4.1. Introduction

The chapter discusses the fuzzification process and the various clustering techniques in fuzzy controller that help in input output space partitioning. Further, the membership types, Fuzzy Inference System (FIS) structures and the rule base structure has been discussed and compared to resolve the various issues that arise in fuzzification process. Section 4.4 discusses the various membership functions used and Section 4.7 describes the important FIS types used. Further, 4.14 discusses the use of Matlab simulator in implementing the Fuzzy controllers.

4.2. Fuzzy Logic as Paradigm of Artificial Intelligence:

In 1965, Lofty Zadeh, an electric engineer, stated the principle of Fuzzy logic as follows: “As the complexity of a system increases, our ability to precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (relevance) become almost mutually exclusive characteristics”. Fuzzy logic provides the generalization and realistic extension of crisp values [Zadeh, 1965].

After Zadeh proposed complete theory of fuzzy sets in 1965, probability theory was applied on crisp logic in 1973 which continued with generalizations. By generalization we mean we can view crisp two valued system as multivalued fuzzy system. Soft decision was given in 1981 and was stated by Zadeh by the statement “In contrast to hard computing, soft computing exploits the tolerance for imprecision, uncertainty and partial truth to achieve tractability, robustness, low solution cost and better rapport with reality”. Application of Fuzzy logic is in industrial systems, decision analysis, geology, pattern recognition, robotics, and diagnostics [Zadeh, 1985] [Zadeh, 1990].

We have seen so far two excellent problem solving techniques – ANN wherein the learning capability results in various fantastic models that are ultimately used in problem solving and Evolutionary computing (Genetic Algorithm) wherein we are concerned with the how to evolve/optimize the system and turn it into a system that is able to solve the unusual problems. The third pillar (algorithm) of softcomputing used is Fuzzy Logic.

By Fuzzy Logic we mean, there is some kind of logic or intelligence (problem-solving methodology) attached to it. There is a rule we all follow in real life and formulation of such kind of rules can help us in decision making using some kind of logical relationships and logical
inductions. In the similar way FIS tries to represent the data (human knowledge) in the form of Fuzzy data set using membership functions and performs the reasoning with that knowledge to produce inferences so as to express its rules. The output fuzzy value is converted into crisp value through the process of Defuzzification.

4.3. Overview of Fuzzy Inference System (FIS):

A technique that plays an important role in decision making taking into consideration the uncertainty and imprecision, is Fuzzy inference system that has the ability to employ if then rules (fuzzy conditional) statements used to capture imprecise modes of reasoning. The basic components in Fuzzy inference system are: [Zanaganeh, 2009] [Abdel, 2011]

- **Rule base** containing fuzzy ‘If-Then’ Rules.
- **Database** defining membership functions.
- **Decision making unit** performing interface operations using fuzzy operators.
- **Fuzzification interface** changing crisp data into fuzzy data (linguistic) using membership degrees.
- **Defuzzification interface** changing fuzzy results into crisp output.

Fig.4.1. Components of Fuzzy Inference System
Fuzzy rule comprises of premise part and the consequent part. The two steps of working in fuzzy inference system are:

1. In the **premise part**, the crisp values are converted into fuzzy values using membership functions.

2. In the **consequent part**, the final output is the weighted average of each rule output.

### 4.4. Statistical Membership Functions:

Fuzzy logic has the ability to make decisions in the same way as the human would make [Gordon, 2005]. The deterministic computer logic has the ability to relate a particular situation to a finite set of states. For example, Color in a black and white image can be defined by the set \{black, white\} in case of the deterministic and rigorous computer logic. However Fuzzy Logic has been introduced in an attempt to offer a way in which a particular situation could be defined as a human logic would do e.g. a black and white image can also be defined by the grey shades in addition to the deterministic black and white color.

