \quad i=1,2,\ldots,J
\]  
(15)

(v) NTS-TTS ratio constraints
To meet the need of assisting the academic and official activities, a certain ratio of NTS and TTS should be provided to each of the departments.
The ratio constraints can be expressed as:
\[
\frac{1}{\sum_{j=1}^{J} f_{ij}} = [R], \quad i=1,2,\ldots,J
\]  
(16)

Now, the executable FGP model formulation of the problem by grafting penalty functions and thereby solving the problem is presented through a demonstrative case example in the Section 5.

5. AN ILLUSTRATIVE EXAMPLE: A CASE STUDY
The academic resource allocation problem of the University of Kalyani, West Bengal, India is considered to demonstrate the application potential of the proposed approach. To illustrate the potential use of the approach, the staff allocation problem of the four new departments: Physiology (PH), Molecular Biology and Bio-Technology (MB-BT), Microbiology (MB) and Geography (GEO) under the faculty of science, University of Kalyani, INDIA is taken into account. The required data for the proposed model was according to the revision of pay scale under 6th pay commission on the basis of the memorandum of the Principal Secretary, Higher Education Department, Bikash Bhavan, Kolkata bearing No. 715 –Edn (U) dated 18.12.2009 for the financial year 2009-2010.

The decision variables and different types of data involved with the problem are summarized in the Tables 1-3.

### Table 1. Data Description of Fuzzy Goal Levels of FTS

<table>
<thead>
<tr>
<th>Department</th>
<th>PH (1)</th>
<th>MB-BT (2)</th>
<th>MB (3)</th>
<th>GEO (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>FTS</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>8</td>
</tr>
</tbody>
</table>

### Table 2. Annual Average Salary for FTS, NTS and Remuneration for PTS

<table>
<thead>
<tr>
<th>Rank</th>
<th>Salary (Rs.)</th>
<th>Remuneration (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>9.17 Lac</td>
<td>-</td>
</tr>
<tr>
<td>Associate Prof.</td>
<td>7.2 Lac</td>
<td>-</td>
</tr>
<tr>
<td>Assistant Prof.</td>
<td>5.16 Lac</td>
<td>-</td>
</tr>
<tr>
<td>PTS (Guest Prof.)</td>
<td>1.5 Lac</td>
<td>-</td>
</tr>
<tr>
<td>NTS</td>
<td>-</td>
<td>0.5 Lac</td>
</tr>
</tbody>
</table>

The total Pay-roll Budget 293.26 Lac.

The Pie Chart of Budget distribution (in percentage) for salary of staff is presented in the Figure 1.

**Fig 1: Budget distribution of salary of staff**

<table>
<thead>
<tr>
<th>Salary In Percentage</th>
<th>Professor</th>
<th>Asst. Prof.</th>
<th>Prof.</th>
<th>FTS</th>
<th>NTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>22%</td>
<td>5%</td>
<td>22%</td>
<td>22%</td>
<td>22%</td>
</tr>
</tbody>
</table>

### Table 3. Data Description of ST and the Ratios of PTS-FTS, TTS-ST, NTS-TTS

<table>
<thead>
<tr>
<th>Department</th>
<th>Number of Students</th>
<th>PTS-FTS</th>
<th>TTS-ST</th>
<th>NTS-TTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PH</td>
<td>30</td>
<td>1:4</td>
<td>1:7</td>
<td>2:5</td>
</tr>
<tr>
<td>MB&amp;BT</td>
<td>24</td>
<td>1:4</td>
<td>1:7</td>
<td>2:5</td>
</tr>
<tr>
<td>MB</td>
<td>32</td>
<td>1:4</td>
<td>1:7</td>
<td>2:5</td>
</tr>
<tr>
<td>GEO</td>
<td>50</td>
<td>1:4</td>
<td>1:7</td>
<td>1:2</td>
</tr>
</tbody>
</table>

The bar diagram of the number of students in different departments is shown in the following Figure 2.

**Fig 2: Graphical Representation of number of students in different departments.**
Now using the data tables and the other necessary collected data, construction of the fuzzy goals and the crisp goals are described in the Section 5.1. Here, since the variables are introduced with running period (t = 1), the time specification (t) is omitted for simplicity during the presentation of the case model.

5.1 FTS Goals
In the fuzzy decision making situation, it is assumed that 80% achievement is the lower tolerance limit as desired by the DM for all the defined FTS goals of the four departments.

Then, following the expressions in (9), the membership goals are obtained as:

\[ 0.05\left(6.67(f_1 + f_2 + f_3) - 80\right) + d^-_1 = 1 \]  (17)
\[ 0.05\left(6.67(f_4 + f_5 + f_6) - 80\right) + d^-_4 = 1 \]  (18)
\[ 0.05\left(12.5(f_7 + f_8) - 80\right) + d^-_7 = 1 \]  (19)
\[ 0.05\left(12.5(f_9 + f_{10}) - 80\right) + d^-_9 = 1 \]  (20)

Now, in the conventional FGP approach, achievement of the highest membership value (unity) by minimizing the under-deviational variables \( d^+_i (i = 1,2,3,4) \) to the extent possible is considered, where any \( d^+_i > 0 \) indicates the measure of non-achievement of an aspired membership value.

But, in a practical decision situation, it is to be observed that achievement of fuzzy objectives in different intervals are to be taken into account [37,38] instead of taking them singled valued as employed in the conventional FGP approach to decision problems. Actually, consideration of this situation arises due to inexactness of the environment of making decision.

To overcome the above situation, relaxation of tolerance ranges of fuzzy goals for their achievement in different intervals up to certain defined goal level for each of them can be considered, and the grafting of penalty functions for measuring the actual degree of achievement of a fuzzy goal can be taken into account.

