Chapter 8

A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System

The content of this chapter is based on the paper published in the


DOI: 10.1504/IJAMS.2012.047678.
Summary

This chapter describes a GP procedure for modelling and solving academic resource planning problems in university management system with interval data uncertainty. In the proposed approach, the interval goals are first converted into the standard goals by using interval arithmetic technique. Certain objectives having the characteristics of fractional programming are transformed into linear goals by using linearization approach to solve the problem by employing linear GP methodology. In the model formulation of the problem, both the aspects of GP, \textit{minsum} and \textit{minmax} approaches, are addressed to construct the goal achievement function for minimising the possible regret towards achieving the goal values within the target intervals specified by the DM in the decision making environment. The potential use of the approach is illustrated by a case example. The model solution is compared with the solutions of the models studied previously.

8.1 Introduction

The interest in higher education in new study areas has increased a lot in the recent years from the viewpoint of emerging academic viability and wide scope for availing of the opportunity of employment in different activity areas. As a matter of fact, initiative for opening of new academic departments has taken into account by most of the higher academic institutions throughout the world.

In the academic planning context, it may be mentioned that in most of the developed countries tuitions and other fees play a major source of financial resources to run the academic organizations. But, in case of developing country, like India, universities are mainly run by the national income of the Government. But, with the increase of educational cost along with the infrastructural development cost, pressure on the national income for budget allocation to run the universities has increased a lot in the recent years. Further, since academic institutions are not profit making organizations, universities are often faced with problem of proper personnel management in the academic units with limited budget allocation in a financial year.

From a historical perspective, early works on education planning were appeared in (Gani, 1963; Platt, 1962) as the applications of quantitative methods in the area of Management Science. In the area of mathematical programming, the concept of formulating LP model of academic planning problem was first studied by Fox in 1967. The survey of works in the field of higher education was conducted by Rath (1968).
Thereafter, different strategic MP models for academic resource management have been investigated in (Hufner, 1968; Marshall and Oliver, 1970) for enrichment of education system. A comprehensive bibliography on the state-of-the art of modelling aspects of academic planning for improving the quality of education studied from 1960s to early '70s was first prepared by Schroeder (1973). Thereafter, worldwide efforts (Walters et al., 1976; Wang, 1996) for implementation of management science models to enrich the various activities of academic institutions were taken place from the view point of potential growth of socio-economic conditions of a nation.

Now, since most of the academic planning problems are multiobjective in nature, the multi-criterion optimization method for university administration towards meeting the different socio-economic goals was introduced by Geoffrion et al. (1971). The contributions made solely for development of effective MP models for educational planning was first surveyed by McNamara in 1971. The GP approach, studied by Ignizio (1976), Kombluth (1973), Romero (1991), and others, as one of the most promising and flexible tool for multiobjective decision analysis has been successfully implemented to university management system by Franz et al. (1981), Keown at al. (1981), Schroeder (1974), Walter et al. (1976), and others. The several other modelling aspects of different higher academic planning problems have also been studied by Ghosh et al. (1992) and Kwak and Lee (1998).

Although GP has been widely used to university planning problems, the main weakness of conventional GP formulation is that all the parameters of the problem need to be specified precisely in the planning environment. But, in most of the practical decision problems, they are often imprecisely defined due to the expert's ambiguous understanding of the nature of them. So, assigning definite aspiration levels to the goals of the problem frequently creates decision trouble in most of the farm planning situations.

To overcome the difficulty, IVP (Bitran, 1980) methods have been developed to solve MODM problems with uncertain parameters, in which the bounds of parameters are only required and it is not necessary to know the probability distributions or membership functions in fuzzy sets. The GP formulation in MODM problems with interval parameter sets known as IVGP has been introduced by Inuiguchi and Kume (1991). The methodological aspects of interval programming studied in the past have been surveyed by Olivera and Antunes (2007). The IVP approaches to real-life
decision problems have been studied Pal and Kumar (2013b) in the recent past. The potential use of IVP has been studied by Assavapokee et al. (2008), Jiang et al. (2008), Kouvelis and Yu (1997), and others. However, the use of IVP approach to academic planning problems is yet to appear in the literature.

In the present chapter, how the IVP approach within the framework of GP can be efficiently used for academic personnel management in university systems is presented. In the proposed model, both the crisp and interval goals are taken into account in the planning horizon. In the model formulation of the problem, the interval goals are first converted into the conventional deterministic goals by employing interval arithmetic technique in IVP approach. Then, the goals having the characteristic of fractional programming are transformed into the linear goals by using the linear transformation technique introduced by Pal et al. (2008) in the recent past. In the executable GP model formulation, both the \textit{minmax} (Ignizio, 1976) and \textit{minsum} (Romero, 1991) aspects of GP approach are considered as the convex combination of them for minimizing the regrets regarding achievement of goals from the optimistic point of view of the DM. In the solution process, goal attainment problem under a preemptive priority structure of GP is considered for achieving the goals on the basis of needs and desires of DM in the decision making context. The potential use of the approach is illustrated by a case example.

The rest of the chapter is organized as follows.

Section 8.2 describes the general framework of the model formulation of the problem, operational steps of formulating the executable model of the problem and a process of linear transformation of fractional goals. In section 8.3, GP model formulation of the problem is described. Section 8.4 provides the descriptions of the variables and parameters associated with the proposed university planning problem. In section 8.5, developments of model goals are described. Section 8.6 demonstrates an illustrative case example in order to establish the feasibility and effectiveness of the proposed approach, and a comparison of the model solution with the solutions of the model discussed previously is made there to show the potential use of the approach studied here. Finally, section 8.7 provides a general conclusion and scope for future research.

Now the formal descriptions of the goals are presented in section 8.2.
8.2 General IVP Formulation

The two types of goals, interval goals and crisp goals, involved with the problem are described as follows:

- **Definition of interval-valued goals**

  Two types of interval goals fractional and linear are associated with the problem. The goals in fractional form appear as

  \[
  Z_k(X): \left[ \frac{a^L_k, a^U_k}{b^L_k, b^U_k} \right] X + \left[ \frac{a^L_k, a^U_k}{\lambda^L_k, \lambda^U_k} \right] = \left[ \frac{t^L_k, t^U_k}{t^L_k, t^U_k} \right], \quad k = 1, 2, \ldots, K_i
  \]

  It is customary to assume that \([b^L_k, b^U_k] X + [\beta^L_k, \beta^U_k] > 0\) to preserve the feasibility of the solution.

  The goals in linear form can be presented as

  \[
  Z_k(X): \left[ a^L_k, a^U_k \right] X + \left[ \alpha^L_k, \alpha^U_k \right] = \left[ t^L_k, t^U_k \right], \quad k = (K_i+1), \ldots, K_2
  \]

  where \(Z_k\) represents the \(k\)-th objective, and \(X\) designates the vector of decision variables. \([a^L_k, a^U_k]\) and \([b^L_k, b^U_k]\) are the vectors of interval-valued coefficients, \([t^L_k, t^U_k]\) represent the target intervals, and \([\alpha^L_k, \alpha^U_k], [\beta^L_k, \beta^U_k]\) are the interval-valued constant associated with the respective goals, where \(L\) and \(U\) stand for the respective lower- and upper-bounds of the interval.

  In IVP approach, the defined intervals of coefficients and goals represent the regions within which the respective values they possibly take.

- **Definition of crisp goals**

  The goals with crisp coefficients and right hand side values appear as

  \[
  Z_k(X) \begin{cases} \geq & t_k, \quad k = (K_2+1), (K_2+2), \ldots, K \end{cases}
  \]

  where \(t_k\) indicates the crisp target value of the \(k\)-th objective \(Z_k(X)\).

  The basic concept of interval arithmetic has been briefly discussed in chapter 1.

  Now, the conversions of planned interval goals into deterministic ones are discussed in the following sections.
8.2.1 Transformation of Interval Goals into Planned Interval Goals

In IVP approach, the interval-valued goals in (8.1) and (8.2) defined in generic form are to be transformed into their standard forms called the planned interval goals by using the basic rules of interval arithmetic (Moore, 1966).

