Chapter 9

A Genetic Algorithm Based Fuzzy Goal Programming Solution Approach to Chance Constrained Bilevel Programming Problems

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9.1 Introduction

The fuzzy goal programming (FGP) formulation and solution methodology for bilevel programming problem (BLPP) have been presented in Chapter 3. The interval programming (IvP) approaches for modelling and solving BLPPs have been discussed in Chapter 5, Chapter 6 and Chapter 7.

In this chapter, a genetic algorithm (GA) based FGP procedure for solving BLPPs having chance constraints in large scale hierarchical decision situations is presented.

In the real-world decision situations, the decision makers (DMs) are often faced with the problem of inexact parameter values due to the imprecision in human judgments as well as inherent uncertainty in the parameters involved with the problem.

The two types of prominent approaches for solving such problems are stochastic programming (SP) which deals with probabilistically uncertain data and fuzzy programming (FP) which deals with imprecisely described data.

The field of SP based on the theory of probability, initially introduced by Charnes and Cooper [117], as chance constrained programming (CCP) has been studied [395] extensively and applied to various real-life problems [90]. The SP deals with the situations where some or all the parameters of the optimization problem are described by stochastic (random / probabilistic) variables rather than deterministic quantities [538, 615]. There are several sources for occurrence of random variables, and that depends on the nature and type of problems. The SP problems arise when certain coefficients of the optimization models are not fixed, i.e., not known with certainty, and they are to some extent probabilistic in nature. In the recent years, the methods of multiobjective stochastic optimization problems have become increasingly important in searching solutions of practical decision problems arising in economics, industry, healthcare, transportation, agriculture, military operations, and others.

Again, the FP based on the theory of fuzzy sets, initially introduced by Zadeh [699], has been studied [284] deeply during the mid '70s to '80s of the last century from the point of view of its potential use to different real-life problems [624] with imprecisely defined data.
In the real-life decision situations, it has been realized that both the probabilistic and fuzzy data are frequently involved in optimization problems, and both the aspects of SP and FP should be taken into account in actual practice of modelling and solving the problems and thereby taking proper decisions. However, consideration of both the aspects in a problem creates a great challenge for developing efficient solution methods to deal with both the stochastic and fuzzy terms in real-life problems.

The constructive modelling aspects on programming problems under randomness and fuzziness have been first studied by Luhandjula [410] in 1983. The methodological development of fuzzy stochastic programming (FSP) approaches for solving linear programming (LP) problems has been surveyed by Luhandjula [414] in 2006.

The BLPP [23, 43, 44, 45, 510] is a special structured mathematical programming problem in a hierarchical decision system. It is actually the most widely used and the simplest version of multilevel programming problem (MLPP) [48, 73, 516] having multiple DMs with multiplicity of objectives in a large hierarchical decision situation.

The bilevel programming (BLP) approaches to hierarchical decision problems have been studied widely since Candler and Townsley [102] demonstrated the use of BLP to large hierarchical decision making and planning organizations.

The GAs [157, 238] as prominent tools to optimization of multiobjective decision problems have been introduced to solve BLPPs [430]. The GA based FGP [710] approaches to linear MLPPs as well fractional BLPPs have been studied by Pal et al. [495, 503] in the recent past. However, the extensive study in this area is at an early stage.

Now, when BLPP is applied to real-world problems, some system parameters are often subject to fluctuations and are difficult to measure exactly. Here, assuming the random in nature of the variables, fuzzy dependent-chance constrained BLP [218] has been studied in the past. But, the extensive study in this area is still at an early stage.

Although, FP approach to chance constrained BLPPs have been investigated [434, 688] in the past, it is too early to the deep study of the field in the area of FSP from
the view point of its potential use in real-life problems. Also, the use of FGP method to BLPPs with chance constraints is rare in the literature.

Further, the use of the GA approach in the solution search process of chance constrained BLPP is yet to appear in literature.