The inclusion of penalty functions to the formulated model by defining penalty scales is presented as follows:

Grafting of Penalty Functions
In the conventional GP approach, the penalty functions are defined by the deviational variables associated with the achievement of goal values in the different ranges, and they are minimized in the goal achievement function on basis of the introduced marginal penalties [39] where marginal penalties indicate the relative weights of importance of achieving the goals by minimizing the associated deviational variables.

In the present decision situation, the deviational variables are expressed in terms of under-achievement of the defined membership values for goal achievement in different specified ranges.

Now, for the defined membership goals in (5), it is to be followed that achievement of highest membership value of a goal is made by minimizing the under-deviational variables. Here, above 100% achievement of a fuzzy goal means that the attainment of a membership value is more than unity.

a situation the marginal penalty is assigned as zero and the DM is overly satisfied here. The other penalty scales for grafting the penalty functions for goal attainment of all the defined FTS goals of the departments are considered the same.

The penalty scale representations to evaluate (17), (18), (19), (20) are summarized in the Table 4.

### Table 4. Penalty Scales for FTS Goals

<table>
<thead>
<tr>
<th>Goal attainment range (in %)</th>
<th>Under-deviation (in %)</th>
<th>(AMV, Under-deviation)</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Over 100</td>
<td>0</td>
<td>(above unity, 0)</td>
<td>0</td>
</tr>
<tr>
<td>100-80</td>
<td>20</td>
<td>(1, 0.2)</td>
<td>0.05</td>
</tr>
<tr>
<td>80-70</td>
<td>10</td>
<td>(0.8, 0.1)</td>
<td>0.1</td>
</tr>
<tr>
<td>Below 70</td>
<td>∞</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

Note: AMV = Aspired membership value, MP = Marginal penalty, ∞ = infinity.

Now, in the decision situation, the aim of the DM is to attain the aspired membership value of a goal by minimizing the associated under-deviational variables. Here, grafting of penalty scales leads to redefine \( d^+_i (i = 1,2,3,4) \) by substituting the sum of different under-deviational variables involved in the context of achieving membership value in different ranges.

Let, \( d^+_i ≥ 0 (i = 1,2,3,4), l = 1,2 \) designate the under-deviational variables for the defined penalty scales in the Table 5.

Then, the membership goals in (17), (18), (19), and (20) can be recast as

\[ 0.05\left(6.67(f_1 + f_2 + f_3) - 80\right) + \frac{2}{l}d^+_i = 1 \]  (21)
\[ 0.05\left(6.67(f_4 + f_5 + f_6) - 80\right) + \frac{2}{l}d^+_i = 1 \]  (22)
\[ 0.05\left(12.5(f_7 + f_8) - 80\right) + \frac{2}{l}d^+_i = 1 \]  (23)
\[ 0.05\left(12.5(f_9 + f_{10}) - 80\right) + \frac{2}{l}d^+_i = 1 \]  (24)

Again, from the view point of minimizing \( d^+_i \) for each of the defined ranges, the penalty function goals in the form of conventional goals can be presented as:

\[ d^+_1 + \eta_1 - \eta_1 = 0.2 \]  (25)
\[ d^+_2 + \eta_2 - \eta_2 = 0.1 \]  (26)
\[ \eta_1, \eta_2 ≥ 0, l = 1,2,3,4, i = 1,2 \]

Now, to make a control of keeping goal achievement within a specified range, \( d^+_1 \) and \( \eta_1, i = 1,2,3,4, l = 1,2 \) are to be minimized in the goal achievement function of the executable FGP model of the problem. Again involvement of \( \eta_1 > 0 \) in the solution indicates further relaxation for goal achievement within an interval.

Here, since below 70% achievement is not acceptable, it can reasonably be assumed that \( \eta_1 ≤ 0.1, i = 1,2,3,4 \) (27) which appear as constraints in the decision making situation.
5.2 Budget Goal
From the fuzzy goal description of budget goal in (11), the upper tolerance ranges for budget allocation are involved in the decision situation. It is assumed that 15% more allocation as relaxation of the budget limit is introduced by the DM. Then, using the prescribed data and following the expression in (5), the membership goal of the fuzzy budget goal in (11) can be obtained as:

\[ 0.07 \left[ 0.15 - (3.12(f_{11} + f_{12} + f_{13}) + 2.44(f_{21} + f_{22} + f_{23} + f_{24}) + 1.7( f_{31} + f_{32} + f_{33} + f_{34}) + 0.17(n_{1} + n_{2} + n_{3} + n_{4}) + 0.51(p_{1} + p_{2} + p_{3} + p_{4}) \right] + d_{3}^{+} - d_{3}^{-} = 1 \] (28)

Now, it is to be followed from the expression in (28) that any over-achievement from 100% of the defined fuzzy goal level is represented by the under-deviation from the highest membership value. Again, for any goal value lower than 100%, the fuzzy goal is overly satisfied which is indicated by the value of the over-deviational variable \( d_{3}^{+} \) and for which attainment of membership value is found to be more than the aspired level (unity). As a matter of fact, similar to the case of membership goal achievement for FTS goals, \( d_{3}^{+} \) is to be minimized here in the goal achievement function, which is not alike the conventional GP with penalty functions where over-deviational variable of a goal is minimized for goal achievement in a given interval.

Now, in the decision situation, consideration of different goal achievement ranges and penalty scale representations of them in terms of membership goal deviations is summarized in the Table 5.

<table>
<thead>
<tr>
<th>Goal attainment range (in %)</th>
<th>Under-deviation (in %)</th>
<th>(AMV, Under-deviation) MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 100</td>
<td>0</td>
<td>(above unity, 0) 0</td>
</tr>
<tr>
<td>100-115</td>
<td>15</td>
<td>(1, 0.15) 0.066</td>
</tr>
<tr>
<td>115-125</td>
<td>10</td>
<td>(0.9, 0.1) 0.1</td>
</tr>
<tr>
<td>Above 125</td>
<td>∞</td>
<td>∞</td>
</tr>
</tbody>
</table>

Note: The meaning of the abbreviations is the same as defined in the Table 4. Now, for the defined penalty scales in the Table 5, grafting of the penalty functions for achievement of budget goal can be described in an analogous way to the case of goal achievement for FTS goals presented in the Section 5.1.