The planned interval goals of the fractional goals expressions in (8.1) can be explicitly presented as

\[
\frac{\sum_{j=1}^{n} a_{ij}^l x_j + \alpha_{ik}^l}{\sum_{j=1}^{n} a_{ij}^u x_j + \alpha_{ik}^u} = [t_k^l, t_k^u], \quad k=1, 2, \ldots, K_2
\]

(8.4)

Again, the linear interval goals in (8.2) appear as

\[
\left[ \sum_{j=1}^{n} b_{ij}^l x_j + \beta_{ik}^l \right] \left[ \sum_{j=1}^{n} b_{ij}^u x_j + \beta_{ik}^u \right] = [t_k^l, t_k^u], \quad k= (K_1+1), (K_1+2), \ldots, K_2
\]

(8.5)

To employ the linear GP methodology, defined goals in (8.3), (8.4) and (8.5) are to be converted into the flexible goals by using interval arithmetic technique introduced by Inuiguchi and Kume (1991).

8.2.2 Conversion of Objective Goals into Deterministic Goals

To formulate the standard GP model of the problem, first each of the defined planned interval goals in (8.4) and (8.5) are to be converted into the two-objective deterministic goals by following the concept of midpoint and width of interval number in interval arithmetic. Then, they are to be made flexible by introducing under- and over-deviational variables to each of them. Again, in the sequel of formulating the GP model of the problem, crisp goals in (8.3) are also to be made flexible only by introducing under- and over-deviational variables to each of them.

8.2.2.1 Conversion of Planned Interval Goals into Flexible Goals

The standard goal expressions of the goals in (8.4) appear as (Pal et al., 2012b):

\[
\frac{\sum_{j=1}^{n} a_{ij}^l x_j + \alpha_{ik}^l}{\sum_{j=1}^{n} a_{ij}^u x_j + \alpha_{ik}^u} + d_{kl}^l - d_{kl}^u = t_k^l, \quad k=1, 2, \ldots, K_1
\]

(8.6)
Similarly, the flexible goal expressions of the goals in (8.5) take the form

\[
\sum_{j=1}^{n} a_{kj}^U x_j + a^U_k + d^L_k - d^U_k = t^L_k, \quad k = (K+1), (K+2), \ldots, K_2
\]  

(8.8)

\[
\sum_{j=1}^{n} b_{kj}^U x_j + b^U_k + d'^L_k - d'^U_k = t'^U_k, \quad k = (K+1), (K+2), \ldots, K_2
\]  

(8.9)

where \((d^L_k, d^U_k)\) and \((d'^L_k, d'^U_k)\) \(\geq 0\) with \(d^L_k, d^U_k = 0\) and \(d'^L_k, d'^U_k = 0\) represent the sets of under- and over-deviational variables, respectively, associated with the respective goals.

### 8.2.2.2 Conversion of Crisp Goals into Flexible Goals

The flexible goal expressions of the goals in (8.3) appear as (Romero, 1991):

\[
Z_k(\mathbf{X}) + \eta^-_k - \eta^+_k = t^-_k, \quad k = (K_2+1), (K_2+2), \ldots, K
\]  

(8.10)

where \(\eta^-_k, \eta^+_k \geq 0\) with \(\eta^-_k, \eta^+_k = 0\) represent the under- and over-deviational variables, respectively, associated with the \(k\)-th goal, \(k = (K_2+1), (K_2+2), \ldots, K\).

Now, it is to be followed that the goal expressions in (8.6) and (8.7) are linear fractional in form. To avoid the computational complexity with the fractional criteria (Hannan, 1977), the use linear transformation approach is discussed in the following section.

### 8.2.3 Linear transformation of fractional goals

Fractional Programming introduced by Charnes and Cooper (1959) is a special field of study in the area of nonlinear programming. The methodological aspects of general fractional programming were studied extensively (Craven, 1988; Schaible, 1981) in the past and widely circulated in the literature. The fractional programming approaches to general fractional MODM problems as well as fractional GP problems were studied by Kornbluth and Steuer (1981a).
In the case of solving problems with linear fractional criteria, it may be mentioned that direct linearization of fractional objectives creates computational difficulty. Here, how decision error occurs for the use of direct linearization of ratio goals was well discussed in (Awerbach et al., 1976; Hannan, 1977) in the context of solving university Planning problems (Lee and Clayton, 1972; Schroeder, 1974). The method of variable change for solving fractional programming problems has also been discussed in (Dutta et al., 1992; Luhandjula, 1984) in the past. The Taylor series approximation approaches to the field have also been investigated by (Pal et al., 2010c) in the recent past. However, the method of variable change for linear transformation of linear fractional goals is widely used for its simplicity. Linear transformation technique presented by Pal et al (Pal et al., 2010b) is addressed here to solve the proposed problem.

The linearization process is defined as follows:

Letting,

\[
\frac{1}{\sum_{j=1}^{n} b_j x_j + \beta_k^L} = y_{k0}, \quad k = 1, \ldots, K_1. \tag{8.11}
\]

where \(y_{k0} > 0\) is a new variable, the goal expression in (8.6) can be defined as

or,

\[
\sum_{j=1}^{n} a_{kj}^U y_{kj} + a_k^U y_{k0} + d^U_{kl} - d^L_{kl} = t^L_k, \tag{8.12}
\]

where \(y_{kj} = x_j y_{k0}, \quad j = 1, 2, \ldots, n; \quad k = 1, 2, \ldots, K_j. \tag{8.13}\]

Now, for \(k = l\), the relation in (13) takes the form

\[
y_{lj} = x_j y_{l0}, \quad j = 1, 2, \ldots, n. \tag{8.14}\]

Combining (8.13) and (8.14), the following ratios are obtained as

\[
\frac{y_{kj}}{y_{lj}} = \frac{y_{k0}}{y_{l0}} = \gamma_k, \quad k = 1, 2, \ldots, K_l
\]

The relational expressions defined for the new variables are obtained as

\[
y_{kj} = \gamma_k y_{lj}, \quad y_{k0} = \gamma_k y_{l0}, \quad \forall j \text{ and } k = 1, 2, \ldots, K_l \tag{8.15}\]

Then, the linear form of the goal expression in (8.6) is obtained as

\[
\sum_{j=1}^{n} a_{kj}^U y_{lj} + a_k^U y_{l0} + \eta^L_{kl} - \eta^L_{kl} = t^L_k \delta, \tag{8.16}\]

or,

\[
\sum_{j=1}^{n} a_{kj}^U y_{lj} + a_k^U y_{l0} - t^L_k \delta_k + \eta^L_{kl} - \eta^L_{kl} = 0, \tag{8.16}\]
A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System

where \( \eta_{kl} = \frac{d_{kl}^-}{\gamma_k} \), \( \eta_{kl}^+ = \frac{d_{kl}^+}{\gamma_k} \), \( \delta_k = \frac{1}{\gamma_k} \),

and \( \eta_{kl}^- \cdot \eta_{kl}^+ \geq 0 \), \( \eta_{kl}^- \cdot \eta_{kl}^- = 0 \), for \( \gamma_k > 0 \).

Again, letting \( \frac{1}{\sum_{j=1}^{n} b_{kj}^i \cdot \gamma_j + \beta_k^i} = \gamma_k^i \), and \( \gamma_k^i > 0 \), \( k = 1, 2, \ldots, K_I \) (8.17)

Following the same variable transformation process, the linear form of the expression in (8.7) can be obtained as

\[
\sum_{j=1}^{n} a_{kj}^i \cdot y_{1j} + \alpha_k^i \cdot y_{10} - t_k^i \cdot \delta^i + \eta_{kl}^i \cdot \eta_{kl}^i = 0,
\]

where, \( \eta_{kl}^i \cdot \eta_{kl}^i \geq 0 \), \( \eta_{kl}^- \cdot \eta_{kl}^i = 0 \), and where \( \eta_{kl}^i = \frac{d_{kl}^i}{\gamma_k^i} \), \( \eta_{kl}^+ = \frac{d_{kl}^+}{\gamma_k^i} \), \( \delta_k^i = \frac{1}{\gamma_k^i} \), \( k = 1, 2, \ldots, K_I \).

