Chapter 6


6.1 Introduction

In an uncertain decision making environment, there are two types of inexactness - probability and fuzzy. The former is expressed by measuring the degree of occurrence of an event out of various trials and the later is employed for measurement of vagueness [291]. These substantially lead to two different branches of mathematical programming known as chance constrained programming (CCP), initially introduced by Charnes and Cooper in [73], and fuzzy programming (FP) based on the concept of fuzzy set theory (FST) introduced by Zadeh in [515] in 1965. Now, due to multidimensional in nature of complex decision systems and imprecise in nature of human judgment, the decision makers (DMs) are often confused with the problems of inexact parameter values (stochastic or fuzzy).

SP deals with the problems where parameters of the problems are described by stochastic or probabilistic variables. SP with continuous random parameter was introduced in [88], and applied to real-life problems [197] by considering different aspects in which the coefficients of the constraints and the parameters in the objectives are inherently random in nature.

During 1960s to mid-1980s, different aspects of CCP problems have been investigated by pioneer researchers of the field [243, 322, 385, 392]. Sullivan et al. [468] suggested an algorithm where goals can be stated in terms of probability of satisfying the aspiration levels. Teghem et al. [480] presented the interactive methods in the framework of the solution approach of SP.

On the other hand, FP offers a powerful approach in solving optimization problem with non-stochastic imprecision and vagueness. FP approaches has been used in different ways in [37, 119, 523] for solving problems in imprecise environment. Hannan [180] presented an FGP approach to decision problems in an imprecisely environment. The use of FGP [484] approaches to different real-life decision problems has been investigated by Pal et al. in [47, 376]. Different approaches of conversion of the original chance constraints using FP were provided in [303, 306]. Slowinski [456] presented the state-of-art of CCP problem in fuzzy environment.

The use of GAs inspired by natural selection and population genetics, initially introduced by Holland [192], to CCP problems was introduced by Iwamura and Liu
[229]. They have proposed a stochastic simulation based GA for solving CCP problems with single as well as multiplicity of objectives. They have suggested this approach to overcome the difficulty in converting the CCP model into deterministic equivalent which is too expensive for complex and large CCP problems. In their approach, stochastic simulation is mainly employed to examine the feasibility of the solution. Liu in [289] provided a GA based approach for solving dependent CCP problems. The stochastic simulation based technique with the use of GA [417] has been studied in [230] in the past.

GA based stochastic simulation approach to FGP formulation of multiobjective CCP problems has been investigated by Jana and Biswal [231] in the recent past. In their study, a CCP problem having discrete random parameters with known probability distributions has been taken into consideration.

Again, in their approach, the conventional GA scheme [164] to the stochastic simulation based on trial and error technique has been incorporated in the process of searching the solution of the problem. However, the use of GA in the FGP formulation of CCP problems is still at an early stage.

In this paper, achievement of fuzzily described objectives of a multiobjective decision making (MODM) problem under a system of chance constraints having continuous random parameters is considered. In the model formulation of the problem, the chance constraints with certain probability distribution of the chance parameters are introduced for selection of candidate solutions by satisficing the prescribed probabilities of constraints in the solution search process. The use of mean and variance is taken into account here to transform the defined chance constraints into their deterministic equivalent [197]. Then, a GA scheme is employed with the operators: fitter-codon selection, two-point crossover and random mutation to obtain individual best and worst solution of the objectives. Then, the objectives are defined fuzzily by introducing fuzzy aspiration level and lower tolerance limit to each of them with the use of the obtained individual solutions. Then, the fuzzy goals are characterized by the membership functions and the GA scheme is again employed for achievement of the highest membership value (unity) of the membership goals on the basis of their weights of importance by minimizing the associated under-deviational variables.
The proposed approach is illustrated by two numerical examples.

6.2 Chance Constrained MODM Problem Formulation

The generic form of chance constrained MODM problem can be stated as

$$\text{Find } X(x_1, x_2, \ldots, x_n) \text{ so as to}$$

$$\text{Maximize } Z_k(X) = \sum_{j=1}^{n} c_{kj} x_j, \quad k = 1, 2, \ldots, K$$

subject to

$$X \in S = \{X \in \mathbb{R}^n \mid \text{Prob}[AX \leq b] \geq \beta, X \geq 0, b \in \mathbb{R}^m\} \quad (6.1)$$

Where $X$ is a vector of deterministically defined decision variables, 'Prob' designates the probabilistically defined constraints, $A$ is an $(m \times n)$ real coefficient matrix, $b$ is a resource vector, and where $\beta (0 < \beta < 1)$ represents the vector of confidence levels of probability for satisfying the defined constraints. It is assumed that the feasible region $S$ ($S \neq \Phi$) is bounded.

