Chapter 4

A Genetic Algorithm Based Fuzzy Goal Programming Approach for Solving Fractional Bilevel Programming Problems

The Content of this Chapter is based on two Papers:

   (International Conference on Operations Research Applications in Engineering and Management).

4.1 Introduction

Bilevel programming (BLP) is a special case of multilevel programming (MLP) for solving hierarchical decision problems. In BLPP, two decision makers (DMs) are located at the two hierarchical decision levels, each independently controlling a vector of decision variables for optimizing the individual objectives which often conflict each other in the decision making situation.

In the hierarchical decision situation, although the lower-level DM (the follower) executes his/her decision power after execution of decision power of the upper-level DM (the leader), the decision of the leader is often affected by the reaction of the follower due to his/her dissatisfaction with the decision. As a consequence, decision deadlock often arises and the problem of distribution of proper decision powers to the DMs is encountered in a hierarchical decision making horizon.

The concept of hierarchical decision problem as a special field of study in the area of mathematical programming was first suggested by Burton et al. [61] for solving decentralized planning problems of large decision making organizations. The concept of mathematical formulation of BLPPs was introduced separately by Fortuny-Amat et al. [141] and Candler et al. [65]. Thereafter, during the 1980s, various version of BLPPs as well as MLPP problems (MPLPPs) in general were studied by Bard [26, 27, 28], Bialas et al. [43, 44], Candler et al. [63, 64], Unlu [487], Wen et al. [497], Ben-Ayed and Blair [38], Yang [505], Anandalingam [9] and the other researchers in the field from the view point of their potential use to different real-world problems as economic systems, warfare, network designs and especially for conflict resolutions. But, in the practical decision situations, they are computationally not efficient, especially for large and complex hierarchical decision problems. Again, most of the classical approaches developed so far for BLPPs often lead to the paradox that the leader's decision power is dominated by the follower.

To overcome the above problem, a multiobjective solution technique with post optimality analysis on the objective values based on the three compromise solutions: ideal point, threat point and ideal threat point, have been introduced by Wen et al. [498]. But, their approach does not always lead to a satisfactory solution in a highly conflicting hierarchical decision situation.
Now, in a hierarchical decision situation, it has been realized that of each of the DMs should have a motivation to cooperate each other to reach a minimum level of satisfaction of each of them and thereby to smoothing the activities of the organization. In such a situation, the concept of membership functions in fuzzy set theory, introduced by Zadeh [515], has been investigated by Lai in [272] to solve BLPPs as well as MLPPs. Thereafter, the supervised search procedure with use of max-min operator introduced by Bellman and Zadeh [37], have been studied by Shih et al.[449]. The conventional fuzzy programming (FP) approaches discussed by Zimmerman [523] have been further extended by Shih and Lee in [450] to solve hierarchical decision problems from the view point of making a balance of decision powers of the DMs in the decision making context. But, the main difficulty with the use a conventional FP approach is that there is a possibility of rejecting the solution again and again by the leader / follower due to his / her dissatisfaction with the solution and involvement of re-evaluation of the problem repeatedly with the elicited membership values in the solution search process.

To overcome the above difficulty, the FGP approach discussed by Pal and Moitra [374] has been extended in [12, 354, 376] and others to solve hierarchical decision problems.

In an FGP approach, the membership functions of the defined fuzzy goals of the DMs are transformed into membership goals by introducing under- and over-deviational variables and assigning the highest membership value (unity) as the aspiration level to each of them. In the decision making process, achievement of highest membership value of each of the membership goals to the extant possible by minimizing the under-deviational variables on the basis of weights of importance of achieving the fuzzy goals and thereby to reach a satisfactory decision is taken into account.

In a hierarchical decision making context, since the interests of the DMs conflict each other, the use of an FGP approach discussed by Pal and Biswas [353] leads to make the decision environment a flexible (soft) one to reach an appropriate decision on the basis of needs and desires of the DMs in the decision making environment.

Further, the computational load for re-evaluation of the problem again and again as involved in the conventional FP approaches to reach the solution does not arise.
here in the use of the FGP approach, due to consideration of highest membership value as the achievement level of the defined membership goals in the decision search process.