Again, the expressions in (8.11) and (8.17) can be represented in linear form as

\[
\sum_{j=1}^{n} b_{kj}^i \cdot y_{1j} + \beta_k^i \cdot y_{10} = \delta_k^i, \quad k = 1, 2, \ldots, K_I.
\]

(8.19)

Thus, in terms of the new variables \( y_{kj} \), the expressions in (8.8) and (8.9) can be obtained as

\[
\sum_{j=1}^{n} a_{kj}^i \cdot y_{1j} + \alpha_k^i \cdot y_{10} - t_k^i \cdot y_{10} + \eta_{kl}^i \cdot \eta_{kl}^i = 0,
\]

(8.21)

\[
\sum_{j=1}^{n} a_{kj}^i \cdot y_{1j} + \alpha_k^i \cdot y_{10} - t_k^i \cdot y_{10} + \eta_{kl}^- \cdot \eta_{kl}^+ = 0,
\]

(8.22)

where \( \eta_{kl} = y_{10} \cdot d_{kl}, \eta_{kl}^+ = y_{10} \cdot d_{kl}^+, \eta_{kl}^- = y_{10} \cdot d_{kl}^-, \eta_{kl}^i = y_{10} \cdot d_{kl}^i \) with \( \eta_{kl} \cdot \eta_{kl}^i \geq 0 \), \( \eta_{kl}^- \cdot \eta_{kl}^+ = 0 \) and \( \eta_{kl}^- \cdot \eta_{kl}^i \geq 0 \), \( \eta_{kl}^+ \cdot \eta_{kl}^i = 0 \) for \( k = (K_I+1), (K_I+2), \ldots, K. \)

Again, for the defined new vector of variables, the linear goal expression in (8.10) can be represented as

\[
\sum_{j=1}^{n} a_{kj} \cdot y_{1j} - t_k \cdot \delta_k + \eta_k^- \cdot \eta_k^+ = 0, \quad k = (K_2+1), (K_2+2), \ldots, K
\]

(8.23)

where \( \eta_{kl} \cdot \eta_{kl}^i \geq 0 \), \( \eta_{kl}^- \cdot \eta_{kl}^+ = 0 \) and \( \eta_k^- = d_k, \eta_k^+ = d_k^+, \delta_k = 1; \) represent the under- and over-deviational variables associated with the \( k \)-th goal, \( k = (K_2+1), (K_2+2), \ldots, K. \)
Now, in the decision situation, DM's objective for achievement of the goal values within the specified ranges means of minimization of the associated deviational variables to the extent possible in the decision making environment. The GP formulation of the problem is presented in section 8.3.

### 8.3 GP Model Formulation

There are different versions of GP (Romero, 2004) for solving real-life problems. The most widely use approaches are minsum GP (Ignizio, 1976) and minmax GP (Romero, 1991).

In minsum GP, minimisation of the (unwanted) deviational variables in the achievement function (regret function) is considered on the basis of weights of importance of achieving the target levels of goals in the decision environment. On the other hand, in case of minmax GP, minimization of maximum deviation of a goal from the target level is considered. This approach provides a solution that gives highest importance to the goal most displaced with respect to its target. Here, the most balanced solution among the achievement of different goals is obtained (Tamiz and Jones, 1998).

An intuitive idea for the use of GP is to take the convex combination of minsum GP and minmax GP models called extended GP (EGP) (Romero, 2004) in order to make a reasonable balance of the solution for aggregated achievement of goals provided by the former model with the balanced solution provided by the latter one.

In the context of using an GP methodology, it may be mentioned that the priority based GP (Ogryczak, 1997) is an extension of the developed GP approaches, where lexicographic ordering of the goals along with their weight structure is considered for achieving the target levels of goals on the basis of their priorities of importance in the decision making context.

In the present GP formulation, achievement of goals under a pre-emptive priority structure using the notion of EGP for attainment of goals in interval form is taken into account to reach a satisfactory solution in the decision making environment.

The priority based GP model of the problem can be presented as:

Find $Y(y_{10}, y_{11}, ..., y_{ln})$ so as to:

$$\text{Minimize } Z = [P_1(\bar{\eta}), P_2(\bar{\eta}), ..., P_r(\bar{\eta})]$$
and satisfy the goal expressions in (8.16), (8.18), (8.21), (8.22) and (8.23),
subject to \( \eta_{kr} + \eta^+_{kr} - V_r \leq 0, \) \( k = 1, 2, \ldots, K_2, \) \( r = 1, 2, \ldots, R, \)

\[
\sum_{j=1}^{n} b_{kj} y_{ij} + \beta_k y_{i1} = \delta_k, \quad k = 1, 2, \ldots, K_i \\
\sum_{j=1}^{n} b_{kj} y_{ij} + \beta_k y_{i1} = \delta'_k, \quad k = 1, 2, \ldots, K_i \\
\text{where } V_r = \max_{k=1}^{K_2} (\eta_{kr} + \eta^+_{kr}),
\]

(8.24)

and \( Z \) represents the vector of regret functions for achievement of goals included in the \( R \) priorities by minimizing the deviational variables on the basis of priorities of importance of achieving the goals within their specified target intervals.

The linear functional form of the \( r \)-th priority level \( P_r(\vec{\eta}) \) is of the form:

\[
P_r(\vec{\eta}) = \sum_{k \in I_r} w_{kr} \eta_{kr} + \{ \lambda \sum_{k \in I_r} w_{kr} (\eta^\text{+}_{kr} + \eta^\text{+}_{klr}) + (1 - \lambda) V_r \}
\]

(8.25)

where \( I_r, I'_r (r = 1, 2, \ldots, R) \) designate the sets of model goals defined for the crisp goals and interval-valued goals, respectively, are included at the \( r \)-th priority level \( P_r \) and where \( \bigcup_{r=1}^{R} I_r = \{(K_2 + 1), \ldots, K_2 \}, \) \( \bigcup_{r=1}^{R} I'_r = \{1, 2, \ldots, K_2 \} \). Also each \( I_r \) and \( I'_r \) pair wise mutually disjoint. \( 0 < \lambda < 1 \) and \( w_{kr} (> 0) (r = 1, 2, \ldots, R) \) associated with deviational variables, represent the numerical weights of importance of achieving the goals at the \( r \)-th priority level, \( \eta_{kr}, \eta^\text{+}_{kr}, \eta^\text{+}_{klr} \) are renamed for \( \eta_{kr}, \eta^\text{+}_{klr}, \eta^\text{+}_{klur} \), respectively, to represent them at the \( r \)-th priority level.

Here, may be noted that the priority factors have the following relationship:

\( P_1 >>> P_2 >>> \ldots >>> P_r >>> \ldots >>> P_R, \) which means that the goal achievement under the priority factor \( P_r \) is preferred most to the next priority factor \( P_{r+1}, \) \( r = 1, 2, \ldots, R-1, \) where \( >>> \) stands for much greater than.

Now, the different types of parameters and variables involved with the problem are defined in section 8.4.
8.4 Definitions of Variables and Parameters

8.4.1 Definition of Decision Variables

\[ f_{ih} = \text{Number of full-time teaching staff (FTS) to be employed in the department}\]
\[ i (i = 1, 2, \ldots, I), \text{rank } h (h = 1, 2, \ldots, H) \text{ at the time period } t \text{ for smooth}\]
\[ \text{functioning of the academic activities of the department.} \]

\[ p_{it} = \text{Number of part time teaching staff (PTS) required in the department } i, \text{ time period } t. \]

\[ N_{it} = \text{Number of non teaching staff (NTS) required in the department } i, \text{ time period } t. \]

The following decision variables are involved in the proposed model.

8.4.2 Definition of Parameters

The two types of parameters, crisp and interval-valued parameters, are defined as follows:

- **Crisp parameter**

\[ [FTS]_{ih} = \text{Minimum number of FTS required in the department } i, \text{ rank } h \text{ at the time period } t. \]

\[ [S]_{it} = \text{Total number of students (TS) at the department } i \text{ at the time period } t. \]

\[ [FS]_{ih} = \text{Annual (average) salary of a FTS in the department } i, \text{ rank } h, \text{ time period } t. \]

\[ [NS]_{it} = \text{Annual (average) salary of a NTS in the department } i, \text{ time period } t. \]

\[ [PS]_{it} = \text{Annual remuneration for a PTS in the department } i \text{ at the time period } t. \]

- **Interval parameter**

\[ [F_{ih}^L, F_{ih}^U] = \text{The specified target in which the total number of FTS must be}\]
\[ \text{employed to the department } i \text{ at the time period } t. \]

\[ [q_i^L, q_i^U] = \text{Target interval to maintain the ratio of PTS and FTS in the department } i. \]

\[ [r_i^L, r_i^U] = \text{Target interval to maintain the ratio of NTS and Total Teaching Staff}\]
\[ \text{(TTS), [FTS and PTS], in the department } i. \]

\[ [s_i^L, s_i^U] = \text{Target interval to maintain the ratio of TS and TTS in the department } i. \]

\[ [b_i^L, b_i^U] = \text{Target interval for pay-roll budget allocated to the department } i \text{ at the time period } t. \]
Now, the crisp goals and goals with target interval involved with the model are described in section 8.5.

