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ABSTRACT

This paper presents a genetic algorithm (GA) based fuzzy goal programming (FGP) solution method to problems with fractional criteria. In the model formulation of the problem, first fractional objectives are imprecise aspiration levels to each of them by employing the proposed GA. Then, the concept of membership measuring the degree of achievement of the fuzzy goals by defining the tolerance limits of them is introduced. The achievement of the highest membership values (unity) of the defined membership functions is iteratively used to the FGP formulation by defining the fitness function and without linearizing the conventional linear transformation approach to reach a satisfactory decision in the decision making environment. Euclidean distance function is used to perform the sensitivity analysis with the change of priorities and structure under which the most satisfactory decision can be reached in the decision situation. Two numerical examples are given and the model solution of a problem is compared with the linear transformation approach strategy.

INDEX TERMS

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  o Controlled Indexing
decision making, decision theory, fuzzy set theory, genetic algorithms, mathematical programming

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Euclidean distance function, GA scheme, fitness function, fractional criteria, fractional membership functions

problems, fractional programming, fuzzy goal programming formulation, fuzzy set theory, genetic algorithms, membership functions

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A Genetic Algorithm Approach to Fuzzy Goal Programming Formulation of Fractional Multiobjective Decision Making Problems

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Abstract—This paper presents a genetic algorithm (GA) based fuzzy goal programming (FGP) solution method to multiobjective decision making (MODM) problems with fractional criteria.

In the model formulation of the problem, first fractional objectives are transformed into fuzzy goals by defining the imprecise aspiration levels to each of them by employing the proposed GA. Then, the concept of membership functions in fuzzy set theory (FST) for measuring the degree of achievement of the fuzzy goals by defining the tolerance limits of them is introduced in the decision making context.

In the executable FGP model, the achievement of the highest membership values (unity) of the defined membership goals to the extent possible by minimizing the associated underdeviational variables on the basis of the priorities of achieving the goals is considered.

In the solution process, the GA scheme is iteratively used to the FGP formulation by defining the fitness function and without linearizing the fractional membership goals unlike the conventional linear transformation approach to reach a satisfactory decision in the decision making environment.

In the decision process, the notion of Euclidean distance function is used to perform the sensitivity analysis with the change of priorities and thereby to identify the appropriate priority structure under which the most satisfactory decision can be reached in the decision situation.

Two numerical examples are solved to illustrate the approach and the model solution of a problem is compared with the linear transformation approach studied previously.

Keywords—Fractional programming, Genetic algorithm, Goal programming, Fuzzy goal programming, Fuzzy set theory.

I. INTRODUCTION

Fractional programming (FP) as a special field of study in non-linear programming (NLP) was initially introduced by Charnes and Cooper\textsuperscript{[1]} in 1962. During the mid-60s and early '70s of the last century, FP for single-objective optimization problems was studied\textsuperscript{[2, 3]} extensively from the viewpoint of its application to several real-life problems. For instance, cost benefit analysis in agricultural production planning, faculty and other staff allocation problems for minimizing certain ratios of students' enrolments and staff structure within academic units of educational institutions, and other optimization problems frequently involve the fractional objectives in a decision situation.

Now, since most of the decision making problems in practice are multiobjective in nature, FP with multiplicity of objectives have also been studied by the pioneer researchers\textsuperscript{[4, 5]} in the field.

The goal programming (GP) approaches\textsuperscript{[6, 7]}, as the prominent tools for multiobjective decision analysis, have been studied\textsuperscript{[8, 9, 10]} for decision analysis with fractional objectives in crisp decision making environment.

But, in contrast to single objective FP problems, multiobjective fractional programming (MOFP) problems has not been discussed that extensively and only few approaches in\textsuperscript{[8, 11]} have been documented in the literature.

However, in most of the real-life multiobjective decision situation, it is to be observed that the decision maker (DM) is often faced with the problem of setting the exact aspiration levels to each objectives due to inherent imprecise in nature of model parameters involved with the practical problems. To overcome such a problem, the FST initially introduced by Zadeh\textsuperscript{[12]} has been used to decision making problems\textsuperscript{[13]} with imprecise data.

Fuzzy programming approaches\textsuperscript{[14]} to FP problems\textsuperscript{[15]} and implementation to real-world problems has been studied\textsuperscript{[16, 17]} in the past.

The FGP approaches\textsuperscript{[18]} in the framework of conventional GP have also been studied for solving general MODM problems\textsuperscript{[19]} as well as problems with fractional criteria\textsuperscript{[20, 21]} in the past.

Now, the linear approximation approaches in\textsuperscript{[3]} are conventionally used to single objective as well as multiobjective decision problems in\textsuperscript{[22]} with fractional objectives to overcome the computational difficulty inherently involved therein in the solution process. Linear transformation approaches to fuzzily described multiobjective fractional programming problems have also been studied by Pal et al.\textsuperscript{[23]} in the recent past. However, the solution approaches to real-life problems with fractional objectives in an imprecise decision environment is at an early stage.
Now, in a decision making environment, GAs [24] based on the natural selection and population genetics, initially introduced by Holland [25], have also appeared as the prominent tools for multiobjective decision analysis. The GA based approaches [26, 27] to different real-world problems have been investigated in the past. The uses of GAs to different frameworks of several problems as well as implementation to real-life problems with fractional criteria have also been studied by Pal et al. [28, 29, 30] in the recent past. But, exploration of potential use of GA to MODM problems is at an early stage. Further, the methodological development of GA based approaches to general MOFP problems is yet to be circulated in the literature.

In this article, how an GA method can be applied to the general framework of FGP formulation of an MOFP problem is presented. In the proposed model, first the fractional objectives are transformed into fuzzy goals by assigning the fuzzy aspiration level to each of them with the use of the proposed GA scheme. Then, the membership functions for measuring the degree of achievement of fuzzy goals by defining the tolerance ranges for goal achievement are constructed.

In the executable FGP model formulation, achievement of the membership goals defined for the membership functions to the highest degree (unity) to the extent possible by minimizing the under-deviational variables associated with the fuzzy goals on the basis of priority and weights of importance of achieving the objective is taken into consideration.

In the solution process, the GA scheme is iteratively used to achieve a priority based solution in the decision making situation.

In the decision making context, the notion of Euclidean distance function is used to perform sensitivity analysis with the solutions under different priority structures and thereby arriving at the most satisfactory solution in the decision making environment.

The proposed approach is illustrated by two numerical examples and the model solution of one is compared with the solution of conventional FP approach studied [20] previously.

II. PROBLEM FORMULATION

The general format of a real valued MOFP problem can be stated as:

Find \( X(x_1, x_2, \ldots, x_n) \) so as to

Maximize \( Z_k(X), \ k \in K_1 \)

and Minimize \( Z_k(X), \ k \in K_2 \)

Subject to \( X \in S = \left\{ X \in \mathbb{R}^n \mid AX \left( \frac{b}{2} \right), X \geq b, b \in \mathbb{R}^m \right\} \)

(1)

Where \( A \) is a coefficient matrix and \( b \) is a resource vector.

It is assumed that the feasible region \( S \) is nonempty \((S \neq \emptyset)\), and \( K_1 \cup K_2 = \{1, 2, \ldots, K\} \) with \( K_1 \cap K_2 = \emptyset \).

Now, in the field of fuzzy programming, an imprecise aspiration level is assigned to each of the objectives and certain tolerance limit for achievement of the respective aspired level is taken into account.

In the proposed problem, since the objectives are fractional in form, an GA scheme is introduced in the solution search process for assigning the fuzzy aspiration level and then the tolerance limit to each of them.

The steps of the GA scheme used in the process of solving the problem are presented in the following Section III.

III. STEPS OF THE PROPOSED GA

Step 1. Representation and Initialization

Let \( V \) denote the binary coded representation of chromosome in a population as \( V = \{e_1, e_2, \ldots, e_a\} \). The population size is defined by \( \text{pop}_1 \) size, and \( \text{pop}_2 \) size chromosomes are randomly initialized in the search domain.

Step 2. Fitness Function

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as

\[ \text{eval}(V_i) = Z_s, \ i = 1, 2, \ldots, \text{pop}_1 \]  

where \( Z_s \) is given by (1).

The best chromosome with largest fitness value at each generation is determined as:

\[ V^* = \max \{\text{eval}(V_i) | i = 1, 2, \ldots, \text{pop}_1 \} \]

or, \( V^* = \min \{\text{eval}(V_i) | i = 1, 2, \ldots, \text{pop}_1 \} \)

which depends on searching of the best (or worst) value of an objective.

Step 3. Selection

The simple roulette-wheel scheme [26] is used for selecting two parents for mating purposes in the genetic search process.

Step 4. Crossover

The parameter \( P_c \) is defined as the probability of crossover. The arithmetic crossover operator (1-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the linear constraints set \( S \). Here, a chromosome is selected as a parent, if for a defined random number \( r \in [0, 1], r < P_c \) is satisfied.

For example, arithmetic crossover for two parents \( V_1, V_2 \in S \) yields two offspring

\[ E_1 = \alpha_1 V_1 + \alpha_2 V_2, \ E_2 = \alpha_2 V_1 + \alpha_1 V_2 \]

where \( \alpha_1, \alpha_2 \geq 0 \) with \( \alpha_1 + \alpha_2 = 1 \), always belong to \( S \) and where \( S \) is a convex set.

Step 5. Mutation

As in the conventional scheme, a parameter \( P_m \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a Random Number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

Step 6. Termination

The execution of the whole process terminates when the number of iterations is reached to the generation number specified in the genetic search process. The generated best chromosome is reported finally in the solution search process.

Now, FGP formulation of the problem by defining the fuzzy goals is presented in the Section IV.
IV. FGP FORMULATION

In the present decision situation, the individual best solution of each of the objectives is considered as the fuzzy aspiration levels of the objectives and they are determined by employing the proposed GA scheme.

Let, $Z_{R1}$ and $Z_{R2}$ be the best solutions of the two types of objectives (max and min), respectively,

where $Z_{R1k} = \max X \in X Z_k(X)$, $k \in K_1$

and $Z_{R2k} = \min X \in X Z_k(X)$, $k \in K_2$

Then, the fuzzy objective goals can be obtained as:

$Z_k(X) \in \{Z_{R1}, Z_{R2}\} \quad k \in K$

and

$Z_k(X) \geq Z_{R1} \quad k \in K_1$

and

$Z_k(X) \leq Z_{R2} \quad k \in K_2$ (3)

where $\geq$ and $\leq$ refers to the fuzziness of the aspiration levels in the sense of Zimmermann [14].

Now, in the multiobjective decision situation, since the objectives often conflict each other for individual goal achievement, a certain tolerance level for goal achievement need be given to make an overall satisfactory decision under the given system constraints in the decision making context.

To make a reasonable balance of goal achievement, the individual worst objective function values are considered as the lower tolerance limit of the objective goals.

Let, $Z_{L1k}$ and $Z_{L2k}$ be the worst objective function values of the respective objectives, where

$Z_{L1k} = \min X \in X Z_k(X)$, $k \in K_1$

and $Z_{L2k} = \max X \in X Z_k(X)$, $k \in K_2$ (4)

Then, characterization of membership functions for goal achievement of the objectives within the tolerance ranges specified in the decision situation is presented in the following Section A.

A. Characterization of Membership Function

Let $\mu_k(X)$ be the membership function representation of the $k$-th fuzzy goal.

Then, for $\geq$ type of restriction, $\mu_k(X)$ takes the form

$\mu_k(X) = \begin{cases} 
1 & \text{if } Z_k(X) \geq Z_{R1k} \\
\frac{Z_k(X) - Z_{L1k}}{t_{ik}} & \text{if } Z_{L1k} \leq Z_k(X) < Z_{R1k} \\
0 & \text{if } Z_k(X) < Z_{L1k}
\end{cases}$ (5)

where, $t_{ik} = (Z_{R1k} - Z_{L1k})$ is the tolerance range for achievement of the $k$-th fuzzy goal, $k \in K_1$.

Similarly, for $\leq$ type of restriction, $\mu_k(X)$ appear as

$\mu_k(X) = \begin{cases} 
0 & \text{if } Z_k(X) \leq Z_{R2k} \\
\frac{Z_{R2k} - Z_k(X)}{t_{2k}} & \text{if } Z_k(X) < Z_{R2k} \leq Z_{L2k} \\
1 & \text{if } Z_k(X) > Z_{L2k}
\end{cases}$ (6)

where, $t_{2k} = (Z_{L2k} - Z_{R2k})$ is the tolerance range for achievement of the $k$-th fuzzy goal, $k \in K_1$.

Now, the FGP model formulation of the problem for the defined membership functions is presented in the following Section B.

B. FGP Model Formulation

The FGP model of the problem under a given pre-emptive priority structure can be obtained as:

Find $X$ so as to:

Minimize $Z = [P_1(d^-), P_2(d^-), \ldots, P_K(d^-)]$

and satisfy

$Z_k(X) - Z_{L1k} + d^- - d^+ = 1$

$Z_{L1k} - Z_{R1k} + d^- - d^+ = 1$

$d^-, d^+ \geq 0$, $k = 1, 2, \ldots, K$ (7)

subject to the given system constraints in (1), where $Z$ represents the vector of $K$ priority goal achievement functions, and $d^-, d^+$ are the under- and over-deviational variables, respectively, of the $k$-th membership goal. $P_i(d^-)$ is a linear function of the weighted under-deviational variables, and where $P_i(d^-)$ is of the form [20]:

$P_i(d^-) = \sum_{\ell=1}^{K} w_{i\ell} d^-_{\ell}$

$k = 1, 2, \ldots, K_i$, $i \leq K$ (8)

where $d^+_i (= 0)$ is renamed for $d^+_i$ to represent it at the $i$-th priority level, $w_{i\ell}$ (=$0$) is the numerical weight associated with $d^+_i$ and represents the weight of importance of achieving the aspired level of the $k$-th goal relative to the others which are grouped together at the $i$-th priority level.

The problem in (7) can be solved by employing the GA method with the associated evaluation function.

In the present decision process, the fitness function appears as:

$\text{eval}(X) = Z = \sum_{\ell=1}^{K} w_{i\ell} d^-_{\ell}$, where $v=1, 2, \ldots, \text{pop\_size}$. (9)
Now, in a decision making situation, achievement of the highest membership value of each fuzzy goal is a trivial one. Again, the DM is often confused with that of assigning the priorities to the conflict in nature of the goals from their achievement in the decision environment.

To overcome the above situation, the concept of Euclidean distance function introduced by Yu [31] can be used to select the appropriate priority structure and thereby to achieve a solution for satisfactory balance of goal achievement of the fuzzily described aspired goal levels.

C. Use of Euclidean Distance Function for Priority Structure Selection

The notion of distance function has been widely used [32, 33] to general MODM problems to arrive at a satisfactory decision. The Euclidean distance function can be presented as [31]:

\[ D_j = \sum_{k=1}^{n} (\mu^* - \mu_k)^2 \]

where, \( D_j \) indicates the distance between the utopia point (ideal point) \( \mu^* \) and the achieved membership value of the \( k \)-th goal under the \( j \)-th priority structure.

In the present decision situation, since the highest goal achievement value is 1, the ideal point would be a vector with each element equal to 1.

Then, with the realization of the fact that the most satisfactory solution is one which is closest to the ideal solution, so minimum of the distances would have to be considered to select the appropriate priority structure for goal achievement in the decision situation.

Let \( D_m = \min \{ D_j | j = 1, 2, \ldots, J \}, \quad 1 \leq m \leq J \)

Then, the \( m \)-th priority structure would be the appropriate one to reach a most satisfactory solution.

Two numerical examples are provided in the Section V to illustrate the potential use of the approach.

V. ILLUSTRATIVE EXAMPLES

**Example 1:**

The following fractional MODM problem is considered: Find \( X(x_1, x_2) \) so as to:

Minimize \( Z_1 = \frac{12x_1 - 10.95x_2 - 19.05}{x_1 - 2x_2 + 1} \)

Minimize \( Z_2 = \frac{5x_1 + 6x_2 + 4}{x_1 + 2x_2} \)

Maximize \( Z_3 = \frac{8x_1 + 5.9x_2}{x_1 - 2x_2 + 2} \)

Maximize \( Z_4 = \frac{12x_1 - x_2 + 2}{x_1 + 1} \)

Subject to

\[ x_1 + 2x_2 \leq 12, \]
\[ x_1 \geq 9, \]
\[ x_1 \leq 6, \]
\[ x_1, x_2 \geq 0. \]

Now, the following GA parameter values are adopted to determine the individual best and worst values of the objectives.

- Probability of crossover \( P_c = 0.8 \)
- Probability of mutation \( P_m = 0.08 \)
- Population size = 50
- Chromosome length = 100

The GA program is developed using the programming language C in the execution process with the hardware support of Intel Pentium IV with 2.66 GHz. Clock-pulse and 1 GB RAM.

Then, following the procedure, the individual best and worst values of the successive objectives are obtained as:

(i) \( Z^*_{11} = 8.5608, \quad Z_{1w} = 10.3607 \)

(ii) \( Z^*_{21} = 4.833, \quad Z_{2w} = 5.4962 \)

(iii) \( Z^*_{31} = 10.1062, \quad Z_{3w} = 6.4108 \)

(iv) \( Z^*_{41} = 11.2308, \quad Z_{4w} = 10.7882 \)

Then, the fuzzy objective goals appear as:

\[ Z_1^* = \frac{12x_1 - 10.95x_2 - 19.05}{x_1 - 2x_2 + 1} \leq 8.5608 \]

\[ Z_2^* = \frac{5x_1 + 6x_2 + 4}{x_1 + 2x_2} \leq 4.833 \]

\[ Z_3^* = \frac{8x_1 + 5.9x_2}{x_1 - 2x_2 + 2} \geq 10.1062 \]

\[ Z_4^* = \frac{12x_1 - x_2 + 2}{x_1 + 1} \geq 11.2308 \]

Now, defining the tolerance limits for the worst values of the objectives and then following the procedure, the membership goals of the fuzzy objectives are successively obtained as:

\[ \mu_{x_1} = \frac{10.3607 - 8.5608}{8.5608} = 1.8 \]

\[ \mu_{x_2} = \frac{5.4962 - 4.833}{4.833} = 0.6632 \]

\[ \mu_{x_3} = \frac{6.4108 - 5.4962}{5.4962} = 0.608 \]

\[ \mu_{x_4} = \frac{10.7882 - 11.2308}{11.2308} = 0.37 \]
Then, in the execution process, the two priority factors, \( P_1 \) and \( P_2 \), are assigned to the membership goals in (11) and the developed FGP model is executed under three different priority structures, where for the defined fitness function in (9), the same GA scheme employed previously is considered here in the decision search process.

The resulting decisions of the three runs are displayed in the following Table I.

<table>
<thead>
<tr>
<th>Run</th>
<th>Priority structure</th>
<th>Decision ( (x_1, x_2) )</th>
<th>Membership Values ( (\mu_1, \mu_2, \mu_3, \mu_4) )</th>
<th>Euclidean Distance ( (D) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( [P_1(d_1^1 + d_1^2), P_2(d_2^1 + d_2^2)] )</td>
<td>(12, 0)</td>
<td>(0.42, 1, 0.12, 0.25)</td>
<td>1.2935</td>
</tr>
<tr>
<td>2</td>
<td>( [P_1(d_1^1 + d_1^2), P_2(d_2^1 + d_2^2)] )</td>
<td>(5.8, 1.24)</td>
<td>(1.0, 0.99, 1)</td>
<td>1.00005</td>
</tr>
<tr>
<td>3</td>
<td>( [P_1(d_1^1 + d_1^2), P_2(d_2^1 + d_2^2)] )</td>
<td>(5.62, 1.34)</td>
<td>(1.0, 1, 1)</td>
<td>1</td>
</tr>
</tbody>
</table>

The results reflect that the minimum distance is \( D_3 = 1 \), which corresponds to the Run3 of the execution process.

Thus, the resultant solution is obtained as:

\[
(x_1, x_2) = (5.62, 1.34) \quad \text{with} \quad (Z_1, Z_2) = (8.5576, 10.287, 10.701, 4.836).
\]

The achieved membership values are \( (1, 0, 1, 1) \). The result indicates that the most satisfactory decision is obtained here from the viewpoint of achieving the aspired goal values on the basis of the needs and desires of the DM in the decision making environment.

**Example 2:**

The more to illustrate the potential use of the approach, the numerical example studied by Pal et al. in [20] is considered. The problem is of the form:

Find \( X(x_1, x_2) \) so as to

Maximize \( Z_1 = \frac{(x_1 - 4)}{(-x_2 + 3)} \),

and Minimize \( Z_2 = \frac{(-x_1 + 4)}{(x_2 + 1)} \),

subject to

\[
-x_1 + 3x_2 \leq 0, \\
x_1 \leq 6, \\
x_1, x_2 \geq 0.
\]

The solution of the problem obtained there by incorporating (2,1) as the aspiration levels of the successive objectives and (0,2) as their tolerance limits, respectively, and using the linearization technique to the minsum FGP formulation of the problem is \( (x_1, x_2) = (6, 2) \) with \( (Z_1, Z_2) = (2, -0.66) \).

The achieved membership values are \( (\mu_1, \mu_2) = (1, 1) \).

Now, to solve the problem by using the proposed approach, the computation environment as considered in the case of Example 1 is employed here.
The individual best and worst values of the successive objectives are obtained as:

(i) \( Z_{B_{i1}} = 2, \quad Z_{L_{i1}} = -4/3 \)

(ii) \( Z_{B_{i2}} = -2, \quad Z_{L_{i2}} = 4 \)

The fuzzy goals are then appear as

\[
Z_1 \leq 2, \quad Z_2 \leq -2.
\]

Then, following the procedure, the priority based FGP model can be constructed by following (7). The resulting decision of the problem under the two different assigned priorities is obtained as:

\( (x_1, x_2) = (6, 2) \) with \( (Z_1, Z_2) = (2, -0.66) \).

The obtained membership values are \( (\mu_1, \mu_2) = (1, 1) \).

The result shows that the solution obtained here is the same as that was obtained previously by using linearization technique. But, it is to be noted that the computational load involved with the linearization approach can be avoided here with the use of the proposed GA method.

VI. CONCLUSION

The main advantage of the proposed approach is that a most satisfactory decision can be obtained here by analyzing the formulated model of the problem under different priority structures using the notion of Euclidean distance function. Again, the computational load with the use of conventional linearization technique can be avoided here with the use of the proposed GA scheme.
The approach can easily be extended to real-life MODM problems with fractional as well as general nonlinear form of objectives in the decision making horizon.

In future study, the proposed approach can be extended to solve hierarchical decision making problems in an imprecise environment.

However, it is hoped that the proposed approach may open up many new areas for study in the current inexact MODM arena.

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GENETIC ALGORITHM BASED HYBRID GOAL PROGRAMMING FOR FUZZY MULTIOBJECTIVE FRACTIONAL PROGRAMMING PROBLEMS

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ABSTRACT

This paper presents a goal programming solution method based on genetic algorithms for fuzzy multiobjective fractional programming problems. In the proposed solution method, called the fuzzy hybrid-goal programming (FHGP), both the features of fuzzy goal programming and genetic algorithms are adopted to make most satisfactory decision for optimizing the objectives of the problem in the decision making environment.

In the proposed approach, first a genetically driven system is employed to develop the FHGP framework of the problem. Then in the decision process, the proposed genetic algorithm is used to achieve the highest membership values (unity) of the objectives to the extent possible on the basis of their relative importance of achieving the fuzzy aspirations.
levels assigned to them in the decision making context. The effectiveness of the approach is illustrated by two numerical examples studied [14] previously.

**KEYWORDS**: Fractional Programming, Genetic Algorithm, Goal Programming, Fuzzy Goal Programming, Fuzzy Hybrid Goal Programming.

1. INTRODUCTION

From the early 1970s to mid 80s of the last century, fractional programming (FP) as a separate area in non-linear programming (NLP) has been deeply studied [1,2,17] by the pioneer researchers and extensively appeared in the literature. The FP approaches to decision making problems with multiplicity of objectives have also been investigated [2] in the past from the viewpoint of their potential use to several real-life problems.