It may be mentioned here that the fuzzification of objectives / constraints as well as randomization of various parameters with different probability distributions depends on the decision making environment.

In this Chapter, the BLPP with random in nature of both the coefficient matrix and the resource vector of the chance constraints with known probability distribution are taken into consideration.

In the proposed solution approach, the notion of the use of mean and variance in CCP is taken into account to convert the defined chance constraints into their equivalent crisp system constraints. In the process of formulating the model of the problem, the individual best and least solutions of the objectives of each of the DMs located at the two different hierarchical decision levels are determined first under the crisply defined system constraints for fuzzy description of the objectives and the control vector of the upper-level DM. Then, the membership functions of the defined fuzzy goals are constructed.

In the FGP model formulation, the membership functions are transformed into membership goals by assigning the highest membership value (unity) as the aspiration level and introducing the under- and over-deviational variables to each of them. In the goal achievement function of the proposed FGP model, achievement of the highest membership value of each of the membership goals to the extent possible by minimizing the associated under-deviational variables on the basis of weights of importance of achieving the fuzzy goals is taken into account.

In the solution process, the GA method is used to solve the BLPP having non-linear characteristics of the defined deterministic constraints. The GA method efficiently searches the decisions of the DMs in the proposed problem without involving any computational complexity which inherently appears in non-linear optimization problems that make use of the traditional solution search methods.

To illustrate the proposed approach, a numerical example is solved.
9.2 Problem Formulation

Let \( X=(x_1, x_2, \ldots, x_n) \) be the vector of deterministic decision variables involved with the two hierarchical decision levels. Then, let \( F_k \) and \( X_k \) be the objective function and the decision vector, respectively, of the leader and follower, \( k = 1, 2 \); where

\[
\bigcup_{k} \{X_k|k=1,2\} = X.
\]

Then, the BLPP in the hierarchical decision structure can be presented as:

Find \( X(X_1, X_2) \) so as to:

\[
\text{Maximize } F_1(X_1, X_2) = c_{11}X_1 + c_{12}X_2 \quad \text{ (leader's problem)}
\]

where, for given \( X_1, X_2 \) solves

\[
\text{Maximize } F_2(X_1, X_2) = c_{21}X_1 + c_{22}X_2 \quad \text{ (follower's problem)}
\]

subject to

\[
X \in S = \{X \in \mathbb{R}^n \mid \Pr[AX \leq b] \geq p, \ X \geq 0, \ b \in \mathbb{R}^n\}
\]

(9.1)

where, \( P_r \) indicates the probabilistically defined constraints, \( A \) is a coefficient matrix and \( b \) is a resource vector and \( p(0 < p < 1) \) is the vector of satisficing probability levels defined for the randomness of the parameters in the constraints set. Again, it is assumed that the feasible region \( S \neq \emptyset \) is bounded.

In the present decision situation, it is assumed that the elements of the coefficient matrix \( A \) and the resource vector \( b \) are independent continuous normally distributed random variables. Then, the conversion to deterministic (crisp) equivalent of the chance constraints in (9.1) is described in the following Section 9.2.1.

9.2.1 Deterministic Equivalent of Chance Constraints

The chance constraints set in (9.1) can be explicitly presented as:

\[
\Pr\left[ \sum_{j=1}^{n} a_{ij}x_j \leq b_i \right] \geq p_i, \ i=1, 2, \ldots, m.
\]

(9.2)

Let, \((E(a_{ij}), \text{Var}(a_{ij}))\) and \((E(b_i), \text{Var}(b_i))\) be the mean and variance pairs of the random variables \( a_{ij} \) and \( b_i \) respectively having the characteristics of normal distribution.
Then, following the notion of distribution function for a random variable, the crisp equivalent of the constraints in (9.2) can be defined. Let $F_i(y)$ be the distribution function of the $i$-th random variable $b_i$. Then, since $F_i(y)$ is a monotonically non-decreasing function, the value of the corresponding variable is determined as

$$F_i^{-1}(e) = \{ \max y \mid P_r(b_i \leq y) \leq e \}, \quad 0 < e < 1$$

(9.3)

where $F_i^{-1}(e)$ is the inverse function of the probability distribution of the random variable $b_i$.