Now, fractional programming, introduced by Charnes and Cooper [75], as a special field of study in the area of non-linear programming has been studied extensively in [50, 90, 439], and others in the past from the viewpoint of potential use to different real-world problems. Then, considering the multiobjective in nature of most of the real-life decision problems, fractional programming with multiplicity of objectives have been studied in [263, 463] and other pioneer researchers in the field. In most of the approaches developed so far in the past, the linearization method discussed by Charnes and Cooper [75] has been extended to solve fractional programming problems

The FP approaches to multiobjective fractional programming problems have been studied by Luhandjula [305] and Dutta et al. [123], in the past. The FGP approaches to MODM problems [48] with fractional criteria discussed by Pal et al. [376] have been extended further in [48, 309, 382] for solving fractional BLPPs (FBLPPs), where linear approximation methods are conventionally used in the process of solving the problems. However, the extensive study in this area for large scale hierarchical decision problems is at an early stage.

Now, GAs based on the natural selection and population genetics established by Holland [192] in 1973 have appeared as robust tools for searching solutions of different real-world decision problems. The GA methods to multiobjective decision problems with fractional objectives have also been suggested in [154, 521] from the viewpoint of avoiding the computational load with the use of linearization techniques to solve fractional programming problems.

The GA approach has also been successfully implemented by Hejazi et al. [188] to solve linear BLPPs. However, the extensive study on the use of GA methods to decision problems is yet to be widely circulated in the literature. Furthermore, the study in the field of using GAs to multiobjective fractional programming problems as well as large scale hierarchical decision problems with linear as well as fractional criteria is at an early stage.
In the present decision situation, since the objectives of both the DMs of the BLPP are inherently nonlinear in nature, computational complexity studied by Hannan [178] arises in the decision search process. Here, the use of conventional linearization methods suggested by Kornbluth and Steuer [263], Pal et al. [376] and others increase the computational load and inherent approximation error occurs in making proper decision.

To overcome the above difficulty, an GA method as the population based global solution search method can be efficiently used to solve the proposed problem without involving any computational complexity.

In this article, the potential use of an GA method to FGP formulation of an BLPP with fractional objectives is presented. In the process of formulating the model of the problem, the individual best and least solutions of the objective of each of the DMs located at the two different hierarchical decision levels are determined first for fuzzy description of the objectives and the control vector of the upper-level DM. Then, the membership functions of the defined fuzzy goals are constructed. In the FGP model formulation, the membership functions are transformed into membership goals by assigning the highest membership value (unity) as the aspiration level and introducing the under- and over-deviational variables to each of them. In the goal achievement function of the proposed FGP model, achievement of the highest membership value of each of the membership goals to the extent possible by minimizing the associated under-deviational variables on the basis of weights of importance of achieving the fuzzy goals is taken into account.

In the proposed GA scheme, the two major operators: Fitter-codon selection suggested [316, 507] and two-point crossover studied by Goldberg [164] which are the enhancement of contemporary approaches discussed by Gen et al. [154] and Zeng et al. [521] for multiobjective optimization problems are adopted here in the decision searching process to make a balance of the executing the decision powers of the DMs in the decision making environment.

A numerical example is solved to illustrate the approach and the obtained solution is compared with the conventional approaches [309, 382] studied previously.
4.2 FBLPP Formulation

Let \( X_1 \) and \( X_2 \) be the vectors of decision variables controlled by the leader and follower, respectively, in the hierarchical decision system, and \( Z_1, Z_2 \) be the objectives to be optimized at the two decision levels.

Then, the FBLPP in generic form can be presented as

Find \((X_1, X_2)\) so as to:

\[
\begin{align*}
\text{Max } Z_1(X_1, X_2) &= \frac{a_{11}X_1 + a_{12}X_2 + \alpha_1}{b_{11}X_1 + b_{12}X_2 + \beta_1} \\
\text{(Leader's problem)}
\end{align*}
\]

and, for given \( X_1, X_2 \) solves

\[
\begin{align*}
\text{Max } Z_2(X_1, X_2) &= \frac{a_{21}X_1 + a_{22}X_2 + \alpha_2}{b_{21}X_1 + b_{22}X_2 + \beta_2} \\
\text{(Follower's problem)}
\end{align*}
\]

subject to

\[
(X_1, X_2) \in S = \{(X_1, X_2) \mid M_1X_1 + M_2X_2 \leq c, \ X_1, X_2 \geq 0 \}
\]

where \( a_{ij}, b_{ij} \) (i, j =1, 2) and \( c \) are constant vectors and \( \alpha_i, \beta_i \) (i =1,2) are scalars and \( M_1, M_2 \) are constant matrices. It is assumed that \( S(\neq \emptyset) \) is bounded, and to preserve the feasibility of the decision in the solution search process

\[
b_{i1}X_1 + b_{i2}X_2 + \beta_i > 0 \quad (i =1, 2),
\]

are taken into consideration.