In most of the classical approaches developed so far for FP problems, linearization technique [1] is used in the solution search process. It may be pointed out here that in case of a non-linear FP problem where either one or both of the ratio objectives are non-linear in nature, computational complexity with the use of further approximation approach [15] to the problem is inevitably involved, and study in this area is very limited in the literature. To overcome such a difficulty, the goal programming (GP) approach [7] to FP problems has been studied [8] in the past. The use of goal satisfying philosophy in GP to multiobjective FP (MOFP) problems has also been investigated by Pal and Basu [11]. But extensive study in this area is yet to appear in the literature. During 1990s, the fuzzy programming approach to FP problems have been investigated [4,9] by the active researchers in the field. The fuzzy goal programming (FGP) approach to MOFP problems using linearization technique has been proposed by Pal et al. [14] in the recent past. The goal approximation approach [12] to bilevel programming problems with quadratic objective functions has also been recently studied by them. However, the extensive study in this area is at an early stage.

In the recent years, Genetic Algorithm (GA), based on the mechanics of natural selection and natural genetics, have emerged as a very promising tools to study a wide variety of optimization problems. During the past twenty years, GA based solution methods
to single-objective as well as multi-objective decision making (MODM) problems in the framework of conventional GP have been investigated [19]. But they are very few in number in the literature. As an intelligent heuristic search method, GAs to multi-objective fuzzy programming problems have also been studied [3, 10, 16] during the last few years. GA approach to FGP problems in the framework of additive model [18] has been studied by Gen et al [5] in 1997. However, genetically driven scheme for fuzzy decision making problems is still at the stage of infancy.

In the present paper, how GAs can be efficiently used for modeling and solving the FHGP formulation of the problem is presented. In the proposed approach, a genetic search process is introduced first to determine the fuzzy aspiration levels of the objectives and their tolerance limits for achievement in the decision situation. In the decision process, the GA search method is employed to achieve the highest membership values of the defined fuzzy goals to the extent possible by minimizing the under-deviational variable of the membership goal constraints of the proposed model.

2. PROBLEM FORMULATION

A real-valued MOFP problem can be presented as [14]:

Find $X$ so as to

Maximize $Z_k(X) = \frac{c_k X + \alpha_k}{d_k X + \beta_k}$, \quad k = 1, 2, \ldots, K.

Subject to,

$X \in S = \left\{ X \in \mathbb{R}^n \mid AX \preceq b, X \succeq 0, b \in \mathbb{R}^m \right\}$ (1)

in which it is assumed that the feasible region $S (\neq \emptyset)$ is bounded,

where $Z_k(X)$ is the $k$-th objective, $c_k$, $d_k$, $\alpha_k$, $\beta_k$ are the associated constant vectors and $A$ is an $(m \times n)$ real matrix. $(c_k X + \alpha_k)$, $(d_k X + \beta_k)$ are given functions with $(d_k X + \beta_k) > 0$, $\forall X \in S$. 

Now, since the objectives of a MODM problem often conflict each other, the decision maker (DM) is always confused with that of assigning the aspiration levels to the objectives and their tolerance limits for achievement in a fuzzy decision environment. Again, since the objectives are non-linear in form, computational difficulty is inevitably involved there in the decision making process, if a classical approach is used as discussed previously. To overcome this situation, GA solution method is used here in the decision search process.

2.1 The Steps of Proposed GA

The genetic search process adopted in the decision-making context is presented in the following steps.

Step - 1 : Representation and Initialization

Let $V$ denote the binary coded representation of chromosome in a population as $V = \{x_1, x_2, \ldots, x_n\}$. The population size is defined by pop-size, and pop-size chromosomes are randomly initialized in its search domain.

Step - 2 : Fitness Function

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as

$$eval(F_j) = Z_k, \quad i = 1, 2, \ldots, \text{pop-size}$$

where $Z_k$ is given by (1).

The best chromosome with largest fitness value at each generation is determined by

$$V^* = \{\max\{eval(V_j) \mid j = 1, 2, \ldots, \text{pop-size}\} \}$$

or

$$V^* = \{\min\{eval(V_j) \mid j = 1, 2, \ldots, \text{pop-size}\} \}$$

depending on searching out the best (or worst) value of an objective.
Step - 3 : Selection

The simple roulette-wheel scheme [6] is used for selecting two parents for mating purposes in the genetic search process.

Step - 4 : Crossover

The parameter $P_c$ is defined as the probability of crossover. The arithmetic crossover operator (1-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the linear constraints set $S$. Here a chromosome is selected as a parent, if for a defined random number $r \in [0, 1]$, $r < P_c$ is satisfied.

For example, arithmetic crossover for two parents $V_1, V_2 \in S$ yields two offspring

$$X_1 = \alpha_1 V_1 + \alpha_2 V_2, \quad X_2 = \alpha_2 V_1 + \alpha_1 V_2,$$

where $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$, always belong to $S$ and where $S$ is a convex set.

Step - 5 : Mutation

As in the conventional scheme, a parameter $P_m$ of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a Random Number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < P_m$.

Step - 6 : Termination

The execution of whole process terminates when the number of iterations is reached to the generation number specified in the genetic process. The best-generated chromosome is reported finally in the genetic search process.
2.2 Characterization of Membership Function

Let \( Z^B_k \) and \( Z^W_k \) be the best and worst values, respectively, of the k-th objective obtained by using the proposed GA. The fuzzy goals then appear as

\[
Z_k(X) \geq Z^B_k, \; k = 1,2,\ldots, K
\]

Then, for the given tolerance values, \( Z^W_k \), the k-th membership function can be algebraically constructed as

\[
\mu_{Z_k}(X) = \begin{cases} 
1 & Z_k(X) > Z^B_k \\
\frac{Z_k(X) - Z^W_k}{T^R_k} & Z^W_k \leq Z_k(X) \leq Z^B_k \\
0 & Z_k(X) < Z^W_k \\
\end{cases}
\]

where, \( T^R_k = (Z^B_k - Z^W_k) \) represents the tolerance interval for achievement of the stated k-th fuzzy goal.

Now, the FHGP formulation is presented in the following Section 3.

3. FHGP FORMULATION

In the field of conventional FGP, a membership goal corresponding to the k-th membership function is of the form [14]

\[
\mu_{Z_k}(X) + d^-_k - d^+_k = 1,
\]

where, \( d^-_k, d^+_k (\geq 0) \) represent the under- and over-deviational variables, respectively, from the aspired level (unity).

Then, in the 'fuzzy goal achievement function', \( d^-_k \) is minimized on the basis of its weight of importance relative to the others in the decision-making context.

Now, it is worthy to mention here that in case of the solution of a general FGP formulation, \( d^+_k \) takes a positive value when the aspired goal level is overly satisfied [13]. This generally happens due to under-estimation of the aspired goal level than what is actually attainable in the environment of making decision.

However, in the present decision situation, due to GA based setting of the individual best value to each of the objectives, a value higher than the estimated one (\( Z^B_k \)) for the k-th
fuzzy goal does not arise in the decision environment. From the above viewpoint, a generalized version of the framework of FGP, i.e., the FHGP can be formulated as follows:

Find $X$ so as to

Minimize: $Z = \sum_{k=1}^{K} w_k d_k$

subject to: $(1 - \mu_k(X)) \leq d_k$

$0 \leq d_k \leq 1$, $k = 0, 1, \ldots, K$  \hspace{1cm} (3)

where $Z$ represents goal achievement function, $d_k$ represents the under-deviation from the highest degree of satisfaction. $w_k (\geq 0)$ indicates the numerical weights of relative importance of achieving the aspired levels of the goal in the decision making context, and they are determined as [14]:

$$w_k = \frac{1}{T_k^R}, \quad k = 1, 2, \ldots K$$

It is to be noted that in the $k$-th membership goal constraint of the problem (3), $(1 - \mu_k(X))$ indicates the degree of dissatisfaction and $d_k$ represents the actual deviation from the aspired level $1$. The logical significance of it is that the DM’s level of dissatisfaction regarding achieving of the $k$-th objective should be kept lower than that actually take place there with the view to make a satisfactory balance of all the objectives of the problem, which is mathematically meaningful in the decision making context and similar to the conventional fuzzy linear programming formulation of a problem. Again, it is to be observed that the given system constraints of the given problem (1) are not taken into consideration in the proposed FHGP problem (3). This can easily be followed from the fact that, since the objectives are specified in the range $[z_k^P, z_k^W]$, $(k = 1, 2, \ldots K)$, the membership values must lie in the range $[0, 1]$. So, violation of system constraints with the solution of the problem (3) does not arise. The consideration of this aspect in formulating the FGP model of a problem with non-linear objectives has also been discussed by Pal and Moitra [12] in the recent past.
3.1 GA for FHGP

Since the membership functions $\mu_k(x)$, $(k = 1, 2, \ldots, K)$, are inherently non-linear in nature, the computational load for the use of linearization technique [14] is involved in the final decision process also. To overcome such a difficulty, the proposed GA is employed by defining here the fitness function as

$$\text{eval}(V_i) = \sum_{k=1}^{K} w_k d_k^i, \quad i = 1, 2, \ldots, \text{pop-size},$$

where $w_k, d_k^i$ are given by (3).

Here, the best chromosome $V^*$ with the largest fitness value at each generation is determined as

$$V^* = \min \{ \text{eval}(V_i) | i = 1, 2, \ldots, \text{pop-size} \}$$

in the genetic search process.

To illustrate the approach, two numerical examples are provided.

4. NUMERICAL EXAMPLES

Example - 1:

The following numerical example studied by Pal and Moitra [14] is solved.

Maximize $Z_1 = (x_1 - 4)/(-x_2 + 3)$ and

Maximize $Z_2 = (-x_1 + 4)/(x_2 + 1)$

subject to

$$-x_1 + 3x_2 \leq 0,$$

$x_1 \leq 6,$

$x_1, x_2 \geq 0.$

(4)

To formulate the fuzzy goals of the objectives of the problem (4), the proposed GA is coded in C and is run over VC++ on a Pentium III PC.
In the genetic search process, the parameter values are set as
\[ \text{pop-size} = 50, P_c = 0.8, P_m = 0.08 \text{ and generation number} = 5000. \]
The individual best objective values are successively obtained as
\[ Z_1^B = 2, Z_2^B = 4. \]
The fuzzy goals are then appear as
\[ Z_1 = 2, Z_2 = 4. \]
Again, the lower tolerance limits of the fuzzy goals, as the worst objective values
are determined as,
\[ Z_1^W = -4/3, Z_2^W = -2. \]
The respective membership functions are then constructed as
\[
\mu_{Z_1} = \frac{(x_1 - 4)/(-x_2 + 3) + 4/3}{10/3}, \mu_{Z_2} = \frac{(-x_1 + 4)/(x_2 + 1) + 2}{6} \tag{5}
\]
Now, following the procedure, the executable FHGP model of the problem is obtained
as
Find \(X(x_1, x_2)\) so as to
\[
\text{Minimize: } Z = \frac{1}{10/3} d_1^- + \frac{1}{6} d_2^- \\
\text{and satisfy } (1 - \mu_{Z_k}(X)) \leq d_k^-, 0 \leq d_k^- \leq 1, k = 1, 2
\]
where \(\mu_{Z_1}\) and \(\mu_{Z_2}\) are given by (5).
\[
\text{Minimize: } Z = \frac{1}{10/3} d_1^- + \frac{1}{6} d_2^- \\
\text{and satisfy } (1 - \mu_{Z_k}(X)) \leq d_k^-, 0 \leq d_k^- \leq 1, k = 1, 2
\]
where \(\mu_{Z_1}\) and \(\mu_{Z_2}\) are given by (5).

Now, using the proposed GA with the same parameter values and in the same
computational environment, the resultant decision of the problem (6) is obtained as
\[ (x_1, x_2) = (6, 2) \text{ with } (Z_1, Z_2) = (2, -2/3). \]
The achieved membership value are \((\mu_{Z_1}, \mu_{Z_2}) = (1, 0.222)\)

Note - 1:
It is to be noted here that the solution of the problem obtained by Pal et al. [14]
using linearization technique and then employing the FGP method is the same as that achieved here.
Example – 2 :

To illustrate more the effectiveness of the proposed approach, another example (example 2) studied by the same authors in which minimization of the second objective ($Z_2$) instead of maximizing it in the same decision environment is considered. To develop the FHGP model of the problem, the proposed GA with the same parameter values and in the same computational environment as defined in case of the Example 1 is used. Here, the FHGP model of the problem appears as:

Find $X(x_1, x_2)$ so as to

Minimize: $Z = \frac{1}{10/3}d_1 - \frac{1}{6}d_2^2$

and satisfy $(1 - \mu_{Z_k}) \leq d_k^1$,

$0 \leq d_k^1 \leq 1, \quad k = 1, 2$ \hspace{1cm}(7)

where the algebraic forms of $\mu_{Z_1}$ and $\mu_{Z_2}$ are

$\mu_{Z_1} = \frac{(x_1 - 4)/(-x_2 + 3) + 4/3}{10/3}, \quad \mu_{Z_2} = \frac{4 - (-x_1 + 4)/(x_2 + 1)}{6}$, respectively.

Following the same decision system, the solution to the problem (7) is obtained as

$(x_1, x_2) = (6, 2)$ with $(Z_1, Z_2) = (2, -2/3)$.

The obtained membership values are

$(\mu_{Z_1}, \mu_{Z_2}) = (1, 0.78)$ with $d_2^1 = 0.22$.

Note – 2 :

It is to be noted that the achieved optimal decision is the same as that obtained under the previous approach. But, it is to be observed here that the under-achievement of the second objective from its aspired level is 1.33, whereas over-achievement of it was identified in the previous study. This happens due to the fact that the fuzzy aspired level of $Z_2$ obtained here is -2/3 using the GA, instead of 1 in the previous study, which was arbitrarily introduced there.
5. CONCLUSION

In this paper, the FHGP formulation of a MOFP problem and the efficient use of GA to solve the problem is presented. The main advantage of the approach presented here is that, if the fractional objectives are non-linear in nature, the computational difficulty for further linearization of them using an approximation approach [15] does not involve here due to the use of GAs in the solution search process.

The proposed approach can be extended to optimization problems in different areas like fuzzy decentralized planning problems with fractional as well as general non-linear objectives at different decision making units, multiobjective non-linear transportation problems, and other multiobjective NLP problems involved with fuzzily described different parameters in the decision making context.

However, it is hoped that the proposed approach may open up new avenue of research for solving real-life complex fuzzy MODM problems.

REFERENCES


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A Genetic Algorithm Based Fuzzy Goal Programming Approach for Solving Fractional Bilevel Programming Problems

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Abstract - This paper presents a genetic algorithm (GA) based fuzzy goal programming (FGP) procedure for modeling and solving bilevel programming problems (BLPP) with fractional objectives in a hierarchical decision system.

In the proposed approach, the individual optimal decision of each of the decision makers (DMs) are determined first by using the GA method adopted in the process of solving the problem. Then the concept of membership functions in fuzzy sets for measuring the degree of satisfaction of the DMs regarding achievements of fuzzily described objective goals as well as the degree of optimality of the decision vector controlled by the upper-level DM are defined in the decision making context.

In the decision process, instead of using the conventional linear transformation approach the GA method is directly employed to the FGP formulation of the problem for achievement of the highest membership value (unity) of the defined membership goals of the problem.

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In the GA search process, the fitter codon selection scheme, two-point crossover and random mutation are adopted to reach a satisfactory solution in the decision making environment.

To illustrate the potential use of the approach, a numerical example is solved. The model solution is compared with the solution of the approaches studied [1, 10] previously.

Keywords: Bilevel programming, Fuzzy goal programming, Fuzzy programming, Genetic algorithm, Membership function.

1. Introduction

Bilevel programming (BLP) is a special case of multilevel programming problem (MLPP) for solving hierarchical decision problems. In BLP problems, two DMs are located at the two hierarchical decision levels each independently controlling a vector of decision variables for optimizing the individual objectives which often conflict each other in the decision situation.

In the hierarchical decision system, although the lower-level DM (the follower) executes his / her decision power after execution of decision power of the top-level DM (the leader), the decision of the leader is often affected by the reaction of the follower due to his/ her dissatisfaction with the decision. As a consequence, decision deadlock often arises and the problem of distribution of proper decision powers is encountered in a hierarchical decision making situation.

The concept of BLP was introduced separately by Fortuny-Amat and McCarl [3] and Candler and Townsley [4]. Thereafter, various version of BLP have been studied in [5, 6,
During the last 30 years, several solution approaches for BLPP [1, 4, 7, 9, 10, 11, 12] as well as MLPPs in have been properly introduced [13, 14, 15] by the pioneer researchers of the field. But, in a practical decision situation, the uses of such approaches lead to the paradox that the leader’s decision power is dominated by the follower.

To overcome the above problem, a multiobjective solution method with post optimality analysis on the objective values based on the three compromise solutions, ideal point, threat point and ideal threat point have been suggested by Wen and Hsu [16]. But, the use of such an approach often leads to an unsatisfactory solution in a highly conflicting hierarchical decision situation.

Now, in a hierarchical decision situation, it has been realized to the fact that there should have a motivation of each of the DMs to cooperate each other with a view to achieve a minimum satisfactory level of the objective of each of them. In such a situation, the concept of membership functions in fuzzy set theory introduced by Zadeh [17] in 1965 has been investigated by Lai [18] in 1996. Thereafter, supervised search procedures with use of max-min operator introduced by Bellman and Zadeh [19] have been studied in [20] from the view point of making a balance of decision powers of the DMs.

But, the main difficulty with the use of fuzzy programming (FP) approaches [21] developed so far is that there is a possibility of rejecting the solution again and again by the leader and re-evaluation of the problem with the elicited membership functions is repeatedly introduced in the solution search process due to conflicting in nature of the objectives.

To overcome the above difficulty, FGP approaches to hierarchical decision problems have been studied by Pal and Moitra [2], Pal and Pal [22], and others in the past.
Now, in a real life hierarchical decision situation, it is to be observed that the objectives with fractional criteria are frequently involved in making a balance of a trade and optimizing benefits in the planning environment, viz. optimization of the profit, cost, inventory, sales rating, and also interest on other ratios arise in the decision making context.

During the last 45 years, fractional programming as a special area of non-linear programming with single as well as multiplicity of objectives has been studied extensively [23, 24, 25] and widely circulated in the literature.

Now, considering the multiobjective in nature of most of the real-life decision problems, and overcoming the shortcoming of using traditional single objective transformation approach [26] to multiobjective fractional programming problems, FP approaches [21] to multiobjective fractional programming problems have been studied [27, 28] in the past.

The methodological aspects of fuzzy programming for solving BLPPs have been investigated in [18, 20] from the viewpoint of their uses to different real-life problems. The FGP approach to fractional BLP problems (FBLPPs) have been studied by Pal et al. [2, 29] in the recent past. But, the extensive study in this area for large scale hierarchical decision problems is at an early stage.

Now, GAs based on the natural selection and population genetics [30] have appeared as robust tools for searching solution of different real-world decision problems [31]. The GA methods to multiobjective decision problems with fractional objectives have also been studied [32, 33] in the recent past from the viewpoint of avoiding the computational load with the use of linearization techniques to solve fractional programming problems.
The GA approach has also been successfully implemented in [34] to solve linear BLP problems. However, the extensive study on the use of GA methods to decision problems is yet to be widely circulated in the literature. Furthermore, the study on GAs to multiobjective fractional programming problems as well as large scale hierarchical decision problems with linear as well as fractional criteria is at an early stage.

In this article, how a GA method can be efficiently used to solve BLP problems with fractional objectives under the framework of FGP is presented. In the process of formulating the model of the problem, the individual best and least objective function values of the DMs are evaluated first by using the proposed GA scheme. Then the membership functions of the fuzzily described objective goals and decision vector of the leader are constructed. In the FGP model formulation, achieving the highest membership value (unity) to the extent possible by minimizing the under deviational variables associated with the membership goals for the defined membership functions on the basis of weights of importance is taken into account.

In the proposed GA scheme, the two major operators: Fitter-codon selection [35, 36], two-point crossover [31], which are enhancement of contemporary approaches [32, 33] for optimization, are adopted in the decision searching process for balancing the decision powers of DMs.

A numerical example is solved to illustrate the approach and the obtained solution is compared with the conventional approaches studied [1, 10] previously.

2. FBLPP formulation
Let $X_1$ and $X_2$ be the vectors of decision variables controlled by the leader and follower, respectively, in the hierarchical decision system, and $Z_1$, $Z_2$ be the objectives to be optimized at the two decision levels.

Then, the FBLPP in generic form can be presented as

Find $(X_1, X_2)$ so as to:

$$\max_{X_1} Z_1(X_1, X_2) = \frac{a_{11}X_1 + a_{12}X_2 + \alpha_1}{b_{11}X_1 + b_{12}X_2 + \beta_1}$$

(Leader’s problem)

and, for given $X_1, X_2$ solves

$$\max_{X_2} Z_2(X_1, X_2) = \frac{a_{21}X_1 + a_{22}X_2 + \alpha_2}{b_{21}X_1 + b_{22}X_2 + \beta_2}$$

(Follower’s problem)

subject to

$$(X_1, X_2) \in S = \{(X_1, X_2) | M_1X_1 + M_2X_2 \leq c, X_1, X_2 \geq 0 \}$$

where $a_{ij}, b_{ij}$ (i, j =1, 2) and $c$ are constant vectors and $\alpha_i, \beta_i$ (i =1,2) are scalars and $M_1, M_2$ are constant matrices. It is assumed that $S (\neq \emptyset)$ is bounded, and to preserve the feasibility of the decision in the solution search process

$$b_{1i}X_1 + b_{2i}X_2 + \beta_i > 0 \ (i =1, 2).$$

Now, to formulate the FGP model of the problem and to reach a satisfactory decision in the notion of goal satisficing philosophy [37], the GA scheme introduced in the solution search process is presented in the Section 3.
3. GA Approach

The two major operational activities in a GA approach are selection and crossover. In the solution search process the Fitter codon selection and Two-point crossover, are used and they are defined as follows:

i. Fitter codon selection: codons are part of the chromosome represented in binary string. In contemporary approaches in the field of optimization problem [32, 33], selection based on Roulette-wheel scheme is used. According to evolutionary computation [30], chromosomes with high fitness from the initial population are most likely to reproduce. The Roulette-wheel scheme [31] provides a computational approach to this principle. The fitness computation in this scheme requires binary to decimal conversion of the entire string representing the chromosome. Whereas, in the proposed fitter codon selection scheme [35, 36], comparison in a portion of comparable strings determines the fitter chromosome. In the proposed problem, codons are considered as a portion of the string from its most significant bit position with an adjustable length. The length of the codon in a particular comparison will be counted from most significant bit to the position of first non matching bit in the comparable chromosome. So in this selection scheme, neither the full comparison nor the conversion into binary values is required. This substantially reduces the computational complexity during selection.

ii. Two-point crossover: In most of the contemporary approaches [38], single point crossover is used. In the presented work, two-point crossover [31] is proposed. This yields a completely changed new population from the initial population in a less number of iteration as compared to single point crossover.

The solution is achieved much faster by introducing the genetic operators.
3.1 Steps of the proposed GA

The algorithmic steps in the genetic search process adopted are presented as follows:

**Step 1.** Representation and Initialization

Let $V_L$ denotes the binary coded representation of chromosome in a population as $V_L = \{ x_{L1}, x_{L2}, x_{L3}, \ldots, x_{Ln} \}$ where 'n' denotes the length of the chromosome and $L = 1, 2, \ldots, \text{pop-size}$, the population size, pop-size chromosomes are randomly initialized in its search domain.

**Step 2.** Fitness function

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as

$$\text{eval}(V_L) = (Z_K)_L, \quad k = 1, 2; \quad L = 1, 2, \ldots, \text{pop-size},$$

where $Z_K$ ($k = 1, 2$) is given by (1).

The best chromosome for the best or least value of the objective is determined by

$$V^* = \max\{\text{eval}(V_L) \mid L = 1, 2, \ldots, \text{pop-size}\}$$

or

$$V^* = \min\{\text{eval}(V_L) \mid L = 1, 2, \ldots, \text{pop-size}\},$$

which depends on the needs and desires in the decision situation.