The membership goal in (28) with penalty functions can be presented as

\[ 0.07 \left[ 0.15 - (3.12(f_{11} + f_{12} + f_{13}) + 2.44(f_{21} + f_{22} + f_{23} + f_{24}) + 1.7( f_{31} + f_{32} + f_{33} + f_{34}) + 0.17(n_{1} + n_{2} + n_{3} + n_{4}) + 0.51(p_{1} + p_{2} + p_{3} + p_{4}) \right] + d_{3}^{+} - d_{3}^{-} = 1 \] (29)

\[ d_{31}^{+} + n_{3}^{+} - n_{3}^{-} = 0.15 \] (30)

\[ d_{34}^{+} + n_{4}^{+} - n_{4}^{-} = 0.1 \] (31)

Then, the deviational variables to be minimized in the achievement function are \( d_{3}^{+} \) and \( n_{i}^{+} (i = 3, 4) \).

Again, since over-deviation of the budget goal is restricted to 25%, similar to the case of achievement of an FTS goal, \( n_{i}^{+} \leq 0.25 \) (32) is introduced as a constraint in the formulated model.

Now, the system constraints of the model are described in the Section 5.3.

5.3 Description of System Constraints
(a) FTS constraints
On the basis of the current work loads of the departments, a minimum number of FTS at each rank need to be employed to each of the departments. The FTS constraints appear as:

\[ f_{21} \geq 1, f_{22} \geq 1, f_{31} \geq 2, f_{32} \geq 1, f_{33} \geq 2, f_{31} \geq 1, f_{32} \geq 2, f_{33} \geq 2, f_{41} \geq 1, f_{42} \geq 3, f_{43} \geq 1. \] (33)

(b) NTS constrains
The constrains can be presented as:

\[ n_{1} \geq 2, n_{2} \geq 2, n_{3} \geq 2, n_{4} \geq 2 \] (34)

(c) Ratio constrains
Using the data in the Table 3, the different ratio constraints are obtained as follows:

(i) PTS-FTS ratio constrains:

\[ \frac{p_{1}}{f_{11} + f_{12} + f_{13}} = 0.25, \frac{p_{2}}{f_{31} + f_{32} + f_{33}} = 0.25, \frac{p_{3}}{f_{11} + f_{12} + f_{13}} = 0.25, \frac{p_{4}}{f_{41} + f_{42} + f_{43}} = 0.25, \] (35)

(ii) TTS-ST ratio constrains:

\[ \frac{f_{11} + f_{12} + f_{13} + p_{1}}{30} \geq 0.14, \frac{f_{21} + f_{22} + f_{23} + p_{2}}{24} \geq 0.14, \frac{f_{31} + f_{32} + f_{33} + p_{3}}{32} \geq 0.14, \frac{f_{41} + f_{42} + f_{43} + p_{4}}{50} \geq 0.14, \] (36)

(iii) NTS-TTS ratio constrains:

\[ \frac{n_{1}}{f_{11} + f_{12} + f_{13} + p_{1}} \geq 0.4, \frac{n_{2}}{f_{21} + f_{22} + f_{23} + p_{2}} \geq 0.4, \frac{n_{3}}{f_{31} + f_{32} + f_{33} + p_{3}} \geq 0.4, \frac{n_{4}}{f_{41} + f_{42} + f_{43} + p_{4}} \geq 0.5, \] (37)

Now, for the defined goals and constraints, formulation of the executable model as an extension of the FGP model in (5) is presented in the following Section 5.4.
5.4 Executable FGP Model Formulation

The two priority factors are introduced for achievement of the model goals in the decision making context.

The first priority (P1) is assigned to the goals with highest membership value and the second one (P2) is assigned to the defined penalty function goals.

The executable model under the framework of priority based FGP with penalty functions appears as

\[
\text{Find} \ (f_{ijP}, n_i | i=1,2,3,4; j=1,2,3) \text{ so as to} \ \\
\text{Minimize} \ Z = \\
\left[ P_1 \{0.05(d_{11} + d_{21} + d_{31} + d_{41}) + 0.1(d_{12} + d_{22} + d_{32} + d_{42}) \} \right. \\
\left. + 0.066d_{13} + 0.1d_{23} \right] \\
+ P_2 \{0.05(n_{11} + n_{21} + n_{31} + n_{41}) + 0.1(n_{12} + n_{22} + n_{32} + n_{42}) \} \\
+ 0.066(n_{13} + 0.1(n_{23})), \\
\text{and satisfy the goal expressions in (21)-(27), (29)-(32) subject to the constraints in (33)-(37)}.
\]

The proposed GA approach is used to solve the problem. Here, the goal achievement function (Z) represents the evaluation sanction in the genetic search process for achieving the goals on the basis of the assigned priorities.

The programming language C is used in the process of coding the evaluation program. The environment of execution is Intel Pentium IV with 2.66 GHz Clock-pulse and 1 GB RAM. The chromosome length = 30 is considered with a view to searching solution in the domain of feasible solution set (S) defined in the decision situation. The population size as in the standard GA method is taken 100. The number of generations = 300 is initially taken to conduct the experiment.

The different experiments with the different values of \( p_c (0 < p_c < 1) \) and \( p_m (0 < p_m < 1) \) in the ranges \( (0.7 < p_m < 0.9) \) and \( (0.03 < p_m < 0.8) \) are made in the proposed GA scheme. It is found that \( p_c = 0.8 \) and \( p_m = 0.08 \) are successful in the decision search process.

The resulting model solution is presented in the Table 6.