Now, in the context of the present formulation, both the elements of the coefficient matrix and resource vector are assumed to be independently random in nature and follow normal distribution.

6.2.1 Chance Constraints and their Deterministic Equivalent

Now, the constraints in (6.1) can be explicitly stated as

$$\text{Prob}\left[\sum_{j=1}^{n} a_{ij} x_j \leq b_i \right] \geq \beta_i, \quad i = 1, 2, \ldots, m. \quad (6.2)$$

where $a_{ij}$ and $b_i$ are normal random variables and $\beta_i$ are the specified probabilities.

Now, the following notion of distribution function for a random variable is defined in the sequel of finding the crisp equivalent of the constraints in (6.2). Let $\phi_i(y)$, be the distribution function of the $i$-th random variable $b_i$. Then, since $\phi_i(y)$ is a monotonically non-decreasing function, the value of the corresponding variable is determined as:
\[
\phi_i^{-1}(\varepsilon) = \{ \text{Max } y \mid \text{Prob}(b_i \geq y) \leq \varepsilon \}, \quad 0 \leq \varepsilon \leq 1 \tag{6.3}
\]

The following two sub-cases for the random variables are considered:

1. \( b_i \) is a random variable.
2. \( a_{ij}, \forall j \), are random variables.

**Case I:** For \( b_i \) is a random variable.

Let, \( b_i \) be a random normal variable with mean and variance \( \mu(b_i) \) and \( \text{Var}(b_i) \), respectively.

Now, the defined constraints set in (6.2) can be expressed as:

\[
\text{Prob}\left[ \sum_{j=1}^{n} a_{ij}x_j \leq b_i \right] = \text{Prob}\left[ \frac{b_i - E(b_i)}{\sqrt{\text{Var}(b_i)}} \geq \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\text{Var}(b_i)}} \right] \tag{6.4}
\]

Then, using the prescribed probability level \( \beta_i \), the expression in (6.3) takes the form

\[
\text{Prob}\left[ \frac{b_i - E(b_i)}{\sqrt{\text{Var}(b_i)}} \geq \frac{\sum_{j=1}^{n} a_{ij}x_j - E(b_i)}{\sqrt{\text{Var}(b_i)}} \right] \geq \beta_i, \quad i=1,2,...,m. \tag{6.5}
\]

Now, using the notion of distribution function in (6.3) to the expression in (6.5), the deterministic equivalent of the constraints in (6.2) with \( \leq \) type can be obtained as a set of linear system constraints as:

\[
\sum_{j=1}^{n} a_{ij}x_j \leq E(b_i) + \phi_i^{-1}(\beta_i)\sqrt{\text{Var}(b_i)}, \quad i=1,2,...,m. \tag{6.6}
\]

**Case II:** For \( a_{ij}, \forall j \), is a random variable.

Let, \( a_{ij} \) are random normal variables with mean and variance \( \mu(a_{ij}) \) and \( \text{Var}(a_{ij}) \), respectively.

Then, let \( Y_i = \sum_{j=1}^{n} a_{ij}x_j \)
The mean and variance of $y_i$ are successively obtained as:

$$E(y_i) = \sum_{j=1}^{n} E(a_{ij}) x_j, \text{ since } x_j, \forall j \text{ are deterministic,}$$

(6.7)

$$Var(y_i) = X^T V_i X, \text{ respectively,}$$

(6.8)

where $V_i$ represents the $i$-th covariance matrix of order $(n \times n)$ for the defined $a_y, \forall j, \text{T means transpose.}$

Then, following the basic rules of probability theory, the expressions of the constraints set in (6.2) can be expressed as:

$$\text{Prob} \left[ \frac{y_i - E(y_i)}{\sqrt{\text{var}(y_i)}} \geq \frac{b_i - E(y_i)}{\sqrt{\text{var}(y_i)}} \right] \geq \beta_i, \ i = 1, 2, ..., m. \quad (6.9)$$

where $\frac{y_i - E(y_i)}{\sqrt{\text{var}(y_i)}}$ is the standard normal variable with mean zero and variance one.