Now, in the decision situation, either $a_{ij}$ or $b_i$ or both of them simultaneously may become independent random variables.

For simplicity, it is assumed that either $a_{ij}, \forall j$ or $b_i, \forall i = 1, 2, ..., m$ is defined randomly in the decision making context.

1) When $a_{ij}, \forall j$ are only normally distributed random variables, then

letting $y_i = \sum_{j=1}^{n} a_{ij}x_j$,

(9.4)

the mean of $y_i$ is obtained as

$$E(y_i) = \sum_{j=1}^{n} E(a_{ij})x_j,$$

(9.5)

since $x_j, \forall j$ are deterministic.

Again, the variance of $y_i$ is found as

$$\text{Var}(y_i) = X^T V_i X,$$

(9.6)

where, $V_i$ represents the $i$-th covariance matrix of the order $(n \times n)$ for the defined $a_{ij}, \forall j, T$ means transpose.

Then, following the basic rules of probability theory, the expressions of the chance constraints in (9.1) can be obtained as

$$\Pr \left( \frac{y_i - E(y_i)}{\sqrt{\text{Var}(y_i)}} \leq \frac{b_i - E(y_i)}{\sqrt{\text{Var}(y_i)}} \right) \geq p_i, \ i = 1, 2, ..., m$$

(9.7)

where, $\frac{y_i - E(y_i)}{\sqrt{\text{Var}(y_i)}}$ is the standard normal variate with mean zero and variance one.
Now, realizing the fact that \( \Pr[y_i \leq b_i] \) and using the notion of the distribution function defined in (9.3), the deterministic equivalent of the given chance constraints appear as:

\[
\frac{b_i - E(y_i)}{\sqrt{\text{var}(y_i)}} \geq F^{-1}_i(p_i),
\]

i.e., \( E(y_i) + F^{-1}_i(p_i)\sqrt{\text{var}(y_i)} \leq b_i, i = 1, 2, \ldots, m \) \hspace{1cm} (9.8)

Here, it is to be observed that, since \( a_y, \forall j \) follow normal distribution, the covariance terms \( \text{cov}(a_y, a_k), \forall j, k; j \neq k, k = 1, 2, \ldots, n \) would be zero.

As such, the expressions in (9.8) in a simplified version and in non-linear form appear as:

\[
\sum_{j=1}^{n} E(a_{y_j})x_j + F^{-1}_i(p_i)\sqrt{\sum_{j=1}^{n} \text{var}(a_{y_j})x_j^2} \leq b_i, i = 1, 2, \ldots, m \] \hspace{1cm} (9.9)

2) When \( i \)-th resource element \( b_i \) in the expression (9.2) is only random in nature, then using the basic probability rules in an analogous to the above, the deterministic equivalent of the expressions in (9.2) can be presented as [284]:

\[
\sum_{j=1}^{n} a_{y_j}x_j - E(b_i) \leq F^{-1}(1-p_i), i = 1, 2, \ldots, m \] \hspace{1cm} (9.10)

The expressions in (9.10) in a simplified linear form appear as

\[
\sum_{j=1}^{n} a_{y_j}x_j \leq E(b_i) + F^{-1}(1-p_i)\sqrt{\text{var}(b_i)} \, , \, i = 1, 2, \ldots, m \] \hspace{1cm} (9.11)

Now, it is worthy to mention here that when non-linearity in objectives and/or constraints occurs then computational complexity [258] is involved in solving optimization problems and computational load as well as inherent error with the use of traditional transformation approaches [659] generally arises in a practical decision situation.