Now, to formulate the FGP model of the problem, the notion of goal satisficing philosophy, introduced by Simon [452], is considered and to reach a satisfactory decision, a GA scheme is employed there in the solution search process.

4.3 GA for BLPP

The two major operational activities in a GA approach are selection and crossover. In the solution search process the Fitter codon selection and Two-point crossover, are used and they are defined as follows:
Fitter codon selection

The codons are parts of a binary coded chromosome in a population of candidate solutions. In using conventional GA methods to optimization problems investigated in [154, 521], the Roulette-wheel scheme discussed in [164] is used for selection of parents to generate new population. In the Roulette-wheel selection scheme, the decimal equivalent of a binary coded chromosome is to be made to determine the fitness value of it at each generation in the selection search process. Whereas, in the fitter codon selection scheme investigated in [316, 507], a comparison of the selected strings by considering the specified string portion of certain length of each string is used to determine the fitter one. In the present selection scheme, the codon selection is made by considering the portion of a string of specific length from its most significant bit to a certain bit position. Here, consideration of the full string length of a chromosome as well as conversion to its decimal value is not required in the selection process. This process substantially reduces the computational load in the genetic search process.

Two-point crossover

In most of the contemporary approaches [154], single point crossover is used. In the presented work, two-point crossover [164] is proposed. This yields a completely changed new population from the initial population in a less number of iteration as compared to single point crossover.

The solution is achieved much faster by introducing the genetic operators.

4.3.1 Steps of the Proposed GA

The algorithmic steps in the genetic search process adopted are presented as follows:

Step 1. Representation and Initialization

Let $V_L$ denotes the binary coded representation of chromosome in a population as $V_L = \{x_1, x_2, ..., x_n\}$ where 'n' denotes the length of the chromosome and $L = 1, 2, ..., \text{pop_size}$, the population size, \text{pop_size} chromosomes are randomly initialized in its search domain.
Step 2. Fitness function

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as:

$$\text{eval} (V_L) = (Z_k)_L, \quad k = 1, 2; \quad L = 1, 2, \ldots, \text{pop\_size},$$

where $Z_k (k = 1, 2)$ is given by (1).

The best and least objective function values of the chromosome can be obtained as:

$$V^* = \max \{\text{eval} (V_L) \mid L = 1, 2, \ldots, \text{pop\_size}\}$$

or

$$V^* = \min \{\text{eval} (V_L) \mid L = 1, 2, \ldots, \text{pop\_size}\},$$

which depends on the needs and desires in the decision situation.

Step 3. Selection

In the fitter codon selection scheme, the selection of chromosomes is made on basis of their fitness scores defined in terms of closeness to the predefined level of fitness value as suggested in [507].

For an instance, the following four chromosomes in a population is considered.

$$111010000$$
$$111101010$$
$$010111010$$
$$101010010$$

Here, codons are selected from the stand point of maximum occurrence of dominant values of the most significant bits, where codon length is defined by the number of bits from most significant bit position to the position of first non-matching bit in the selected pair of chromosomes. It is to be observed here that the chromosomes in (i) and (ii) with codon length 4 are the fitter ones in comparison to the others defined in (iii) and (iv). Again, the chromosome in (ii) is fitter than the chromosome in (i).

From the above discussion, it is clear that the decimal equivalent of chromosomes is not required here for selection of fitter ones.
Step 4. Crossover

The probability of crossover is defined by the parameter $P_c$. Here, in a two-point crossover in the genetic system, the mating chromosomes interchange their middle portion in the process of reproduction. Again, a chromosome is selected as a parent, if for two defined random number $r$, $r_1 \in [0, 1]$; $r, r_1 < P_c$ with $r + r_1 < 1$ is satisfied.