**Step 3.** Selection

The fitter codon selection scheme is used in the proposed GA [35]. Chromosomes are likely to be selected from the population depending on their fitness score. Though the merit of the fitter codon selection is to reach a solution with predefined level of fitness [36], it is used to reduce the computational complexity in the presented work.

Viz. in the following population,

i) $111010000$
Here, codons are determined by means of maximum occurrence of dominant values in the most significant bits. Codon length is the most significant bit to the position of first non-matching bits in the compared pair. E.g. the length is 4, when chromosome (i) and chromosome (ii) are compared. In the proposed selection scheme, it is not required to compute the strings from their most to least significant bits in order to convert them into corresponding decimal values and compare. The codon provides the fitter chromosome with less computational burden. E.g. in the above comparison, chromosome (ii) is fitter than chromosome (i), because all its entry inside the codon is dominant (one). However, chromosome (i) has one non-dominant (zero) value at its least position inside the codon. Similarly, if chromosome (iii) and (iv) are considered, codon with length 2 can decide the fitter one.

**Step 4. Crossover**

The parameter $P_c$ is defined as the probability of crossover. Two-point crossover of a genetic system is applied, where mating chromosomes interchange their middle portion in the process of reproduction. Here a chromosome is selected as a parent, if for a defined random number $r, r_1 \in [0, 1]; \ r, r_1 < P_c$ and further $r + r_1 < 1$ is satisfied.

Let $r_2 = 1 - r - r_1$

For example, two-point crossover for two parents $V_1, V_2 \in S$ yields two offspring as:

$X_1 = (r + r_2). V_1 + r_1 V_2, \ X_2 = r_1 V_1 + (r + r_2). V_2,$

**Step 5. Mutation**
As in the conventional GA scheme, the parameter $P_m$ of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < P_m$.

**Step 6. Termination**

The execution of the whole process terminates when the number of iterations is reached to the specified generation number in the genetic process. The best chromosome generated finally is reported in the genetic search process as the decision.

4. FGP Model Formulation

To formulate the Fuzzy Goal programming model of the problem (1), the imprecise aspiration levels of the objectives of both the DMs and the decision vector $X_i$ controlled by the leader are to be determined first. Then, the defined fuzzy goals are characterized by the membership functions to measure the degree of goal achievements in terms of membership values.

Let, the individual best and least solutions of the leader are $(X_{1}^{b}, X_{2}^{b}, Z_{i}^{b})$ and $(X_{1}^{n}, X_{2}^{n}, Z_{i}^{l})$ respectively which can be obtained by using the proposed GA

where $Z_{i}^{b} = \max_{(X_{1}, X_{2}) \in S} Z_{i}(X_{1}, X_{2})$ and $Z_{i}^{l} = \min_{(X_{1}, X_{2}) \in S} Z_{i}(X_{1}, X_{2})$.

Similarly, let the best and least solutions of the follower are $(X_{1}^{f}, X_{2}^{f}, Z_{2}^{b})$ and $(X_{1}^{n}, X_{2}^{n}, Z_{2}^{l})$, respectively where
\[ Z_b^i = \max_{(x_1, x_2) \in \mathcal{S}} Z_i (X_1, X_2) \] and \[ Z_i^l = \min_{(x_1, x_2) \in \mathcal{S}} Z_i (X_1, X_2) \]

Then, the fuzzy goals of the leader and follower can be defined as

\[ Z_i & Z_i^b, \quad Z_2 & Z_2^b \text{ with the control vector } X_i & X_i^b. \]

Now, in the fuzzy decision making context, the lower tolerance limits of the leader and follower can be defined as \[ Z_i^l (Z_i^l < Z_i^b) \] and \[ Z_2^l (Z_2^l < Z_2^b) \], respectively.

Further, since leader has a higher power of making decision, a certain relaxation of \( X_i^b \) as a lower tolerance limit should be given for searching a better decision by the follower.

Let \( X_i^b ((X_i^b < X_i^p < X_i^b)) \) be the lower tolerance limit of \( X_i^b \).

Then, characterizations of membership functions of the defined fuzzy goals are presented in the following Section 4.1

### 4.1. Characterization of membership function

The tolerance membership function for the defined fuzzy goals can be expressed as:

\[
\mu_{Z_i} (Z_i (X_1, X_2)) = \begin{cases} 
1 & \text{if } Z_i (X_1, X_2) > Z_i^b \\
\frac{Z_i (X_1, X_2) - Z_i^l}{Z_i^b - Z_i^l} & \text{if } Z_i^l \leq Z_i (X_1, X_2) \leq Z_i^b \\
0 & \text{if } Z_i (X_1, X_2) < Z_i^l 
\end{cases} \quad (2)
\]

\[
\mu_{Z_2} (Z_2 (X_1, X_2)) = \begin{cases} 
1 & \text{if } Z_2 (X_1, X_2) > Z_2^b \\
\frac{Z_2 (X_1, X_2) - Z_2^l}{Z_2^b - Z_2^l} & \text{if } Z_2^l \leq Z_2 (X_1, X_2) \leq Z_2^b \\
0 & \text{if } Z_2 (X_1, X_2) < Z_2^l 
\end{cases} \quad (3)
\]
4.2 FGP model formulation

Now, in a fuzzy decision making situation, the aim of each of the DMs is to achieve the highest membership values of each of the membership functions defined for his/her in the decision environment.

In FGP formulation of the problem, the membership functions are transformed into membership goals by assigning the aspiration level unity and under- and over-deviational variable to each of them.

Then, in the fuzzy goal achievement function, minimization of the sum of the under deviational variables, on the basis of relative weights of importance is considered.

The minsum FGP formulation can be stated as:

Find $X(X_1, X_2)$ so as to

Minimize: $Z = \sum_{i=1}^{2} w_i d_i^- + w_3 d_3^-$

and satisfy $\frac{Z_1(X_1, X_2) - Z_1^l}{Z_1^b - Z_1^l} + d_i^- - d_i^+ = 1$
where \( d_i^- , d_i^+ \geq 0 \), with \( d_i^- . d_i^+ = 0 \) \((i = 1, 2)\) represent the under- and over-deviational variables, associated with the \( i \)-th membership goals respectively. \( d_{j_1}^- , d_{j_1}^+ \geq 0 \) with \( d_{j_1}^- . d_{j_1}^+ = 0 \) represent the vector of under- and over-deviational variables associated with the membership goals defined for the decision vector \( X_i \), and \( I \) is a column vector with all elements equal to 1 and the dimension of it depends on the decision vector \( X_i \).

\( Z \) represents goal achievement function, \( w_k^- (\geq 0), k = 1, 2, \) indicates the numerical weights of relative importance of achieving the aspired goal levels, and \( w_3^- (\geq 0) \) is the vector of numerical weights associated with \( d_{j_1}^- \), and they are determined as \([22, 29]\):

\[
w_k^- = \frac{1}{Z_k^b - Z_k^a}, \quad (k = 1, 2) \quad \text{and} \quad w_3^- = \frac{1}{X_{i_k}^b - X_{i_k}^p}
\]

Now, it is to be followed that the first two membership goals associated with the fuzzy goal achievement of the leader and follower, respectively, are fractional in form. Here, the use of conventional linearization approach leads to a local optimal solution inherent in such solution approach. Also, computational load with the linearization method is involved there.

To overcome the above situation, the GA scheme described in the Section 3.1 is introduced hereto reach a satisfactory decision for both of the DMs.
5. GA for FGP

The goal achievement functions $Z$ here appears as the fitness function in the evaluation process of using the GA. The evaluation function for judging the fitness of a chromosome can be represented as:

$$\text{eval} (V_L) = (Z_k)_L = \left( \sum_{k=1}^{2} w_k d_1^k + w_3 d_3^k \right)_L, \quad k = 1, 2; \quad L = 1, 2, \ldots, \text{pop-size}. \quad (6)$$

Here, the chromosome $V^*$ with the best fitness value at each generation is determined as

$$V^* = \min \{ \text{eval} (V_L) \mid L = 1, 2, \ldots, \text{pop-size} \} \text{ in the genetic search process.}$$

To illustrate the approach, a numerical example studied previously [1, 10] is considered.

6. Numerical example

The problem solved by Pal et al. [1] using linear approximation approach is as follows:

Find $(x_1, x_2)$ so as to

Maximize $Z_1(x_1, x_2) = \frac{(2x_1 + x_2)}{(2x_1 + 3x_2 + 1)} \quad \text{(Leader's problem)}$

and for given $x_1, x_2$ solves

Maximize $Z_2(x_1, x_2) = \frac{(x_1 + 2x_2)}{(x_1 + x_2 + 1)} \quad \text{(Follower's problem)}$

subject to

$$-x_1 + 2x_2 \leq 3$$
$$2x_1 - x_2 \leq 3$$
$$x_1 + 2x_2 \geq 3$$
$$x_1, x_2 \geq 0 \quad (7)$$

Now, in the evaluation process of GA scheme, the following genetic parameters are considered.
probability of crossover $P_c = 0.8$
probability of mutation $P_m = 0.08$
population size 100
Chromosome length =100.
The GA is implemented using Programming Language C. The execution is made in Intel Pentium IV with 2.66 GHz. Clock-pulse and 1 GB RAM.
The individual best and least solution of the leader are obtained by the proposed GA as
$$(x_1, x_2, Z_f^L) = (1.80, 0.60, 0.6562), \quad \text{and} \quad (x_1, x_2, Z_f^U) = (0, 1.50, 0.2727)$$
and the solutions for them of the follower are obtained as
$$(x_1, x_2, Z_f^L) = (2.9809, 2.9904, 1.2855), \quad \text{and} \quad (x_1, x_2, Z_f^U) = (1.7989, 0.6006, 0.8825).$$
Then the fuzzy goals can be defined as:
$$Z_1 \leq 0.6562, \quad Z_2 \leq 1.2855 \quad \text{and} \quad x_1 \leq 1.80$$
The lower tolerance limits of $Z_1$ and $Z_2$ are obtained as 0.2727 and 0.8825, respectively.
Again the lower tolerance limit of $x_1$ is taken as 1.70
Then, the membership functions are determined as
$$\mu_{Z_1} = \frac{Z_1 - 0.2727}{0.6562 - 0.2727}$$
$$\mu_{Z_2} = \frac{Z_2 - 0.8825}{1.2855 - 0.8825}$$
$$\mu_{x_1} = \frac{x_1 - 1.70}{1.7491 - 1.70} \quad \text{(8)}$$
The executable FGP model of the problem is obtained as
Find $(x_1, x_2)$ so as to minimize
$$Z = 2.6076 \ d_1^+ + 2.4814 \ d_2^+ + 20.366 \ d_3^+$$
and satisfy

\[
2.6075 \frac{(2x_1 + x_2)}{(2x_1 + 3x_2 + 1)} + d^-_i - d^+_i = 1
\]

\[
2.4813 \frac{(x_1 + 2x_2)}{(x_1 + x_2 + 1)} + d^-_i - d^+_i = 1
\]

\[
20.366 (x_1 - 1.70) + d^-_i - d^+_i = 1
\]

\[d^-_i, d^+_i \geq 0, \ i = 1,2,3.\]

Now, following the GA scheme with the evaluation function defined in (6), the resulting solution is obtained as \((x_1, x_2) = (1.80, 0.60)\) with \((Z_1, Z_2) = (0.6562, 0.8825)\).

The achieved membership values are \(\mu_{Z_1} = 1, \mu_{Z_2} = 0, \mu_{Z_3} = 1.029\).

A comparative study of proposed approach with the solution approaches considered previously is discussed as follows:

i) The solution of the problem obtained in [10] by using conventional goal programming approach is \((x_1, x_2) = (3, 3)\) with \((Z_1, Z_2) = (0.5625, 1)\)

ii) The solution of the problem obtained by Pal et al.[1] by using FGP approach with linearization technique is \((x_1, x_2) = (2, 1)\) with \((Z_1, Z_2) = (0.625, 1)\)

A graphical representation of the goal values achieved by using the different approaches is displayed in the figure 1.
7. Conclusion

This paper presents how the GA method can be used to solve a FBLPP in the framework of FGP. The main advantage of the approach presented here is that the computational load with the linearization of the goals can be avoided with the use of the proposed GA. Further, upon the flexible nature of FGP, several other restrictions on the basis of needs and desires of the DMs can be accommodated to reach a satisfactory decision in the decision making environment.

The efficiency of the proposed GA method may be improved by means of proper modification of the genetic operators in the solution searching process, which may be the
problem for the future research. In future studies, the proposed method may be extended to solve multiobjective FBLPPs as well as FMLPPs with different fuzzy input parameters in the hierarchical decentralized decision making problems.

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A Goal Programming approach for solving Interval valued Multiobjective Fractional Programming problems using Genetic Algorithm

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Abstract

In this article, the efficient use of a genetic algorithm (GA) to the goal programming (GP) formulation of interval valued multiobjective fractional programming problems (MOFPPs) is presented. In the proposed approach, first the interval arithmetic technique [1] is used to transform the fractional objectives with interval coefficients into the standard form of an Interval programming problem with fractional criteria. Then, the redefined problem is converted into the conventional fractional goal objectives by using Interval programming approach [2] and then introducing under-and over-deviational variables to each of the objectives. In the model formulation of the problem, both the aspects of GP methodologies, minsum GP and minimax GP [3] are taken into consideration to construct the interval function (achievement function) for accommodation within the ranges of the goal Intervals specified in the decision situation where minimization of the regrets (deviations from the goal levels) to the extent possible within the decision environment is considered. In the solution process, instead of using conventional transformation approaches [4, 5, 6] to fractional programming, a GA approach is introduced directly into the GP framework of the proposed problem. In using the proposed GA, based on mechanism of natural selection and natural genetics, the conventional roulette wheel selection scheme and arithmetic crossover are used for achievement of the goal levels in the solution space specified in the decision environment. Here the chromosome representation of a candidate solution in the population of the GA method is encoded in binary form.

Index Terms

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A Goal Programming approach for solving Interval valued Multiobjective Fractional Programming problems using Genetic Algorithm

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Abstract— In this article, the efficient use of a genetic algorithm (GA) to the goal programming (GP) formulation of interval valued multiobjective fractional programming problems (MOFPs) is presented.

In the proposed approach, first the interval arithmetic technique [1] is used to transform the fractional objectives with interval coefficients into the standard form of an interval programming problem with fractional criteria. Then, the redefined problem is converted into the conventional fractional goal objectives by using interval programming approach [2] and then introducing under- and over-deviational variables to each of the objectives. In the model formulation of the problem, both the aspects of GP methodologies, minsum GP and minimax GP [3] are taken into consideration to construct the interval function (achievement function) for accommodation within the ranges of the goal intervals specified in the decision situation where minimization of the regrets (deviations from the goal levels) to the extent possible within the decision environment is considered.

In the solution process, instead of using conventional transformation approaches [4, 5, 6] to fractional programming, a GA approach is introduced directly into the GP framework of the proposed problem. In using the proposed GA, based on mechanism of natural selection and natural genetics, the conventional roulette wheel selection scheme and arithmetic crossover are used for achievement of the goal levels in the solution space specified in the decision environment. Here the chromosome representation of a candidate solution in the population of the GA method is encoded in binary form. Again, the interval function defined for the achievement of the fractional goal objectives is considered the fitness function in the reproduction process of the proposed GA.

A numerical example is solved to illustrate the proposed approach and the model solution is compared with the solutions of the approaches [6, 7] studied previously.

Keywords-genetic algorithm; goal programming; multiobjective fractional programming; fuzzy goal programming; interval programming

I. INTRODUCTION

Uncertainty in describing decision parameters of problems is inherently involved in different ways due to imprecise in nature of human judgment in real decision situations. In the context of making decision in uncertain environment, stochastic programming [8] has been studied extensively in the past. To cope with the decision situations in an imprecise environment, Fuzzy programming (FP) [9] has appeared as a prominent tool for decision analysis to real-world problems.

FP approaches to Multiobjective Decision Making (MODM) problems have been studied extensively [10, 11, 12] by the pioneer researchers in the field and they are implemented to several real-life problems. The methodological development of FP as well as Fuzzy goal programming (FGP) [13] as an extension of conventional GP [14, 15] is still the area for extensive study.

In the recent years, inexact programming [16, 17] has appeared as one of the promising tools for decision making to problems in an imprecise environment where setting of imprecise values to the parameters involved with the problem is not possible in the decision situation. In inexact programming, instead of assigning single parameter values (crisp or fuzzy), interval valued parameters are introduced depending on the decision making environment.

The methodological aspects of interval programming have been studied in [18, 19] at lot in the past. The interval programming approaches to decision problems within the frameworks GP have also been studied [20] in the recent past. The methodological aspects of interval programming studied previously have been surveyed by Oliviera and Anunes [21] in 2007.

The efficient use of interval programming approaches to mobile robot path planning [22] and portfolio selection [23] have been investigated in the recent past. However, methodological development of interval programming is at an early stage and its implementation to real life problems are not reported adequately in the literature. Furthermore, interval-
programming methodology for MOFPPs is so far yet to appear in the literature.

The fractional programming as a special field of study in non-linear programming have been studied extensively [24] in the literature. The linearization approaches [25, 26] are the best known and widely used for solving linear fractional programming problems with crisp or fuzzy parameter values. But computational load is inherently involved to apply conventional approaches to practical problems.

In the current computational world, GAs [27, 28], have been introduced into the field of mathematical programming as a prominent and most flexible search tools for complex decision analysis.

During the last twenty-five years, the power of GAs to solving optimization problems have been investigated [29, 30, 31, 32, 33, 34] by the pioneer researchers in the field. The GA based approaches to decision making problems in the framework of GP have presented by Poulos et. al.[35], Stewart et. al.[36] and Zheng et. al [37], and others in the recent past.

The GAs to interval programming problems is yet to be circulated in the areas of MODM problems. Again, GAs to interval valued MOFPPs are so far yet to appear in the literature.

In the present study, a GA method is introduced to the GP formulation of the interval valued MOFPPs. In using the GA process, the satisficing philosophy in GP exposed by Simon [38] is for goal attainment of the fractional goals defined for the interval objective functions and thereby arriving at a satisfactory decision. The proposed approach is illustrated by a numerical example, and the GA solution is compared with the adopted model solutions of the linearization approaches studied in the past.

II. PROBLEM FORMULATION

A. Interval valued MOFP problem formulation

The generic form of a interval valued MOFPPs can be stated as:

\begin{equation}
\begin{aligned}
\text{Optimize:} & \quad [c_{IL}, c_{IU}]X + [\alpha_{IL}, \alpha_{IU}] \\
\text{subject to} & \quad X \in S = \left\{ X \in \mathbb{R}^n \mid AX = b, X \geq 0, b \in \mathbb{R}^m \right\}
\end{aligned}
\end{equation}

where L and U denote the lower and upper bound, respectively, of the defined intervals.

Also, it is assumed that

\[ [d_{IL}^L, d_{LU}^U]X + [\beta_{IL}^L, \beta_{LU}^U] > 0, \quad \forall X \in S \]

Now, using the rules of interval arithmetic operation [35], the interval valued objectives in (1) can be explicitly expressed as

\[ \left[ \sum_{j=1}^{n} c_{UL}^j x_j + \alpha_{UL}^j, \sum_{j=1}^{n} c_{LU}^j x_j + \alpha_{LU}^j \right] \]

\[ \left[ \sum_{j=1}^{n} d_{UL}^j x_j + \beta_{UL}^j, \sum_{j=1}^{n} d_{LU}^j x_j + \beta_{LU}^j \right] = [l_{UL}, u_{UL}] \]

In the standard form of interval valued objectives, the expression in (3) can be presented as [39]:

\[ \left[ \sum_{j=1}^{n} c_{UL}^j x_j + \alpha_{UL}, \sum_{j=1}^{n} c_{LU}^j x_j + \alpha_{LU} \right] \]

\[ \left[ \sum_{j=1}^{n} d_{UL}^j x_j + \beta_{UL}, \sum_{j=1}^{n} d_{LU}^j x_j + \beta_{LU} \right] = [l_{UL}, u_{UL}] \]

where \( x_j \) is the j th component of the decision vector \( X = (x_1, x_2, ..., x_n) \).

Now, GP formulation of the problem with the interval valued fractional objectives in (4) is discussed in the following Section III.

III. FORMULATION OF GP PROBLEM

To represent the problem in the framework of GP, the objectives in (4) are transformed into goals by using the interval arithmetic operation rules and introducing the under-and over-deviational variables to each of the single objective expressions obtained from (4).

The two objective goal expressions are obtained as:

\begin{equation}
\begin{aligned}
\sum_{j=1}^{n} c_{UL}^j x_j + \alpha_{UL} \\
\sum_{j=1}^{n} d_{UL}^j x_j + \beta_{UL} + \rho_{UL} - \eta_{UL} = t_{UL}, \quad k = 1, 2, ..., K
\end{aligned}
\end{equation}

\begin{equation}
\begin{aligned}
\sum_{j=1}^{n} c_{LU}^j x_j + \alpha_{LU} \\
\sum_{j=1}^{n} d_{LU}^j x_j + \beta_{LU} + \rho_{LU} - \eta_{LU} = t_{LU}, \quad k = 1, 2, ..., K
\end{aligned}
\end{equation}
where \( \rho_{UL}, \rho_{UL} \geq 0 \) represent under-deviational variables and \( \eta_{UL}, \eta_{UL} \geq 0 \) indicate over-deviational variables with \( \rho_{UL} \cdot \eta_{UL} = 0 \) and \( \rho_{UL} \cdot \eta_{UL} = 0 \) (\( k=1,2, \ldots, K \)) of the respective goal expressions.

Now, construction of the objective function for the problem of goal achievement is presented as follows:

A. Objective function construction

In GP, the objective function is termed as 'achievement function' in which minimization of the (unwanted) deviational variables on the basis of needs and desires of the DM is considered.

In the context of present formulation, both the prominent aspects of GP (minsum GP and minimax GP) are taken together as a convex combination of them to reach a satisfactory decision by minimizing the deviational variables of the defined goals within their specified target intervals. Here, the term 'regret function' instead of achievement function is used in the sense of minimizing the regrets (See [21]).

The regret function can be defined as follows:

From the viewpoint of minimizing the possible regrets to the extent possible, both \( \rho_{UL} \) and \( \eta_{UL} \) associated with the goal expression in (5) and (6) respectively are considered. This aspect is taken into account with a view to reach the proper decision in the decision making environment.

The regret function appears as:

\[
\minimize Z = \lambda \sum_{k=1}^{K} (w_{UL} \cdot \rho_{UL} + w_{UL} \cdot \eta_{UL}) + (1-\lambda) \max (\rho_{UL} + \eta_{UL}) ,
\]

\[
0 \leq \lambda \leq 1 \tag{7}
\]

where \( w_{UL}, w_{UL} \geq 0 \) represent the relative numerical weights of importance of minimizing the respective deviational variables for goal achievement in the decision situation, and where

\[
\sum_{k=1}^{K} (w_{UL} + w_{UL}) = 1
\]

Letting, \( \max (\rho_{UL} + \eta_{UL}) = d \), the executable GP model of the problem can be presented as:

Find \( X \) so as to

\[
\minimize Z = \lambda \sum_{k=1}^{K} (w_{UL} \cdot \rho_{UL} + w_{UL} \cdot \eta_{UL}) + (1-\lambda) d,
\]

Subject to \( (\rho_{UL} + \eta_{UL}) \leq d \), \( k=1,2, \ldots, K \),

and the given system constraints in (2).

Now, the proposed GA procedure for solving the problem in (8) is presented in the following Section IV.

IV. DESIGN OF THE GENETIC ALGORITHM

For the given GP structure of the proposed problem, the task of the Decision Maker (DM) is to search the solution which satisfies the fractional goals to the extent possible by evaluating the defined regret function for minimizing the deviational variables of the goals. As such, GAs as the global search algorithms [27] can be efficiently used to achieve the most satisfactory decision in the decision making environment.

In the proposed GA, a binary representation is used in coding each candidate solution as genetic representation of it. The initial population (the initial feasible solution individuals) is generated randomly with the condition of satisfying the system constraints in the solution search process.

The feasible solution individuals are then evaluated for fitness with the view to minimizing the given regret function. Here, the lower the evaluation value indicates the better score of an individual.