To overcome the above difficulties, the GA approach as the decision satisficers [157] rather than optimizers can be efficiently used in the decision search process.

To formulate the FGP model of the problem and to reach a satisfactory decision using the notion of goal satisficing philosophy [621] in GP, an GA scheme is adopted in the Section 9.3.
9.3 Design of the GA Scheme

In this chapter, the basic steps of the GA scheme with the core functions adopted in the solution search process are same as those of the algorithmic steps described in Chapter 2 and Chapter 3.

For the present problem, the fitness function is defined as:

$$\text{eval}(E_v) = (F_k)_v, \quad k = 1, 2; \quad v = 1, 2, \ldots, \text{pop}_\text{size}. $$

where $F_k$ represents the objective function of the k-th level DM given in (9.1), and the subscript $v$ is used to indicate the fitness value of the $v$-th chromosome, $v = 1, 2, \ldots, \text{pop}_\text{size}$.

The best chromosome with largest fitness value at each generation is determined as:

$$E^* = \max\{\text{eval}(E_v) \mid v = 1, 2, \ldots, \text{pop}_\text{size}\}, \quad k = 1, 2$$

or, $$E^* = \min\{\text{eval}(E_v) \mid v = 1, 2, \ldots, \text{pop}_\text{size}\}, \quad k = 1, 2$$

which depends on searching of the maximum or minimum value of an objective function.

Now, the model formulation of the problem is described in the Section 9.4.

9.4 FGP Problem Formulation

In FGP formulation of the problems, both the objectives $F_1$ and $F_2$ and the control vectors $X_1$ are to be transformed into fuzzy goals by means of assigning an imprecise aspiration level to each of them. Then, the defined fuzzy goals are characterized by the membership functions to measure the degree of goal achievement in terms of membership values.

In the present decision situation, the individual best decisions of the DMs are taken into consideration and they are evaluated by using the proposed GA scheme. Let, $(X_1^l, X_2^l; F_1^l)$ and $(X_1^f, X_2^f, F_2^f)$ be the optimal solutions of the leader and follower, respectively, when calculated in isolation over the feasible solution space $S$, where

$$F_1^l = \max_{(X_1, X_2)} F_1(X_1, X_2)$$

and

$$F_2^f = \max_{(X_1, X_2)} F_2(X_1, X_2)$$
Then, the fuzzy objective goals appear as:

\[ F_1 \geq F'_1, \quad F_2 \geq F'_2 \]

Also the fuzzy goal for the control vector \( X \) is obtained as:

\[ X_i \geq X'_i. \]

where ' \( \geq \) ' refers to the fuzziness of an aspiration level and it is to be understood as 'essentially greater than' in the sense of Zimmermann [712].

Now, in the decision situation, it is assumed that both the DMs have a motivation to cooperate with each other to make a balance of decision powers, and they agree to give a possible relaxation of their individual optimal decision. Then, lower-tolerance limits of the respective fuzzy objective goals for the leader and follower can be determined as \( F'_f[X_1, X_2] \) and \( F'_2[X_1, X_2] \), respectively.

Further, since the leader has a higher power of making decision, a certain relaxation of \( X'_i \) as a lower-tolerance limit should be given for searching a better decision by the follower.

Let, \( X'_f(X'_f < X'_p < X'_i) \) be the lower tolerance limit of \( X'_i \).

Then, characterization of membership functions of the defined fuzzy goals are presented in the following Section 9.4.1.