Table 6. Solution for Staff Allocation under the proposed Model

<table>
<thead>
<tr>
<th>Department</th>
<th>PH</th>
<th>MB-BT</th>
<th>MB</th>
<th>GEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>3</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>PTS (Guest Prof.)</td>
<td>2</td>
<td>4</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>NTS</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

The data of the existing staff structure of the departments are presented in the Table 7.

Table 7. Existing Staff Allocation Structure (2009-2010)

<table>
<thead>
<tr>
<th>Department</th>
<th>PH</th>
<th>MB-BT</th>
<th>MB</th>
<th>GEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>PTS (Guest Prof.)</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>NTS</td>
<td>8</td>
<td>35</td>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

A comparison of the model solution with the existing staff allocation displayed in the Table 7 shows that a satisfactory solution is achieved under the proposed model in the decision making environment.

Note: If the defined penalty functions are not taken under consideration and achievement of the defined membership goals in (17)-(20) and (28) are only taken into account in minsum FGP formulation of the problem and if all the goals are treated at the same priority level, then the obtained solution of the problem is presented in the Table 8.

Table 8. Model Solution under the Minsum FGP Approach

<table>
<thead>
<tr>
<th>Department</th>
<th>PH</th>
<th>MB-BT</th>
<th>MB</th>
<th>GEO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>PTS (Guest Prof.)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>NTS</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

The following Figure shows a comparison of the solutions obtained by the three different cases.

![Solution Comparison](Fig3.png)

A further comparison of the model solution with the solution in the Table 8 shows that the proposed approach is a superior one from the view point of achieving the desired staff levels for smooth functioning of the academic activities of the departments.

6. CONCLUSION

The main advantage of using the proposed GA approach is that the computational load involved with the traditional approaches for linearization of the real-life problems with fractional criteria can be avoided here in the solution process. Moreover, the most satisfactory decision can easily be reached here in the solution search process of the proposed GA method without involving extra computational burden with redefining the model as involved in the decision process of using the traditional approaches.

Further, the FGP with penalty function approach to academic personnel planning problem in a University system demonstrated in the paper provides a new look into the way of analyzing the achievement of the fuzzily described objective goal levels in different intervals on the basis of needs and desires of the departments towards enrichment of academic activities of a university. The main advantage of using the proposed approach is that the grafting of penalty functions makes the model a flexible one to reach a satisfactory decision in the academic planning horizon.

The proposed approach can be extended to solve different other university management problems with imprecise
parameter values involved in both the objectives and
constraints in the decision making environment. The IGP
with penalty function method to university planning in inexact
decision environment may be a problem for future study.
Finally, it is expected that the modelling aspects of the
academic planning problem presented here can contribute to
future research of different real-life problems for managerial
decision making.

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problems with interval-valued resource goals in
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399.


USING GENETIC ALGORITHM TO FUZZY GOAL PROGRAMMING FORMULATION OF OPTIMAL ELECTRIC POWER GENERATION AND DISPATCH

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Abstract: This paper describes how the genetic algorithms (GAs) can be efficiently used to fuzzy goal programming (FGP) formulation of optimal power flow in power system operation and planning phases by solving a multiobjective optimization problem. In the proposed approach the objectives of optimal power flow calculations are fuzzily described in the model formulation of the problem, the membership functions of the defined fuzzy goals are characterized first for measuring the degree of achievement of the aspiration levels of the goals specified in the decision making context. Then, the achievement function for minimizing the regret for under-deviations from the highest membership value (unity) of the defined membership goals to the extent possible is constructed for optimal power flow problems.

In the solution process, the proposed GA method is used in an iterative manner for satisficing the goal levels on the basis of needs and desires of the decision maker (DM).

The developed method has been tested on IEEE 6-generator 30-bus System. To illustrate the potential use of the approach, numerical results have been shown to reflect that this method is promising for handling uncertain constraints in practical power systems.

Keywords: Fuzzy goal programming. Goal programming, Genetic algorithm. Membership function. Optimal power flow problems.

1 INTRODUCTION

The Optimal Load Flow problem was introduced in the early 1960s by Carpentier [1] and has grown into a powerful tool for power system operation and planning. This formulation was later named the Optimal Power Flow problem (OPF) [2]. It has since been generalized to include many other problems.

Today any problem that involves the determination of the instantaneous "optimal" steady state of an electric power system is an OPF problem [3], [4], [5].

Throughout the years, the OPF problem has been solved using a variety of nonlinear optimization techniques, as discussed in [6].

Optimal power flow calculation used to optimize specific aspects of power system operations, usually employ standard mathematical programming techniques. These techniques are not suitable to handle many practical considerations encountered in power systems, including the uncertainty of the operational constraints.

Generally, in most real-world situations, the necessary information to specify the exact model coefficients is not available, because data are scarce, difficult to obtain, uncertain and the system being modeled may be subject to changes. Therefore, mathematical programming models for decision support must take explicitly into account, besides multiple and conflicting objective functions, the treatment of the uncertainty associated with the model coefficients.

The conventional OPF problem is formulated as an optimization problem with crisp constraints.

Constraint limits are given fixed values that have to be met at all times. However, the constraints of a real-life system, in terms of their physical realization, can be divided into two groups: a) physical limits and b) operating limits. Physical limits on the control variables cannot be violated. However, operating limits are imposed to enhance security and do not represent physical bounds. They can be relaxed temporarily, if necessary, to obtain feasible solutions. This kind of infeasibility is encountered in several OPF. Crisp treatment of the constraints in the OPF problem usually leads to over-conservative solutions.
However in practice, there are two types of inequality constraints: hard constraints and soft constraints. For example, the limits of the generating unit outputs are hard constraints because there are physical limitations on the capacity of the generating units to produce active power. On the other hand, the limits for the transmission line flows are soft. Small violations of these limits sometimes are acceptable, especially during stressed (e.g., emergency or peak loaded) situations of the system. There are usually two flow limits for each transmission line, namely, normal and emergency limits. In general, operators desire to economically operate the system within the normal limits. When there is a real need, small violations of the normal limits are allowed. However, the emergency limits can never be violated and are considered as hard limits. These practical considerations of constraint limits are not formulated satisfactorily in a conventional OPF.