In the selection of two parents, another random number $r_2$ is defined such that $r_2 = 1 - r_1$. Then, two parents $V_1, V_2 \in S$ yield two offspring as:

$E_1 = (r + r_2). V_1 + r_2V_2, \quad E_2 = r_1. V_1 + (r + r_2). V_2$, where $E_1, E_2 \in S$

Step 5. Mutation

The parameter $P_m$, as in the conventional GA scheme, is defined as the probability of mutation. The mutation operation is performed randomly on a bit-by-bit basis, where for a random number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < P_m$.

Step 6. Termination

The execution of the whole process terminates when the number of iterations is reached to the generation number specified in the genetic search process. The generated best chromosome is reported as the finally decision in the genetic search process.

Now, the FGP formulation is presented in the section 4.4.

4.4 FGP Formulation

To formulate the Fuzzy Goal programming model of the problem (4.1), the imprecise aspiration levels of the objectives of both the DMs and the decision vector $X$, controlled by the leader are to be determined first. Then, the defined fuzzy goals are characterized by the membership functions to measure the degree of goal achievements in terms of membership values.

Let, the individual best and least solutions of the leader are $(X_1^*, X_2^*, Z_1^*)$ and $(X_1^n, X_2^n, Z_1^n)$ respectively which can be obtained by using the proposed GA.
where $Z_i^b = \max_{(X_1, X_2) \in S} Z_i(X_1, X_2)$ and $Z_i^l = \min_{(X_1, X_2) \in S} Z_i(X_1, X_2)$.

Similarly, let the best and least solutions of the follower are $(X_1^b, X_2^b, Z_2^b)$ and $(X_1^l, X_2^l, Z_2^l)$, respectively where

$$Z_2^b = \max_{(x_1, x_2) \in S} Z_2(X_1, X_2) \quad \text{and} \quad Z_2^l = \min_{(x_1, x_2) \in S} Z_2(X_1, X_2)$$

Then, the fuzzy goals of the leader and follower can be defined as

$$Z_i \geq Z_i^b, \quad Z_2 \geq Z_2^b$$

with the control vector $X_i \geq X_i^b$.

Now, in the fuzzy decision making context, the lower tolerance limits of the leader and follower can be defined as $Z_i^b (Z_i^l < Z_i^b)$ and $Z_2^b (Z_2^l < Z_2^b)$, respectively.

Further, since leader has a higher power of making decision, a certain relaxation of $X_i^b$ as a lower tolerance limit should be given for searching a better decision by the follower.

Let $X_i^p ((X_i^b < X_i^p < X_i^b)$ be the lower tolerance limit of the decision $X_i$.

Then, characterizations of membership functions of the defined fuzzy goals are presented in the following Section 4.4.1.

### 4.4.1 Characterization of Membership Function

The tolerance membership function for the defined fuzzy goals can be expressed as:

$$\mu_{Z_i}(Z_i(X_1, X_2)) = \begin{cases} 
1 & \text{if } Z_i(X_1, X_2) \geq Z_i^b \\
\frac{Z_i(X_1, X_2) - Z_i^l}{Z_i^b - Z_i^l} & \text{if } Z_i^l \leq Z_i(X_1, X_2) < Z_i^b \\
0 & \text{if } Z_i(X_1, X_2) < Z_i^l 
\end{cases} \quad (4.2)$$

$$\mu_{Z_2}(Z_2(X_1, X_2)) = \begin{cases} 
1 & \text{if } Z_2(X_1, X_2) \geq Z_2^b \\
\frac{Z_2(X_1, X_2) - Z_2^l}{Z_2^b - Z_2^l} & \text{if } Z_2^l \leq Z_2(X_1, X_2) < Z_2^b \\
0 & \text{if } Z_2(X_1, X_2) < Z_2^l 
\end{cases} \quad (4.3)$$
In a hierarchical decision system, since execution of the decision powers of the DMs is sequential, i.e., the follower executes his/her decision power after execution of decision of the leader, a hierarchical decision problem cannot be considered as a general MODM problem. As such, a special structured mathematical programming problem, called BLPP, has been developed and several solution approaches have been investigated in the past.