8.5 Description of Crisp and Interval Goals:

- **FTS goals**

  Both the crisp and interval-valued goals are involved for employment of FTS in a department.

  (i) Crisp goals

  For smooth running of the academic departments, a university should always have to provide a minimum number of FTS at each rank to each of the departments at the time period $t$.

  The goals with their aspiration levels can be presented as

  $$f_{iht} + d_s^- - d_s^+ = [FTS]_{iht}, \quad i = 1, 2, ..., I; \quad h = 1, 2, ..., H; \quad s = 1, 2, ..., IH.$$  

  (ii) Interval-valued goals

  For potential academic performance, a total number FTS within a specified interval must be employed to each of the departments.

  The goals with target intervals appear as:

  $$\sum_{h=1}^{H} f_{iht} = [F_{it}^L, F_{it}^U], \quad i = 1, 2, ..., I$$

- **Budget goal**

  Due to limitation of total pay-roll budget to run the university, the pay-roll budget allocation for each of the departments is defined as interval-valued.

  The budget goal expression appears as:

  $$\sum_{h=1}^{H} [FS]_{iht} f_{iht} + [N_{it}^L, N_{it}^U] N_{it} + [PS_{it}] p_{it} = [b_{it}^L, b_{it}^U], \quad i = 1, 2, ..., I$$

- **Ratio goals**

  For enrichment of academic activities, certain ratios of faculty members, other staff and students should be maintained within the specified intervals. The ratio goals are defined as follows:

  (i) PTS-FTS ratio goals
Due to the limitation of the pay-roll budget, if the required number of FTS cannot be employed, then PTS at a certain ratio of the total FTS would have to be employed to each of the departments.

The goal expressions appear as:

\[
\frac{P_{ht}}{\sum_{h=1}^{H} f_{ih}} = [q^L_i, q^U_i], \quad i = 1, 2, ..., I
\]

(ii) NTS-TTS ratio goals:

A minimum number of NTS at a certain ratio of the TTS of a department need be employed for performing the official and technical jobs.

The goal expressions with the target intervals take the form

\[
\frac{N_{ht}}{(\sum_{h=1}^{H} f_{ih} + p_{ht})} = [r^L_i, r^U_i], \quad i = 1, 2, ..., I
\]

(iii) TS-TTS ratio goal:

For smooth running of the academic activities, a certain ratio of students and TTS should be maintained.

The goal expression appears as:

\[
\frac{[S]_i}{(\sum_{h=1}^{H} f_{ih} + p_i)} = [s^L_i, s^U_i], \quad i = 1, 2, ..., I
\]

Now, GP formulation of the problem is demonstrated via a case example presented in section 8.6.

8.6 An Illustrative Case Example

The academic resource allocation problem of the University of Kalyani, West Bengal (WB), India, is considered to demonstrate the application potential of the proposed approach. The resource allocation problems of six newly established departments: Microbiology (MB), Computer Science & Engineering. (CSE), Master of Business Administration (MBA), Geography (GEO), Molecular Biology & Bio-Technology (MB & BT), Physiology (PHY) as well as two previously established departments: Mathematics (MATH) and Statistics (STAT) are considered to expound the model.

The data for the proposed model were collected from the Budget allocation programme published by the University of Kalyani (2010–2011). The pay-roll budget
allocation to each of the eight departments for the financial year 2009 – 2010 (say, the period 1) is considered as the lower bound \( (b_i^L) \) and the estimated budget allocation for the financial year 2010-2011 is considered as the upper bound \( (b_i^U) \) of the interval-valued pay-roll budget goal of the department \( i (i = 1, 2, 3, 4, 5, 6, 7, 8) \). Then, the data associated with the departments are presented in the following Tables 8.1 - 8.4.

### Table 8.1: Pay-roll budget data

<table>
<thead>
<tr>
<th>Department</th>
<th>MB (1)</th>
<th>CSE (2)</th>
<th>MBA (3)</th>
<th>GEO (4)</th>
<th>MB&amp;BT (5)</th>
<th>PHY(6)</th>
<th>MATH (7)</th>
<th>STAT (8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b_i^L )</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
<td>[b_i^L, b_i^U ]</td>
</tr>
<tr>
<td>Pay-roll Budget (in Rs.lac)</td>
<td>[25.36, 62.37]</td>
<td>[59.89, 96.68]</td>
<td>[37.60, 88.50]</td>
<td>[25.36, 62.42]</td>
<td>[59.92, 105.69]</td>
<td>[55.32, 109.79]</td>
<td>[25.86, 109.79]</td>
<td>[55.32, 109.79]</td>
</tr>
</tbody>
</table>

### Table 8.2: Annual average salaries/remuneration of FTS, NTS and PTS

<table>
<thead>
<tr>
<th>Rank</th>
<th>Professor (1)</th>
<th>Associate Professor (2)</th>
<th>Assistant Professor (3)</th>
<th>PTS</th>
<th>NTS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Salary / Remuneration (Rs. Lac.)</td>
<td>9.17</td>
<td>7.2</td>
<td>5.16</td>
<td>1.5</td>
</tr>
</tbody>
</table>

### Table 8.3: Target intervals for the ratio goals

<table>
<thead>
<tr>
<th>Department (i)</th>
<th>Number of students</th>
<th>PTS-FTS ( [q_i^L, q_i^U] )</th>
<th>NTS-TTS ( [r_i^L, r_i^U] )</th>
<th>TS-TTS ( [s_i^L, s_i^U] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>32</td>
<td>[0.25,0.35]</td>
<td>[0.38,0.40]</td>
<td>[0.18,0.22]</td>
</tr>
<tr>
<td>2</td>
<td>110</td>
<td>[0.25,0.35]</td>
<td>[0.38,0.40]</td>
<td>[0.03,0.06]</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>[0.25,0.35]</td>
<td>[0.33,0.35]</td>
<td>[0.10,0.14]</td>
</tr>
<tr>
<td>4</td>
<td>50</td>
<td>[0.25,0.35]</td>
<td>[0.45,0.50]</td>
<td>[0.10,0.14]</td>
</tr>
<tr>
<td>5</td>
<td>24</td>
<td>[0.25,0.35]</td>
<td>[0.38,0.40]</td>
<td>[0.25,0.29]</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>[0.25,0.35]</td>
<td>[0.38,0.40]</td>
<td>[0.20,0.23]</td>
</tr>
<tr>
<td>7</td>
<td>150</td>
<td>[0.25,0.35]</td>
<td>[0.38,0.40]</td>
<td>[0.25,0.29]</td>
</tr>
<tr>
<td>8</td>
<td>120</td>
<td>[0.25,0.35]</td>
<td>[0.40,0.42]</td>
<td>[0.20,0.23]</td>
</tr>
</tbody>
</table>

### Table 8.4: Target intervals for full-time teaching staff

<table>
<thead>
<tr>
<th>Department</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target intervals for total FTS ( [F_i^L, F_i^U] )</td>
<td>[8,10]</td>
<td>[10,12]</td>
<td>[10,12]</td>
<td>[8,10]</td>
<td>[8,12]</td>
<td>[8,12]</td>
<td>[8,12]</td>
<td>[8,12]</td>
</tr>
</tbody>
</table>

156
Now, using the data Tables and the other necessary collected data, the crisp model goals are described in section 8.6.1.

Here, since the data for a running period \( t = 1 \) is considered, the time specification \( (t) \) is omitted for simplicity during representation of the case model.