However, if the case is nearly feasible (or nearly infeasible), the solutions from conventional OPF may become unrealistic. Sometimes, in order to enforce a soft constraint with small violations, the control variables may have to move significantly and also the cost may increase sharply. Even though the solutions are mathematically correct for the formulated OPF problem, they are not consistent with practical operational practices. For infeasible cases, the conventional OPF usually cannot produce acceptable solutions even with the assistance of available relaxation mechanisms.

Recently, fuzzy set methods have been applied to obtain more realistic models. Fuzzy set methods have already been used in many applications such as control, scheduling, robotics, artificial intelligence, etc. In the field of power engineering, they have been applied to some areas including optimal power flow [7], [8], [9], [10].

Since most of the OPF problems are multiobjective in nature, the goal programming (GP) methodology in [11] can be used as an efficient tool for solving the problem. Although, the GP has appeared as a rich field of solving multiobjective decision making (MODM) problems, the main weakness of using the conventional GP to decision problems is that in the real-world MODM situations, the decision maker (DM) is often faced with the problem of assigning the precise aspiration levels to the goals due to inherent inexactness in nature of the decision parameters as well as imprecision in human judgments.

To overcome the above difficulty, FGP [12] in the framework of conventional GP and as an extension of fuzzy programming (FP) [13] have been studied in the past, and implemented to different real-world decision making problems [14], [15].

Now, in practical decision situations, it is found that nonlinearity in general form as well as in fractional form are frequently involved with the defining of various relationships among the parameters and decision variables. In such a case, the use of conventional approximation approaches to FGP problems [14] involve computation load and often lead to local optimal solutions.

To overcome the computational complexity in practical decision problems, GAs [16] appear as a robust tool for searching satisfactory decisions for MODM problems. GAs to real-world multiobjective decision problems have been studied in [15] in the past. However, the study of GA based FGP approaches to real-life problems are at an early stage. Moreover, the use of GA based FGP technique to Load Flow problems is yet to appear in the literature.

In this article, an FGP formulation of multiobjective optimal planning of electric power generation dispatch with the various constraint functions are considered. A solution scheme based on GA is introduced to reach a satisfactory decision of achieving the defined objectives in the decision making environment.

The simulation results of IEEE 6-generator 30-bus System expound the potential use of the proposed approach.

II. FGP PROBLEM FORMULATION
In a fuzzy decision making environment, instead of crisp description of the objectives and constraints, fuzzification of them depend on the needs and desires of the DM in the decision situation.

In the present FGP formulation of the problem, a full fuzzy version of goal achievement is considered to make the model a flexible one in the decision making context.

Now, fuzzy goal description is presented in the following Section A.

A. Definition of Fuzzy Goal
Let } be the imprecise aspiration level of the k-th objective } (k = 1, 2, ..., K). Then the fuzzy goals may appear in one of the forms:

\[ F_k(X) \geq b_k \quad \text{and} \quad F_k(X) \leq b_k \]

where X is the vector of decision variables, and where \( \geq \) and \( \leq \) indicate the fuzziness of the aspiration levels, and is to be understood as 'essentially more than' and 'essentially less than', respectively, in the sense of Zimmermann [13].

Now, in the field of FP, the fuzzy goals are characterized by their respective membership functions.

B. Characterization of Membership Function
Let } and } be the lower- and upper-tolerance ranges, respectively, for achievement of the aspired level } of the k-th fuzzy goal. Then, the membership function, say } (X), for the fuzzy goal } (X) can be characterized as follows [14].
Using Genetic Algorithm to Fuzzy Goal Programming Formulation of Optimal Electric Power Generation...

For \( \gtrsim \) type of restriction, \( \mu_4(X) \) takes the form:

\[
\mu_4(X) = \begin{cases} 
1 & \text{if } F_t(X) \gtrsim b_k, \\
\frac{F_t(X) - (b_k - t_a)}{t_a} & \text{if } b_k - t_a \leq F_t(X) < b_k, \\
0 & \text{if } F_t(X) < b_k - t_a,
\end{cases}
\]  

(1)

where \((b_k - t_a)\) represents the lower-tolerance limit for achievement of the stated fuzzy goal.

Again, for \( \lesssim \) type of restriction, \( \mu_4(X) \) becomes:

\[
\mu_4(X) = \begin{cases} 
1 & \text{if } F_t(X) \lesssim b_k, \\
\frac{(b_k + t_u) - F_t(X)}{t_u} & \text{if } F_t(X) \leq b_k + t_u, \\
0 & \text{if } F_t(X) > b_k + t_u,
\end{cases}
\]  

(2)

where \((b_k + t_u)\) represents the upper-tolerance limit for achievement of the stated fuzzy goal.

Then, the FGP model formulation for the defined membership functions is presented in Section C.

C. FGP Model Formulation

In FGP model formulation, the membership functions are transformed into membership goals by assigning the highest degree (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. Then, in the goal achievement function, the under-deviational variables are minimized on the basis of importance of achieving the aspired goal levels in the decision making context.

Now, since multiple goals are involved with the problem, and they often conflict each other for achievement of their aspired goal levels, a \textit{minsum} FGP model for goal achievement is considered in the decision making situation.