Now, using the notion of distribution function defined in (6.6), and realizing the fact that $\text{Prob}[y_i \leq b_i]$, the deterministic equivalent of the given chance constraints appear as:

$$\frac{b_i - E(y_i)}{\sqrt{\text{var}(y_i)}} \geq F^{-1}(\beta_i)$$

i.e.,

$$E(y_i) + \varphi_i^{-1}(\beta_i) \sqrt{\text{var}(y_i)} \leq b_i, \quad i = 1, 2, ..., m. \quad (6.10)$$

Here, since $a_y, \forall j, \text{ follow normal distribution, the covariance terms } \text{cov}(a_y, a_{ik}), \forall j, j \neq k, k = 1, 2, ..., n \text{ would be zero.}$

The expression in (6.10) in a simplified form can be presented as:

$$\sum_{j=1}^{n} E(a_{ij}) x_j + \varphi_i^{-1}(\beta_i) \sqrt{\left( \sum_{j=1}^{n} \text{var}(a_{ij}) x_j^2 \right)} \leq b_i, \quad i = 1, 2, ..., m. \quad (6.11)$$

Now, the GA scheme adopted in process of FGP formulation of the problem in (6.1) and thereby solving the executable model of the problem is presented in the Section 6.3.
6.3 Design of the GA Scheme

Step 1. Representation and Initialization

Let $V_L$ denote the binary coded representation of chromosome in a population as

$$V_L = \{x_1, x_2, x_3, \ldots, x_n\}_L,$$

where $L = 1, 2, \ldots, \text{pop}_\text{size}$, represents the population size, and where $\text{pop}_\text{size}$ chromosomes are randomly initialized in its search domain.

Step 2. Fitness function

The fitness value of each chromosome is determined by the value of an objective function. The fitness function is defined as:

$$\text{eval} (V_L) = (Z_k)_L, \quad k = 1, 2; \quad L = 1, 2, \ldots, \text{pop}_\text{size},$$

The best and least objective function values of the fittest chromosome can be obtained as:

$$V^* = \max \{\text{eval} (V_L) | L = 1, 2, \ldots, \text{pop}_\text{size}\},$$

and

$$V^* = \min \{\text{eval} (V_L) | L = 1, 2, \ldots, \text{pop}_\text{size}\},$$

respectively.

Step 3. Selection

In the fitter codon selection scheme [507], the selection of chromosomes is made on the basis of their fitness scores, where nearer the fitness score of a chromosome to a predefined level of fitness value indicates the fitter one in the selection process.

For instance, the following four chromosomes in a population are considered.

(i) 111010 000
(ii) 111101 010
(iii) 010111010
(iv) 101010010

Here, codons are selected from the stand point of maximum occurrence of dominant values of the most significant bits, where codon length is defined by the number of bits from most significant bit position to the bit position of the first non-matching bit in the selected pair. It is to be observed here that the chromosomes in (i) and (ii) with codon length 4 each are the fitter ones in
comparison to the that in (iii) and (iv). Again, the chromosome in (ii) is fitter than the chromosome in (i). It is clear from the above that the decimal equivalents of chromosomes are not needed here for selection of them as the fitter ones.

Step 4. Crossover

The probability of crossover is defined by the parameter $P_c$. Here in a two-point crossover genetic system, the mating chromosomes interchange their middle portion in the process of reproduction. Again, a chromosome is selected as a parent, if for two defined random number $r, r_1 \in [0, 1]$; $r, r_1 < P_c$ with $r + r_1 < 1$ is satisfied.

In the selection of two parents, another random number $r_2$ is defined such that $r_2 = 1 - r - r_1$. Then, two parents $V_1, V_2 \in S$ yield two offspring as:

$$U_1 = (r + r_2), V_1 + r_1 V_2, \quad U_2 = r_1, V_1 + (r + r_2), V_2,$$

where $U_1, U_2 \in S$.

Step 5. Mutation

The parameter $P_m$, as in the conventional GA scheme, is defined as the probability of mutation. The mutation operation is performed on a bit-by-bit basis, where for a random number $r \in [0, 1]$, a chromosome is selected for mutation provided that $r < P_m$.

Step 6. Termination

The execution of the whole process terminates when the number of iterations is reached to the specified generation number in the genetic process. The generated best chromosome is reported finally as the decision in the genetic search process.

Now, the FGP formulation is presented in the following Section 6.4.

6.4 FGP Problem Formulation

In the field of FP, an imprecise aspiration level is assigned to each of the objectives and the tolerance limit for achievement of each of them is taken into consideration.