Now, in solving BLPPs, the main shortcoming of using most of the previous approaches is that the leader’s decision often dominates the follower, and in no way his/her decision is sacrificed for a benefit of follower. As a matter of fact, decision deadlock arises due to dissatisfaction of the follower with the decision. In such a case, relaxation of the decision of the leader is inevitably needed for sustainable growth and overall benefit of the organization.

In the above situation, FGP as a robust and flexible technique can be potentially used to make a compromise decision with a certain relaxation of decision of the leader in the decision making context.

Now, in a fuzzy decision making environment, the aim of each of the DMs is to achieve the highest membership value (unity) of each of the fuzzy goals defined for his/her in the decision situation. To cope with the situation, FGP formulation is fruitful to apply to minimize the regrets of the DMs regarding achievement of the aspired fuzzy goal levels.

In FGP model formulation of the problem, the membership functions are transformed into membership goals by assigning the highest membership value as the aspiration level and introducing under- and over-deviational variable to each of them. Then, minimization of the under-deviational variables of the stated goals on the basis of their weights of importance of achieving the goal level is taken into account.

\[
\mu_{x_i}(x_i) = \begin{cases} 
1 & \text{if } x_i \geq x_i^b \\
\frac{x_i - x_i^p}{x_i^b - x_i^p} & \text{if } x_i^p \leq x_i < x_i^b \\
0 & \text{if } x_i < x_i^p 
\end{cases}
\]
In the present FGP formulation, the minsum FGP approach as the most flexible and widely used technique for MODM analysis studied by Pal et al. [371] as an extension of the conventional goal programming (GP) approach discussed by Romero [407] is considered.

In the goal achievement function of the minsum FGP, minimization of the sum of the under-deviational variables on the basis of the relative weights of importance of achieving the goals is taken into consideration.

The minsum FGP formulation can be stated as:

Find $X(X_1, X_2)$ so as to

Minimize: $Z = \sum_{k=1}^{2} w_k d_k^+ + w_3 d_3^-$

and satisfy

$\frac{Z_1(X_1, X_2) - Z_1^i}{Z_1^b - Z_1^i} + d_i^- - d_i^+ = 1,$

$\frac{Z_2(X_1, X_2) - Z_2^i}{Z_2^b - Z_2^i} + d_2^- - d_2^+ = 1,$

$\frac{X_i - X_i^p}{X_i^b - X_i^p} + d_3^- - d_3^+ = I.$

(4.5)

and the set of constraints in (4.1),

where $d_i^-, d_i^+ \geq 0, (i = 1, 2)$ represent the under- and over-deviational variables, associated with the $i$-th membership goals respectively. $d_3^-, d_3^+ \geq 0$ represent the vector of under- and over-deviational variables associated with the membership goals defined for the decision vector $X_1$, and $I$ is a column vector with all elements equal to 1 and the dimension of it depends on the decision vector $X_1$.

$Z$ represents goal achievement function, $w_k^+ > 0, k = 1, 2$, indicates the numerical weights of relative importance of achieving the aspired goal levels, and $w_3^+ > 0$ is the vector of numerical weights associated with $d_3^-$, and they are determined as [374]:

$w_k^+ = \frac{1}{Z_k^b - Z_k^w}, (k = 1, 2)$ and $w_3^+ = \frac{1}{X_i^b - X_i^w}$

Now, it is to be observed that the first two membership goals in (4.5) are fractional in form. Here, the use of a classical optimization method for fractional programming

account. In the present FGP formulation, the minsum FGP approach as the most flexible and widely used technique for MODM analysis studied by Pal et al. [371] as an extension of the conventional goal programming (GP) approach discussed by Romero [407] is considered.

In the goal achievement function of the minsum FGP, minimization of the sum of the under-deviational variables on the basis of the relative weights of importance of achieving the goals is taken into consideration.

The minsum FGP formulation can be stated as:

Find $X(X_1, X_2)$ so as to

Minimize: $Z = \sum_{k=1}^{2} w_k d_k^+ + w_3 d_3^-$

and satisfy $\frac{Z_1(X_1, X_2) - Z_1^i}{Z_1^b - Z_1^i} + d_i^- - d_i^+ = 1,$

$\frac{Z_2(X_1, X_2) - Z_2^i}{Z_2^b - Z_2^i} + d_2^- - d_2^+ = 1,$

$\frac{X_i - X_i^p}{X_i^b - X_i^p} + d_3^- - d_3^+ = I.$

(4.5)

and the set of constraints in (4.1),

where $d_i^-, d_i^+ \geq 0, (i = 1, 2)$ represent the under- and over-deviational variables, associated with the $i$-th membership goals respectively. $d_3^-, d_3^+ \geq 0$ represent the vector of under- and over-deviational variables associated with the membership goals defined for the decision vector $X_1$, and $I$ is a column vector with all elements equal to 1 and the dimension of it depends on the decision vector $X_1$.