The \textit{minsum} FGP formulation of the problem appears as:

Find \( X(x_1, x_2, \ldots, x_n) \) so as to:

Minimize

\[
z = w_1^* d_1^* + w_2^* d_2^* + \ldots + w_k^* d_k^* + \ldots + w_k^* d_k^*
\]

and satisfy

\[
\frac{F_t(X) - (b_k - t_a)}{t_a} + d_1^* - d_k^* = 1
\]

\[
\frac{(b_k + t_u) - F_t(X)}{t_u} + d_1^* - d_k^* = 1
\]

subject to

\[
AX \begin{bmatrix} \leq \\ \geq \end{bmatrix} b, \quad X \geq 0,
\]

(3)

where \( Z \) represents the fuzzy achievement function consisting of the weighted under-deviational variables \( d_i^* \), and where \( d_i^*, d_i^* \) represent the under- and over-deviational variables associated with the \( k \)-th membership goal. \( w_k^* (\geq 0) \) represents the relative importance for achieving the \( k \)-th fuzzy goal in a decision-making environment and is determined as:

\[
W_k^* = \begin{bmatrix} \frac{1}{t_a} & \text{for the defined } \mu_4 \text{ in (1)} \\
\frac{1}{t_u} & \text{for the defined } \mu_4 \text{ in (2)}
\end{bmatrix}
\]

(4)

When some of the objectives, \( F_t(X), k = 1, 2, \ldots, K, \) are non-linear in form, then conventionally the traditional linearization approach is used in the solution process of the MODM problems which involve huge computational complexity. But to avoid the computational load involved in linearization of the objectives as well as the inherent decision error involved in the approximation approach, a GA procedure is used in the process of solving the FGP model in (3).

The GA scheme used in the process of solving the problem in (3) is presented in the following Section III.

III. DESIGN OF GA SCHEME

For the given FGP structure of the proposed problem, the task of the DM is to search the solution which satisfies fuzzy linear as well as fractional goals to the extent possible by evaluating the defined goal achievement function. As such, GAs as the global search algorithms can be efficiently used to achieve the most satisfactory decision in the planning environment.

Now, in the literature of GAs, there is large number of schemes [16] for generating new populations with the use of different operators: selection, crossover and mutation. However, the basic steps of the GA procedure with the core functions adopted in the solution process are presented via the following steps.

Step 1. Representation and Initialization

Let \( E \) denote the binary coded representation of a chromosome in a population as \( E = \{x_1, x_2, \ldots, x_n\} \). The population size is defined by \( \text{pop\_size} \), and \( \text{pop\_size} \) chromosomes are randomly initialized in its search domain.

Step 2. Fitness Function

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as

\[
eval(E_v) = \sum_{i=1}^{k} w_i^* d_i^*
\]

(5)

where \( Z_i \) represents the \( i \)-th priority factor of the goal achievement of the function \( Z \) in (3), and where the subscript \( 'v' \) refers to the fitness value of the selected \( v \)-th chromosome, \( v = 1, 2, \ldots, \text{pop\_size} \). The best chromosome
with largest fitness value at each generation is determined as

\[ E^* = \min \{ \text{eval}(E_v) \mid v = 1, 2, \ldots, \text{pop}_\text{size} \}, \]

in searching out the best value of the objective.

**Step 3. Selection**

The simple roulette-wheel scheme [16] is used for selecting two parents for mating purposes in the genetic search process.

**Step 4. Crossover**

The parameter \( P_c \) is defined as the probability of crossover. The arithmetic crossover operator (single-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the linear constraints set \( S(\neq) \). Here a chromosome is selected as a parent, if for a defined random number \( r \in [0, 1] \), \( r < P_c \) is satisfied.

Here single-point crossover for two parents \( E_1, E_2 \in S \) is defined as:

\[ X_i = \alpha_1 E_1 + \alpha_2 E_2, \quad X_i = \alpha_1 E_1 + \alpha_2 E_2, \]

for producing two offspring \( X_1 \) and \( X_2 \), where, \( \alpha_1, \alpha_2 \geq 0 \) with \( \alpha_1 + \alpha_2 = 1 \) always belong to \( S \), and where \( S \) is a convex set.

**Step 5. Mutation**

As in the conventional GA scheme, a parameter \( P_m \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

**Step 6. Termination**

The execution of the whole process terminates when the fittest chromosome is reported at a certain generation number in the solution search process. Now, the proposed multiobjective optimal planning of electric power generation dispatch problem is described in Section IV.

### IV. PROBLEM DESCRIPTION

The environmental and economic optimal operation planning problem with transmission constraints and load characteristic is to minimize two competing objective functions, pool purchase cost and emission, while satisfying several equality and inequality constraints, that is, Market Operator purchase low-price and low-emission power to supply load demand considering transmission constraints and load characteristic of distribution system. Generally the problem is formulated as follows.

**A. Objective Functions**

1) **Cost-minimization Objective**

The fuel cost of a generator is usually described with a quadratic function of real power output:

\[ f_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \]  

where, \( P_i \) is the real power output of the \( i \)-th generator. Then the objective function of dispatch with fuel cost minimized is as follows:

\[ \text{Min } F_i = \sum_{i=1}^{N} (\alpha_i P_i^2 + \beta_i P_i + \gamma_i) \]

\( N \) is the total number of generators in the power system.