### 8.6.1 Construction of Model Goals

- **FTS Goals**
  
  (i) Crisp goal
  The goals with the aspired levels appear as:
  
  \[
  \begin{align*}
  f_{11} + \eta_{11}^* - \eta_{11}^- &= 1, \\
  f_{12} + \eta_{22}^* - \eta_{22}^- &= 2, \\
  f_{13} + \eta_{33}^* - \eta_{33}^- &= 3, \\
  f_{14} + \eta_{44}^* - \eta_{44}^- &= 1, \\
  f_{21} + \eta_{21}^* - \eta_{21}^- &= 2, \\
  f_{22} + \eta_{22}^* - \eta_{22}^- &= 3, \\
  f_{31} + \eta_{31}^* - \eta_{31}^- &= 1, \\
  f_{32} + \eta_{32}^* - \eta_{32}^- &= 2, \\
  f_{33} + \eta_{33}^* - \eta_{33}^- &= 3, \\
  f_{41} + \eta_{41}^* - \eta_{41}^- &= 1, \\
  f_{42} + \eta_{42}^* - \eta_{42}^- &= 2, \\
  f_{43} + \eta_{43}^* - \eta_{43}^- &= 1, \\
  f_{44} + \eta_{44}^* - \eta_{44}^- &= 2, \\
  f_{51} + \eta_{51}^* - \eta_{51}^- &= 1, \\
  f_{52} + \eta_{52}^* - \eta_{52}^- &= 2, \\
  f_{53} + \eta_{53}^* - \eta_{53}^- &= 3, \\
  f_{54} + \eta_{54}^* - \eta_{54}^- &= 1, \\
  f_{61} + \eta_{61}^* - \eta_{61}^- &= 1, \\
  f_{62} + \eta_{62}^* - \eta_{62}^- &= 2, \\
  f_{63} + \eta_{63}^* - \eta_{63}^- &= 2, \\
  f_{64} + \eta_{64}^* - \eta_{64}^- &= 4, \\
  f_{71} + \eta_{71}^* - \eta_{71}^- &= 2, \\
  f_{72} + \eta_{72}^* - \eta_{72}^- &= 3, \\
  f_{73} + \eta_{73}^* - \eta_{73}^- &= 4, \\
  f_{74} + \eta_{74}^* - \eta_{74}^- &= 3, \\
  f_{81} + \eta_{81}^* - \eta_{81}^- &= 2, \\
  f_{82} + \eta_{82}^* - \eta_{82}^- &= 3, \\
  f_{83} + \eta_{83}^* - \eta_{83}^- &= 4, \\
  f_{84} + \eta_{84}^* - \eta_{84}^- &= 4.
  
  \end{align*}
  
  \tag{8.26}
  
  (ii) Interval-valued goals:
  Using the data in Table 4 and following the procedure, the goals for the total FTS for each of the departments are described as follows:
  
  \[
  \begin{align*}
  \sum_{h=1}^{3} f_{1h} + d_{1L}^* - d_{1U}^* &= 8, & \sum_{h=1}^{3} f_{1h} + d_{1L}^* - d_{1U}^* &= 10, \\
  \sum_{h=1}^{3} f_{2h} + d_{2L}^* - d_{2U}^* &= 10, & \sum_{h=1}^{3} f_{2h} + d_{2L}^* - d_{2U}^* &= 12, \\
  \sum_{h=1}^{3} f_{3h} + d_{3L}^* - d_{3U}^* &= 10, & \sum_{h=1}^{3} f_{3h} + d_{3L}^* - d_{3U}^* &= 12,
  \end{align*}
  \]
A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System

\[
\begin{align*}
\sum_{h=1}^{3} f_{4h} + d_{4L} - d_{4U}^* &= 8, & \sum_{h=1}^{3} f_{4h} + d_{4U} - d_{4L}^* &= 10, \\
\sum_{h=1}^{3} f_{5h} + d_{5L} - d_{5U}^* &= 8, & \sum_{h=1}^{3} f_{5h} + d_{5U} - d_{5L}^* &= 12, \\
\sum_{h=1}^{3} f_{6h} + d_{6L} - d_{6U}^* &= 8, & \sum_{h=1}^{3} f_{6h} + d_{6U} - d_{6L}^* &= 12, \\
\sum_{h=1}^{3} f_{7h} + d_{7L} - d_{7U}^* &= 8, & \sum_{h=1}^{3} f_{7h} + d_{7U} - d_{7L}^* &= 12, \\
\sum_{h=1}^{3} f_{8h} + d_{8L} - d_{8U}^* &= 8, & \sum_{h=1}^{3} f_{8h} + d_{8U} - d_{8L}^* &= 12.
\end{align*}
\]

\[ (8.27) \]

- **Budget goals**

Using the data described in Tables 1 and 2, the budget goals can be presented as

\[
\begin{align*}
9.17 f_{11} + 7.2 f_{12} + 5.16 f_{13} + 1.8 N_1 + 1.5 p_1 + d_{1L} - d_{1U}^* &= 62.37, \\
9.17 f_{21} + 7.2 f_{22} + 5.16 f_{23} + 1.8 N_2 + 1.5 p_2 + d_{2L} - d_{2U}^* &= 59.89, \\
9.17 f_{31} + 7.2 f_{32} + 5.16 f_{33} + 1.8 N_3 + 1.5 p_3 + d_{3L} - d_{3U}^* &= 96.68, \\
9.17 f_{31} + 7.2 f_{32} + 5.16 f_{33} + 1.8 N_3 + 1.5 p_3 + d_{3L} - d_{3U}^* &= 37.6, \\
9.17 f_{31} + 7.2 f_{32} + 5.16 f_{33} + 1.8 N_3 + 1.5 p_3 + d_{3L} - d_{3U}^* &= 88.5, \\
9.17 f_{41} + 7.2 f_{42} + 5.16 f_{43} + 1.8 N_4 + 1.5 p_4 + d_{4L} - d_{4U}^* &= 25.36, \\
9.17 f_{41} + 7.2 f_{42} + 5.16 f_{43} + 1.8 N_4 + 1.5 p_4 + d_{4L} - d_{4U}^* &= 62.42, \\
9.17 f_{51} + 7.2 f_{52} + 5.16 f_{53} + 1.8 N_5 + 1.5 p_5 + d_{5L} - d_{5U}^* &= 59.92, \\
9.17 f_{51} + 7.2 f_{52} + 5.16 f_{53} + 1.8 N_5 + 1.5 p_5 + d_{5L} - d_{5U}^* &= 105.69, \\
9.17 f_{61} + 7.2 f_{62} + 5.16 f_{63} + 1.8 N_6 + 1.5 p_6 + d_{6L} - d_{6U}^* &= 25.86, \\
9.17 f_{61} + 7.2 f_{62} + 5.16 f_{63} + 1.8 N_6 + 1.5 p_6 + d_{6L} - d_{6U}^* &= 62.87, \\
9.17 f_{71} + 7.2 f_{72} + 5.16 f_{73} + 1.8 N_7 + 1.5 p_7 + d_{7L} - d_{7U}^* &= 49.19, \\
9.17 f_{71} + 7.2 f_{72} + 5.16 f_{73} + 1.8 N_7 + 1.5 p_7 + d_{7L} - d_{7U}^* &= 121.74, \\
9.17 f_{81} + 7.2 f_{82} + 5.16 f_{83} + 1.8 N_8 + 1.5 p_8 + d_{8L} - d_{8U}^* &= 55.32, \\
9.17 f_{81} + 7.2 f_{82} + 5.16 f_{83} + 1.8 N_8 + 1.5 p_8 + d_{8L} - d_{8U}^* &= 109.79.
\end{align*}
\]

\[ (8.28) \]

- **Ratio goals**

Using the data in Table 3, the different ratio goals corresponding to lower- and upper-bounds of the interval-valued goals can be obtained as follows.

(i) PTS - FTS ratio goals:

The PTS and FTS ratio objective goals are obtained as

\[
\frac{p_1}{\sum_{h=1}^{3} r_{1h}} + d_{17L} - d_{17U}^+ = 0.25, \\
\frac{p_2}{\sum_{h=1}^{3} r_{2h}} + d_{18L} - d_{18U}^+ = 0.25, \\
\frac{p_3}{\sum_{h=1}^{3} r_{3h}} + d_{19L} - d_{19U}^+ = 0.25, \\
\frac{p_4}{\sum_{h=1}^{3} r_{4h}} + d_{20L} - d_{20U}^+ = 0.25, \\
\frac{p_5}{\sum_{h=1}^{3} r_{5h}} + d_{21L} - d_{21U}^+ = 0.25, \\
\frac{p_6}{\sum_{h=1}^{3} r_{6h}} + d_{22L} - d_{22U}^+ = 0.25, \\
\frac{p_7}{\sum_{h=1}^{3} r_{7h}} + d_{23L} - d_{23U}^+ = 0.25, \\
\frac{p_8}{\sum_{h=1}^{3} r_{8h}} + d_{24L} - d_{24U}^+ = 0.25.
\]

(ii) NTS - TTS ratio goals:

The goal expressions take the form

\[
\frac{N_1}{[(\sum_{h=1}^{3} f_{1h}) + p_1]} + d_{25L} - d_{25U}^+ = 0.38, \\
\frac{N_2}{[(\sum_{h=1}^{3} f_{2h}) + p_2]} + d_{26L} - d_{26U}^+ = 0.38, \\
\frac{N_3}{[(\sum_{h=1}^{3} f_{3h}) + p_3]} + d_{27L} - d_{27U}^+ = 0.30, \\
\frac{N_4}{[(\sum_{h=1}^{3} f_{4h}) + p_4]} + d_{28L} - d_{28U}^+ = 0.35, \\
\frac{N_5}{[(\sum_{h=1}^{3} f_{5h}) + p_5]} + d_{29L} - d_{29U}^+ = 0.35, \\
\frac{N_6}{[(\sum_{h=1}^{3} f_{6h}) + p_6]} + d_{30L} - d_{30U}^+ = 0.35.
\]
A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System

\[
\frac{N_4}{3} + d_{28L}^{-} - d_{28L}^{+} = 0.10,
\]
\[
\left(\sum_{h=1}^{3} f_{4h} + p_4\right)
\]

\[
\frac{N_5}{3} + d_{29L}^{-} - d_{29L}^{+} = 0.10,
\]
\[
\left(\sum_{h=1}^{3} f_{5h} + p_5\right)
\]

\[
\frac{N_6}{3} + d_{30L}^{-} - d_{30L}^{+} = 0.20,
\]
\[
\left(\sum_{h=1}^{3} f_{6h} + p_6\right)
\]

\[
\frac{N_7}{3} + d_{31L}^{-} - d_{31L}^{+} = 0.38,
\]
\[
\left(\sum_{h=1}^{3} f_{7h} + p_7\right)
\]

\[
\frac{N_8}{3} + d_{32L}^{-} - d_{32L}^{+} = 0.40,
\]
\[
\left(\sum_{h=1}^{3} f_{8h} + p_8\right)
\]

(iii) TS - TTS ratio goals:

The goal expression can be obtained as

\[
\frac{1}{3} + d_{33L}^{-} - d_{33L}^{+} = 0.18,
\]
\[
\left(\sum_{h=1}^{3} f_{1h} + p_1\right)
\]

\[
\frac{1}{3} + d_{34L}^{-} - d_{34L}^{+} = 0.03,
\]
\[
\left(\sum_{h=1}^{3} f_{2h} + p_2\right)
\]

\[
\frac{1}{3} + d_{35L}^{-} - d_{35L}^{+} = 0.25,
\]
\[
\left(\sum_{h=1}^{3} f_{3h} + p_3\right)
\]

\[
\frac{1}{3} + d_{36L}^{-} - d_{36L}^{+} = 0.10,
\]
\[
\left(\sum_{h=1}^{3} f_{4h} + p_4\right)
\]

\[
\frac{1}{3} + d_{37L}^{-} - d_{37L}^{+} = 0.10,
\]
\[
\left(\sum_{h=1}^{3} f_{5h} + p_5\right)
\]

\[
\frac{1}{3} + d_{38L}^{-} - d_{38L}^{+} = 0.20,
\]
\[
\left(\sum_{h=1}^{3} f_{6h} + p_6\right)
\]

\[
\frac{1}{3} + d_{39L}^{-} - d_{39L}^{+} = 0.25,
\]
\[
\left(\sum_{h=1}^{3} f_{7h} + p_7\right)
\]
A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System

\[
\frac{1}{3} + d_{40L}^* - d_{40L}^* = 0.20,
\frac{1}{3} + d_{40U}^* - d_{40U}^* = 0.23.
\]

(8.31)

8.6.2 Linear Form of Model Goals

Employing the linear transformation process (described in section 8.2.3), linear form of the executable model goals in section 8.6.1 can be converted in terms of the defined variables \(y_{ij}\) as follows:

- **FTS goals**
  
  (i) Crisp goals:
  
The goals in (8.26) can be expressed in terms of \(y_{ij}\) as:
  
  \[
  \begin{align*}
  y_{11} + \eta_{1}^+ - \eta_{1}^- &= 1, \\
  y_{12} + \eta_{2}^+ - \eta_{2}^- &= 2, \\
  y_{13} + \eta_{3}^+ - \eta_{3}^- &= 3, \\
  y_{14} + \eta_{4}^+ - \eta_{4}^- &= 1, \\
  y_{15} + \eta_{5}^+ - \eta_{5}^- &= 2, \\
  y_{16} + \eta_{6}^+ - \eta_{6}^- &= 3, \\
  y_{17} + \eta_{7}^+ - \eta_{7}^- &= 1, \\
  y_{18} + \eta_{8}^+ - \eta_{8}^- &= 2, \\
  y_{19} + \eta_{9}^+ - \eta_{9}^- &= 3, \\
  y_{110} + \eta_{110}^- - \eta_{110}^+ &= 1, \\
  y_{111} + \eta_{111}^- - \eta_{111}^+ &= 2, \\
  y_{112} + \eta_{112}^- - \eta_{112}^+ &= 3, \\
  y_{113} + \eta_{113}^- - \eta_{113}^+ &= 1, \\
  y_{114} + \eta_{114}^- - \eta_{114}^+ &= 2, \\
  y_{115} + \eta_{115}^- - \eta_{115}^+ &= 3, \\
  y_{116} + \eta_{116}^- - \eta_{116}^+ &= 1, \\
  y_{117} + \eta_{117}^- - \eta_{117}^+ &= 3, \\
  y_{118} + \eta_{118}^- - \eta_{118}^+ &= 2, \\
  y_{119} + \eta_{119}^- - \eta_{119}^+ &= 2, \\
  y_{122} + \eta_{122}^- - \eta_{122}^+ &= 3, \\
  y_{123} + \eta_{123}^- - \eta_{123}^+ &= 3, \\
  y_{124} + \eta_{124}^- - \eta_{124}^+ &= 4.
  \end{align*}
\]

(ii) Interval-valued goals:

According to the linear transformation, goals in (8.27) can be represented in terms of \(y_{ij}\) as:

\[
\begin{align*}
\sum_{j=1}^{3} y_{ij} - 8y_{10} + \eta_{1L}^- - \eta_{1L}^+ &= 0, \\
\sum_{j=1}^{6} y_{ij} - 10y_{10} + \eta_{2L}^- - \eta_{2L}^+ &= 0, \\
\sum_{j=4}^{9} y_{ij} - 10y_{10} + \eta_{3L}^- - \eta_{3L}^+ &= 0, \\
\sum_{j=7}^{12} y_{ij} - 10y_{10} + \eta_{4L}^- - \eta_{4L}^+ &= 8y_{10}, \\
\sum_{j=10}^{15} y_{ij} - 8y_{10} + \eta_{5L}^- - \eta_{5L}^+ &= 0.
\end{align*}
\]

\[
\begin{align*}
\sum_{j=1}^{3} y_{ij} - 10y_{10} + \eta_{1U}^- - \eta_{1U}^+ &= 0, \\
\sum_{j=1}^{6} y_{ij} - 12y_{10} + \eta_{2U}^- - \eta_{2U}^+ &= 0, \\
\sum_{j=4}^{9} y_{ij} - 12y_{10} + \eta_{3U}^- - \eta_{3U}^+ &= 0, \\
\sum_{j=7}^{12} y_{ij} - 12y_{10} + \eta_{4U}^- - \eta_{4U}^+ &= 10y_{10}, \\
\sum_{j=10}^{15} y_{ij} - 12y_{10} + \eta_{5U}^- - \eta_{5U}^+ &= 0.
\end{align*}
\]

(8.32)

\[
\begin{align*}
\sum_{j=16}^{18} y_{ij} - 8y_{10} + \eta_{6L} - \eta_{6L}^* = 0, \\
\sum_{j=19}^{21} y_{ij} - 8y_{10} + \eta_{7L} - \eta_{7L}^* = 0, \\
\sum_{j=22}^{24} y_{ij} - 8y_{10} + \eta_{8L} - \eta_{8L}^* = 0, \\
\sum_{j=16}^{18} y_{ij} - 12y_{10} + \eta_{6U} - \eta_{6U}^* = 0, \\
\sum_{j=19}^{21} y_{ij} - 12y_{10} + \eta_{7U} - \eta_{7U}^* = 0, \\
\sum_{j=22}^{24} y_{ij} - 12y_{10} + \eta_{8U} - \eta_{8U}^* = 0.
\end{align*}
\]

(8.33)