Now, in the literature of GAs, there is a large number of schemes [28] for generating new population with the use of different operators; selection, crossover and mutation. However, the basic steps of the GA procedure with the core functions adopted in the solution process are presented via the following steps.

A. Steps of the GA scheme

Step 1: Representation and Initialization

Let \( \tilde{V} \) denote the binary coded representation of chromosome in a population, where \( \tilde{V} = (x_1, x_2, \ldots, x_n) \). The population size is defined by \( \text{pop size} \), and \( \text{pop size} \) number of chromosomes is randomly initialized in the search domain.

Step 2: Fitness function

The fitness score of each individual is evaluated by the defined objective function \( Z \) given in (8) of the GP formulation of the problem. The fitness value of each individual is defined as follows:

\[
\text{Let } \text{eval}(V_i) = Z = \lambda \sum_{k=1}^{K} (w_{UL} \cdot \rho_{UL} + w_{UL} \cdot \eta_{UL}) + (1-\lambda) d
\]

\[
\text{where the subscript } i \text{ refers to the fitness value of the selected } i-th \text{ chromosome, } i=1,2, \ldots, \text{pop size}.
\]

Then, in the search process, the best chromosome with the highest fitness value at a generation is determined as

\[
V^* = \min(\text{eval}(V_i) | i = 1, 2, \ldots, \text{pop size})
\]

Here, in the evaluation of fitness function, the choice of weights \( w_{UL}, w_{UL} \) and \( \lambda \) in the convex combination is made in a deterministic way.
Step 3: Selection

The selection process defines the choosing of individuals (the parent solutions) from the population for the mating purpose. The selection procedure is biased towards best solutions using a simple roulette wheel scheme [27]. In this scheme, the selection of two individuals is made on the basis of their successive highest fitness scores.

Step 4: Crossover

The parameter \( P_c \) is defined as the probability of crossover. The conventional arithmetic crossover operator (Single-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the given system constraints set. In the crossover mechanism, a chromosome (solution individual) is selected as a parent for producing offspring, if for a defined random number \( r \in [0, 1] \), \( r < P_c \) is satisfied.

Here for the two selected parents \( X_1 \) and \( X_2 \in S \) (the feasible search space), the arithmetic crossover yields the offspring \( X'_1 \) as

\[
X'_1 = \alpha X_1 + (1- \alpha) X_2, \quad 0 \leq \alpha \leq 1,
\]

where \( X'_1 \) always belongs to \( S \).

Step 5: Mutation

Mutation mechanism is applied over the population after performing the crossover operation. It alters one or more genes of a selected chromosome according to mutation process in order to introduce some extra variability and reintroduce the genetic material in the population. As in the conventional scheme, the parameter \( P_m \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a Random Number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

Step 6: Termination

A loop of the Step 3 to Step 5 is executed for a certain number of times. Each iteration of the loop is called a generation. The search process terminates after a number of generations where the most satisfactory solution is achieved.

V. NUMERICAL EXAMPLE

The following numerical example is solved to illustrate the proposed approach.

Maximize \( Z_1: \quad [2, 4] x_1 + [3, 5] x_2 = [0.41, 1.295] \)

Maximize \( Z_2: \quad [1, 3] x_1 + [4, 7] x_2 = [0.295, 3.29] \)

Subject to \( x_1 + 2 x_2 \leq 5.6, \quad x_1, x_2 \geq 0 \) \( (9) \)

Now, following the procedure, the fractional objective goals are obtained as

\[
\begin{align*}
\frac{2x_1 + 3x_2 + \rho_{11} - \eta_{11}}{5x_1 + 7x_2} &= 1.295 \\
\frac{4x_1 + 5x_2 + \rho_{12} - \eta_{12}}{3x_1 + 4x_2} &= 0.41 \\
\frac{x_1 + 4x_2 + \rho_{12} - \eta_{12}}{7x_1 + 8x_2} &= 3.29 \\
\frac{3x_1 + 7x_2 + \rho_{12} - \eta_{12}}{x_1 + 2x_2} &= 0.295
\end{align*}
\]

Then using (8), and taking \( (w_{1a}, w_{1b}) = (0.5, 0.5) \), \( (w_{2a}, w_{2b}) = (0.5, 0.5) \) and \( \lambda = 0.5 \), the executable regret function can be obtained.

Now the proposed GA is applied to solve the problem.

The genetic parameters are taken as:

- probability of crossover \( P_c = 0.8 \)
- probability of mutation \( P_m = 0.08 \)
- population size 100
- Chromosome length = 30.

The GA is implemented using in the Programming Language C. The execution is done in a Intel Pentium IV with 2.66 GHz, Clock-pulse and 1 GB RAM. At the final stage of GA search process, it is found that

- the required number of generation is 100
- the solution run-time is approximately 10 minutes, to reach the solution.

The resulting decision of the problem is obtained as

\( (x_1, x_2) = (2.418, 1.565) \)

with \( Z_1 = [0.4135, 1.2947] \), \( Z_2 = [0.2947, 3.2829] \)

The result shows that the achieved solution is the most satisfactory in the decision-making environment.

NOTE 1: It may be noted here that for the obtained values of \( (x_1, x_2) \), the objective function values of the problem is always achieved within their specified target intervals for the given values of the associated objective function coefficients within their specified ranges.

NOTE 2: If the linearization approach presented by Pal et al. [7] is adopted to solve the problem, the solution is found as
(x₁, x₂) = (2.42, 1.56)
with Z₁ = [0.402, 0.87], Z₂ = [0.22, 2.291]

Again, using the linearization technique [6], the objective
solution of the problem is obtained as
(x₁, x₂) = (2.42, 1.56)
with Z₃ = [0.39, 0.95], Z₄ = [0.25, 2.45]

A comparison shows that almost the same solution is
obtained here in terms of achieving the goal values of the
objectives of the problem within the ranges specified in the
decision making environment.

Here, it worth mentioning that the computational load with
the linearization of fractional objectives as involved in the
previous conventional approaches does not arise due to the
direct use of the goal satisfying philosophy for achievement of
the goal values within the ranges specified in the decision
situation.

Further, the proposed GA method is more fruitful to apply
from the view points of its flexible nature of accommodating
the changes in the specified ranges of the parameters on the
basis of the needs and desires of the DM as well as into
changing the different other inexact parameters inherent to
large scale MOFFPs within the framework of the model and
without involving extra computational burden.

VI. CONCLUSION

In this article, how the proposed GA method can be used to
solve an interval valued MOFPP without linearization of the
fractional objective goals of the problems is presented. In the
process of the GA method, the weight structure as well as the
value of the parameters introduced for convex combination in
the framework of the proposed GP model is considered in a
deterministic way.

If the choice of them be made in a stochastic way, then
searching of efficiency can be improved by means of proper
modification of genetic operators of the proposed GA, which
may be a problem for future study.

Future research in the given field may be the
hybridization of the GA method introduced here with the other
artificial intelligence techniques [40] for better computational
efficiency and thereby improve the quality of solutions. Again,
applications of the proposed method to the real-life interval
valued decision problems would also be an important research
problem in future.

However, it is hoped that the method presented in this
study will open up many new vistas of research on interval
programming from its actual implementation to the real-world
decision making problems.

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REFERENCES

[1] S. Tong, “Interval number and fuzzy number linear programming”,
objectives function coefficients”, Management Science, vol. 26, 1981,
pp. 333-348.
functions”, Naval Res. Quart. 9, 1962, pp. 181-186.
programming problems using fuzzy goal programming”, in Proceedings
of the First Indian Conference on Trends in Modern Engineering
fuzzy multiobjective linear fractional programming problem”, Fuzzy
optimizing and satisfying under uncertainty”, Operational Research 11,
Technique to Land Use Planning in Agricultural System”, Omega,
water supply system development planning”, Fuzzy sets and systems 19,
problems with multiple fuzzy goals using dynamic programming”, Euro.
[15] W.T. Lin, “A survey of goal programming applications”, Omega 8,
1980, pp. 115-117.
programming problems with an interval objective function”, Euro. J.
optimization problem with interval valued objective function”, Euro. J.
345-361.
models with interval coefficients – an illustrated overview”, Euro. J.
[22] M. Ida, “Interval multiobjective programming and mobile robot path
planning”, In Mohammadian, M., Mohammadian, M., (Eds), New
Frontiers in Computational Intelligence and its Applications, IOS Press,
[23] M. Ida, “Goal programming problems with interval coefficients and


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ABSTRACT
This paper presents how genetic algorithm (GA) can be used in fuzzy goal programming (FGP) formula (SP) problems.

INDEX TERMS
- Author Keywords
  Chance constrained programming, Fuzzy goal programming, Fuzzy programming, Genetic algorithm, Stx

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Fuzzy Goal Programming Approach to Chance Constrained Multiobjective Decision Making Problems using Genetic Algorithm

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Abstract—This paper presents how genetic algorithm (GA) can be used in fuzzy goal programming (FGP) formulation of multiobjective stochastic programming (SP) problems.

In the proposed approach, the individual optimal decision of each of the objectives are determined by using the GA scheme adopted in the process of solving the problem after converting the chance constraints into their deterministic equivalent in [1,2]. Then, the FGP model of the problem is formulated by introducing the concept of tolerance membership functions in fuzzy sets.

In the solution process, the GA method is employed to the FGP formulation of the problem for achievement of the highest membership value (unity) of the defined membership functions to the extent possible in the decision making environment.

Two numerical examples are solved to illustrate the approach. The model solution of the first example is compared with the solution of the conventional approach studied previously.

Keywords—Chance constrained programming, Fuzzy programming, Fuzzy goal programming, Genetic algorithm, Stochastic programming.

I. INTRODUCTION

In an uncertain decision making environment, there are two types of inexactness—probability and fuzzy. The former is expressed by measuring the degree of occurrence of an event out of various trials and the later is employed for measurement of vagueness [3]. These substantially lead to two different branches of mathematical programming known as chance constrained programming (CCP), initially introduced by Charnes and Cooper in [4], and fuzzy programming (FP) based on the concept of fuzzy set theory (FST) introduced by Zadeh in [5] in 1965.

Now, due to multidimensional in nature of complex decision systems and imprecise in nature of human judgment, the decision makers (DMs) are often confused with the problems of inexact parameter values (stochastic or fuzzy). SP deals with the problems where parameters of the problems are described by stochastic or probabilistic variables. SP with continuous random parameter was introduced in [1], and applied to real-life problems [2] by considering different aspects in which the coefficients of the constraints and the parameters in the objectives are inherently random in nature.

During 1960s to mid-1980s, different aspects of CCP problems have been investigated by pioneer researchers of the field [6, 7, 8, 9]. Sullivan et al. [10] suggested an algorithm where goals can be stated in terms of probability of satisfying the aspiration levels. Teghem et al. [11] presented the interactive methods in the framework of the solution approach of SP.

On the other hand, FP offers a powerful approach in solving optimization problem with non-stochastic imprecision and vagueness. FP approaches has been used in different ways in [12, 13, 14] for solving problems in imprecise environment. Hannan [15] presented an FGP approach to decision problems in an imprecisely environment. The use of FGP [16] approaches to different real-life decision problems has been investigated by Pal et al. in [17, 18]. Different approaches of conversion of the original chance constraints using FP were provided in [19, 20]. Słomnicki [21] presented the state-of-art of CCP problem in fuzzy environment.

The use of GAs inspired by natural selection and population genetics, initially introduced by Holland [22], to CCP problems was introduced by Liu et al. [23]. They have proposed a stochastic simulation based GA for solving CCP problems with single as well as multiplicity of objectives. They have suggested this approach to overcome the difficulty in converting the CCP model into deterministic equivalent which is too expensive for complex and large CCP problems. In their approach, stochastic simulation is mainly employed to examine the feasibility of the solution. Liu in [24] provided a GA based approach for solving dependent CCP problems. The stochastic simulation based technique with the use of GA [25] has been studied in [26] in the past.

An GA based stochastic simulation approach to FGP formulation of multiobjective CCP problems has been investigated by Jana and Sharma [27] in the recent past. In their study, a CCP problem having discrete random parameters with known probability distributions has been taken into consideration. Again, in their approach, the conventional GA scheme [28] to the stochastic simulation based on trial and error technique
has been incorporated in the process of searching the solution of the problem. However, the use of GA in the FGP formulation of CCP problems is still at an early stage.

In this paper, achievement of fuzzily described objectives of a multiobjective decision making (MODM) problem under a system of chance constraints having continuous random parameters is considered. In the model formulation of the problem, the chance constraints with certain probability distribution of the chance parameters are introduced for selection of candidate solutions by satisfying the prescribed probabilities of constraints in the solution search process. The use of mean and variance is taken into account here to transform the defined chance constraints into their deterministic equivalent [2]. Then, a GA scheme is employed with the operators: fitter-codon selection, two-point crossover and random mutation to obtain individual best and worst solution of the objectives. Then, the objectives are defined fuzzily by introducing fuzzy aspiration level and lower tolerance limit to each of them with the use of the obtained individual solutions. Then, the fuzzy goals are characterized by the membership functions and the GA scheme is again employed for achievement of the highest membership value (unity) of the membership goals on the basis of their weights of importance by minimizing the associated under-deviational variables.

The proposed approach is illustrated by two numerical examples.

II. CHANCE CONSTRAINED MODM PROBLEM FORMULATION

The generic form of chance constrained MODM problem can be stated as:

Find \( X(x_1, x_2, ..., x_n) \) so as to

Maximize \( Z_k(X) = \sum_{j=1}^{n} c_{kj}x_j, \quad k = 1,2, ..., K \)

subject to

\( X \in S = \{ x \in \mathbb{R}^n \mid \text{Prob}[AX \leq b] \geq \beta, X \geq 0, b \in \mathbb{R}^m \} \) \hspace{1cm} (1)

Where \( X \) is a vector of deterministically defined decision variables, \( \text{Prob} \) designates the probabilistically defined constraints, \( A \) is an \((m \times n)\) real coefficient matrix, \( b \) is a resource vector, and \( \beta (0 < \beta < 1) \) represents the vector of confidence levels of probability for satisfying the defined constraints. It is assumed that the feasible region \( S \) (of \( S \neq \emptyset \)) is bounded.

Now, in the context of the present formulation, both the elements of the coefficient matrix and resource vector are assumed to be independently random in nature and follow normal distribution.

A. Chance constraints and their deterministic equivalent

Now, the constraints in (1) can be explicitly stated as

\[
\text{Prob}\left[ \sum_{j=1}^{n} a_{ij}x_j \leq b_i \right] \geq \beta_i, \quad i=1,2, ..., m. \hspace{1cm} (2)
\]

where \( a_{ij} \) and \( b_i \) are normal random variables and \( \beta_i \) are the specified probabilities.

Now, the following notion of distribution function for a random variable is defined in the sequel of finding the crisp equivalent of the constraints in (2). Let \( \phi(y) \) be the distribution function of the \( i \)-th random variable \( b_i \). Then, since \( \phi(y) \) is a monotonically non-decreasing function, the value of the corresponding variable is determined as

\[
\phi^{-1}(e) = \left\{ \max y \mid \text{Prob}(b_i > y) < e \right\}, \quad 0 < e < 1 \hspace{1cm} (3)
\]

The following two sub-cases for the random variables are considered:

(i) \( b_i \) is a random variable.

(ii) \( a_{ij}, \forall j \), are random variables.

Case I: For \( b_i \) is a random variable.

Let, \( b_i \) be a random normal variable with mean and variance \( \mu(b_i) \) and \( \sigma^2(b_i) \), respectively.

Now, the defined constraints set in (2) can be expressed as

\[
\text{Prob}\left[ \sum_{j=1}^{n} a_{ij}x_j - \mu(b_i) \right] \leq \frac{ \sum_{j=1}^{n} a_{ij}x_j - \mu(b_i) }{ \sqrt{\sigma^2(b_i)} } \geq \beta_i, \quad i=1,2, ..., m. \hspace{1cm} (4)
\]

Then, using the prescribed probability level \( \beta_i \), the expression in (3) takes the form

\[
\text{Prob}\left[ \frac{ \sum_{j=1}^{n} a_{ij}x_j - \mu(b_i) }{ \sqrt{\sigma^2(b_i)} } \geq \beta_i, \quad i=1,2, ..., m. \hspace{1cm} (5)
\]

Now, using the notion of distribution function in (3) to the expression in (5), the deterministic equivalent of the constraints in (2) with \( \leq \) type can be obtained as a set of linear system constraints as

\[
\sum_{j=1}^{n} a_{ij}x_j \leq \mu(b_i) + \phi^{-1}(\beta_i)\sqrt{\sigma^2(b_i)}, \quad i=1,2, ..., m. \hspace{1cm} (6)
\]

Case II: For \( a_{ij}, \forall j \), is a random variable.

Let, \( a_{ij} \) are random normal variables with mean and variance \( \mu(a_{ij}) \) and \( \sigma^2(a_{ij}) \), respectively.

Then, let \( y_i = \sum_{j=1}^{n} a_{ij}x_j \).

The mean and variance of \( y_i \) are successively obtained as:

\[
\text{E}(y_i) = \sum_{j=1}^{n} \text{E}(a_{ij})x_j, \quad \text{since } x_j, \forall j \text{ are deterministic,} \hspace{1cm} (7)
\]

\[
\text{Var}(y_i) = X^\top V_i X, \quad \text{respectively,} \hspace{1cm} (8)
\]
where \( V_j \) represents the \( j \)-th covariance matrix of order \((n \times n)\) for the defined \( a_j \), \( V_j \), \( T \) means transpose.

Then, following the basic rules of probability theory, the expressions of the constraints set in (2) can be expressed as:

\[
\text{Prob}\left( \frac{y_i - E(y_j)}{\sqrt{\text{var}(y_j)}} > \frac{b_i - E(y_j)}{\sqrt{\text{var}(y_j)}} \right) \geq \beta_i, \quad i = 1, 2, ..., m. \tag{9}
\]

where \( \frac{y_i - E(y_j)}{\sqrt{\text{var}(y_j)}} \) is the standard normal variable with mean zero and variance one.

Now, using the notion of distribution function defined in (6), and realizing the fact that \( \text{Prob}[y_i \leq b_i] \), the deterministic equivalent of the given chance constraints appear as:

\[
b_i - E(y_j) \geq \Phi^{-1}(\beta_i), \quad i = 1, 2, ..., m. \tag{10}
\]

Here, since \( a_j \), \( V_j \) follow normal distribution, the covariance terms \( \text{cov}(a_j, a_k), V_j, j \neq k, k = 1, 2, ..., n \) would be zero.

The expression in (10) in a simplified form can be presented as:

\[
\sum_{j=1}^{n} E(a_j) x_j + \Phi^{-1}(\beta_i) \sqrt{\sum_{j=1}^{n} \text{var}(a_j) x_j^2} \leq b_i, \quad i = 1, 2, ..., m. \tag{11}
\]

Now, the GA scheme adopted in process of FGP formulation of the problem in (1) and thereby solving the executable model of the problem is presented in the Section III.

III. DESIGN OF THE GA SCHEME

Step 1. Representation and Initialization

Let \( V_L \) denotes the binary coded representation of chromosome in a population as

\[ V_L = \{x_1, x_2, x_3, ..., x_n\}_L, \]

where \( L = 1, 2, ..., \text{pop\_size} \), represents the population size, and where \( \text{pop\_size} \) chromosomes are randomly initialized in its search domain.

Step 2. Fitness function

The fitness value of each chromosome is determined by the value of an objective function.

The fitness function is defined as:

\[ \text{eval}(V_L) = \left(Z_k\right)_k, \quad k = 1, 2, L = 1, 2, ..., \text{pop\_size}, \]

where \( Z_k \) is the value of the objective function for the best and least value of the objective function are determined as

\[ \text{F}^* = \max\{\text{eval}(V_L) \mid L = 1, 2, ..., \text{pop\_size}\}, \]

and \( \text{F}^* = \min\{\text{eval}(V_L) \mid L = 1, 2, ..., \text{pop\_size}\}, \) respectively.

Step 3. Selection

In the fitter codon selection scheme [29], the selection of chromosomes is made on the basis of their fitness scores, where nearer the fitness score of a chromosome to a predefined level of fitness value indicates the fitter one in the selection process.

For instance, the following four chromosomes in a population are considered:

(i) \( \{1, 1, 0, 1, 0, 0\} \)

(ii) \( \{1, 1, 1, 1, 0, 1\} \)

(iii) \( \{0, 1, 0, 1, 1, 0\} \)

(iv) \( \{1, 0, 1, 0, 1, 0\} \)

Here, codons are selected from the stand point of maximum occurrence of dominant values of the most significant bits, where codon length is defined by the number of bits from most significant bit position to the bit position of the first non-matching bit in the selected pair. It is to be observed here that the chromosomes in (i) and (ii) with codon length 4 each are the fitter ones in comparison to the that in (iii) and (iv).

Again, the chromosome in (ii) is fitter than the chromosome in (i). It is clear from the above that the decimal equivalents of chromosomes are not needed here for selection of them as the fitter ones.

Step 4. Crossover

The probability of crossover is defined by the parameter \( P_c \).

Here in a two-point crossover genetic system, the mating chromosomes interchange their middle portion in the process of reproduction. Again, a chromosome is selected as a parent, if for two defined random number \( r, r_1 \in [0, 1]; \ r, r_1 < P_c \)

with \( r + r_1 < 1 \) is satisfied.

In the selection of two parents, another random number \( r_2 \) is defined such that \( r_2 = 1 - r - r_1 \). Then, two parents \( V_1, V_2 \in S \) yield two offspring as:

\[ U_1 = (r + r_2). V_1 + r_1 V_2, \]

\[ U_2 = r_1 V_1 + (r + r_2). V_2, \]

where \( U_1, U_2 \in S \)

Step 5. Mutation

The parameter \( P_m \) as in the conventional GA scheme, is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

Step 6. Termination

The execution of the whole process terminates when the number of iterations is reached to the specified generation number in the genetic process. The generated best chromosome is reported finally as the decision in the genetic search process.

Now, the FGP formulation is presented in the following Section IV.

IV. FGP PROBLEM FORMULATION

In the field of FP, an imprecise aspiration level is assigned to each of the objectives and the tolerance limit for achievement of each of them is taken into consideration.
Let \((X^b_k, Z_k^b)\) and \((X^w_k, Z_k^w)\) be the best and worst solutions for maximization of the \(k\)-th objective \(Z_k\), \(k = 1, 2, ..., K\), subject to the constraints either in (6) or (11) or both obtained by employing the proposed GA scheme, where \(B\) and \(W\) are used to indicate the best and worst solutions, respectively.

Then, the fuzzy goals of the problem appear as

\[ Z_k \geq Z_k^b, \quad k = 1, 2, ..., K, \tag{12} \]

with the lower tolerance limits \(Z_k^w\), \(k = 1, 2, ..., K\), where \(\geq\) indicates the fuzzified version of \(\geq\) in the sense of Zimmermann [13].

Now, in the field of FP, the fuzzy goals are characterized by their respective membership functions.

### A. Characterization of membership function

Let \(\mu_k(X)\) be the membership function representation of the \(k\)-th fuzzy goal.

Then, for \(\geq\) type of restriction, \(\mu_k(X)\) takes the form

\[
\mu_k(X) = \begin{cases} 
    1 & \text{if } Z_k(X) > Z_k^b \\
    \frac{Z_k^w - Z_k^w}{t_k} & \text{if } Z_k^w < Z_k(X) \leq Z_k^b \\
    0 & \text{if } Z_k(X) < Z_k^w 
\end{cases}
\]

Where \(t_k = (Z_k^b - Z_k^w)\) is the tolerance range for achievement of the \(k\)-th fuzzy goal, \(k = 1, 2, ..., K\).

Now, in the field of FP, achievement of the highest membership value (unity) of each of the objectives is highly desired by the DM in a decision situation. In such a case, the FGP [24] as an extension of conventional GP and a robust tool for fuzzy multiobjective decision analysis is taken into consideration in the decision making context.

### B. FGP model formulation

In FGP, the membership functions are considered as goals by assigning the highest degree (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. In the goal achievement function, the under-deviational variables of the membership goals are minimized on the basis of their priorities and weights of importance in the decision situation.