This is possible using the degree of member ship, wherein each value in the fuzzy set has an associated degree (value) that will determine its strength or weakness towards a particular feature concerned. The degree of membership is any number between 0 and 1, wherein ‘1’ will demote full membership. So the fuzzy set for defining the color is shown as below:

Fuzzy Set for White Color= \{1/g1, 0.97/g2, 0.84/g3,\ldots, 0/g9 \}

Here in the above case , Full White Color is indicated by ‘g1’ having ‘1’ as the associated degree of membership (100% membership towards white color) and the Full Black Color is indicated by ‘g9’ having ‘0’ as the associated degree of membership (0% membership towards the white color). The elements in the fuzzy set above ‘g2’ and ‘g3’ have the 97% and 84% membership, respectively, towards the color white. Several operations can be performed on the Fuzzy sets like union (maximum), Intersection (minimum) or Inversion. Further the process of de-fuzzification is performed that helps in moving from a fuzzy set towards a point where a control decision could be made. Fuzzy logic has helped in simulating the biological intelligence. Its application has been presented by [Berson and Smith, 2004] in correcting the shortcomings of the rule-based systems.
Fuzzy logic provides the generalization and realistic extension crisp values using membership functions. There can be numerous membership functions that can be used for modelling the problem and we are able to model a problem if we are able to understand how the input variables behave and what are the twinges in MFs, that when changed, behaves in different way. Following table 4.1 lists out some of the membership types used:

<table>
<thead>
<tr>
<th>Statistical Membership Functions</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular Membership Function</td>
<td>$trimf(x; a, b, c) = \max(\min(\frac{x - a}{b - a}, \frac{c - x}{c - b}), 0)$</td>
</tr>
<tr>
<td>Trapezoidal Membership Function</td>
<td>$trimf(x; a, b, c, d) = \max(\min(\frac{x - a}{b - a}, 1, \frac{d - x}{d - c}), 0)$</td>
</tr>
<tr>
<td>Gaussian Membership Function</td>
<td>$gaussmf(x; c, \sigma) = e^{\frac{-1}{2} \left( \frac{x - c}{\sigma} \right)^2}$</td>
</tr>
<tr>
<td>Generalized bell Membership Function</td>
<td>$gbellmf(x; a, b, c) = \frac{1}{1 + \left</td>
</tr>
</tbody>
</table>

There is a cure that we get from both of the functions. For example we have s shaped curve and z-shaped curve. We can combine those two and then form Pi shaped curve. We can play around with these membership functions and get many kinds of possibilities of problem solving using some kind of production rules. So, normally we do not try all the membership functions one by one, it’s just if we have some kind of initial idea like how the input behaves, we use a suitable function or we go with the assumption using Gaussian Function. As a black box approach we may assume if we don’t have any initial idea of how the inputs behave and try to adapt the rules for such a system (See Fig 4.2).

So under FIS, we have set of conditions that model the system and we model not only the system but the underlying mathematics behind the system. We are able to convert antecedent part of the rule into a number and that number denotes a degree to which the rule would be fired. Thus, we
can make somehow a kind of system which can be given set of inputs and produce some kind of output that may be correct or wrong. If wrong, then we need to perform some kind of actions so that output can be better.

Fig 4.2. Graphical representations of membership functions

4.5. Fuzzy Set:

Fuzzy sets do not have distinct boundaries as the Crisp sets have. The main difference between Crisp data and Fuzzy data is that the latter is capable of handling quantifiers and predicates to define the crisp data with realistic extension and generalization using the degree of belongingness (membership degree) to the domain set. Such membership degree (function) provides a fuzzy descriptor value to point out the approximate situation like fuzzy values ‘slow’ or ‘fast’ for crisp variable ‘speed’.
The membership values that lie between 0 and 1, represent the fuzzy data (i.e., not crisp). Mathematically, a membership function for a fuzzy set $F$ is represented as follows [George, 1988]:

$$m_r [x \in F]= \mu_F(x) : \text{Range} \to [0,1]$$

So, in Crisp Sets, we have crisp boundaries e.g. Set ‘A’ of middle age people and in Fuzzy sets, expressed as a set of ordered pairs, we have fuzzy boundaries represented by membership functions.