$Z$ represents goal achievement function, $w_k^+ > 0, k = 1, 2$, indicates the numerical weights of relative importance of achieving the aspired goal levels, and $w_3^+ > 0$ is the vector of numerical weights associated with $d_3^-$, and they are determined as [374]:

$w_k^+ = \frac{1}{Z_k^b - Z_k^w}, (k = 1, 2)$ and $w_3^+ = \frac{1}{X_i^b - X_i^w}$

Now, it is to be observed that the first two membership goals in (4.5) are fractional in form. Here, the use of a classical optimization method for fractional programming
studied by Bitran et al. [50] often leads to a local optimal solution which is inherent to such a traditional method. Again, the use of conventional linearization approaches discussed in [75, 263] and others involve computational load and create decision trouble due to involvement of approximation error in linearization of the nonlinear form of goals.

To overcome the above situation, an GA method described in the Section 3.1 as the promising and volume-oriented (global one) search method and as a goal satisficer discussed by Deb in [105] rather than objective optimizer can be effectively employed to make a reasonable balance for execution of decision powers of the DMs in the decision making environment.

**Note 4.1:** It is worthy to note here that if more than one objective (linear / nonlinear) is involved at any of the hierarchical decision levels, then that can easily be incorporated within the framework of the proposed model without facing any computational difficulty.

Again, under the flexible nature of the proposed model, if relaxation of decision of the leader is further needed to make sensitivity analysis on the solution for arriving at a compromise decision, then that can be made in the decision search process without involving any computational complexity. Further, it may be noted that if some or all of the system constraints in (4.1) are nonlinear in nature, then computational complexity does not occur here for the use of GA scheme in the decision search process. Moreover, if fuzzy description of the system constraints is also needed in the decision situation, then that can easily be made there within the framework of the proposed FGP model formulation.

Finally, from the above view points, it may be concluded that the solution procedure presented here might be the workhorse for solving hierarchical decentralized planning problems.

Now, in the genetic search process of the proposed GA scheme, the fitness function for goal achievement is described in the following Section 4.5.
4.5 GA for FGP Model

The goal achievement functions $Z$ in (4.5) appears as the fitness function in the evaluation process of using the GA. The evaluation function for judging the fitness of a chromosome can be represented as:

$$\text{eval}(V_L) = (Z_k) = \left( \sum_{d=1}^{k} w_{k}^{d} + w_{k}^{d} \right), \quad k = 1, 2; \quad L = 1, 2, \ldots, \text{pop\_size}. \quad (4.6)$$

Here, the chromosome $V^*$ with the best fitness value at each generation is determined as

$$V^* = \min \{ \text{eval}(V_i) \mid L = 1, 2, \ldots, \text{pop\_size} \}$$

in the genetic search process.

To illustrate the proposed approach, a numerical example studied previously by Pal et al. [382] is solved.

4.6 Numerical Example

The problem solved by Pal et al. [382] using linear approximation approach is as follows:

Find $X(x_1, x_2)$ so as to

Maximize: $Z_1 (x_1, x_2) = \frac{(2x_1 + x_2)}{(2x_1 + 3x_2 + 1)}$ \quad (Leader's problem)

and for given $x_1, x_2$ solves

Maximize $Z_2 (x_1, x_2) = \frac{(x_1 + 2x_2)}{(x_1 + x_2 + 1)}$ \quad (Follower's problem)

subject to $-x_1 + 2x_2 \leq 3$, $2x_1 - x_2 \leq 3$, $x_1 + 2x_2 \geq 3$, \quad (4.7)

$x_1, x_2 \geq 0$.

Now, in the evaluation process of GA scheme, the following genetic parameters are considered.

- probability of crossover $P_c = 0.8$
- probability of mutation $P_m = 0.08$
- population size $= 100$
Chromosome length = 30.