Let $(X^B_k, Z^B_k)$ and $(X^W_k, Z^W_k)$ be the best and worst solutions for maximization of the $k$-th objective $Z_k$, $k = 1, 2, \ldots, K$, subject to the constraints either in (6.6) or
(6.11) or both obtained by employing the proposed GA scheme, where B and W are used to indicate the best and worst solutions, respectively.

Then, the fuzzy goals of the problem appear as

$$Z_k \geq Z_k^B, \quad k=1,2,...,K,$$

(6.12)

with the lower tolerance limits $Z_k^W$, $k=1,2,...,K$, where $\geq$ indicates the fuzzified version of $\geq$ in the sense of Zimmermann [526].

Now, in the field of FP, the fuzzy goals are characterized by their respective membership functions.

### 6.4.1 Characterization of Membership Function

Let $\mu_k(X)$ be the membership function representation of the $k$-th fuzzy goal.

Then, for $\geq$ type of restriction, $\mu_k(X)$ takes the form

$$\mu_k(X)=\begin{cases} 1 & \text{if } Z_k(X) \geq Z_k^B \\ \frac{Z_k(X)-Z_k^W}{t_k} & \text{if } Z_k^W \leq Z_k(X) < Z_k^B \\ 0 & \text{if } Z_k(X) < Z_k^W \end{cases}$$

(6.13)

where $t_k = (Z_k^B - Z_k^W)$ is the tolerance range for achievement of the $k$-th fuzzy goal, $k=1,2,...,K$.

Now, in the field of FP, achievement of the highest membership value (unity) of each of the objectives is highly desired by the DM in a decision situation. In such a case, the FGP [484] as an extension of conventional GP and a robust tool for fuzzy multiobjective decision analysis is taken into consideration in the decision making context.

### 6.4.2 FGP Model Formulation

In FGP, the membership functions are considered as goals by assigning the highest degree (unity) as the aspiration level and introducing under- and over-deviational variables to each of them. In the goal achievement function, the under-deviational variables of the membership goals are minimized on the basis of their priorities and weights of importance in the decision situation.
The FGP model of the problem under a pre-emptive priority structure can be presented as:
Find $X$ so as to:

Minimize $Z = [P_1(d^-), P_2(d^-), \ldots, P_r(d^-), \ldots, P_R(d^-)]$

and satisfy

$$\frac{Z_k(X) - Z_{k}^{w}}{t_k} + d_k^- - d_k^+ = 1, \quad k = 1, 2, \ldots, K$$

subject to the system constraints in (6.6) or (6.11). Here $Z$ represents the vector of $R$ priority goal achievement functions and $d_k^-$ and $d_k^+$ are the under- and over-deviational variables, respectively, of the $k$-th membership goal. $P_r(d^-)$ is a linear function of the weighted under-deviational variables, and where $P_r(d^-)$ is of the form:

$$P_r(d^-) = \sum_{k=1}^{K} w_{rk} d_{rk}^{-}, \quad R \leq K$$

where $d_{rk}^{-}$ is renamed for $d_k^-$ to represent it at the $r$-th priority level, $w_{rk} (\geq 0)$ is the numerical weight associated with $d_{rk}^-$ and represents the weight of importance of achieving the aspired level of the $k$-th goal relative to the others which are grouped together at the $r$-th priority level.

It may be noted here that the priority factors have the relationship:

$$P_1 >> P_2 >> \ldots >> P_r >> \ldots >> P_R$$

which implies that the goal achievement for the priority factor $P_r$ is preferred most to the next priority factor $P_{r+1}$, $r = 1, 2, \ldots, R$.

### 6.5 GA for FGP

The goal achievement functions $Z$ in (6.14) here appears as the fitness function in the evaluation process of using the GA scheme. The evaluation function for determination of the fitness of a chromosome can be represented as:

$$\text{eval}(V_L) = (Z)_L, \quad k = 1, 2, \ldots, k; \quad L = 1, 2, \ldots, \text{pop}_\text{size}.$$
Here, the chromosome $V^*$ with the best fitness value at each generation is determined as $V^* = \min \{\text{eval}(F_l) | L = 1, 2, \ldots, \text{pop\_size}\}$ in the genetic search process.

To illustrate the approach, two numerical examples are solved.

### 6.6 Numerical Example

**Example 1:**

The following chance constrained MODM problem studied previously [379] is considered to illustrate the potential use of the proposed approach.