**Pay-roll Budget goals**

The budget goals in (8.28) with respect to the variable \(y_{ij}\) as can be presented as:

\[
\begin{align*}
9.17y_{11} + 7.2y_{12} + 5.16y_{13} + 1.8y_{125} + 1.5y_{133} - 25.36y_{10} + \eta_{6L}^* - \eta_{6L} = 0, \\
9.17y_{11} + 7.2y_{12} + 5.16y_{13} + 1.8y_{125} + 1.5y_{133} - 62.37y_{10} + \eta_{6U}^* - \eta_{6U} = 0, \\
9.17y_{14} + 7.2y_{15} + 5.16y_{16} + 1.8y_{126} + 1.5y_{134} - 59.89y_{10} + \eta_{7L}^* - \eta_{7L} = 0, \\
9.17y_{14} + 7.2y_{15} + 5.16y_{16} + 1.8y_{126} + 1.5y_{134} - 96.68y_{10} + \eta_{7U}^* - \eta_{7U} = 0, \\
9.17y_{17} + 7.2y_{18} + 5.16y_{19} + 1.8y_{127} + 1.5y_{135} - 36.7y_{10} + \eta_{8L}^* - \eta_{8L} = 0, \\
9.17y_{17} + 7.2y_{18} + 5.16y_{19} + 1.8y_{127} + 1.5y_{135} - 88.5y_{10} + \eta_{8U}^* - \eta_{8U} = 0, \\
9.17y_{20} + 7.2y_{21} + 5.16y_{22} + 1.8y_{128} + 1.5y_{136} - 25.36y_{10} + \eta_{12L}^* - \eta_{12L} = 0, \\
9.17y_{20} + 7.2y_{21} + 5.16y_{22} + 1.8y_{128} + 1.5y_{136} - 62.42y_{10} + \eta_{12U}^* - \eta_{12U} = 0, \\
9.17y_{13} + 7.2y_{14} + 5.16y_{15} + 1.8y_{129} + 1.5y_{137} - 34.27y_{10} + \eta_{9L}^* - \eta_{9L} = 0, \\
9.17y_{13} + 7.2y_{14} + 5.16y_{15} + 1.8y_{129} + 1.5y_{137} - 105.69y_{10} + \eta_{9U}^* - \eta_{9U} = 0, \\
9.17y_{16} + 7.2y_{17} + 5.16y_{18} + 1.8y_{130} + 1.5y_{138} - 36.7y_{10} + \eta_{14L}^* - \eta_{14L} = 0, \\
9.17y_{16} + 7.2y_{17} + 5.16y_{18} + 1.8y_{130} + 1.5y_{138} - 88.5y_{10} + \eta_{14U}^* - \eta_{14U} = 0, \\
9.17y_{19} + 7.2y_{20} + 5.16y_{21} + 1.8y_{131} + 1.5y_{139} - 34.27y_{10} + \eta_{15L}^* - \eta_{15L} = 0, \\
9.17y_{19} + 7.2y_{20} + 5.16y_{21} + 1.8y_{131} + 1.5y_{139} - 105.69y_{10} + \eta_{15U}^* - \eta_{15U} = 0, \\
9.17y_{22} + 7.2y_{23} + 5.16y_{24} + 1.8y_{132} + 1.5y_{140} - 55.32y_{10} + \eta_{16L}^* - \eta_{16L} = 0, \\
9.17y_{22} + 7.2y_{23} + 5.16y_{24} + 1.8y_{132} + 1.5y_{140} - 109.79y_{10} + \eta_{16U}^* - \eta_{16U} = 0.
\end{align*}
\]

(8.34)

**Linear form of the ratio goals**

(i) PTS-FTS ratio goals:

\[
\begin{align*}
y_{133} - 0.25\delta_1 + \eta_{7L}^* - \eta_{7L} = 0, \\
y_{133} - 0.35\delta_1 + \eta_{7U}^* - \eta_{7U} = 0, \\
y_{134} - 0.25\delta_2 + \eta_{8L}^* - \eta_{8L} = 0, \\
y_{134} - 0.35\delta_2 + \eta_{8U}^* - \eta_{8U} = 0, \\
y_{135} - 0.25\delta_3 + \eta_{9L}^* - \eta_{9L} = 0, \\
y_{135} - 0.35\delta_3 + \eta_{9U}^* - \eta_{9U} = 0, \\
y_{136} - 0.25\delta_4 + \eta_{10L}^* - \eta_{10L} = 0, \\
y_{136} - 0.35\delta_4 + \eta_{10U}^* - \eta_{10U} = 0.
\end{align*}
\]
"A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System"

\[
y_{1,37} - 0.25\delta_5 + \eta_{21L}^- - \eta_{21U}^- = 0, \quad y_{1,37} - 0.35\delta_5 + \eta_{21U}^- - \eta_{21L}^- = 0,
\]
\[
y_{1,38} - 0.25\delta_6 + \eta_{22L}^- - \eta_{22U}^- = 0, \quad y_{1,38} - 0.25\delta_6 + \eta_{22U}^- - \eta_{22L}^- = 0,
\]
\[
y_{1,39} - 0.25\delta_7 + \eta_{23L}^- - \eta_{23U}^- = 0, \quad y_{1,39} - 0.35\delta_7 + \eta_{23U}^- - \eta_{23L}^- = 0,
\]
\[
y_{1,40} - 0.25\delta_8 + \eta_{24L}^- - \eta_{24U}^- = 0, \quad y_{1,40} - 0.35\delta_8 + \eta_{24U}^- - \eta_{24L}^- = 0,
\]

(8.35)  

(ii) NTS-TTS ratio goals:

\[
y_{1,25} - 0.38\delta_9 + \eta_{25L}^- - \eta_{25U}^+ = 0, \quad y_{1,25} - 0.40\delta_9 + \eta_{25U}^- - \eta_{25L}^+ = 0,
\]
\[
y_{1,26} - 0.38\delta_{10} + \eta_{26L}^- - \eta_{26U}^+ = 0, \quad y_{1,26} - 0.40\delta_{10} + \eta_{26U}^- - \eta_{26L}^+ = 0,
\]
\[
y_{1,27} - 0.33\delta_{11} + \eta_{27L}^- - \eta_{27U}^+ = 0, \quad y_{1,27} - 0.35\delta_{11} + \eta_{27U}^- - \eta_{27L}^+ = 0,
\]
\[
y_{1,28} - 0.45\delta_{12} + \eta_{28L}^- - \eta_{28U}^+ = 0, \quad y_{1,28} - 0.50\delta_{12} + \eta_{28U}^- - \eta_{28L}^+ = 0,
\]
\[
y_{1,29} - 0.38\delta_{13} + \eta_{29L}^- - \eta_{29U}^+ = 0, \quad y_{1,29} - 0.40\delta_{13} + \eta_{29U}^- - \eta_{29L}^+ = 0,
\]
\[
y_{1,30} - 0.38\delta_{14} + \eta_{30L}^- - \eta_{30U}^+ = 0, \quad y_{1,30} - 0.40\delta_{14} + \eta_{30U}^- - \eta_{30L}^+ = 0,
\]
\[
y_{1,31} - 0.38\delta_{15} + \eta_{31L}^- - \eta_{31U}^+ = 0, \quad y_{1,31} - 0.40\delta_{15} + \eta_{31U}^- - \eta_{31L}^+ = 0,
\]
\[
y_{1,32} - 0.40\delta_{16} + \eta_{32L}^- - \eta_{32U}^+ = 0, \quad y_{1,32} - 0.42\delta_{16} + \eta_{32U}^- - \eta_{32L}^+ = 0.
\]

(8.36)  

(iii) ST-TTS ratio goals:

\[
\delta_9 + \eta_{33L}^- - \eta_{33U}^+ = 0.18, \quad \delta_9 + \eta_{33U}^- - \eta_{33L}^+ = 0.22,
\]
\[
\delta_{10} + \eta_{34L}^- - \eta_{34U}^+ = 0.03, \quad \delta_{10} + \eta_{34U}^- - \eta_{34L}^+ = 0.064,
\]
\[
\delta_{11} + \eta_{35L}^- - \eta_{35U}^+ = 0.25, \quad \delta_{11} + \eta_{35U}^- - \eta_{35L}^+ = 0.292,
\]
\[
\delta_{12} + \eta_{36L}^- - \eta_{36U}^+ = 0.10, \quad \delta_{12} + \eta_{36U}^- - \eta_{36L}^+ = 0.14,
\]
\[
\delta_{13} + \eta_{37L}^- - \eta_{37U}^+ = 0.10, \quad \delta_{13} + \eta_{37U}^- - \eta_{37L}^+ = 0.14,
\]
\[
\delta_{14} + \eta_{38L}^- - \eta_{38U}^+ = 0.20, \quad \delta_{14} + \eta_{38U}^- - \eta_{38L}^+ = 0.233,
\]
\[
\delta_{15} + \eta_{39L}^- - \eta_{39U}^+ = 0.25, \quad \delta_{15} + \eta_{39U}^- - \eta_{39L}^+ = 0.29,
\]
\[
\delta_{16} + \eta_{40L}^- - \eta_{40U}^+ = 0.20, \quad \delta_{16} + \eta_{40U}^- - \eta_{40L}^+ = 0.23.
\]

(8.37)  

Again, in the sequel of linear transformation additional linear constraints are obtained by (8.19) and (8.20).