The GA is implemented using Programming Language C. The execution is made in Intel Pentium IV with 2.66 GHz Clock-pulse and 1 GB RAM.

The individual best and least solution of the leader are obtained by the proposed GA as

\[(x_1^*, x_2^*, Z_1^*) = (1.80, 0.60, 0.6562)\]

and \[(x_1^*, x_2^*, Z_1^*) = (0, 1.50, 0.2727)\],

and the solutions for them of the follower are obtained as

\[(x_1^f, x_2^f, Z_2^f) = (2.9809, 2.9904, 1.2855)\]

and \[(x_1^f, x_2^f, Z_2^f) = (1.7989, 0.6006, 0.8825)\].

Then the fuzzy goals can be defined as:

\[Z_1 \geq 0.6562, \quad Z_2 \geq 1.2855\] and \[x_1 \geq 1.80\].

The lower tolerance limits of \(Z_1\) and \(Z_2\) are obtained as \(0.2727\) and \(0.8825\), respectively. Again the lower tolerance limit of \(x_1\) is taken as \(1.70\).

Then, the membership functions are determined as;

\[
\mu_{Z_1} = \frac{Z_1 - 0.2727}{0.6562 - 0.2727} \\
\mu_{Z_2} = \frac{Z_2 - 0.8825}{1.2855 - 0.8825} \\
\mu_{x_1} = \frac{x_1 - 1.70}{1.80 - 1.70}
\]

(4.8)

The executable FGP model of the problem is obtained as

Find \((x_1, x_2)\) so as to minimize

\[Z = 2.6076 \ d_1^- + 2.4814 \ d_2^- + 10 \ d_3^- \]

and satisfy

\[
\frac{2.6076(2x_1 + x_2)}{(2x_1 + 3x_2 + 1)} + d_1^- - d_1^* = 1, \\
\frac{2.4814(x_1 + 2x_2)}{(x_1 + x_2 + 1)} + d_2^- - d_2^* = 1, \\
10(x_1 - 1.70) + d_3^- - d_3^* = 1.
\]

\[d_i^-, d_i^* \geq 0, \ i = 1, 2, 3.\]

(4.9)

subject to the given system constraints in (4.7).
Now, following the GA scheme with the evaluation function defined in (4.6), the resulting solution is obtained as

\[(x_1, x_2) = (1.80, 0.60) \text{ with } (Z_1, Z_2) = (0.6562, 0.8825).\]

The achieved membership values are \(\mu_{Z_1} = 1\), \(\mu_{Z_2} = 0\) and \(\mu_{x_1} = 1\).

The result shows that a satisfactory decision is reached here on the basis of the hierarchical execution process of the DMs in the decision making situation.

**Note 4.2:** A comparative study of the proposed solution with the solutions of the approaches studied by Malhotra et al. [309] and Pal et al. [382] is made as follows:

(i) The solution of the problem obtained by Malhotra et al. [309] by using conventional GP approach is \((x_1, x_2) = (3, 3) \text{ with } (Z_1, Z_2) = (0.5625, 1).\)

(ii) The solution of the problem obtained by Pal et al. [382] by using FGP approach with linearization technique is \((x_1, x_2) = (2, 1) \text{ with } (Z_1, Z_2) = (0.625, 1)\)

Here, in both the cases, it is to be noted that the leader’s decision power is dominated by the follower in terms of achieving the objective function values in the order of hierarchy of execution of their decision powers.

A graphical representation of the achieved goal values for the use of the proposed approach and two different approaches of the previous study is displayed in the Figure 4.1.

![Figure 4.1: Goal achievement under different approaches](image-url)
It is apparent from the results that the proposed GA approach offers the best decision from the viewpoint of satisfaction of both the DMs with the solution in the decision making environment.

4.7 Performance Comparison

More to explore the potential use of the proposed solution approach, a modified version of the problem presented in the Section 4.6 is considered.

In this problem, the objectives of the leader and follower are those of the follower and leader, respectively. Actually, the objectives of the leader and follower are considered here as:

Maximize: \( Z_1(x_1, x_2) = \frac{(x_1 + 2x_2)}{(x_1 + x_2 + 1)} \) (Leader’s problem)

and for given \( x_1, x_2 \) solves

Maximize \( Z_2(x_1, x_2) = \frac{(2x_1 + x_2)}{(2x_1 + 3x_2 + 1)} \) (Follower’s problem)

The decision variable \( x_1 \) is also considered under the control of the leader.