The FGP model of the problem under a pre-emptive priority structure can be presented as:

\[
\text{Find } X \text{ so as to:} \\
\text{Minimize } Z = [P_1(d^-), P_2(d^-), ..., P_R(d^-)] \\
\text{and satisfy} \\
\frac{Z_k(X) - Z_k^w}{t_k} + d_k^- - d_k^+ = 1, \quad k = 1, 2, ..., K 
\]

subject to the system constraints in (6) or (11). Here \(Z\) represents the vector of \(R\) priority goal achievement functions and \(d_k^-\) and \(d_k^+\) are the under- and over-deviational variables, respectively, of the \(k\)-th membership goal. \(P_r(d^-)\) is a linear function of the weighted under-deviational variables, and where \(P_r(d^-)\) is of the form:

\[
P_r(d^-) = \sum_{k=1}^{R} w_k d_k^- 
\tag{15}

where \(d_k^-\) is renamed for \(d_k^+\) to represent it at the \(r\)-th priority level, \(w_k \geq 0\) is the numerical weight associated with \(d_k^-\) and represents the weight of importance of achieving the aspirered level of the \(k\)-th goal relative to the others which are grouped together at the \(r\)-th priority level.

It may be noted here that the priority factors have the relationship:

\[
P_1 >> P_2 >> ... >> P_r >> ... >> P_R 
\]

which implies that the goal achievement for the priority factor \(P_r\) is preferred most to the next priority factor \(P_{r+1}\), \(r = 1, 2, ..., R\).

### V. GA FOR FGP

The goal achievement functions \(Z\) in (14) here appears as the fitness function in the evaluation process of using the GA scheme. The evaluation function for determination of the fitness of a chromosome can be represented as:

\[
\text{eval}(V_L) = (Z)_L, \quad k = 1, 2, ..., R; \quad L = 1, 2, ..., \text{pop_size}. \tag{16}
\]

Here, the chromosome \(V^*\) with the best fitness value at each generation is determined as

\[
V^* = \min \{ \text{eval}(V_L) \mid L = 1, 2, ..., \text{pop_size} \}
\]

in the genetic search process.

To illustrate the approach, two numerical examples are solved.

### VI. NUMERICAL EXAMPLES

#### Example 1

The following chance constrained MODM problem studied previously [30] is considered to illustrate the potential use of the proposed approach.

Find \(X(x_1, x_2, x_3)\) so as to

Maximize \(Z_1 = 6x_1 + 2x_2 + 3x_3\).

Maximize \(Z_2 = 3x_1 + 5x_2 + 4x_3\).

Maximize \(Z_3 = 2x_1 + 7x_2 + x_3\)

subject to

\[
\begin{align*}
\text{Prob}[x_1 + x_2 + x_3 \leq b_1] & \geq 0.05 \\
\text{Prob}[5x_1 + x_2 + 6x_3 \leq b_2] & \geq 0.90 \\
\text{Prob}[2x_1 + 3x_2 + x_3 \leq b_3] & \geq 0.85
\end{align*}
\]

\[
\tag{17}
\]
Here, $b_1$, $b_2$, $b_3$ are normally distributed random variables with the mean and variance $(2.5, 2)$, $(8, 9)$, and $(10, 12)$ respectively.

Now, by employing the procedure mentioned in Section II, the constraints in (17) appears in their deterministic form as:

\[
\begin{align*}
&x_1 + x_2 + x_3 \leq 5.174 \\
&5x_1 + x_2 + 6x_3 \leq 11.855 \\
&2x_1 + 3x_2 + x_3 \leq 13.592
\end{align*}
\]

(18)

Now the proposed GA scheme, developed using Programming Language C with hardware support of Dell Power Edge R900 Server with 2 GB RAM is used to solve the problem. The following GA parameter values are adopted to determine the individual best and least values of the objectives.

- Probability of crossover $P_c = 0.8$
- Probability of mutation $P_m = 0.08$
- Population size = 100

The number of generations $= 300$ is initially taken to conduct the experiment. The different experiments with the different values of $P_c$ ($0<P_c<1$) and $P_m$ ($0<P_m<1$), in the ranges $(0.7<P_c<0.9)$ and $(0.03<P_m<0.09)$ are made in the proposed GA scheme. It is found that $P_c = 0.8$ and $P_m = 0.08$ are most acceptable in the decision search process.

Then following the procedure, the individual best and least values of the successive objectives are obtained as:

\[
\begin{align*}
&\langle x_1^*, x_2^*, x_3^*; Z_1^* \rangle = (1.7, 3.4, 0; 13.6), \\
&\langle x_1^*, x_2^*, x_3^*; Z_2^* \rangle = (0, 4.5, 0.5; 5), \\
&\langle x_1^*, x_2^*, x_3^*; Z_3^* \rangle = (1.8, 3.0, 0; 20.4), \\
&\langle x_1^*, x_2^*, x_3^*; Z_4^* \rangle = (0, 4.5, 0.5; 32.0), \\
&\langle x_1^*, x_2^*, x_3^*; Z_5^* \rangle = (1.75, 3.0, 0; 24.5).
\end{align*}
\]

The fuzzy goals can be constructed as

\[
\begin{align*}
Z_1 &\geq 13.6, \\
Z_2 &\geq 20.4, \text{ and } Z_3 \geq 32.0.
\end{align*}
\]

Now, based on the above numerical values, the resulting FGP model can be obtained by following (14) and (15):

The model appears as:

Find $X (x_1, x_2, x_3)$ so as to

\[
\begin{align*}
\text{Maximize } Z_1 &= 5x_1 + 3x_2 + 2x_3, \\
\text{Maximize } Z_2 &= 7x_1 + 2x_2 + 4x_3, \\
\text{Maximize } Z_3 &= 2x_1 + 3x_2 + 8x_3
\end{align*}
\]

subject to

\[
\begin{align*}
0.076[6x_1 + 2x_2 + 3x_3 - 0.5] + d_1^- - d_1^+ &= 1, \\
0.078[3x_1 + 5x_2 + 4x_3 - 7.7] + d_2^- - d_2^+ &= 1, \\
0.133[2x_1 + 7x_2 + x_3 - 24.5] + d_3^- - d_3^+ &= 1
\end{align*}
\]

(19)

subject to the given constraints in (18).

The resultant solution of the problem (19) by employing the proposed GA scheme is obtained as:

\[
\langle x_1, x_2, x_3 \rangle = (0.7, 4.0, 0.8)
\]

with $\langle Z_1, Z_2, Z_3 \rangle = (14.6, 25.3, 30.2)$

Note: The solution of the problem using the linearization approach studied in [28] previously, is found as

\[
\langle x_1, x_2, x_3 \rangle = (0.68, 3.81, 0.76)
\]

with $\langle Z_1, Z_2, Z_3 \rangle = (14.05, 24.21, 28.86)$.

A comparison shows that the proposed solution approach is superior one from the viewpoint of goal achievement of the specified goal levels of the objectives in the decision making environment.

**Example 2**

To expound the effectiveness of the approach more, the following example with three objectives and two chance constraints with normally distributed random variables is considered.

Find $X (x_1, x_2, x_3)$ so as to

\[
\begin{align*}
\text{Maximize } Z_1 &= 5x_1 + 6x_2 + 3x_3, \\
\text{Maximize } Z_2 &= 7x_1 + 2x_2 + 4x_3, \\
\text{Maximize } Z_3 &= 2x_1 + 3x_2 + 8x_3
\end{align*}
\]

subject to

\[
\begin{align*}
\text{Prob}[a_1x_1 + a_2x_2 + a_3x_3 \leq b_1] &\geq 0.95, \\
\text{Prob}[5x_1 + x_2 + 6x_3 \leq b_2] &\geq 0.10
\end{align*}
\]

(20)

Here, $a_i$ and $b_i$ are independently normally distributed random variables with the means and variances defined as follows:

Here, the means and variances of $a_{11}, a_{12}, a_{13}$ are successively defined as: $(3, 6), (9, 4)$, and $(1, 2)$ respectively. Again, the means and variances of $b_1, b_2$ are defined as: $(5, 8)$, and $(7, 9)$, respectively.

Now, following the procedure, the deterministic equivalent of the successive constraints in (20) are found as:

\[
\begin{align*}
3x_1 + 9x_2 + x_3 + 1.645(6x_1^2 + 4x_2^2 + 2x_3^2)^{1/2} &\leq 8, \\
5x_1 + x_2 + 6x_3 &\leq 10.9
\end{align*}
\]

(21)

The same computational environment of Dell Server and C language and, the genetic parameter values adopted in the case of Example 1 are also considered here to solve the problem.

Now, the individual best and least values of the successive objectives are obtained as:

\[
\begin{align*}
&\langle x_1^*, x_2^*, x_3^*; Z_1^* \rangle = (0.5, 0.7, 0; 6.7), \\
&\langle x_1^*, x_2^*, x_3^*; Z_2^* \rangle = (0.5, 0, 0.5; 4), \\
&\langle x_1^*, x_2^*, x_3^*; Z_3^* \rangle = (0, 0.9, 0; 6.3), \\
&\langle x_1^*, x_2^*, x_3^*; Z_4^* \rangle = (0, 1.25, 0.5; 4.5), \\
&\langle x_1^*, x_2^*, x_3^*; Z_5^* \rangle = (0, 0.06, 0.07, 0.62; 5.25), \\
&\langle x_1^*, x_2^*, x_3^*; Z_6^* \rangle = (0.75, 0.67, 0; 3.51).
\end{align*}
\]

Then, following the procedure, the resulting FGP model is obtained as:

Authorised licensed use limited to: JIS College of Engineering. Downloaded on March 15, 2010 at 08:51:03 EDT from IEEE Xplore. Restrictions apply.
Find $X(x_1,x_2,x_3)$ so as to

\[
\text{Minimize } Z = 0.37d_1 + 0.45d_2 + 0.575d_3
\]

and satisfy:

\[
0.37[5x_1 + 6x_2 + 3x_3 - 4.0] + d_1 - d_1^* = 1
\]

\[
0.45[7x_1 + 2x_2 + 4x_3 - 4.5] + d_2 - d_2^* = 1
\]

\[
0.575[2x_1 + 3x_2 + 8x_3 - 3.5] + d_3 - d_3^* = 1
\]

subject to

\[
x_1 + 9x_2 + x_1 + 1.645(6x_1^2 + 4x_2^2 + 2x_3^2)^{1/2} \leq 8
\]

\[
x_1 + x_2 + 6x_3 \leq 10.9
\]

\[
x_1, x_2, x_3 \geq 0
\]

The resulting decision of the problem (22) by employing the GA scheme is obtained as:

\[
(x_1, x_2, x_3) = (0.542, 0.418, 0.401)
\]

with \( (Z_1, Z_2, Z_3) = (6.421, 6.234, 5.546) \).

The result shows how the satisfactory decision can be obtained with the change of the random parameters involved with problems in the decision making environment.

VII. CONCLUSION

In this paper, how the GA scheme can be effectively used in the FGP formulation of MODM problems under the chance constraints with different continuously distributed random parameters is presented.

In the proposed approach, computational complexity associated with conventional approaches and inherent computational error associated with linearization technique does not arise. Again, within the framework of the proposed FGP model, crisp objectives as well as constraints can easily be incorporated without involving any computational difficulty.

The proposed approach can be extended to solve general non-linear and different types of fractional MODM problems in uncertain environment.

An extension of the approach to different chance constrained hierarchical decision problems may be the problem for future research. Finally, it is expected that the approach presented here may lead to further research for its implementation to real-life MODM problems in the current complex uncertain decision making environment.

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REFERENCES


A genetic algorithm approach for fuzzy goal programming formulation using stochastic simulation

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ABSTRACT
This paper presents how the stochastic simulation based genetic algorithm (GA) can be used to the fuzzy chance constrained multiobjective decision making (MODM) problem.

INDEX TERMS
- Author Keywords
  Chance constrained programming, Fuzzy goal programming, Fuzzy programming, Genetic algorithm, St...
A Genetic Algorithm Approach for Fuzzy Goal Programming Formulation of Chance Constrained Problems Using Stochastic Simulation

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Abstract— This paper presents how the stochastic simulation based genetic algorithm (GA) can be used to the fuzzy goal programming (FGP) formulation of a chance constrained multiobjective decision making (MODM) problem.

In the proposed approach, a stochastic simulation to the chance constraints having the continuous random parameters is introduced first to determine the candidate solutions in the decision making context.

Then, in the model formulation, the fuzzy goal descriptions of the objective are defined by employing the proposed GA method.

In the solution process, achievement of the membership goals of the defined fuzzy goals to the highest membership value (unity) by minimizing the associated under-deviational variables to the extent possible by using the GA scheme is taken into consideration.

A numerical example is solved and a comparison of the model solution with the conventional fuzzy programming (FP) approach is made to illustrate the potential use of the approach.

Keywords— Chance constrained programming, Fuzzy programming, Fuzzy goal programming, Genetic algorithm, Stochastic programming.

I. INTRODUCTION

The two types of prominent approaches for solving decision making problems in an uncertain environment are the stochastic programming (SP) and fuzzy programming (FP). In the case of SP, inexactness of the parameter values is inherent to problem description in the decision making situations.

On the other hand, imprecision in data description is inherent due to inexactness of human judgments.

The field of SP based on the probability theory, initially introduced by Charnes and Cooper [1] as the chance constrained programming (CCP), has been studied [2] extensively in the past and applied to several real-life problems in [3,4].

The study on single objective SP problems made in the past has been surveyed by Luhandjula [5] in 2006.

Now considering the multiobjective nature of the real-life decision problems, various approaches including the Goal Programming (GP) approach [6] as a prominent tool to solve MODM problems have been studied [7] to solve SP problems with multiplicity of objectives in the past.

However, in most of the studies [8], the CCP problems with known distribution of the stochastic parameters are converted into their deterministic equivalents in using the well established crisp approaches to solve the problems. But, with the realization of the fact that there are different types of distribution functions involved with the real-world CCP problems in which deterministic equivalent become computationally expensive and computational complexity often arises [9] in actual practice.

To overcome the above difficulty, the stochastic simulation [10, 11] based approaches to various versions of CCP problems have been studied [12, 13] in the past.

Now, another type of prominent approach for solving decision problems in an imprecise environment is the FP approach based on the theory of fuzzy sets introduced by Zadeh [14] in 1965.

Several FP approaches [15] including FGP approach [16] within the framework of conventional GP [6, 17] have been studied extensively in the past and applied to different real-life problems in [18, 19].

Now, in the real-life decision situations, it has been realized that both the probabilistic and fuzzy data would have to be simultaneously taken into account to make more realistic decision in an inexact decision making environment. The study on the area of CCP has been made by the pioneer researchers [20, 21, 22] in the past. The use of GAs to stochastic decision problems by using stochastic simulation [23] as well as consideration of deterministic equivalent of them in [24] has been studied [25] in the recent past.

Here, it may be mentioned that the consideration of both the aspects discussed previously to MODM problems is a great challenge to the researchers in this field and the methodological development of it is at an early stage from
the viewpoint of their implementation to the real-world complex decision problems.

In this article, achievement of fuzzily described objectives of MODM problems under a system of chance constraints is considered. In the model formulation of the problem, instead of converting the constraints set to the deterministic one, a stochastic simulation to the defined goals and with certain probability distributions of the chance parameters is introduced in the solution search process.

In the solution process, an GA scheme is employed to reach a satisfactory decision by minimizing the underdeviational variables of the defined membership goals of the fuzzy objective goals to the highest degree (unity) to the extent possible on the basis of the weight of importance of achieving the goals is taken into consideration in the decision making environment.

A numerical example is solved and the model solution of proposed FGP framework is compared with the solution of the conventional FP approach to expound the potential use of the method presented here in the decision making context.

II. Problem Formulation

The generic form of a chance constrained MODM problem can be presented as:

Find $X(x_1, x_2, ..., x_n)$ so as to

Maximize $Z_k(X), \ k \in K_1$

and Minimize $Z_k(X), \ k \in K_2$

Subject to

$X \in S = \{X \in R^n | \text{Prob}[A X \leq b] \geq \beta, X \geq 0, b \in R^n \}$

$L \leq X \leq U,$

where $X$ is a vector of deterministically defined decision variables, $L$ and $U$ denote upper- and lower- bounds, respectively, of the vector $X$, and dimension of them depends on the dimension of $X$, $\text{Prob}$ designates the probabilistically defined constraints, $A$ is an $(m \times n)$ real coefficient matrix, $b$ is a resource vector, and where $\beta (0 < \beta < 1)$ represents the vector of confidence level of probabilities for satisfying the defined constraints.

It is assumed that the feasible region $S (\text{Swp})$ is bounded, and where $K_1 \cup K_2 = \{1, 2, ..., K\}$ with $K_1 \cap K_2 = \phi$.

Now, in the present problem formulation, it is assumed that the coefficient vectors as well as the resource vector are independently random in nature and follow different continuous probability distributions in the decision making context.

Then, the stochastic simulation approach to search the solutions subject to the defined chance constraints is presented in the following Section A.

A. Stochastic Simulation for solution estimation

The generation of random numbers for simulation run and thereby estimation of probability for feasible solutions have been well documented by Rubenstein [26], and widely used in [13] in the context of making decision under chance constraints.

Now, the constraints in (1) can be explicitly stated as

$$\text{Prob} \left[ \sum_{i=1}^{n} a_{ij} x_j \leq b_i \right] \geq \beta_i, \ i=1,2,..,m.$$ (2)

Then, the simulation process can be defined as follows:

Let, $\text{Prob} \left[ \sum_{i=1}^{n} a_{ij} x_j \leq b_i \right] = \text{Prob}[g_i(x,r) \leq 0 \text{ for } i=1,2,..,m.$

where, $r = (r_1, r_2, ..., r_{n+1})$ is the $(n+1)$ dimensional random vector, and where $r_j^T = (a_{j1}, a_{j2}, ..., a_{jm})$, $j = 1,2,..,n$ and $r_{n+1}^T = (b_1, b_2, ..., b_m)$.

Then, for a given $X$, $P$ independent random vectors are generated in such a way that

$r^{(p)} = (r_1^{(p)}, r_2^{(p)}, ..., r_{n+1}^{(p)})$, $p=1,2,..,P.$

for the given distributions of the defined vectors of random variables.

Then, let $P'$ be the number of occasions on which

$g_i(x,r^{(p)}) \leq 0, \ i=1,2,..,m; \ p=1,2,..,P.$

are satisfied.

Then, the probability of occurrence of constraint satisfaction is defined by

$$V = \frac{P'}{P}.$$ (3)

The simulation process is summarized in the following steps:

Step 1. Initialize $P' = 0$.

Step 2. Generate the vectors of random numbers according to their defined distribution functions.

Step 3. If the constraints $g_i(x,r) \leq 0, \ i=1,2,..,m$ then set $P' = P' + 1$.

Step 4. Repeat the steps 2 and 3 $P$ times.

Step 5. Compute $V = \frac{P'}{P}$.

Now, the proposed simulation process for feasibility verification of a candidate solution (a chromosome) in a genetic search process is briefly described as follows [23]:

B. Use of Stochastic Simulation to the GA scheme

- Determine the initial candidate solutions (the chromosome)
- Verify the feasibility criteria using the stochastic simulation and determine the fitness scores (the objective function values)
III. DESIGN OF THE GA SCHEME

The algorithm steps in the genetic search process adopted are presented as follows:

Step 1: Representation and initialization
Let \( V_L \) denotes the binary coded representation of chromosome in a population as \( V_L = \{x_1, x_2, ..., x_n\} \), \( L = 1, 2, ..., \) pop size, the population size pop size number of chromosomes are randomly initialized in its search domain.

Step 2: Feasibility Check
The proposed stochastic simulation approach is used to check the feasibility of the candidate solution.

Step 3: Fitness function
The fitness value of each chromosome is determined by the value of an objective function. The fitness function is defined as

\[
\text{eval}(V_L) = (Z_K)_L, \quad k \in K_1 \quad \text{(maximization)}
\]

or

\[
\text{eval}(V_L) = (Z_K)_L, \quad k \in K_2 \quad \text{(minimization)}
\]

where \( Z_K \) represents the \( k \)-th objective function value.

The best chromosome for the best or least value of an objective is determined by

\[
V^* = \max \{\text{eval}(V_L)| L = 1, 2, ..., \text{pop size}\},
\]

or

\[
V^* = \min \{\text{eval}(V_L)| L = 1, 2, ..., \text{pop size}\},
\]

which depends on the needs and desires of the DM in the decision situation.

Step 4: Selection
The simple roulette-wheel scheme [27] is used for selecting two parents for mating purposes in the genetic search process.

Step 5: Crossover
The parameter \( P_c \) is defined as the probability of crossover. The arithmetic crossover operator (1-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the linear constraints set \( S \). Here a chromosome is selected as a parent, if for a defined random number \( r \in [0, 1] \), \( r < P_c \) is satisfied.

For example, arithmetic crossover for two parents \( V_1, V_2 \in S \) yields two offspring

\[
E_1 = \alpha_1 V_1 + \alpha_2 V_2, \quad E_2 = \alpha_3 V_1 + \alpha_4 V_2,
\]

where \( \alpha_1, \alpha_2 \geq 0 \) with \( \alpha_1 + \alpha_2 = 1 \), always belong to \( S \) and where \( S \) is a convex set.

Step 6. Mutation
As in the conventional scheme, a parameter \( P_m \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a Random Number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

Step 6. Termination
The execution of whole process terminates when the number of iterations is reached to the generation number specified in the genetic search process. Now, representation of each type of the objectives in (1) in fuzzy versions of them and thereby FGP formulation of the problem by using the proposed GA scheme is presented in the following Section IV.

IV. FGP FORMULATION

In the field of FP, an imprecise aspiration level is assigned to each of the objectives and certain tolerance limit for achievement of aspired level of each of them is taken into consideration.

Let \( (X^B_k, Z^B_k) \) and \( (X^W_k, Z^W_k) \) be the best and worst solutions for maximization of the \( k \)-th objective \( Z^k \), \( k \in K_1 \), subjects to the constraints in (1) where \( B \) and \( W \) are used to indicate the best and worst solutions, respectively. Again, let \( (X^*_B, Z^*_B) \) and \( (X^*_W, Z^*_W) \) be the best and worst solutions respectively for minimization of the \( k \)-th objective \( Z^k \), \( k \in K_2 \).

Then, the fuzzy goals of the problem appear as:

\[
Z_k \geq Z^*_B, \quad k \in K_1
\]

and

\[
Z_k \leq Z^*_B, \quad k \in K_2
\]

with the lower and upper tolerance limits \( Z^*_B \) (\( k \in K_1 \)) and \( Z^*_W \) (\( k \in K_2 \)) , respectively, where \( \geq \) and \( \leq \) indicates the fuzzy version of \( \geq \) and \( \leq \) respectively.

Now, in the field of FP, the fuzzy goals are characterized by their respective membership functions.
Where, \( t_{lk} = (z_{wk} - z_{wk}) \) is the tolerance range for achievement of the k-th fuzzy goal, \( k \in K_1 \).

Similarly, for \( \leq \) type of restriction, \( \mu_k(x) \) appear as

\[
\mu_k(x) = \begin{cases} 
1 & \text{if } z_k(x) \leq z_k^* \\
\frac{z_k^* - z_k(x)}{t_{2k}} & \text{if } z_k^* < z_k(x) \leq z_k^* \\
0 & \text{if } z_k(x) > z_k^*. 
\end{cases}
\]

(7)

Where, \( t_{2k} = (z_{wk} - z_{wk}) \) is the tolerance range for achievement of the k-th fuzzy goal, \( k \in K_2 \).

The FGP model of the problem is presented in the following Section B.

**B. FGP model formulation**

In FGP model formulation, the defined membership functions are converted into the membership -goals by introducing the under- and over-deviational variables and assigning unity as the aspiration level to each of them. In the 'goal achievement function', under-deviational variables on the basis of weights of importance of achieving the goals are taken into consideration. Such a version of FGP called the minsum FGP is the simplest and widely used approach [19] to solve fuzzy MODM problems.