Find $X(x, x_2, x_3)$ so as to

Maximize $Z_1 = 6x_1 + 2x_2 + 3x_3$

Maximize $Z_2 = 3x_1 + 5x_2 + 4x_3$

Maximize $Z_3 = 2x_1 + 7x_2 + x_3$

subject to

\[
\begin{align*}
\text{Prob} \left[ x_1 + x_2 + x_3 < b_1 \right] &\geq 0.05 \\
\text{Prob} \left[ 5x_1 + x_2 + 6x_3 < b_2 \right] &\geq 0.90 \\
\text{Prob} \left[ 2x_1 + 3x_2 + x_3 < b_3 \right] &\geq 0.85
\end{align*}
\]

(6.17)

Here, $b_1, b_2, b_3$ are normally distributed random variables with the mean and variance $(2.5, 2), (8, 9), \text{and} (10, 12)$ respectively.

Now, by employing the procedure mentioned in Section II, the constraints in (6.17) appears in their deterministic form as:

\[
\begin{align*}
x_1 + x_2 + x_3 &\leq 5.174 \\
5x_1 + x_2 + 6x_3 &\leq 11.855 \\
2x_1 + 3x_2 + x_3 &\leq 13.592
\end{align*}
\]

(6.18)

Now the proposed GA scheme, developed using Programming Language C with hardware support of Dell Power Edge R900 Server with 2 GB RAM is used to solve the problem. The following GA parameter values are adopted to determine the individual best and least values of the objectives.
• Probability of crossover $P_c = 0.8$
• Probability of mutation $P_m = 0.08$
• Population size = 100

The number of generations = 300 is initially taken to conduct the experiment. The different experiments with the different values of $P_c$ (0 < $P_c$ < 1) and $P_m$ (0 < $P_m$ < 1), in the ranges (0.7 < $P_c$ < 0.9) and (0.03 < $P_m$ < 0.09) are made in the proposed GA scheme. It is found that $P_c = 0.8$ and $P_m = 0.08$ are most acceptable in the decision search process. Then following the procedure, the individual best and least values of the successive objectives are obtained as:

$$
(x_1, x_2, x_3; Z_f^g) = (1.7, 3.4, 0; 13.6),
$$

$$
(x_1, x_2, x_3; Z_f^w) = (0, 4.5, 0.5; 0.5),
$$

$$
(x_1, x_2, x_3; Z_f^g) = (1.8, 3.0, 0; 20.4),
$$

$$
(x_1, x_2, x_3; Z_f^w) = (0, 0.5, 1.3; 7.7),
$$

$$
(x_1, x_2, x_3; Z_f^g) = (0, 4.5, 0.5; 32.0),
$$

$$
(x_1, x_2, x_3; Z_f^w) = (1.75, 3.0, 0; 24.5).
$$

The fuzzy goals can be constructed as

$$
Z_f > 13.6, \ Z_f^w > 20.4, \text{ and } Z_f^g > 32.0.
$$

Now, based on the above numerical values, the resulting FGP model can be obtained by following (6.14) and (6.15):

The model appears as:

Find $X(x_1, x_2, x_3)$ so as to

Minimize $Z = 0.076d_1^- + 0.078d_2^- + 0.133d_3^-$

and satisfy:

$$
0.076[6x_1 + 2x_2 + 3x_3 - 0.5] + d_1^- - d_1^+ = 1
$$

$$
0.078[3x_1 + 5x_2 + 4x_3 - 7.7] + d_2^- - d_2^+ = 1
$$

$$
0.133[2x_1 + 7x_2 + x_3 - 24.5] + d_3^- - d_3^+ = 1
$$

Subject to the given constraints in (6.18).
The resultant solution of the problem (6.19) by employing the proposed GA scheme is obtained as:

\[(x_1, x_2, x_3) = (0.7, 4.0, 0.8)\]

with \((Z_1, Z_2, Z_3) = (14.6, 25.3, 30.2)\)

The following Figure 6.1 used to represent the model solution diagrammatically.

![Figure 6.1: Representation of Model solution](image)

**Note 6.1:** The solution of the problem using the linearization approach studied in [379] previously, is found as

\[(x_1, x_2, x_3) = (0.68, 3.81, 0.76)\]

with \((Z_1, Z_2, Z_3) = (14.05, 24.21, 28.86)\)

A comparison shows that the proposed solution approach is superior one from the viewpoint of goal achievement of the specified goal levels of the objectives in the decision making environment.
The result comparison is projected pictorially in the following Figure 6.2.