2) **Emission-minimized Objective**

The atmospheric pollutants such as sulphur oxides SOx and nitrogen oxides NOx caused by fossil-fueled thermal units can be modeled separately. However, for comparison purposes, the total ton/h emission of these pollutants can be expressed as:

\[ f_i(P_i) = 10^{-2}(a_i P_i^3 + b_i P_i + c_i) + d_i \exp(e_i P_i) \]

And the objective function of dispatch with emission quantity minimized is as follows:

\[ \text{Min } F_2 = \sum_{i=1}^{N} (a_i P_i^3 + b_i P_i + c_i) + d_i \exp(e_i P_i) \]

**Table 1**

<table>
<thead>
<tr>
<th>List of Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{oi} ) : Active output of generator ( i )</td>
</tr>
<tr>
<td>( Q_{oi} ) : Reactive output of generator ( i )</td>
</tr>
<tr>
<td>( P_i ) : Active power at the bus ( i )</td>
</tr>
<tr>
<td>( Q_i ) : Reactive power at the bus ( i )</td>
</tr>
<tr>
<td>( P_{r} ) : Active power specified value at the bus ( i )</td>
</tr>
<tr>
<td>( Q_{r} ) : Reactive power specified value at the bus ( i )</td>
</tr>
<tr>
<td>( V_{i} ) : Voltage magnitude of bus ( i )</td>
</tr>
<tr>
<td>( V_{i, r} ) : Voltage specified value of bus ( i )</td>
</tr>
<tr>
<td>( N ) : Number of generators</td>
</tr>
<tr>
<td>( L ) : Number of lines</td>
</tr>
<tr>
<td>( P_{t} ) : Total load capacity</td>
</tr>
<tr>
<td>( P_{t} ) : Total transmission loss</td>
</tr>
<tr>
<td>( \alpha, \beta, \gamma ) : Coefficients of generator cost characteristic</td>
</tr>
<tr>
<td>( a, b, c, d, e ) : Coefficients of generator's NOx emission characteristic</td>
</tr>
</tbody>
</table>

### B. Objective Constraints

1) **Equality and Inequality Constraints**

The static optimization of a power system can be formulated as a mathematical programming problem which minimizes a given evaluation index under equality constraints given below:

At a load bus,

\[ P_i - P_{r} = 0 \]  

\[ Q_i - Q_{r} = 0 \]
Using Genetic Algorithm to Fuzzy Goal Programming Formulation of Optimal Electric Power Generation...

At a generator bus,
\[ |v_i| - |v_i'| = 0 \]  

(12)

To ensure the active power balance,
\[ \sum_{i=1}^{n} P_{G_i} - (P_d + P_f) = 0 \]  

(13)

Here, transmission loss \( P_f \) is expressed as a nonlinear function of bus voltages to allow for its fluctuation:
\[ P_f = P_L(x) \]  

(14)

It should be noted that the active and reactive power outputs of a generator are expressed as nonlinear functions of bus voltages. Equality constraints (10) to (13) can succinctly be expressed in a vector form as:
\[ h(x) = 0. \]  

(15)

The real power balance is as follows:
\[ P_{G_i} - P_{L_i} - \sum_{j=1}^{n} V_j(V_i \cos \theta_{ij} + B_j \sin \theta_{ij}) = 0 \]  

(16)

The reactive power balance is:
\[ Q_{G_i} - Q_{L_i} - \sum_{j=1}^{n} V_j(G_i \sin \theta_{ij} + B_j \cos \theta_{ij}) = 0 \]  

(17)

where \( P_{G_i}, Q_{G_i} \) is the real and reactive power demand of bus \( i \) respectively, \( V_i \) is the rating voltage of bus \( i \), \( \theta_{ij} = \theta_i - \theta_j \) is the difference of voltage angle between bus \( i \) and bus \( j \) respectively.

2) Inequality Constraints

Next it follows that the inequality constraints must be imposed on generator outputs and bus voltage magnitudes as:
\[ P_{G_i \min} \leq P_{G_i} \leq P_{G_i \max} \]  

(18)

\[ Q_{G_i \min} \leq Q_{G_i} \leq Q_{G_i \max} \]  

(19)

\[ V_{i \min} \leq V_i \leq V_{i \max} \]  

(20)

Now, the FGP model of the problem is described as follows.

V. FGP MODEL OF THE PROBLEM

The membership functions for each of the objectives (7) and (9) must be defined for fuzzy description of them. Since, the pool purchase cost and emission are all cost type index, the smaller the objective value of them, the better is the result of operation planning.

Here, the fuzzy goal for cost-minimization objective-function takes the form:
\[ F_1 = \sum_{i=1}^{n} 10^{-3}(\alpha_i P_{G_i}^2 + \beta_i P_{G_i} + \gamma_i) \leq b_1 \]  

(22)

Here, \( b_1 \) and \( b_2 \) are the imprecise aspiration values of the cost-minimization and emission-minimization objectives respectively.

Therefore, let \( F_1(P_{G_i}) \leq b_1 + t_1 \) represent an imprecise upper bound on the maximum permissible pool purchase cost and \( t_1 \) be the "tolerance" parameter, that is, a measure of fuzziness in this constraint. So the mathematical formulation of the membership function is as follows:
\[ \mu(F_1) = \begin{cases} 1 & \text{if } F_1 \leq b_1, \\ \frac{(b_1 + t_1) - F_1}{t_1} & \text{if } b_1 < F_1 \leq b_1 + t_1, \\ 0 & \text{if } F_1 > b_1 + t_1. \end{cases} \]  

(23)

In the same way, let \( F_2(P_{G_i}) \leq b_2 + t_2 \) represent an imprecise upper bound on the maximum permissible emission and \( t_2 \) be the "tolerance" parameter. So, a linear membership function can be defined as follows:
\[ \mu(F_2) = \begin{cases} 1 & \text{if } F_2 \leq b_2, \\ \frac{(b_2 + t_2) - F_2}{t_2} & \text{if } b_2 < F_2 \leq b_2 + t_2, \\ 0 & \text{if } F_2 > b_2 + t_2. \end{cases} \]  

(24)

In fuzzy programming approaches, the highest degree of membership function is 1. Thus, for the defined membership functions in (23) and (24), the flexible membership goals with the aspired level 1 can be presented as:
\[ \frac{(b_1 + t_1) - F_1}{t_1} + d_1 - d_1^* = 1 \]  

\[ \frac{(b_2 + t_2) - F_2}{t_2} + d_2 - d_2^* = 1 \]  

(25)

\( d_1^* (\geq 0) \) and \( d_2^* (\geq 0) \) with \( d_1^* \cdot d_2^* = 0, k = 1, 2 \) represent the under- and over-deviational variables, respectively, from the aspired levels.