Then, the executable minsum FGP formulation appears as:

Find \( x \) so as to:

\[
\text{Minimize } Z = \sum_{k=1}^{K} w_k d_k^i,
\]

and satisfy

\[
\frac{Z_k(x) - Z_{wk}}{t_{1k}} + d_k^i - d_k^r = 1, \quad k = 1, 2, ..., K
\]

subject to the system constraints in (1) and (2), where \( Z \) represents the fuzzy goal achievement function, \( d_k^i (\geq 0) \) represents the under- and over-deviational variables associated with the k-th membership goal, \( w_k (\geq 0), k = 1, 2, ..., K \), represent the numerical weights associated with the respective under deviational variables, and where \( w_k, k = 1, 2, ..., K \), are determined as [19]:

\[
w_k = \begin{cases} 
\frac{1}{t_{1k}} & \text{for the } \mu_k \text{ defined in (6)} \\
\frac{1}{t_{2k}} & \text{for the } \mu_k \text{ defined in (7)}
\end{cases}
\]

Now, the fitness function in using the GA scheme to the problem (8) is defined in the following Section C.

**C. Definition of Fitness function**

The goal achievement function \( Z \) in (8) appears as the fitness function in the evaluation process of using the GA scheme. The evaluation function for measuring the fitness of a chromosome can be presented as:

\[
eval(V_L) = (Z_L)^L = \left( \sum_{k=1}^{K} w_k d_k^i \right)^L,
\]

where, \( k = 1, 2, ..., K \) and \( L = 1, 2, ..., \text{pop_size} \). (9)

Here, the chromosome \( V^* \) with the best fitness value at each generation is defined as:

\[
V^* = \min \{ \eval(V_L) | L = 1, 2, ..., \text{pop_size} \},
\]

in the genetic search process.

To illustrate the proposed approach, a numerical example is solved.

**V. ILLUSTRATIVE EXAMPLE**

To illustrate the use of the proposed procedure, the following MOCCP problem having two objective functions are considered.

Find \( (x_1, x_2, x_3) \) so as to

Maximize: \( Z_1(x) = 4x_1^2 + 3x_2 + 5x_3 \) (10)

and minimize: \( Z_2(x) = 5x_1^2 + 4x_2 + x_3 \) (11)

subject to,

\[
\begin{align*}
\text{Prob} & \{ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 < b_3 \} > 0.90 \\
\text{Prob} & \{ a_{32}x_1 + a_{32}x_2 + a_{33}x_3 < b_3 \} > 0.80 \\
\text{Prob} & \{ a_{32}x_1 + a_{32}x_2 + a_{33}x_3 < b_3 \} > 0.90 \\
1 \leq x_i \leq 2, & \quad i = 1, 2, 3.
\end{align*}
\]

In the decision situation, the distribution of parameters, of the problem and their values are defined as follows:

(i) \( a_{11}, a_{12}, a_{13}, b_1 \) follow beta distribution with parameters \((1,50), (2,100), (4,90), (8,5)\), respectively.

(ii) \( a_{21}, a_{22}, a_{23}, b_2 \) follow gamma distribution with parameters \((2,0.01), (2,1), (3,0.1), (3,5)\) respectively.

(iii) \( a_{31}, a_{32}, a_{33}, b_3 \) follow Weibull distribution with parameters \((2,3), (3,2), (3,1), (5,30)\) respectively.

Now, to formulate the fuzzy goals of the objectives of the problem and their respective tolerance limits, the proposed GA is coded in Programming Language C, and the values \( p_{\text{size}} = 100, p_c = 0.8, p_m = 0.08 \) and the generation number = 300 are adopted in the genetic search process. The obtained best and least values of the objectives are presented in the Table I.
TABLE I
THE BEST AND THE LEAST OBJECTIVE VALUES

<table>
<thead>
<tr>
<th>Best value</th>
<th>Least value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (x_1, x_2, x_3) )</td>
<td>( (x_1, x_2, x_3) )</td>
</tr>
<tr>
<td>( Z_1^* = 55 )</td>
<td>( Z_1^* = 12 )</td>
</tr>
<tr>
<td>( (3.0, 3.0, 3.0) )</td>
<td>( (1.0, 1.0, 1.0) )</td>
</tr>
<tr>
<td>( Z_2^* = 10 )</td>
<td>( Z_2^* = 30 )</td>
</tr>
<tr>
<td>( (1.0, 1.0, 1.0) )</td>
<td>( (3.0, 3.0, 3.0) )</td>
</tr>
</tbody>
</table>

Now, based on the numerical values presented in the Table I, the fuzzy goals can be constructed as

\[ Z_1 \geq 36, \]
\[ Z_2 \leq 10 \]

The membership functions are then found as:

\[ \mu_1(x) = \frac{4x_1 + 3x_2 + 5x_3 - 12}{24}, \]  
\[ \mu_2(x) = \frac{30 - (5x_1 + 4x_2 + x_3)}{20} \]

Now, following the procedure, the resulting FGP model of the problem appears as:

Find \( (x_1, x_2, x_3) \) so as to

Minimize \( Z = 0.04167d_1^2 + 0.05d_2^2 \)

and satisfy

\[ 0.04167(4x_1 + 3x_2 + 5x_3) - 0.5 + d_1 - d_1^2 = 1 \]
\[ 1.5 - 0.05(5x_1 + 4x_2 + x_3) + d_2 - d_2^2 = 1 \]  
\[ \text{subject to the constraints in (12) - (15)}. \]

Now, employing the proposed stochastic simulation based GA method by addressing the fitness function defined in (18) and using the same genetic parameters set discussed previously, the model solution is obtained as

\( (x_1, x_2, x_3) = (1.0, 1.0, 3.0) \)

with \( (z_1, z_2) = (22.0, 12.0) \).

The achieved membership values are

\[ \mu_1(z_1, z_2) = (0.41, 0.90) \]

The result shows that a satisfactory decision is achieved here in the decision environment.

Note 1: In the decision making context, if the max-min fuzzy operator [15] is used to solve the problem of optimizing the defined membership functions of the fuzzy goals, where the objective is to maximize \( \lambda \) (say), subject to \( \lambda \) 'less than or equal to' \( \lambda \) each of the defined membership functions with \( 0 \leq \lambda \leq 1 \) and also the given system constraints then in the same decision environment the solution of the problem is obtained as:

\( (x_1, x_2, x_3) = (1.8529, 1.5285, 1.6345) \)

with \( (z_1, z_2) = (20.1696, 17.013) \).

Note 2: It is to be noted that when the random parameters of the chance constraints follow normal distribution, then deterministic equivalent of the constraints can easily be obtained by using the means and variances of the of the parameter values which also appear with the characteristic of normal distribution. In such a case, the conventional SP can be efficiently used to solve the problem.

But, in a decision situation, when the parameters follow any other distribution, the corresponding means and variances may not always agree with the distribution pattern followed by the random parameters. As a matter of fact, computational complexity arises there to convert the chance constraints into their deterministic equivalent in the decision making process. Here, proposed stochastic simulation as the 'trial and error' approach is the most fruitful to employ to any type of probability distribution followed by the random parameters of the problem.

VI. CONCLUSION

In this paper, how the stochastic simulation based GA can be efficiently used for solving fuzzily described MODM problems under chance constraints with different continuously distributed random parameters are presented.

The main advantage of the approach is that the computational complexity in converting the chance constraints with different distribution of the parameters to deterministic equivalent does not involve here due to the use of stochastic simulation in the solution search process. Again, since the objective of a MODM problem often conflict each other for goal achievement, the use of the GA method as GA is a goal satisficer rather than objective optimizer, to the FGP formulation of the problem always provides a satisfactory decision and the proposed approach may be claimed as a superior one over the conventional crisp / fuzzy approaches for solving CCP problems.

Further, the proposed approach can easily be extended to solve non-linear including fractional MODM problems without involving linearization of the objectives as employed in the conventional approaches [28, 29] to the decision problems in both the crisp and uncertain environment.

An extension of the approach to different chance constrained hierarchical decision making problems may be a problem for future research. Finally, it is expected that the approach presented here may lead to future research for its implementation to real-life MODM problems in the current complex uncertain decision making environment.

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REFERENCES


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Dept. of Environment, Govt. of West Bengal
AN APPLICATION OF GENETIC ALGORITHM METHOD TO FUZZY GOAL PROGRAMMING MODEL FOR ACADEMIC PERSONNEL PLANNING PROBLEMS IN UNIVERSITY MANAGEMENT SYSTEMS

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INTRODUCTION

With the increase of social awareness for betterment of social life as well as uplift in technological aspects, interest in higher education in new study areas has increased a lot in the recent years from the viewpoint of job prospective to different activity areas. As a matter of fact, initiative for opening of new academic departments has been taken into account by most of the higher academic institutions. But, most of the teaching organizations are faced with the problem of proper allocation of teaching personnel in the academic departments.

Since the mid-1960s, the different strategic mathematical programming models developed for university management was surveyed by Schroeder [1] in 1973. Since, most of the academic planning problems are multiobjective in nature, the conventional goal programming (GP) approach [2, 3], as one of the most promising tool for multiobjective decision analysis has been introduced to the field of academic planning problems in [4, 5].

Now, in most of the real-life decision situations, it is to be observed that the fuzziness is frequently involved in different ways to different decision making problems. The methodological aspects of conventional Fuzzy Programming (FP) [6] as well as FGP [7] as an extension of the conventional GP method have been studied in the past and implemented to different real-life problems.

In the decision making environment, GAs based on natural selection and population genetics, initially introduced by Holland [8], have also appeared as robust computational tools in the field of optimization problems [9]. The use of GAs in the framework of multiobjective decision making (MODM) problems have been investigated by the pioneer researchers in the field and implemented to practical problems [10]. But exploration of the potential use of GAs to real-life MODM problems (crisp or fuzzy) is at an early stage.

Now, it is to be observed that non-linearity in fractional form appears in most of the decision situations due to consideration of different ratios involved with model objectives as well as structural constraints. The fractional programming has been studied in [11,12, 13,14] extensively in the past. A linearization approach to fuzzy fractional programming problems has been studied in [15, 16] in the recent past and implemented to a real life problem in [17]. But is inherently involved to apply the conventional approaches to practical problems.

To overcome computational load involved with the use of conventional approaches to such problems, a GA approach to interval programming (IVP) problems with fractional criteria has been recently introduced by Pal and Gupta [18]. However, the use of GAs to such real-life problems is at an early stage.
In this article, a GA method is introduced to the FGP formulation of academic personnel management problems with fractional criteria in a university system. In the solution process, the proposed GA is used in an iterative manner to the preemptive priority based FGP model of the problem. In the decision process, the goal satisfying philosophy [9] in FGP is used for achievement of the linear as well as the ratio goals to their aspired levels (unity) to the extent possible in the decision making environment.

Now, construction of FGP model of the problem is presented in the Section 2.

2. FGP Problem formulation

The generic form of a multiobjective FP problem can be presented as:

Find X

so as to satisfy

\[ Z_k(X) \begin{cases} \geq & g_k, \\ \leq & b_k \end{cases}, \quad k = 1, 2, \ldots, K \]

subject to

\[ X \in S = \left\{ X \in \mathbb{R}^n \mid AX \left( \begin{array}{c} \mathbf{s} \\ \mathbf{b} \end{array} \right) \geq \mathbf{b}, X \geq 0, b \in \mathbb{R}^m \right\} \]...

where X is the vector of decision variables, \( g_k \) is the imprecise aspiration level of the k-th objective \( Z_k(X) \) (k = 1, 2, ..., K), A is the technological coefficient matrix, b is the vector of right-hand side values, and where ≥ and ≤ refer to the fuzziness of the aspiration levels [6].

Now, in a fuzzy decision-making situation, the fuzzy goals are to be characterized by their respective membership functions.

2.1 Characterization of membership Function

Let \( t_{lk} \) and \( t_{uk} \) be the lower and upper tolerance ranges, respectively, for achievement of the aspired goal level \( g_k \) of a fuzzy goal. Then the membership function, \( \mu_k(X) \), for the fuzzy goal \( Z_k(X) \) can be characterized as [7]:

For ≥ type of restriction, \( \mu_k(X) \) takes the form

\[ \mu_k(X) = \begin{cases} 1 & \text{if } Z_k(X) \geq g_k, \\ \frac{Z_k(X) - (g_k - t_{lk})}{t_{uk}} & \text{if } g_k - t_{lk} \leq Z_k(X) < g_k, \\ 0 & \text{if } Z_k(X) < g_k - t_{lk}, \quad k = 1, 2, \ldots, K \end{cases} \]...

where \((g_k - t_{lk})\) represents the lower-tolerance limit for the achievement of the stated fuzzy goal.

Again, for ≤ type restriction, \( \mu_k(X) \) becomes

\[ \mu_k(X) = \begin{cases} 1 & \text{if } Z_k(X) \leq g_k, \\ \frac{(g_k + t_{uk}) - Z_k(X)}{t_{uk}} & \text{if } g_k \leq Z_k(X) \leq (g_k + t_{uk}), \\ 0 & \text{if } Z_k(X) > (g_k + t_{uk}), \quad k = 1, 2, \ldots, K \end{cases} \]...

where \((g_k + t_{uk})\) represents the upper-tolerance limit for the achievement of the stated fuzzy goal.
2.2 Construction of FGP model

In FGP, the membership functions are considered as fuzzy goals by assigning the highest degree (unity) as the aspiration level and by introducing under- and over-deviational variables to each of them. In the achievement function, the under-deviational variables of the goals are minimized on the basis of their priorities and weights of importance in the decision situation.

The FGP model of the problem under a pre-emptive priority structure can be presented as:

Find $X$ so as to:

Minimize $Z = \langle p_1(d^-), p_2(d^-), \ldots, p_i(d^-), \ldots, p_t(d^-) \rangle$

and satisfy

$$\frac{Z_k(X) - g_{uk}}{t_k} + d_k - d_k^u = 1,$$

$$\frac{g_{lk} - Z_k(X)}{t_k} + d_k - d_k^l = 1,$$

subject to the given system constraints in (4),

where $g_{lk} = (g_k - t_k)$ and $g_{uk} = (g_k + t_k)$ represent lower- and upper-tolerance values respectively,

$Z$ represents the vector of $I$ priority goal achievement functions and $d_k^l, d_k^u$ are the under- and over-deviational variables, respectively, of the $k$-th membership goal. $p_i(d^-)$ is a linear function of the weighted under-deviational variables, and where $p_i$ is of the form:

$$p_i(d^-) = \sum_{k=1}^{K} w_k^i d_k^i, \quad k = 1, 2, \ldots, K.$$ 

where $d_k^i (\geq 0)$ is renamed for $d_k$ to represent its $i$-th priority level, $w_k^i (\geq 0)$ is the numerical weight associated with $d_k^i$ and represents the weight of importance of achieving the specified level of the $k$-th goal relative to the others which are grouped together at the $i$-th priority level.

When some of the objective $Z_k(x) (k = 1, 2, \ldots, K)$ are linear fractional in form, then the linearization approach [15] is conventionally used in the solution process. To avoid the computational load involved with the linearization of the objectives, a GA procedure can be used in the process of solving the FGP model in (4).

The GA scheme used in the process of solving the problem (4) is presented in the following Section 3.

3. Design of the GA scheme

The basic steps of the GA scheme:

Step 1. Representation and Initialization

Let $E$ denote the binary coded representation of chromosome in a population as $E = \{x_1, x_2, \ldots, x_n\}$. The population size is defined by pop_size, and pop-size chromosomes are randomly initialized in its search domain.

Step 2. Fitness Function

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as

$$\text{eval}(E_n) = Z_n = \left( \sum_{k=1}^{K} w_k^i d_k^i \right),$$

where $d_k^i (\geq 0)$ is the numerical weight associated with $d_k^i$ and represents the weight of importance of achieving the specified level of the $k$-th goal relative to the others which are grouped together at the $i$-th priority level.
where \( Z_i \) represents the \( i \)-th priority factor of the goal achievement of the function \( Z \) in (4), and where the subscript \( v \) in (5) refers to the fitness value of the selected \( v \)-th chromosome, \( v = 1, 2, \ldots, \text{pop}\_\text{size} \). The best chromosome with the largest fitness value at each generation is determined as

\[
E^* = \{ \min \{ \text{eval}(E_v) \mid v = 1, 2, \ldots, \text{pop}\_\text{size} \} \},
\]

depending on searching out the best value of an objective.

**Step 3. Selection**

The simple roulette-wheel scheme [9] is used for selecting two parents for mating purposes in the genetic search process.

**Step 4. Crossover**

The parameter \( P_c \) is defined as the probability of crossover. The arithmetic crossover operator (single-point crossover) of a genetic system is applied here in the sense that the resulting offspring always satisfy the linear constraints set \( S \). Here a chromosome is selected as a parent, if for a defined random number \( r \in [0, 1] \), \( r < P_c \) is satisfied.

Here arithmetic crossover for two parents \( E_1, E_2 \in S \) is defined as

\[
X_1 = \alpha_1 E_1 + \alpha_2 E_2, \quad X_2 = \alpha_2 E_1 + \alpha_1 E_2,
\]

for producing two offspring \( X_1 \) and \( X_2 \), where \( \alpha_1, \alpha_2 \geq 0 \) with \( \alpha_1 + \alpha_2 = 1 \) always belong to \( S \), and where \( S \) is a convex set.

**Step 5. Mutation**

As in the conventional GA scheme, a parameter \( P_m \) of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number \( r \in [0, 1] \), a chromosome is selected for mutation provided that \( r < P_m \).

**Step 6. Termination**

The execution of the whole process terminates when the fittest chromosome is reported at a certain generation number in the solution search process.

**Remark:** Since GAs are satisficers rather than optimizers [9], in the process of using the GA method to the problem, the system constraints are also made flexible by introducing under- and over-deviation variables to each of them in the notion of searching the most satisfactory solution in the expanded flexible region \( S \) in goal satisficing philosophy of conventional GP [2]. The system constraints are then termed as the crisp goals.

The crisp goals in explicit form can be presented as [3]:

\[
a_j x_j + d_j^n - d_j^s = b_j, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n
\]

Here, inclusion of \( d_j^n \geq 0 \) or \( d_j^s \geq 0 \) or both of them for minimizing them in the framework of the objective function \( Z \) in (4) depends on the \( \geq \) or \( \leq \) or \( = \) type of restrictions imposed in the decision-making environment.

Now, FGP formulation of the problem by defining the membership goals of the fuzzy goals as well as the crisp goals is presented in the Section 4.

**4. FGP Model Formulation of the problem**

The different types of parameters and decision variables are defined first in the context of developing the model of the problem.

**4.1 Definition of parameters:**

- \( m_i \): minimum number of total FTS required in the department \( i \), at the time period \( t \).
- \( m_{ft} \): Minimum number of full-time teaching staff (FTS) required in the department \( i \) (\( i = 1, 2, \ldots, I \)), rank \( j \) (\( j = 1, 2, \ldots, J \)) at the time period \( t \).
- \( s_i \): Total number of students (TS) at department \( i \) at the time period \( t \).
AS_{ijt} = Annual (average) salary of a FTS in the department \(i\), rank \(j\) at the time period \(t\).

AN_{it} = Annual (average) salary of a NTS in the department \(i\) at the time period \(t\).

AF_{ijt} = Annual remuneration of a part-time teaching staff (PTS) in the department \(i\) at time period \(t\).

Q_{it} = ratio of PTS and FTS in the department \(i\) at time period \(t\).

R_{it} = ratio of NTS and total teaching staff (TTS), - total FTS and PTS, ratio in the department \(i\) at time period \(t\).

T_{ijt} = ratio of total Students and TTS in the department \(i\).

B_{it} = budget allocated to the department \(i\) at time period \(t\).

B_{it+1} = estimated budget allocation to the department \(i\) at time period \(t+1\).

4.2 Definition of decision variables:

- \(F_{ijt}\) = number of FTS in the department \(i\), rank \(j\) at the time period \(t\).
- \(N_{it}\) = number of NTS in the department \(i\) at the time period \(t\).
- \(P_{it}\) = number of PTS in the department \(i\) at the time period \(t\).

Now, the fuzzy and crisp goals of the model are defined as in the following Section 4.3.

4.3 Definition of Fuzzy and Crisp goals:
The fuzzy goals as well as crisp goals of the problem are described as follow:

(i) FTS Goals:
- a) For potential academic performance, a minimum number of total FTS should be employed to each of the departments. But, due to the financial resource constraint, employment of such FTS to a department is fuzzy in nature.

The fuzzy goal expression appears as:

\[
\sum_{j=1}^{J} F_{ijt} \geq m_{ijt}, \quad i = 1, 2, \ldots, I \quad \ldots (6)
\]

- b) For smooth functioning of the academic activities, university should always have to try to provide a minimum number of FTS at each rank to each of the departments at the time period \(t\).

The crisp goals for FTS can be presented as:

\[
F_{ijt} + d^- - d^+ = m_{ijt} \quad i = 1, 2, \ldots, I; \quad j = 1, 2, \ldots, J; \quad p = 1, 2, \ldots, U \quad \ldots (7)
\]

(ii) PTS - FTS Ratio Goals:
Due to the limitation of pay-roll budget, if the required number of FTS can not be employed, then PTS at a certain ratio of the FTS need be employment to the departments.

The crisp ratio goal equations can be presented as:

\[
\frac{P_{it}}{\sum_{j=1}^{J} F_{ijt}} + d^- - d^+ = Q_{it} \quad i = 1, 2, \ldots, I; \quad p = I+1, I+2, \ldots, I(J+1) \quad \ldots (8)
\]

(iii) TS-TTS Ratio Goals:
For enrichment of academic performance at each department, a certain ratio of students and TTS should be maintained.

The crisp ratio goal equation can be presented as:

\[
\frac{S_{it}}{\sum_{j=1}^{J} F_{ijt} + P_{it}} + d^- - d^+ = T_{it} \quad i = 1, 2, \ldots, I; \quad p = I+1, I+2, \ldots, I+2J \quad \ldots (9)
\]

(iv) NTS - TTS Ratio Goals:
A certain number of NTS need be employed each department for the clerical and technical jobs, at a certain ratio of the TTS of a department.
The goal expression can be presented as:
\[ N_i \cdot d_p + d_p^* = R_i, \quad i = 1, 2, \ldots, I; \]
\[ \sum_{j=1}^{j} F_{ji} + P_{ji} \]

(v) Budget Goal:
Due to limitation of total available budget to run the university, the pay-roll budget for each of the departments is fuzzily described.

The fuzzy goal can be represented as:
\[ \sum_{j=1}^{j} (A S_{ji} \cdot F_{ji} + A N_{ji} \cdot N_{ji} + A P_{ji} \cdot P_{ji}) \leq B_{ji}, \quad i = 1, 2, \ldots, I. \]

Then, construction of the membership goals of the defined fuzzy goals and then formulation of the FGP model of the problem are demonstrated via the case example of the University of Kalyani presented in the Section 5.

5. A Case Study
The academic resource allocation problem of the University of Kalyani, W.B., India is considered to demonstrate the application potential of the proposed approach. The problem of six departments: Microbiology (MB), Computer Science and Engineering (CSE), Master of Business Administration (MBA), Geography (GEO), Molecular Biology and Bio-Technology (MB & BT) and Physiology (PHY) established in the recent past are considered.

The required data for the proposed model was collected from the Budget Allocation Programme published by the Finance Officer, University of Kalyani (in the financial year 2008–2009), Kalyani, W.B., India.

The pay-roll budget allocation to each of the six departments for the financial year (2008 – 2009), say the time period 1 (t=1), is considered as the aspiration level (A^T) of the associated fuzzy budget goal. The estimated budget allocation to the individual department for the financial year (2009 – 2010), say the time period 2 (t=2), is considered as upper tolerance limit (U^T) of defined fuzzy goal. The model data are presented in the Tables 1 – 3.