### 9.4.1 Characterization of Membership Function

The membership function for the fuzzy objective goal of the leader appears as [511, 713]:

\[
\mu_{f_i}(F_i(X_1, X_2)) = \begin{cases} 
1 & \text{if } F_i(X_1, X_2) \geq F'_i, \\
\frac{F_i(X_1, X_2) - F'_i}{F'_i - F'_f} & \text{if } F'_f \leq F_i(X_1, X_2) < F'_i, \\
0 & \text{if } F_i(X_1, X_2) < F'_f 
\end{cases}
\]

(9.12)

Similarly, the membership function for the fuzzy objective goal of the follower takes the form:
The membership function for the fuzzy decision of the leader appears as:

\[
\mu_{x_1}[x_1] = \begin{cases} 
1 & \text{if } X_1 \geq X_1^l, \\
\frac{X_1 - X_1^p}{X_1^f - X_1^p} & \text{if } X_1^p \leq X_1 < X_1^f, \\
0 & \text{if } X_1 < X_1^p
\end{cases}
\]

(9.14)

Now, the FGP model formulation is presented in the following Section 9.4.2.

### 9.4.2 FGP Model Formulation

In a fuzzy decision making environment, the aim of each of the DMs is to achieve the highest membership value (unity) of each of the membership functions defined for them in the decision situation.

Then, in the fuzzy goal achievement function, minimization of the sum of the under deviational variables on the basis of the relative weights of importance of achieving the goals is taken into consideration. The minsum FGP model can be presented as:

Find \( X(X_1, X_2) \) so as to

\[
\begin{align*}
\text{Minimize: } & \sum_{k=1}^{2} w_i d_i^- + w_3 d_3^- \\
\text{and satisfy } & \frac{F_1(X_1, X_2) - F_1^f}{F_1^f - F_1^i} + d_1^+ - d_1^- = 1, \\
& \frac{F_2(X_1, X_2) - F_2^f}{F_2^f - F_2^i} + d_2^+ - d_2^- = 1, \\
& \frac{X_1 - X_1^p}{X_1^f - X_1^p} + d_1^+ - d_1^- = 1
\end{align*}
\]

subject to the system constraints (9.9) and (9.11).

(9.15)
Here, $d_k^+, d_k^- > 0$, with $d_k^+ d_k^- = 0$ (k = 1,2) represent the under- and over-deviational variables, respectively, associated with the k-th membership goals and $d_j^+, d_j^- > 0$ with $d_j^+ d_j^- = 0$ represent the vector of under- and over-deviational variables associated with the membership goals defined for the decision vector $X_i$, and I is a column vector with all elements equal to 1 and the dimension of it depends on the decision vector $X_i$, $Z$ represents the goal achievement function consisting of the weighted under-deviational variables and vectors of weighted under-deviational variables, where the numerical weights $w_k (> 0)$, $k = 1,2$ and the vector of the numerical weights $w_j (> 0)$, represent the relative weights of importance of achieving the goals to their aspired levels, and they are determined as [511]:

$$w_k^+ = \frac{1}{F_1^+ - F_1^-, \quad w_k^- = \frac{1}{F_2^+ - F_2^-}, \quad w_j^+ = \frac{1}{X_i^+ - X_i^-}}$$

Now, since BLPP is a hierarchical decision structured problem and the objectives at the two decision levels often conflict each other in the decision making situation, the use of conventional FGP solution approach may not always provide the satisfactory decision from the view point of distribution of proper decision powers to the DMs in the decision making environment.

In such a situation the GA method as a decision satisficer [157] rather than optimizer can be employed in the solution search process of the problem to find a satisfactory solution for the DMs.

The fitness function under the GA scheme appears as:

$$\text{Eval}(E_v) = (Z)_v = (\sum_{k=1}^{2} w_k^+ d_k^- + w_j^+ d_j^-), \quad v = 1,2,\ldots, \text{pop.size}.$$  

In the decision search process, the best chromosome $E^*$ with the highest score at a generation is determined as

$$E^* = \min \{ \text{eval}(E_v) \mid v = 1,2,\ldots, \text{pop.size} \}$$

in the genetic search process.

The efficient use of the proposed approach is illustrated by a numerical example presented in the Section 9.5.
9.5 Numerical Example

Let $x_1, x_2$ be the decision variables under the control of the leader and $x_3$ be the decision variable under the control of the follower.