![Result Comparison](image)

**Figure 6.2: Result comparison between Linearization technique and GA**

*Example 2:*

To expound the effectiveness of the approach more, the following example with three objectives and two chance constraints with normally distributed random variables is considered.

Find $X(x_1, x_2, x_3)$ so as to

Maximize $Z_1 = 5x_1 + 6x_2 + 3x_3$.

Maximize $Z_2 = 7x_1 + 2x_2 + 4x_3$

Maximize $Z_3 = 2x_1 + 3x_2 + 8x_3$

subject to

$\text{Prob}[a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 \leq b_i] \geq 0.95$

$\text{Prob} [5x_1 + x_2 + 6x_3 \leq b_2] \geq 0.10$, \hspace{1cm} (6.20)
Here, \( a_y \) and \( b_i \) are independently normally distributed random variables with the means and variances defined as follows:

Here, the means and variances of \( a_1, a_2, a_3 \) are successively defined as: \((3, 6), (9, 4), \) and \((1, 2)\) respectively.

Again, the means and variances of \( b_1, b_2 \) are defined as: \((5, 8), \) and \((7, 9), \) respectively.

Now, following the procedure, the deterministic equivalent of the successive constraints in (6.20) are found as:

\[
3x_1 + 9x_2 + x_3 + 1.645(6x_1^2 + 4x_2^2 + 2x_3^2)^{1/2} \leq 8
\]

\[
5x_1 + x_2 + 6x_3 \leq 10.9
\]

The same computational environment of Dell Server and C language and, the genetic parameter values adopted in the case of Example 1 are also considered here to solve the problem.

Now, the individual best and least values of the successive objectives are obtained as:

\[
(x_1, x_2, x_3; Z_1^b) = (0.5, 0.7, 0; 6.7),
\]

\[
(x_1, x_2, x_3; Z_1^w) = (0.5, 0, 0.5; 4),
\]

\[
(x_1, x_2, x_3; Z_2^b) = (0.9, 0, 0; 6.3),
\]

\[
(x_1, x_2, x_3; Z_2^w) = (0, 1.25, 0.5; 4.5),
\]

\[
(x_1, x_2, x_3; Z_3^b) = (0.06, 0.07, 0.62; 5.25),
\]

\[
(x_1, x_2, x_3; Z_3^w) = (0.75, 0.67, 0; 3.51).
\]

Then, following the procedure, the resulting FGP model is obtained as:

Find \( X(x_1, x_2, x_3) \) so as to

Minimize \( Z = 0.37d_1^- + 0.45d_2^- + 0.575d_3^- \)

and satisfy:

\( 0.37[5x_1 + 6x_2 + 3x_3 - 4.0] + d_1^- - d_1^+ = 1 \)
subject to

\[ 3x_1 + 9x_2 + x_3 + 1.645(6x_1^2 + 4x_2^2 + 2x_3^2)^{1/2} \leq 8 \]
\[ 5x_1 + x_2 + 6x_3 \leq 10.9 \]
\[ x_1, x_2, x_3 \geq 0 \]  

(6.22)

The resulting decision of the problem (6.22) by employing the GA scheme is obtained as:

\[ (x_1, x_2, x_3) = (0.542, 0.418, 0.401) \]

with \( (Z_1, Z_2, Z_3) = (6.421, 6.234, 5.546) \).

The pictorial representation of the model solution is shown in the following Figure 6.3.

![Model Solution](image)

**Figure 6.3: Pictorial representation of Model solution**

The result shows how the satisfactory decision can be obtained with the change of the random parameters involved with problems in the decision making environment.
6.7 Conclusion

In this paper, how the GA scheme can be effectively used in the FGP formulation of MODM problems under the chance constraints with different continuously distributed random parameters is presented.

In the proposed approach, computational complexity associated with conventional approaches and inherent computational error associated with linearization technique does not arise. Again, within the framework of the proposed FGP model, crisp objectives as well as constraints can easily be incorporated without involving any computational difficulty.

The proposed approach can be extended to solve general non-linear and different types of fractional MODM problems in uncertain environment. An extension of the approach to different chance constrained hierarchical decision problems may be the problem for future research. Finally, it is expected that the approach presented here may lead to further research for its implementation to real-life MODM problems in the current complex uncertain decision making environment.