**Table 1: Data description of pay-roll budget**

<table>
<thead>
<tr>
<th>Departments</th>
<th>MB(1)</th>
<th>CSE(2)</th>
<th>MBA(3)</th>
<th>GEO(4)</th>
<th>MB&amp;BT (5)</th>
<th>PHY(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pay-roll Budget (in Rs. Lac)</td>
<td>A^T</td>
<td>U^T</td>
<td>A^T</td>
<td>U^T</td>
<td>A^T</td>
<td>U^T</td>
</tr>
<tr>
<td>11.41</td>
<td>15.45</td>
<td>31.10</td>
<td>36.60</td>
<td>16.11</td>
<td>18.53</td>
<td>14.07</td>
</tr>
</tbody>
</table>

**Table 2: Annual average salary for FTS, NTS**

<table>
<thead>
<tr>
<th>Rank</th>
<th>Salary (Rs.)</th>
<th>Remuneration (Rs.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>3.30 Lac</td>
<td>-</td>
</tr>
<tr>
<td>Reader</td>
<td>2.40 Lac</td>
<td>-</td>
</tr>
<tr>
<td>Lecturer</td>
<td>2.00 Lac</td>
<td>-</td>
</tr>
<tr>
<td>PTS</td>
<td>0.705 Lac</td>
<td>-</td>
</tr>
<tr>
<td>NTS</td>
<td>-</td>
<td>0.05 Lac</td>
</tr>
</tbody>
</table>

**Table 3: Number of TS, NTS-FTS ratio, PTS-FTS ratio and TS-FTS ratio**

<table>
<thead>
<tr>
<th>Department</th>
<th>Number of Students</th>
<th>NTS-FTS</th>
<th>PTS-FTS</th>
<th>TS-FTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>MB</td>
<td>32</td>
<td>2:5</td>
<td>1:4</td>
<td>7:1</td>
</tr>
<tr>
<td>CSE</td>
<td>110</td>
<td>2:5</td>
<td>1:4</td>
<td>7:1</td>
</tr>
<tr>
<td>MBA</td>
<td>50</td>
<td>1:3</td>
<td>1:4</td>
<td>7:1</td>
</tr>
<tr>
<td>GEO</td>
<td>50</td>
<td>1:2</td>
<td>1:4</td>
<td>7:1</td>
</tr>
<tr>
<td>MB&amp;BT</td>
<td>24</td>
<td>2:5</td>
<td>1:4</td>
<td>7:1</td>
</tr>
<tr>
<td>PHY</td>
<td>30</td>
<td>2:5</td>
<td>1:4</td>
<td>7:1</td>
</tr>
</tbody>
</table>
Now using the data tables and the other necessary collected data, construction of the fuzzy goals and the crisp goals are described in the Section 5.1.

Here, since the variables are introduced with the running time period \((t-1)\), the time specification \((t)\) is omitted for simplicity during the presentation of the case model.

### 5.1 Description of fuzzy and crisp goals

- **The Fuzzy goals:**
  - a) FTS goals:
    
    Here, \(A_i\) for MB, CSE, MBA, GEO, MB&BT and PHY are considered as \(8, 12, 12, 8, 10\) and \(8\), respectively, as the minimum requirement of the respective departments on the basis of their workload.
    
    Again, the lower tolerance limit \((L_i)\) for each of them is considered as \(0.6\) on the basis of the general norm to avail the financial assistance from the UGC, India and the other funding agencies in connection to upgrading the departments. Then, the membership goals for the defined fuzzy goals in (6) can be constructed by following the first goal expression in (4).
    
    The membership goals appear as:
    
    $$0.5 \left( (F_{t_1} + F_{t_2} + F_{t_3}) - 6 \right) + d_{t_1}^* - d_{t_2}^* = 1$$
    $$0.166 \left( (F_{t_1} + F_{t_2} + F_{t_3}) - 6 \right) + d_{t_4}^* - d_{t_5}^* = 1$$
    $$0.25 \left( (F_{t_1} + F_{t_2} + F_{t_3}) - 6 \right) + d_{t_6}^* - d_{t_7}^* = 1$$

  - b) Budget goals:
    
    Using the data in the Tables 1 and 2 the membership goals of the defined fuzzy goals in (11) are constructed by following the second goal expression in (4).
    
    The membership goals of the successive departments as presented in the Table 1 are obtained as:
    
    $$3.824 - (0.817 F_{t_1} + 0.594 F_{t_2} + 0.495 F_{t_3} + 0.012 F_{t_4} + 0.174 N_i) + d_{t_1}^* - d_{t_2}^* = 1$$
    $$6.654 - (0.6 F_{t_1} + 0.436 F_{t_2} + 0.364 F_{t_3} + 0.009 F_{t_4} + 0.128 N_i) + d_{t_3}^* - d_{t_4}^* = 1$$
    $$7.657 - (1.364 F_{t_1} + 0.991 F_{t_2} + 0.826 F_{t_3} + 0.020 F_{t_4} + 0.291 N_i) + d_{t_5}^* - d_{t_6}^* = 1$$
    $$5.885 - (1.146 F_{t_1} + 0.833 F_{t_2} + 0.694 F_{t_3} + 0.017 F_{t_4} + 0.245 N_i) + d_{t_7}^* - d_{t_8}^* = 1$$
    $$5.175 - (1.071 F_{t_1} + 0.779 F_{t_2} + 0.649 F_{t_3} + 0.016 F_{t_4} + 0.229 N_i) + d_{t_9}^* - d_{t_{10}}^* = 1$$
    $$5.429 - (1.24 F_{t_1} + 0.902 F_{t_2} + 0.752 F_{t_3} + 0.019 F_{t_4} + 0.265 N_i) + d_{t_11}^* - d_{t_{12}}^* = 1$$

  - **The Crisp goals:**
    
    a) FTS goals:
    
    The goals for minimum number of FTS in each rank to the departments areas follows:
    
    $$F_{t_1} + d_{t_1}^* - d_{t_2}^* = 1$$
    $$F_{t_2} + d_{t_2}^* - d_{t_3}^* = 1$$
    $$F_{t_3} + d_{t_3}^* - d_{t_4}^* = 2$$
    $$F_{t_4} + d_{t_4}^* - d_{t_5}^* = 2$$
    $$F_{t_5} + d_{t_5}^* - d_{t_6}^* = 2$$
    $$F_{t_6} + d_{t_6}^* - d_{t_7}^* = 2$$
    $$F_{t_7} + d_{t_7}^* - d_{t_8}^* = 2$$
    $$F_{t_8} + d_{t_8}^* - d_{t_9}^* = 2$$
    $$F_{t_9} + d_{t_9}^* - d_{t_{10}}^* = 2$$
    $$F_{t_{10}} + d_{t_{10}}^* - d_{t_{11}}^* = 1$$
    $$F_{t_{11}} + d_{t_{11}}^* - d_{t_{12}}^* = 1$$
    $$F_{t_{12}} + d_{t_{12}}^* - d_{t_{13}}^* = 1$$
    $$F_{t_{13}} + d_{t_{13}}^* - d_{t_{14}}^* = 1$$
    $$F_{t_{14}} + d_{t_{14}}^* - d_{t_{15}}^* = 1$$
    $$F_{t_{15}} + d_{t_{15}}^* - d_{t_{16}}^* = 2$$
    $$F_{t_{16}} + d_{t_{16}}^* - d_{t_{17}}^* = 2$$
    $$F_{t_{17}} + d_{t_{17}}^* - d_{t_{18}}^* = 2$$
    $$F_{t_{18}} + d_{t_{18}}^* - d_{t_{19}}^* = 2$$
    $$F_{t_{19}} + d_{t_{19}}^* - d_{t_{20}}^* = 2$$
    $$F_{t_{20}} + d_{t_{20}}^* - d_{t_{21}}^* = 2$$
    $$F_{t_{21}} + d_{t_{21}}^* - d_{t_{22}}^* = 3$$
    $$F_{t_{22}} + d_{t_{22}}^* - d_{t_{23}}^* = 1$$
    $$F_{t_{23}} + d_{t_{23}}^* - d_{t_{24}}^* = 1$$
    $$F_{t_{24}} + d_{t_{24}}^* - d_{t_{25}}^* = 1$$

  - b) Ratio goals:
i) PTS - FTS ratio goals:
\[ \frac{F_1}{(F_{11} + F_{12} + F_{13})} + d_{17} - d_{17}^* = 0.25, \quad \frac{F_2}{(F_{21} + F_{22} + F_{23})} + d_{17}^* - d_{33}^* = 0.25, \]
\[ \frac{F_3}{(F_{31} + F_{32} + F_{33})} + d_{18} - d_{18}^* = 0.25, \quad \frac{F_4}{(F_{41} + F_{42} + F_{43})} + d_{38}^* - d_{38} = 0.25, \]
\[ \frac{F_5}{(F_{51} + F_{52} + F_{53})} + d_{19} - d_{19}^* = 0.25, \quad \frac{F_6}{(F_{61} + F_{62} + F_{63})} + d_{39}^* - d_{39} = 0.25 \]

ii) TS - TTS ratio goals:
\[ \frac{1}{[F_{11} + F_{12} + F_{13}] + P_1} + d_{17}^* - d_{17} = 0.22, \quad \frac{1}{[F_{21} + F_{22} + F_{23}] + P_2} + d_{33}^* - d_{33} = 0.064, \]
\[ \frac{1}{[F_{31} + F_{32} + F_{33}] + P_3} + d_{18}^* - d_{18} = 0.14, \quad \frac{1}{[F_{41} + F_{42} + F_{43}] + P_4} + d_{38} - d_{38} = 0.14, \]
\[ \frac{1}{[F_{51} + F_{52} + F_{53}] + P_5} + d_{19}^* - d_{19} = 0.292, \quad \frac{1}{[F_{61} + F_{62} + F_{63}] + P_6} + d_{39} - d_{39} = 0.233. \]

iii) NTS - TTS ratio goals:
\[ N_1 [(F_{11} + F_{12} + F_{13}) + P_1] + d_{17}^* - d_{17} = 0.4, \quad N_2 [(F_{21} + F_{22} + F_{23}) + P_2] + d_{33}^* - d_{33} = 0.4, \]
\[ N_3 [(F_{31} + F_{32} + F_{33}) + P_3] + d_{18}^* - d_{18} = 0.33, \quad N_4 [(F_{41} + F_{42} + F_{43}) + P_4] + d_{38} - d_{38} = 0.5, \]
\[ N_5 [(F_{51} + F_{52} + F_{53}) + P_5] + d_{19}^* - d_{19} = 0.4, \quad N_6 [(F_{61} + F_{62} + F_{63}) + P_6] + d_{39} - d_{39} = 0.4. \]

Now, following the proposed procedure the resulting executable FGP model under the two given priority levels can be obtained as follows:

Find \( \{F_{ij}, N_i, P_i\} \ i=1,2,...,6; j=1,2,3 \) so as to

Minimize:
\[ Z = p_1 \left( d_{17}^* + d_{17} + d_{18} + 0.5d_{19} + d_{20} + 0.166d_{21} + d_{22} + 0.166d_{23} + d_{24} + 0.166d_{25} + d_{26} + 0.166d_{27} + d_{28} + d_{29} + 0.5d_{30} + d_{31} + 0.25d_{32} + d_{33} + 0.5d_{34} + d_{35} + d_{36} + d_{37} + d_{38} \right) \]
\[ + d_{18} + d_{39} + d_{40} + d_{41} + d_{42} + (d_{31} + d_{32} + d_{33} + d_{34} + d_{35} + d_{36} + d_{37} + d_{38}) \]
\[ + d_{39} + d_{40} + d_{41} + d_{42} + (d_{31} + d_{32} + d_{33} + d_{34} + d_{35} + d_{36} + d_{37} + d_{38}) \]
\[ + p_1 (0.247d_{43} + 0.181d_{44} + 0.413d_{45} + 0.347d_{46} + 0.325d_{47} + 0.376d_{48}) \]

satisfy the goal expressions given above in the Section 5.1.

\[ F_{ij}, N_i, P_i \geq 0, \quad p_i \geq 0, \quad i=1,2,3,4,5,6; \quad j=1,2,3; \]
\[ d_{p}, d_{p}^* \geq 0 \quad \text{with} \quad d_{p}^*-d_{p}^* = 0, \quad p=1,2,..,48. \]  \( \text{(12)} \)

Then, the proposed GA approach presented in the Section 3 is used to solve the problem in (12). The objective function of the model appears as the fitness function in the solution search process.

The following genetic parameter values are introduced in the search process:
- probability of crossover \( P_c = 0.8 \)
- probability of mutation \( P_m = 0.08 \)
- population size 100
- chromosome length 30.

The GA based Program is designed in Programming Language C++. The execution is done in a Intel Pentium IV with 2.66 GHz Clock-pulse and 1 GB RAM. The optimal solution is reached after 200 generations. The achieved result is presented in the Table 4.
A comparison of the model solution with the existing allocation structure shows that a satisfactory solution is achieved here in terms of the needs and desires of the departments.

Note: If the linearization approach to the academic planning problem studied by Pal and Sen [17] is used, then the obtained solution for staff allocation is displayed in Table 6.

Table 6: Staff allocation structure under linear FGP model

<table>
<thead>
<tr>
<th>Department</th>
<th>MB</th>
<th>CSE</th>
<th>MBA</th>
<th>GEO</th>
<th>MB&amp;BT</th>
<th>PHY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Reader</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Lecturer</td>
<td>6</td>
<td>9</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>PTS</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>NTS</td>
<td>4</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

The existing staff allocation structure is given in the Table 5:

Table 5: Existing staff allocation structure (2008-2009)

<table>
<thead>
<tr>
<th>Department</th>
<th>MB</th>
<th>CSE</th>
<th>MBA</th>
<th>GEO</th>
<th>MB&amp;BT</th>
<th>PHY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Reader</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Lecturer</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>NTS</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

6. Conclusion

The main advantage of the proposed GA approach is that the computational load involved with the traditional approaches for linearization of the real-life problems with fractional criteria can be avoided here in the solution process. Moreover, the most satisfactory decision can easily be reached here in the solution search process of the proposed GA method without involving extra computational burden with redefining the model as involved in the decision process of using the traditional approaches.

However, it is hoped that the approach presented here can contribute to future research in the area of real-life planning problems for managerial decision making.

References


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The Use of Genetic Algorithm for Solving a Long-Term Land Allocation Problem for Optimal Cropping Plan in Agricultural System

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Abstract - This paper presents how the genetic algorithms (GAs) can be used to the fuzzy goal programming (FGP) formulation of land allocation problems for optimal production of seasonal crops in agricultural system.

In the proposed approach, utilization of total available land for cultivation, aspiration levels of the production of crops, expected profit from the farm as well as certain ratios in fractional form for crops production and profit achievement are fuzzily described in the decision making context.

In the model formulation, achievement of highest membership value (unity) of the defined fuzzy goals to the extent possible by minimizing the under-deviational variables of the defined membership goals on the basis of priorities and thereby measuring the

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degree of optimality of the aspired goal levels are considered.

In the solution process, the proposed GA method is used in an iterative manner for satisficing the goal levels on the basis of needs and desires of the decision maker (DM).

To illustrate the potential use of the approach, the case example of the Nadia district, West Bengal, India is considered. The obtained solution is compared with the existing cropping plan of the district as well as the solution of the FGP approach studied previously.

**Keywords:** Cropping plan, Fuzzy goal, Fuzzy goal programming, Genetic algorithm, Membership function

1. Introduction

In an agricultural planning situation, optimal production of seasonal crops highly depends on the proper allocation of land and adequate supply of the productive resources for cultivating the crops in different season of the planning period.

Mathematical programming (MP) model for proper allocation of cultivable land to cropping plan have been studied in [1] since 1954. During 1970s, linear programming (LP) models of farm planning problems were studied by the active researchers in the field. The use of LP approaches to farm management problems have been surveyed by Nix [2].

Although, LP models have been successfully used to the farm planning problems, there is a difficulty to implement them with regard to meeting the different socio-economic goals due to the limitation of optimizing only a single objective associated with the LP methods developed in the past in the farm planning context.

Now, since most of the agricultural planning problems are multiobjective in nature, the goal programming (GP) approach [3], as a most prominent tool for multiobjective
decision analysis to agricultural production planning, has been introduced by Wheller and Russell [4] in 1977. The extensive study on GP formulations of farm planning problem has been surveyed by Glen [5] in 1987.

The efficient use of priority based GP approach to land use planning problems has been studied by Pal et al. [6, 7, 8] in the past. However the main weakness of GP formulation of real-life problems is that the different resource parameters involved with the problem need be precisely defined. But, in most of the decision situations, they are found to be imprecise (fuzzy) in nature due to expert’s ambiguous understanding of the nature of them.

To overcome the above difficulties, fuzzy programming (FP) approaches to farm planning problems in the framework of LP has been studied by Slowinski [9] in 1986. The FGP approach [10] to farm planning problems have been studied by Pal et al [11] in the recent past.

Now, it is to be observed that non-linearity in fractional form appears in most of the farm planning decision situation due to consideration of different ratios involved with the problems.

The fractional programming as a special field of non-linear programming has been studied [12, 13] extensively in the past for both the single objective and multiobjectivve programming problems. The linearization approach to FP as well as FGP problems with linear fractional criteria have been studied [14] in the past.

In the decision making environment, GAs based on natural selection and population genetics, initially introduced by Holland [15] have also appeared as a robust computational tools in the field of optimization problems [16]. The use of GAs in the framework of multiobjective decision making (MODM) problems have been investigated.
by the pioneer researcher s in the field and implemented to practical problems [17]. But exploration of potential use of GA to real life MODM problems (crisp or fuzzy) is at an early stage.

To overcome computational load involved with the use of conventional approaches to the problems with fractional objectives, a GA approach to interval programming (IVP) problems with fractional criteria has been recently introduced by Pal and Gupta [18]. GA approaches to real-life problems with fractional criteria have been studied [19] in the recent past. However, extensive study in this area is yet to be circulated in the literature.

In this article, a GA method is introduced to the FGP formulation of land allocation problem on a long-term basis for optimal cropping plan in agriculture systems. In the context of using GA method, the roulette-wheel selection scheme, arithmetic crossover and random mutation are used as genetic operators.

In the solution process, the proposed GA is used in an iterative manner to the preemptive priority based FGP model of the problem. In the decision process, the goal satisficing philosophy [20] in FGP is used for achievement of linear as well as ratio goals to their aspired levels (unity) to the extent possible in the decision making context. A case example is considered to illustrate the potential use of the approach.

2. FGP formulation

In a fuzzy decision making environment, the objective are always described fuzzily, whereas the resource constraints as well as the other structural constraints may be crisp or fuzzy and that depend upon the fuzzification of them in the decision situation. In the present decision context, a fully fuzzified version of the system is taken into account to explore the potential solution of the problem in the decision environment.
The generic form of a multiobjective FP problem can be expressed as:

Find \( X \) so as to

\[
\text{satisfy } Z_k(X) = \begin{cases} 
  \geq & g_k \text{ } k = 1, 2, ..., K.
\end{cases}
\]

where \( X \) is the vector of decision variables, \( g_k \) is the imprecise aspiration level of the \( k \)-th objective \( Z_k(X) \), \( k=1, 2, ..., K \), and where \&, \ refer to the fuzziness of the aspiration levels [21].

Now, in the fuzzy decision making context, the defined fuzzy goal objectives in (1) are characterized by their associated membership functions [7].

2.1 Characterization of membership function:

In a fuzzy decision making environment, certain tolerance limits are introduced to the objectives concerning achievements of the respective aspired goal levels.

Let \( t_{lk} \) and \( t_{uk} \) be the lower and upper tolerance ranges, respectively, for achievement of the aspired goal level \( g_k \) of a fuzzy goal. Then the membership function, \( \mu_k(X) \), for the fuzzy goal \( Z_k(X) \) can be characterized as [15]:

For \( \geq \) type of restriction, \( \mu_k(X) \) takes the form

\[
\mu_k(X) = \begin{cases} 
  1 & \text{if } Z_k(X) \geq g_k \\
  \frac{Z_k(X) - (g_k - t_{uk})}{t_{uk}} & \text{if } g_k - t_{uk} \leq Z_k(X) < g_k \\
  0 & \text{if } Z_k(X) < g_k - t_{uk}
\end{cases}
\]

\hspace{1cm} k = 1, 2, ..., K. \hspace{1cm} (2)

where \( (g_k - t_{uk}) \) represents the lower-tolerance limit for the achievement of the stated fuzzy goal.
Again, for \( \leq \) type restriction, \( \mu_k(X) \) becomes

\[
\mu_k(X) = \begin{cases} 
1 & \text{if } Z_k(X) \leq g_k \\
\frac{(g_k + t_{uk}) - Z_k(X)}{t_{uk}} & \text{if } g_k < Z_k(X) \leq g_k + t_{uk} \\
0 & \text{if } Z_k(X) > g_k + t_{uk}
\end{cases}
\]

(3)

where \( (g_k + t_{uk}) \) represents the upper-tolerance limit for the achievement of the stated fuzzy goal.

Now, the general FGP formulation of a problem is presented in the following Section 2.2.

2.2 FGP problem formulation

In FGP, the membership functions are considered as fuzzy goals by assigning the highest degree (unity) as the aspiration level and by introducing under- and over-deviational variables to each of them. In the achievement function, the under-deviational variables of the goals are minimized on the basis of their priorities and weights of importance in the decision situation.

The FGP model of the problem under a pre-emptive priority structure can be presented as:

Find \( X \) so as to:

Minimize \( Z = [p_1(d^-), p_2(d^-), \ldots, p_i(d^-), \ldots, p_l(d^-)] \)

and satisfy

\[
\frac{Z_k(X) - g_{ik}}{t_{ik}} + d_k^- - d_k^+ = 1
\]

\[
\frac{g_{ik} - Z_k(X)}{t_{ik}} + d_k^- - d_k^+ = 1
\]

\[
d_{k}^-, d_{k}^+ \geq 0, \quad k = 1, 2, \ldots, K
\]

(4)
where \( g_{lk} = (g_k - t_{lk}) \) and \( g_{uk} = (g_k + t_{uk}) \) represent the lower- and upper-tolerance values, respectively; \( Z \) represents the vector of \( I \) priority goal achievement functions and \( d_k^-, d_k^+ \) are the under- and over-deviational variables, respectively, of the \( k \)-th membership goal. \( p_i(d^-) \) is a linear function of the weighted under-deviational variables, and where \( p_i(d^-) \) is of the form:

\[
p_i(d^-) = \sum_{k=1}^{K} w^k_i d^-_k; \quad k = 1, 2, \ldots, K; \quad I \leq K,
\]

where \( d^-_k \geq 0 \) is renamed for \( d^-_k \) to represent it at the \( i \)th priority level, \( w^k_i > 0 \) is the numerical weight associated with \( d^-_k \) and represents the weight of importance of achieving the aspired level of the \( k \)-th goal relative to the others which are grouped together at the \( i \)-th priority level.

It may be noted here that the priority factors have the relationship:

\[
p_1 > p_2 > \ldots > p_i > \ldots > p_n,
\]

which implies that the goal achievement for the priority factor \( P_i \) is preferred most to the next priority factor \( P_{i+1} \); \( i = 1, 2, \ldots, I \).

When some of the objective \( Z_k(x) \), \( k = 1, 2, \ldots, K \), are linear fractional in form, then the linearization approach [11] is conventionally used in the solution process. To avoid the computational load involved with the linearization of the objectives as well as decision error involves in approximation approach, a GA procedure is used in the process of solving the FGP model in (4).

The GA scheme used in the process of solving the problem (4) is presented in the following Section 3.
3. Design of the GA scheme

The basic steps of the GA scheme are defined as follows:

**Step 1. Representation and Initialization**

Let $E$ denote the binary coded representation of chromosome in a population as $E = \{x_1, x_2, ..., x_n\}$. The population size is defined by pop_size, and pop-size chromosomes are randomly initialized in its search domain.

**Step 2. Fitness Function**

The fitness value of each chromosome is judged by the value of an objective function. The fitness function is defined as

$$
\text{eval} (E_v) = (Z_i)_v
$$

$$
= \{ \sum_{k=1}^{K} w_k d_{ik} \}_v
$$

(5)

where $Z_i$ represents the $i$-th priority factor of the goal achievement of the function $Z$ in (4), and where the subscript $v$ in (5) refers to the fitness value of the selected $v$-th chromosome, $v=1,2,\ldots,\text{pop\_size}$. The best chromosome with largest fitness value at each generation is determined as

$$
E^* = \{ \min \{ \text{eval} (E_v) \mid v = 1, 2, \ldots, \text{pop\_size} \}
$$

in searching out the best value of the objective.

**Step 3. Selection**

The simple roulette-wheel scheme [16] is used for selecting two parents for mating purposes in the genetic search process.