Then, the BLPP is of the form:

Maximize $F_1(x_1, x_2, x_3) = 5x_1 + 6x_2 + 3x_3$ (leader’s problem)

and, for given $x_1, x_2; x_3$ solves

Maximize $F_2(x_1, x_2, x_3) = 2x_1 + 3x_2 + 8x_3$ (follower’s problem)

subject to

$\Pr \left[ anx_1 + a_{12}x_2 + a_{13}x_3 < 8 \right] > 0.95$

$\Pr \left[ 5x_1 + x_2 + 6x_3 < b_1 \right] > 0.10$

$\Pr \left[ x_1 + x_2 + x_3 < b_2 \right] > 0.05$

$\Pr \left[ 2x_1 + 3x_2 + x_3 < b_3 \right] > 0.85$

(9.16)

where, $a_{11}, a_{12}, a_{13}$ and $b_1, b_2, b_3$ are independent normally distributed random variables.

The means and variances of the random variables are determined as follows:

$E(a_{11}) = 1, \text{Var}(a_{11}) = 5$;

$E(a_{12}) = 3, \text{Var}(a_{12}) = 16$;

$E(a_{13}) = 9, \text{Var}(a_{13}) = 4$;

$E(b_1) = 7, \text{Var}(b_1) = 9$;

$E(b_2) = 2.5, \text{Var}(b_2) = 2$;

$E(b_3) = 10, \text{Var}(b_3) = 2$;

Then, following the procedure, the deterministic equivalent of the successive constraints in (9.16) are obtained as

$x_1 + 3x_2 + 9x_3 + 1.645(25x_1^2 + 16x_2^2 + 4x_3^2)^{\frac{1}{2}} \leq 8$

$5x_1 + x_2 + 6x_3 \leq 10.855$

$x_1 + x_2 + x_3 \leq 5.174$

$2x_1 + 3x_2 + x_3 \leq 13.592$

(9.17)

Now, to solve the problem by employing the proposed GA scheme, the following genetic parameter values are incorporated in the decision search process.
• population size = 100  
• probability of crossover $p_c = 0.8$  
• probability of mutation $p_m = 0.08$  
• chromosome length = 30.

Here, it may be mentioned that the above parameter values are found successful during conduct of the experiments to find solution in the search domain specified in the decision environment.

The GA is implemented using the Programming Language C. The execution is performed in an Intel Pentium IV PC with 2.66 GHz Clock-pulse and 1 GB RAM.

Now, following the procedure, the individual optimal solutions of the two successive levels are obtained as

\[ (x_1^l, x_2^l, x_3^l; F_1^l) = (0.4360, 0.6541, 0; 6.1042) \]
\[ (x_1^l, x_2^l, x_3^l; F_2^l) = (0.0717, 0.0904, 0.6094; 5.2896), \]

respectively.

Then, the fuzzy goals can be defined as:

\[ F_1 > 6.1042, \quad F_2 > 5.2896, \quad \text{and} \quad x_1 > 0.4360, \quad x_2 > 0.6541 \]

The lower-tolerance limits of the objective goals are determined as:

\[ F_1^l = 2.7291, \quad F_2^l = 2.8343. \]

The leader feels that his/her control variables $x_1$ and $x_2$ can be relaxed up to 0.2 and 0.1, respectively, for the benefit of the follower, and not beyond of them. So, $x_1^p = 0.2 (x_1^l < 0.2 < x_1^f)$ and $x_2^p = 0.1(x_2^l < 0.1 < x_2^f)$

act as lower-tolerance limits of the decisions $x_1$ and $x_2$, respectively.