The constraints appear as:

\[
\sum_{j=1}^{3} y_{1j} = \delta_1, \quad \sum_{j=4}^{6} y_{1j} = \delta_2, \quad \sum_{j=7}^{9} y_{1j} = \delta_3, \quad \sum_{j=10}^{12} y_{1j} = \delta_4,
\]

163
The interval-valued GP model is described in section 8.6.3.

**8.6.3 GP Model Formulation**

Now, following the procedure, the GP model with the regret function under the four assigned priorities of the goals are considered in the present academic problem. Priorities are structured with the view of the management's needs and desires.

- **P1**: Topmost priority is to maintain appropriate student-teacher ratio and fulfilling the required number of full time teaching staffs at each department for potential academic activities.
- **P2**: The second priority is to provide the PTS in some ratio depending on the number of the full time teaching staffs in different department.
- **P3**: The third priority is to maintain the NTS-TTS ratio in different department.
- **P4**: The fourth priority is to maintain the budget goals.

The executable GP model is given by

Find \( \{f_h, p_i, N_i\}_{i=1, 2, 3, 4, 5, 6, 7, 8; h=1, 2, 3} \) so as to:

**Minimize** \( Z = \)

\[
\begin{align*}
P_1 & \left[ \left( \sum_{i=1}^{24} \eta_i^+ \right) + \left( \sum_{k=1}^{8} w_{k1} (\eta_{kL}^+ + \eta_{kU}^-) \right) + \lambda_1 \left( \sum_{k=1}^{33} w_{k1} (\eta_{kL}^+ + \eta_{kU}^-) + (1 - \lambda_1) V_1 \right) \right], \\
P_2 & \left[ \lambda_2 \left( \sum_{k=1}^{24} w_{k2} (\eta_{kL}^- + \eta_{kU}^+) \right) + (1 - \lambda_2) V_2 \right], \\
P_3 & \left[ \lambda_3 \left( \sum_{k=1}^{33} w_{k3} (\eta_{kL}^- + \eta_{kU}^+) \right) + (1 - \lambda_3) V_3 \right], \\
P_4 & \left[ \lambda_4 \left( \sum_{k=1}^{16} w_{k4} (\eta_{kL}^- + \eta_{kU}^+) \right) + (1 - \lambda_4) V_4 \right],
\end{align*}
\]

and satisfy the defined goal expressions (8.32) – (8.37),

subject to the constraints in (8.38),

together with \( \eta_{kL} + \eta_{kU}^- - V_j \leq 0, \) for \( k = 1, 2, \ldots, 8, \) and \( k = 33, 34, \ldots, 40; \)

\( \eta_{kL} + \eta_{kU}^- - V_2 \leq 0, \) for \( k = 17, 18, \ldots, 24; \)

\( \eta_{kL} + \eta_{kU}^- - V_3 \leq 0, \) for \( k = 25, 26, \ldots, 32; \)
A Priority Based Goal Programming Method for Solving Academic Personnel Planning Problems with Interval-Valued Resource Goals in University Management System

\[ \eta_{k1} + \eta_{kU} - V_d \leq 0, \ k = 9, 10, \ldots, 16; \]

where \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \in [0, 1] \), \( \eta_r^- \eta_r^+ \geq 0 \), \( \eta_r^- \eta_r^+ = 0 \), \( r = 1, 2, \ldots, 24 \) and

\[ \eta_{k1}^-, \eta_{kU}^+ \geq 0, \ \text{with} \ \eta_{k1}^- \eta_{kU}^+ = 0 \ \text{for} \ k = 1, 2, \ldots, 40. \] \hspace{1cm} (8.39)

Now, for simplicity, giving the equal weights to the goals at the same priority level and taking \( w_{k1} = 1/16, \ k = 1, 2, \ldots, 8 \), \( w_{k2} = 1/8, \ k = 17, 18, \ldots, 24 \); \( w_{k3} = 1/8, \ k = 25, 26, \ldots, 32 \); and \( w_{k4} = 1/8, \ k = 9, 10, \ldots, 16 \) for the goals at the priority levels \( P_1, P_2, P_3 \) and \( P_4 \), respectively, and letting \( \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.5 \), the final executable GP model can be obtained by following (8.24).

The LINGO (ver. 12.0) solver (the permissible size of instance is 500 variables and 250 constraints) is used to solve the problem. The model (variable size 190, constraint size 200) is executed in Pentium IV CPU with 2.66 GHz Clock-pulse and 3GB RAM. The required CPU time is 0.01 second.

The resultant solution is presented in Table 8.5.

Table 8.5: Model Solution under the proposed approach

<table>
<thead>
<tr>
<th>Department</th>
<th>MB</th>
<th>CSE</th>
<th>MBA</th>
<th>GEO</th>
<th>MB&amp;BT</th>
<th>PHY</th>
<th>MATH</th>
<th>STAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Professor</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Proposed</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Approach</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Assistant</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>NTS</td>
<td>2</td>
<td>6</td>
<td>2</td>
<td>7</td>
<td>12</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

The result shows that a satisfactory decision is achieved here in the decision making environment.

8.6.3.1 Illustration for Performance Comparison

The obtained solutions for the instances of varying the proposed structure of the regret function and the existing staff pattern of the university are presented in Table 8.6.

The following variations are made to expound the efficiency of the proposed model.

(i) The model without any priority structure is taken into account, i.e., WGP model is considered.

(ii) The two priority factors, \( P_1 \) and \( P_2 \), are interchanged.
(iii) The fractional goals (8.29) – (8.31) are directly considered in the model without linearizing them and incorporating the associated deviational-variables in the regret function redefined there in the same decision environment.

Table 8.6: Solutions under different structures of regret function and existing staff allocation

<table>
<thead>
<tr>
<th>Department</th>
<th>MB</th>
<th>CSE</th>
<th>MBA</th>
<th>GEO</th>
<th>MB&amp;BT</th>
<th>PHY</th>
<th>MATH</th>
<th>STAT</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Weighted Goal Programming</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professor</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Associate Professor</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>PTS</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>NTS</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td><strong>Interchanging priority factors</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professor</td>
<td>0</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>3</td>
<td>6</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>8</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>PTS</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>NTS</td>
<td>2</td>
<td>6</td>
<td>4</td>
<td>7</td>
<td>15</td>
<td>5</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td><strong>Using without Linearization Technique</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professor</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>1</td>
<td>9</td>
<td>1</td>
<td>7</td>
<td>2</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>PTS</td>
<td>4</td>
<td>6</td>
<td>11</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>NTS</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td><strong>Existing Staff Allocation Structure (2009-2010)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Professor</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Associate Professor</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Assistant Professor</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>PTS</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>NTS</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

The comparisons of different solutions for FTS and Budget-utilization obtained under different structures of regret function are represented via bar-diagrams in Figure 8.1 and Figure 8.2, respectively.
8.7 Conclusions and Scope for Future Research

In this Chapter, a priority based GP model for academic personnel management problem is presented. Here, the main merit of using the IVP approach to the proposed problem is that all possible instances of interval data can be taken into account without knowing the fuzziness / probability distribution of model data. The major
advantage of using the linearization technique is that the computational complexity (Hannan, 1977) does not arise here in the process of solving the problem. In the framework of the proposed model, the other goals /constraints involved with academic personnel management problems can easily be incorporated on the basis of the needs and desires of the DM and without involving any computational complexity.

The limitations of the proposed approach are

- If tuition and other fees play the major role to run a university, then salary structure of staff depends upon student enrolments in different academic units. Here, some parameters mainly those which are associated with pay-roll budget might be random in nature. As a matter of fact, there is a need to extend the proposed model for incorporation of random variables, which may be a problem for future study.

- The proposed model is highly beneficial for proper allocation of academic resources towards enrichment of educational activities. In the academic planning context, it is necessary to measure outputs or benefits of educational programmes. But, output measurement in education is one of the most difficult problems. Although, some works on input-output analysis in education have been made by Wang (1996) and Zhu et al. (2007) in the past, better tools for such measurement are needed as an immediate future research.

However, it is hoped that the approach presented here can contribute to future research in the academic management problems for making managerial decision towards enrichment of higher education globally in society.