**Step 4. Crossover**

The parameter $P_c$ is defined as the probability of crossover. The arithmetic crossover operator (single-point crossover) of a genetic system is applied here in the sense that the
resulting offspring always satisfy the linear constraints set $S (\neq \emptyset)$. Here a chromosome is selected as a parent, if for a defined random number $r \in [0, 1]$, $r < P_c$ is satisfied.

Here single-point crossover for two parents $E_1, E_2 \in S$ is defined as

$$
X_1 = \alpha_1 E_1 + \alpha_2 E_2, \quad X_2 = \alpha_2 E_1 + \alpha_1 E_2,
$$

for producing two offspring $X_1$ and $X_2$, where $\alpha_1, \alpha_2 \geq 0$ with $\alpha_1 + \alpha_2 = 1$ always belong to $S$, and where $S$ is a convex set.

**Step 5. Mutation**

As in the conventional GA scheme, a parameter $P_m$ of the genetic system is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < P_m$.

**Step 6. Termination**

The execution of the whole process terminates when the fittest chromosome is reported at a certain generation number in the solution search process.

Now, the FGP model formulation of the proposed problem is presented in the Section 4.

### 4. FGP model formulation of the problem

The decision variables and different types of parameters are defined first in the context of developing the model of the problem.

#### 4.1 Definition of decision variables and parameters

(a) *Decision variable:*

$$
x_{cs} = \text{Allocation of land for cultivating the crop } c \text{ during the season } s,
$$

$c = 1, 2, \ldots, C; s = 1, 2, \ldots, S.$
(b) **Productive resource parameters:**

- \( L_s \) = Total area of land (in hectares (ha)) currently in use for cultivating the crops in the season \( s \),
- \( MH_s \) = Estimated total machine hours (converted into rupees (Rs.)) required during the season \( s \),
- \( MD_s \) = Estimated total mandays (in days) required during the season \( s \),
- \( W_s \) = Estimated total amount of water (in inch / ha) required during the season \( s \),
- \( F_f \) = Estimated total amount of the fertilizer \( f (f = 1, 2, \ldots, F) \) (in quintals (qtls.)) required during the year,
- \( R_s \) = Estimated total amount of cash (in Rupees (Rs.)) required per annum for supply of the productive resources,

(c) **Aspiration level:**

- \( P_c \) = Annual production level (in quintals) of the crop \( c \),
- \( MP \) = Estimated total market value (in Rs.) of all the yield crops during the year,
- \( R_{ij} \) = Ratio of annual production of the \( i \)-th and \( j \)-th crop \( (i, j = 1, 2, \ldots, C; i \neq j) \),
- \( r_{ij} \) = Ratio of annual profits obtained from the \( i \)-th and the \( j \)-th crops \( (i, j=1,2,\ldots,C; i \neq j) \),

(d) **Crisp coefficients:**

- \( MH_{cs} \) = Average machine hours (in hrs.) required for tillage per ha of land for cultivating the crop \( c \) during the season \( s \),
- \( MD_{cs} \) = Mandays (in days) required per ha of land for cultivating the crop \( c \) during the season \( s \),
- \( W_{cs} \) = Amount of water consumed (in inch) per ha of land for cultivating the crop \( c \) during the season \( s \),
$F_{cs} = \text{Amount of the fertilizer } f \text{ required per ha of land for cultivating the crop } c$

during the season $s$,

$P_{cs} = \text{Estimated production of the crop } c \text{ per ha of land cultivated during the}$

season $s$,

$A_{cs} = \text{Average cost for purchasing seeds and different farm assisting materials}$

per ha of land cultivated for the crop $c$ during the season $s$,

$MP_{cs} = \text{Market price (Rs. / qtl.) at the time of harvest of the crop } c \text{ cultivated}$

during the season.

The fuzzy goals of the problem are now described in the following Section 4.2.

4.2 Description of fuzzy goals:

For the defined variables and different types of parameters involved with the problem, the
algebraic structures of the fuzzy goals appear as follows.

(i) Land utilization goal:

The fuzzy goals for utilization of total cultivable land in different seasons take the forms:

$$\sum_{c=1}^{C} x_{cs} \leq L_s, \quad s = 1, 2, ..., S.$$  

(ii) Productive resource goals:

(a) Machine-hour goal:

An estimated number of machine hours is to be provided to the land in different seasons of the year.

The fuzzy goal appears as:

$$\sum_{s=1}^{S} \sum_{c=1}^{C} x_{cs} \cdot MH_{cs} \geq MH_s.$$  

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(b) Man-power requirement goal:

A number of labourers are to be employed throughout the year to avoid the trouble with hiring of more labourers at the peak times and involvement of extra cost for it.

The fuzzy goal takes the form:

$$\sum_{c=1}^{C} x_{cs} \cdot MD_{cs} \geq MD_s.$$ 

(c) Water consumption goal:

Water is an essential input for yielding the crops. So, a minimum supply level of water is needed in each cropping season.

The fuzzy goal appears as:

$$\sum_{c=1}^{C} x_{cs} \cdot W_{cs} \geq W_s, \quad s = 1, 2, ..., S.$$ 

(d) Fertilizer requirement goal:

To maintain the productivity of the soil, different types of fertilizer are to be used in different seasons in the year.

The fuzzy takes the form:

$$\sum_{s=1}^{S} \sum_{c=1}^{C} x_{cs} \cdot F_{fc} \geq F, \quad f = 1, 2, ..., F.$$ 

(iii) Cash expenditure goal:

An estimated amount of cash (Rs.) is involved for the purpose of purchasing the seeds, fertilizers and other productive resources.

The fuzzy goal takes the form:

$$\sum_{s=1}^{S} \sum_{c=1}^{C} x_{cs} \cdot A_{cs} \leq R_s.$$
(iv) Production goals:

(i) Production achievement goal:

To meet the demand of agricultural products in society, a minimum achievement level of production of each type of the crops is needed.

The fuzzy goal appears as:

\[ \sum_{c=1}^{C} x_{c} \cdot P_{c} \geq P_{c^*}, \quad c = 1, 2, ..., C. \]

(ii) Production ratio goals:

To meet the demand of the primary food products in the society, allocation of land for the major crops in different seasons, should be made in such a way that certain ratios of total production of different crops can be maintained.

The production ratio goals appear as:

\[ \frac{\sum_{i=1}^{S} T_{ij} \cdot P_{ij}}{\sum_{j=1}^{S} T_{ij} \cdot P_{ij}} \geq R_{ij}, \quad i, j = 1, 2, ..., C \text{ and } i \neq j. \]

(v) Profit goals:

(i) Profit achievement goal:

A certain level of profit from the farm is highly expected by the farm manager. The fuzzy profit goal appears as:

\[ \sum_{i=1}^{C} \sum_{j=1}^{C} (MP_{ij} \cdot P_{ij} - A_{ij}) \cdot x_{ij} \geq MP \]

(ii) Profit ratio goal:

From the socio-economic point of view, beyond the meeting of demand of food products in society, the attention for cultivation of the other profitable crops need be given in the farm planning context. As such, certain ratios of crops production are to be maintained with a view of making profit from the farm.
The profit ratio goal takes the form

\[
\frac{\sum_{c=1}^{C} (\sum_{i=1}^{i} MP_{cis} - A_{cis}) x_{cis}}{\sum_{c=1}^{C} (\sum_{j=1}^{j} MP_{jcs} P_{jcs} - A_{jcs}) x_{jcs}} = r_{ij}, \quad i, j = 1, 2, \ldots, C \text{ and } i \neq j.
\]

Now, the FGP formulation of the problem is demonstrated through the following case example presented in the Section 5.

5. An illustrative case example

The land-use planning problem for production of the principal crops of the Nadia District in West Bengal, India is considered to illustrate the proposed FGP model.

The data of the planning year 2005-2006 were collected from different agricultural planning units. The sources are: District Statistical Hand Book, Nadia, 2005-2006 [22]; Action Plan Records (2005-2006 and 2004-2005) [23]; Soil Testing and Fertilizer Recommendation [24]; The Nadia Gramin Bank; Department of Agri-Irrigation [25].

Now, the decision variables and different types of data involved with the problem are summarized in the following Tables I–III.

### Table I: Summary of the seasonal crops and the associated decision variables

<table>
<thead>
<tr>
<th>Crop (c)</th>
<th>Prekharif</th>
<th>Kharif</th>
<th>Rabi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Jute</td>
<td>Sugarcane</td>
<td>Aus</td>
</tr>
<tr>
<td></td>
<td>x_{11}</td>
<td>x_{21}</td>
<td>x_{31}</td>
</tr>
<tr>
<td></td>
<td>Aman</td>
<td>Boro</td>
<td>Wheat</td>
</tr>
<tr>
<td></td>
<td>x_{42}</td>
<td>x_{53}</td>
<td>x_{63}</td>
</tr>
<tr>
<td></td>
<td>Mustard</td>
<td>Potato</td>
<td>Pulses</td>
</tr>
<tr>
<td></td>
<td>x_{73}</td>
<td>x_{83}</td>
<td>x_{93}</td>
</tr>
</tbody>
</table>

### Table II: Data description of the aspiration levels of goals and tolerance limits

<table>
<thead>
<tr>
<th>Goal</th>
<th>Aspiration Level</th>
<th>Tolerance Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower</td>
<td>Upper</td>
</tr>
<tr>
<td>1. Land utilization</td>
<td>272.135</td>
<td>-</td>
</tr>
<tr>
<td>(i) Prekharif season ('000 hectares)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(ii) Kharif season ('000 hectares)</td>
<td>272.135</td>
<td>-</td>
</tr>
<tr>
<td>(iii) Rabi season ('000 hectares)</td>
<td>272.135</td>
<td>-</td>
</tr>
<tr>
<td>2. a) Machine hours (Rs. Lac.):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Prekharif season</td>
<td>1189.4210</td>
<td>1103.6919</td>
</tr>
<tr>
<td>(ii) Kharif season</td>
<td>602.8849</td>
<td>563.0618</td>
</tr>
<tr>
<td>(iii) Rabi season</td>
<td>2896.6847</td>
<td>2822.7913</td>
</tr>
<tr>
<td>b) Man-days (days):</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Prekharif season</td>
<td>345.2753</td>
<td>340.3678</td>
</tr>
<tr>
<td>(ii) Kharif season</td>
<td>177.3189</td>
<td>165.6064</td>
</tr>
</tbody>
</table>
### Table III: Data description for utilization of productive resources, cash expenditure and market price

<table>
<thead>
<tr>
<th>Crops</th>
<th>MHS</th>
<th>MDS</th>
<th>WC</th>
<th>FR</th>
<th>PA</th>
<th>CE</th>
<th>MP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jute</td>
<td>204</td>
<td>90</td>
<td>20</td>
<td>40</td>
<td>2</td>
<td>0</td>
<td>2693.266</td>
</tr>
<tr>
<td>Sugarcane</td>
<td>510</td>
<td>123</td>
<td>60</td>
<td>200</td>
<td>1</td>
<td>00</td>
<td>78666.6</td>
</tr>
<tr>
<td>Aus</td>
<td>425</td>
<td>60</td>
<td>34</td>
<td>40</td>
<td>2</td>
<td>0</td>
<td>2203.463</td>
</tr>
<tr>
<td>Aman</td>
<td>204</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>2</td>
<td>0</td>
<td>2513.876</td>
</tr>
<tr>
<td>Boro</td>
<td>816</td>
<td>60</td>
<td>70</td>
<td>100</td>
<td>5</td>
<td>0</td>
<td>3253.953</td>
</tr>
<tr>
<td>Wheat</td>
<td>204</td>
<td>39</td>
<td>15</td>
<td>100</td>
<td>5</td>
<td>0</td>
<td>2131.634</td>
</tr>
<tr>
<td>Mustard</td>
<td>102</td>
<td>30</td>
<td>10</td>
<td>80</td>
<td>4</td>
<td>0</td>
<td>901.515</td>
</tr>
<tr>
<td>Potato</td>
<td>340</td>
<td>70</td>
<td>18</td>
<td>150</td>
<td>7</td>
<td>5</td>
<td>26818.18</td>
</tr>
</tbody>
</table>

**Goal**

<table>
<thead>
<tr>
<th>Goal</th>
<th>Aspiration Level</th>
<th>Tolerance Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(iii) Rabi season</td>
<td>379.3503</td>
<td>363.0368</td>
</tr>
<tr>
<td>c) Water consumption (inch)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Prekharif season</td>
<td>100.4412</td>
<td>99.9964</td>
</tr>
<tr>
<td>(ii) Kharif season</td>
<td>147.7658</td>
<td>124.2048</td>
</tr>
<tr>
<td>(iii) Rabi season</td>
<td>243.4898</td>
<td>236.8824</td>
</tr>
<tr>
<td>d) Fertilizer requirement</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) Nitrogen</td>
<td>34.973</td>
<td>34.7</td>
</tr>
<tr>
<td>(ii) Phosphate</td>
<td>19.6175</td>
<td>18.8</td>
</tr>
<tr>
<td>(iii) Potash</td>
<td>16.72</td>
<td>15.2</td>
</tr>
</tbody>
</table>

3. Production ('000 metric ton): (a) Jute 325.152
   (b) Sugarcane 139.0
   (c) Rice 800.0
   (d) Wheat 111.78
   (e) Mustard 78.5
   (f) Potato 147.5
   (g) Rabi pulse 42.46

4. Production ratio value (Rice-Wheat ) 6

5. Cash expenditure (Rs. Lac.) 83965.3158

5. (i) Profit (Rs. Lac.) 93849.4200

(ii) Profit ratio value (Jute-Aus) 5.5
Now using the data tables, the membership goals for the fuzzy goals defined in the Section 4.2 can be constructed by using the goal expressions in (4).

The fuzzy goals appear as follows:

1. Land utilization goals

It may be noted that most of the crops are single-season based, but three consecutive seasons are required for yielding the crops sugarcane

\[
\mu_1: 8.316 - 0.027(x_{11} + x_{21} + x_{31}) + d^-_1 - d^+_1 = 1 \\
\mu_2: 8.316 - 0.027(x_{21} + x_{22}) + d^-_2 - d^+_2 = 1 \\
\mu_3: 8.316 - 0.027(x_{21} + x_{23} + x_{31} + x_{73} + x_{83} + x_{93}) + d^-_3 - d^+_3 = 1
\]

(Prekharif) (Kharif) (Rabi)

2. Productive resource goals

(a) Machine hour goal (\(\mu_4\)):

\[
\mu_4: 0.0634 x_{11} + 0.1586 x_{21} + 0.1322 x_{31} - 13.9748 + d^-_4 - d^+_4 = 1 \\
\mu_5: 1.2658 x_{42} - 14.1390 + d^-_5 - d^+_5 = 1 \\
\mu_6: 0.2728 x_{53} + 0.0682 x_{63} + 0.0342 x_{73} + 0.11378 x_{83} + 0.0501 x_{93} + 13.9748 + d^-_6 - d^+_6 = 1
\]

(Prekharif) (Kharif) (Rabi)

(b) Man power goal:

\[
\mu_7: 0.4530 x_{11} + 0.6191 x_{21} + 0.3020 x_{31} - 69.3329 + d^-_7 - d^+_7 = 1 \\
\mu_8: 0.1265 x_{42} - 14.1378 + d^-_8 - d^+_8 = 1 \\
\mu_9: 0.0928 x_{53} + 0.0602 x_{63} + 0.1081 x_{73} + 0.0463 x_{83} + 0.0231 x_{93} - 22.2534 + d^-_9 - d^+_9 = 1
\]

(Prekharif) (Kharif) (Rabi)

(c) Water consumption goals:
\[ \mu_{10} : 1.1109x_{11} + 3.3328x_{21} + 1.8886x_{31} - 224.9917 + d_{10} - d_{10}^+ = 1 \]  
(Prekharif)

\[ \mu_{11} : 0.1524x_{42} - 5.2712 + d_{11} - d_{11}^+ = 1 \]  
(Kharif)

\[ \mu_{12} : 0.2618x_{33} + 0.0560x_{63} + 0.0374x_{73} + 0.0673x_{83} + 0.0374x_{93} - 35.8497 + d_{12} - d_{12}^+ = 1 \]  
(Rabi)

(d) Fertilizer requirement goals:

\[ \mu_{13} : 0.1465x_{11} + 0.7326x_{21} + 0.1465x_{31} + 0.1465x_{42} + 0.0366x_{53} + 0.0366x_{63} + 0.2930x_{73} 
+ 0.5494x_{83} + 0.0732x_{93} - 127.1061 + d_{13} - d_{13}^+ = 1 \]  
(N)

\[ \mu_{14} : 0.0244x_{11} + 0.0122x_{21} + 0.0244x_{31} + 0.0244x_{42} + 0.0611x_{53} + 0.0611x_{63} + 0.0489x_{73} 
+ 0.1529x_{83} + 0.0611x_{93} - 22.9961 + d_{14} - d_{14}^+ = 1 \]  
(P)

\[ \mu_{15} : 0.0102x_{11} + 0.0256x_{21} + 0.0051x_{31} + 0.0051x_{42} + 0.0128x_{53} + 0.0128x_{63} + 0.0102x_{73} 
+ 0.0318x_{83} + 0.0051x_{93} - 4.2771 + d_{15} - d_{15}^+ = 1 \]  
(K)

3. Cash expenditure goal:

\[ \mu_{16} : 10.9998 - (0.0206x_{11} + 0.0367x_{21} + 0.0171x_{31} + 0.0153x_{42} + 0.0282x_{53} + 0.0132x_{63} + 0.0444x_{73} 
+ 0.0100x_{83} + 0.0059x_{93}) + d_{16} - d_{16}^+ = 1 \]

4. Production goals:

(i) Production achievement goals:

\[ \mu_{17} : 0.0326x_{31} + 0.0372x_{42} + 0.0481x_{53} - 10.8343 + d_{17} - d_{17}^+ = 1 \]  
(Rice)

\[ \mu_{18} : 0.1635x_{11} - 18.7419 + d_{18} - d_{18}^+ = 1 \]  
(Jute)

\[ \mu_{19} : 0.1873x_{43} - 8.8224 + d_{19} - d_{19}^+ = 1 \]  
(Wheat)

\[ \mu_{20} : 3.93x_{21} - 5.9 + d_{20} - d_{20}^+ = 1 \]  
(Sugarcane)

\[ \mu_{21} : 1.269x_{73} - 10.0559 + d_{21} - d_{21}^+ = 1 \]  
(Mustard)

\[ \mu_{22} : 0.7506x_{83} - 3.1284 + d_{22} - d_{22}^+ = 1 \]  
(Potato)

\[ \mu_{23} : 0.2155x_{93} - 9.9999 + d_{23} - d_{23}^+ = 1 \]  
(Pulses)

(ii) Production ratio goal:

\[ \mu_{24} : 0.66 \big( (2.2034x_{31} + 2.5138x_{42} + 3.2539x_{31})/(2.1313x_{63} - 4.5) \big) + d_{24} - d_{24}^+ = 1 \]
5. Profit goals:

(i) Profit achievement goal:

\[
\mu_{25} : 0.0278x_1 + 0.1037x_{21} + 0.0106x_{31} + 0.0212x_{42} + 0.0198x_{63} + 0.0086x_{73} + 0.0834x_{73} + 0.0098x_{93} - 8.9947 + d_{25} - d_{25}^* = 1
\]

(ii) Profit ratio goal:

The crop rice and wheat are the major crops in India and they are used as substitute of other as main food grains.

The ratio of rice wheat goal appears as:

\[
\mu_{26} : 0.66 \left( \frac{259.8446x_{11}}{713.7916x_{21} + 99.06x_{31}} - 4 \right) + d_{26} - d_{26}^* = 1
\]

Now for the stated membership goals of the problem, using the expression in (4), the achievement function of the executable FGP model under the three assigned priorities is obtained as:

Find \( \{x_s \mid c=1,2,...,9; s=1,2,3\} \) so as to:

\[
\text{Minimize } Z = \left[ P_1 \left( \sum_{k=1}^{26} w_{1k}d_{1k}^* \right), P_2 \left( \sum_{k=1}^{3} w_{2k}d_{2k}^* \right), P_3 \left( \sum_{k=4}^{16} w_{3k}d_{3k}^* \right) \right] \quad (6)
\]

Then, the proposed GA approach presented in the Section 3 is used to solve the problem in (6). The objective function of the model appears as the fitness function in the solution search process.

The following genetic parameter values are introduced in the search process:

- Probability of crossover \( P_c = 0.8 \)
- Probability of mutation \( P_m = 0.08 \)
- Population size = 100
- Chromosome length = 30

The GA based program is designed in Programming Language C. The execution is done...
in a Intel Pentium IV PC with 2.66 GHz CPS and 1 GB RAM.

The optimal solution is reached after 200 generations. The achieved result is presented in the following Table IV.

**Table IV: Land allocation and production of crops under the proposed model**

<table>
<thead>
<tr>
<th>Crop (c)</th>
<th>Jute</th>
<th>Sugarcane</th>
<th>Rice</th>
<th>Wheat</th>
<th>Mustard</th>
<th>Potato</th>
<th>Pulses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land allocation</td>
<td>129.6267</td>
<td>1.7557</td>
<td>297.89</td>
<td>52.4444</td>
<td>87.1229</td>
<td>78.0233</td>
<td>51.043</td>
</tr>
<tr>
<td>Production</td>
<td>349.084</td>
<td>138.115</td>
<td>799.827</td>
<td>111.7581</td>
<td>78.498</td>
<td>2092.429</td>
<td>42.417</td>
</tr>
</tbody>
</table>

The total profit obtained under the proposed cropping plan is Rs. 153319.33 Lac.

The production structure for the existing cropping plan (2005–2006) of the District is presented in the Table V.

**Table V: Land allocation and production of crops recorded in the year 2005 – 2006**

<table>
<thead>
<tr>
<th>Crop (c)</th>
<th>Jute</th>
<th>Sugarcane</th>
<th>Rice</th>
<th>Wheat</th>
<th>Mustard</th>
<th>Potato</th>
<th>Pulses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land allocation</td>
<td>120.2</td>
<td>1.5</td>
<td>265.4</td>
<td>47.1</td>
<td>79.2</td>
<td>5.5</td>
<td>46.4</td>
</tr>
<tr>
<td>Production</td>
<td>325.15</td>
<td>118.0</td>
<td>732.4</td>
<td>100.4</td>
<td>71.4</td>
<td>147.5</td>
<td>38.6</td>
</tr>
</tbody>
</table>

The total profit obtained under the existing cropping plan is Rs. 95803.0117 Lac.

A comparison of the model solution with the results in the Table IV and Table V shows that the solution under the proposed approach is better for the view-point of achieving the aspired goal levels defined in the decision making context.

**Note:**

(i) If the FGP approach [6] with crisp resource and ratio constraints is used, then the obtained solution for production achievement is presented in the Table VI.

**Table VI: Production achievement with crisp resource and ratio constraints**

<table>
<thead>
<tr>
<th>Crop (c)</th>
<th>Jute</th>
<th>Sugarcane</th>
<th>Rice</th>
<th>Wheat</th>
<th>Mustard</th>
<th>Potato</th>
<th>Pulses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Land allocation</td>
<td>112.85</td>
<td>1.766</td>
<td>277.34</td>
<td>63.85</td>
<td>60.10</td>
<td>5.50</td>
<td>51.20</td>
</tr>
</tbody>
</table>
Production 325.48 138.92 742.04 136.10 54.18 147.50 42.45

The total profit obtained here is Rs. 136180.0 Lac.

5. Conclusion

The main advantage of the GA based FGP approach presented here is that the proper decision for allocation of cultivable land regarding optimal production of crops on the basis of the needs and desires of the society can be made in the decision making situation. Again, under the framework of proposed approach the other different parameters can easily be incorporated without introducing any computational difficulty.

The proposed approach can be extended to farm planning problems with interval valued data, instead of fuzzy description of parameters in an inexact environment, which is a problem for future research.

In future studies, the proposed approach can be extended to solve agricultural production planning problems with probabilistic data in an uncertain decision environment.

Finally, it is hoped that the solution concept presented here can contribute to future studies in farm planning and other MODM problems in the current multiobjective decision making arena.

References


[25] Department of Agri-irrigation, Office of the Executive Engineer, Krishnanagar, Nadia, Govt. of W. B., India.