The system constraints and the other restrictions are identical to those given in (4.7). Then, the best and least solutions of the leader are obtained by using the proposed GA scheme are

\[ (x_1, x_2, Z_1^*) = (2.9809, 2.9904, 1.2855) \]

and \( (x_1, x_2, Z_1^*) = (1.7989, 0.6006, 0.8825) \), respectively, which are actually the best and least solutions, respectively, obtained for the follower in case of the problem in (4.7).

Similarly, the best and least solutions of the follower are found as

\[ (x_1, x_2, Z_2^*) = (1.80, 0.60, 0.6562) \]

and \( (x_1, x_2, Z_2^*) = (0, 1.50, 0.2727) \), respectively.

Here, the lower tolerance limit of the decision \( x_1 \) is considered 2.

Now, in an analogous to the model formulation in (4.9), the executable FGP model of the problem can easily be obtained.
Then, employing the same GA scheme, the resultant decision is obtained as 
\[(x_1, x_2) = (2.9809, 2.9904) \text{ with } (Z_1, Z_2) = (1.2855, 0.5619)\].
The achieved membership values are \(\mu_{Z_1} = 1, \mu_{Z_2} = 0.7540\) and \(\mu_{x_1} = 0.99\).

Here, the result reveals how the solution changes with the change of the hierarchical structure of the decision organization.
The solutions of the problem obtained by using the approaches studied previously are now discussed as follows:

(i) The solution of the problem studied by Pal et al. [382] by using conventional linear approximation approach is 
\[(x_1, x_2) = (3, 3) \text{ with } (Z_1, Z_2) = (1.2857, 0.5625)\].
The obtained membership values are \(\mu_{x_1} = 0.6538\) and \(\mu_{x_2} = 1\).
The result reflects that the better decision is actually achieved by using the proposed GA scheme from the viewpoint of more satisfaction of the follower with the achieved membership value of the fuzzy objective goal defined for him/her.

(ii) If the linear approximation technique studied by Pal et al. [374] is used to solve the problem, then the resulting decision obtained by using Software LINGO (version.6.0) is 
\[(x_1, x_2) = (2, 1) \text{ with } (Z_1, Z_2) = (1, 0.625)\].
The achieved membership values are \(\mu_{x_1} = 0.29, \mu_{x_2} = 0.91\) and \(\mu_{x_1} = 1\).
The result shows that the leader's decision power is here dominated by the follower. As a matter of fact, the solution is quite unacceptable in the decision situation due to violation of hierarchical ordering of the decision structure of the organization.

(iii) If the traditional min-max fuzzy operator studied by Zimmermann [523] is used without linearization of the fractional membership goals, then the solution is found as 
\[(x_1, x_2) = (2.71, 2.42) \text{ with } (Z_1, Z_2) = (1.232, 0.573)\].
The resulting membership values are \(\mu_{z_1} = 0.81, \mu_{z_2} = 0.71\) and \(\mu_{x_1} = 0.71\).
The result indicates that, although the hierarchical order of the decision powers of the DMs is preserved here, the solution is inferior in comparison to the solutions obtained by using the proposed GA based FGP approach and the conventional FGP approach studied previously by Pal et al. [382].

Remark: From the above discussion and solution comparison, it may be claimed that the proposed approach is superior over the other approaches studied previously with regard to the view points of proper distribution of decision powers to the DMs as well as arriving at the most satisfactory decision in the decision making environment.

4.8 Conclusion

This paper presents how the GA method can be used to solve a FBLPP in the framework of FGP. The main advantage of the approach presented here is that the computational load with the linearization of the goals can be avoided with the use of the proposed GA. Further, upon the flexible nature of FGP, several other restrictions on the basis of needs and desires of the DMs can be accommodated to reach a satisfactory decision in the decision making environment.

The efficiency of the proposed GA method may be improved by means of proper modification of the genetic operators in the solution searching process, which may be the problem for the future research. In future studies, the proposed method may be extended to solve multiobjective FBLPPs as well as FMLPPs with different fuzzy input parameters in the hierarchical decentralized decision making problems.