Following the procedure and using the above numerical values, the membership functions of the defined fuzzy goals can be constructed by using (9.12), (9.13) and (9.14). Then, the resultant FGP model appears as:

Find $(x_1, x_2, x_3)$ so as to:

Minimize $Z = \frac{1}{3.3751}d^-_1 + \frac{1}{2.4553}d^-_2 + \frac{1}{0.2360}d^-_3 + \frac{1}{0.5541}d^-_4$

and satisfy:

\[ \mu_{F_1} : \frac{5x_1 + 6x_2 + 3x_3 - 2.7291}{3.3751} + d^-_1 - d^+_1 = 1, \]
subject to the system constraints in (9.17)

\[ \mu_{F_1} : \frac{2x_1 + 3x_2 + 8x_3 - 2.8343}{2.4553} + d_2^- - d_2^+ = 1, \]

\[ \mu_{x_1} : \frac{x_1 - 0.2}{0.2360} + d_3^- - d_3^+ = 1, \]

\[ \mu_{x_2} : \frac{x_2 - 0.1}{0.5541} + d_4^- - d_4^+ = 1, \]

\[ d_q^+, d_q^- \geq 0, q = 1,2,3,4 \]  

(9.18)

The obtained solution of the problem in (9.18) is

\( (x_1, x_2, x_3) = (0.4360, 0.6530, 0.0009) \) with \( (F_1, F_2) = (6.1007, 2.8382) \)

The achieved membership values are

\( \mu_{F_1} = 0.9990, \mu_{F_2} = 0.0016, \mu_{x_1} = 1 \) and \( \mu_{x_2} = 0.9980. \)

The result shows that a most satisfactory decision is achieved from the point of view of hierarchical execution of the decision powers of the DMs in the decision making environment.

**Note 9.1:** If the conventional fuzzy min-max approach [710] is used to solve the problem in the same decision making environment, where maximization of \( \lambda \) subject to \( \lambda \) 'less than equal to' for all the defined membership functions with \( 0 \leq \lambda \leq 1 \) is considered, then the solution using the Software LINGO (version 11.0) is found as \( (x_1, x_2, x_3) = (0.3327, 0.4116, 0.2753) \) with \( (F_1, F_2) = (4.9590, 4.1026) \).

The achieved membership values are

\( \mu_{F_1} = 0.6607, \mu_{F_2} = 0.5166, \mu_{x_1} = 0.5623 \) and \( \mu_{x_2} = 0.5624. \)

The result indicates that, although the hierarchical order of the decision powers of the DMs is preserved here, the solution is inferior in comparison to the solution obtained by using the proposed GA based FGP approach in terms of obtaining a better decision of the leader using the proposed approach.
It is to be noted that there is a paradox that the leader's decision is often dominated by the follower from the viewpoint of achievement of the aspired goal levels. But, in case of the model solution, under the proposed approach, the leader's higher power of making decision in the hierarchical decision system is preserved and a satisfactory solution for the follower within the specified tolerance range is achieved here in the decision situation.

A graphical representation of the goal values achieved with the use of the two different approaches is displayed in the Figure 9.1.

A comparison of the model solutions shows that the solution under the proposed model is superior over the conventional FP approach [710] in the decision making environment.

9.6 Conclusion

The main advantage of the proposed approach is that a compromise decision for achievement of the aspired goal levels of the objectives defined individually for each of them can be made on the basis of their weights of importance and the admissible tolerance values of the aspired goal levels of the objectives. Further, the proposed FGP model is flexible enough to accommodate different other aspiration levels and
associated tolerance ranges defined in the decision situation and that depends on the DMs' needs and desires in the decision making context.

Here, under the proposed approach, the defined random parameters can easily be accommodated within the framework of the proposed model without involving any computational complexity.

The coefficients of the fuzzy objective goals may also be considered random, which is a problem for future study. An extension of the approach to BLPPs having non-linear objective functions involving both the fuzziness and randomness may be one of the current research problems. However, it is expected that the approach presented here can contribute to future study in the field of real-life multiobjective hierarchical decentralized decision problems in uncertain decision environment.