In the model formulation, the under-and / or over-deviational variables are included in the achievement function for minimizing them and that depend upon the type of the objective functions to be optimized.

In the proposed approach, only the under-deviational variables \( d_i^* \), \( i = 1, 2 \) are required to be minimized to achieve the desired levels of the fuzzy goals. It may be noted that any over-deviation from a fuzzy goal indicates the full achievement of the membership value.

The minsum FGP model of the problem can be presented as:
\[ \text{Minimize } Z = [w_1 d_1^* + w_2 d_2^*] \]
and satisfy
\[
\begin{align*}
\frac{(b_1 + t_1)}{t_1} - F_1 &= d_1^* - d_1^* = 1 \\
\frac{(b_2 + t_2)}{t_2} - F_2 &= d_2^* - d_2^* = 1 \\
\end{align*}
\]
Subject to all equality and inequality system constraints in (10)-(17) and (18)-(20).

Now, since GA is a goal satisficer in [16] rather than optimizer, the proposed GA scheme can be employed here to minimize the achievement function ‘Z’ in (26) and thereby to reach a satisfactory solution. Here the goal achievement functions ‘Z’ appears as the fitness function in the evaluation process of using the GA.

The efficient use of the proposed approach is illustrated by a demonstrative case example in the Section VI.

VI. DEMONSTRATIVE CASE EXAMPLE

In this paper, a standard IEEE 30-bus 6-generator test system is used to illustrate the potential use of the approach.

![Figure 1: IEEE 6-generator 30-bus System](image)

The flexible membership goals of the two competitive objectives are as follows;

**Goal 1: Cost-minimization**

\[
\frac{640 - F_1}{40} + d_1^* - d_1^* = 1,
\]

where,

\[
F_1 = 100P_{G1}^2 + 200P_{G1} + 10 + 120P_{G2}^2 + 150P_{G2} + 10 + 40P_{G3}^2 + 180P_{G3} + 20 + 60P_{G4}^2 + 100P_{G4} + 10 + 40P_{G5}^2 + 180P_{G5} + 10 + 100P_{G6} + 150P_{G6} + 10
\]

**Goal 2: Emission-minimization**

\[
\frac{0.23 - F_2}{0.06} + d_2^* - d_2^* = 1,
\]

where,

\[
F_2 = 10^{-2}[4.091P_{G1}^2 - 5.554P_{G1} + 6.490 + 2.4 \times 10^{-6}]
\exp(2.857P_{G1}) + 2.243P_{G2}^3 - 6.047P_{G2} + 5.638 + 5.0 \times 10^{-4}
\exp(3.333P_{G3}) + 4.258P_{G3}^2 - 5.094P_{G3} + 4.586 + 1.0 \times 10^{-6}
\exp(8.000P_{G4}) + 5.326P_{G4}^2 - 3.55P_{G4} + 3.380 + 2.0 \times 10^{-3}
\exp(2.000P_{G4}) + 4.258P_{G5}^2 - 5.094P_{G5} + 4.586 + 1.0 \times 10^{-6}
\exp(8.000P_{G6}) + 6.131P_{G6}^2 - 5.55P_{G6} + 5.515 + 1.0 \times 10^{-3}
\exp(6.667P_{G6})
\]

Now for the stated membership goals of the problem, using the expression in (26), the achievement function of the executable minimum FGP model is obtained as:

Find X so as to:

\[
\text{Minimize } Z = [w_1 d_1^* + w_2 d_2^*]
\]

Then, the proposed GA approach presented in the Section III is used to solve the problem in (27) using the data given in [17, 18] subject to the goal constraints in (26) and the system constraints in (10)-(17) and (18)-(20).

The objective function of the model appears as the fitness function in the solution search process.

<table>
<thead>
<tr>
<th>Generator No.</th>
<th>(a_i^2)</th>
<th>(b_i)</th>
<th>(c_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>150</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>40</td>
<td>180</td>
<td>20</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>180</td>
<td>20</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>150</td>
<td>10</td>
</tr>
</tbody>
</table>
In the genetic search process, the following parameter values are introduced:
- probability of crossover $P_c = 0.8$
- probability of mutation $P_m = 0.08$
- population size = 100
- chromosome length = 200

The GA based program is designed in Programming Language C++. The execution is done in an Intel Pentium IV with 2.66 GHz Clock-pulse and 1GB RAM. The optimal solution is reached after 200 generations.

The model solution is presented in the Table III.

### Table III
**: Solutions Under The Proposed Model**

<table>
<thead>
<tr>
<th>Objective</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(w_1 = 0.61, w_2 = 0.39)$</td>
<td>$II(w_1 = 0.6, w_2 = 0.4)$</td>
</tr>
<tr>
<td>Cost in $$ $</td>
<td>601.0100</td>
</tr>
<tr>
<td>Emission/ton</td>
<td>0.2048148</td>
</tr>
</tbody>
</table>

The solution obtained using the $\epsilon$-constrained technique as in [17] is as follows:
- Total generation cost (Cost/ $\$ = 612.87
- Total emission (ton/h) $= 0.206$

A comparison shows that a better solution is obtained here in terms of achieving the goal values of the objectives of the problem.

### VII. CONCLUSIONS

In this paper, an GA based FGP approach is presented to solve the multiobjective optimal planning of electric power generation dispatch.

The main advantage of the proposed approach is that the computational load and approximation error inherent to conventional linearization approaches can be avoided here with the use of the GA based solution method. Further, the proposed approach is flexible enough to accommodate different other restrictions as and when needed in the decision making context.

Again, since the various objectives involved with the problem often conflict each other in achieving the aspired goal levels, the use of GA search method as a goal satisfier offers the most satisfactory decision in decision making environment.

### References