Fuzzy set ‘$F$’ for Crisp Set ‘$A$’ is represented as:

Crisp Set: $A = \{ x | x \text{ is middle aged} \}$

Fuzzy Set: $F = \{ (x, \mu_A(x)) | x \in A \}$

![Graphical Comparison between Crisp and Fuzzy dataset](image)

Fig 4.3. Graphical Comparison between Crisp and Fuzzy dataset

4.6. Fuzzy Operators:

Fuzzy sets can be manipulated using fuzzy operators. The various operations on Fuzzy Sets performed by the fuzzy operators are listed below [Jang et al., 1997] [Sivanandam and Deepa, 2007] [Kannan.G. 2008] [John, 2011]:

$i. \quad \text{And /Intersection /Min/Prod Operation:}$

The Intersection of two sets contains those elements that are common to both the sets and thus for each element in the set, the minimum value is being selected. The Intersection of fuzzy sets ‘$A$’ and ‘$B$’ is represented as:
\[ \mu_{A \cap B} = \text{Min} (\mu_A (x), \mu_B (x)) = \mu_A(x) \land \mu_B(x) ; \text{for all } x \in U \]

ii. **OR/Union/Max Operation:**

The union of two fuzzy sets contains all the elements in both the sets and thus for each element in the set, the maximum value is being selected. The Union of fuzzy sets ‘A’ and ‘B’ is represented as:

\[ \mu_{A \cup B} = \text{Max} (\mu_A (x), \mu_B (x)) = \mu_A(x) \lor \mu_B(x) ; \text{for all } x \in U \]

iii. **NOT/Complement Operation:** A continuous function ‘f(x)’ that meets the boundary requirements such that f (0) =1 and f (1) =0. For relation ‘R’ and 'S' on Cartesian space (X x Y), the complement of a relation is represented as :

\[ \mu_A^c = 1 - \mu_A (x) ; \text{for all } x \in U \]

**Example:**

Fuzzy Set elements are represented as: x| µ(x), x being crisp input, and µ(x) is its membership degree (fuzziness of input ‘x’). e.g. for input ‘5’, fuzzy element is represented as (5|0.1). The Fuzzy Addition and Multiplication of aµ₁ and bµ₂ is shown in the figures below:

**Addition Operation of Fuzzy Sets:** a+b, \(\mu_{a+b}(x)\): \(\text{max} [\mu_a, \mu_b]\)

![Addition Operation in FIS](image-url)

Fig 4.4. Addition Operation in FIS
**Multiplication Operation of Fuzzy Sets:** $a \times b, \mu_{a \times b}(x) : \min[\mu_a, \mu_b]$

![Diagram of Multiplication Operation in FIS](image)

Fig 4.5. Multiplication Operation in FIS

**4.7. Types of Fuzzy Inference system:**

The Fuzzy Inference System can be classified under two important categories i.e. Takagi Sugeno and Mamdani FIS system. The use of the membership functions in Fuzzy Inference System is represented by Fig 4.6 below.

![Diagram of Fuzzy Inference System](image)

Fig 4.6. Processing of fuzzy input towards the fuzzy output

Example of fuzzy variable: An input variable ‘I’ can defined by ‘n’ fuzzy variables (e.g. temperature defined by fuzzy variable- cold, warm and hot). Example of input membership function for variable distance ‘d’ (in meters) is: short, medium, long and example of output membership function for variable breaking power ‘b’ (in percent) is: large, medium, none. The two important categories of Fuzzy inference system defined below are:
1. Takagi Sugeno Model

2. Mamdani Model

The two categories have been defined in sections 4.7.1 and 4.7.2 along with respective rule base structure.

### 4.7.1. Mamdani Fuzzy Inference System

In case of Mamdani Inference model, the output membership function is in the form of fuzzy set and the main focus is on interpretability. It applies the Linguistic fuzzy Modelling. The rule base in Fuzzy reasoning is of the form:

If $x$ is $A_1$ and $y$ is $B_1$, then $z_1 = c_1$ and

If $x$ is $A_2$ and $y$ is $B_2$, then $z_2 = c_2$ then

$Z = \text{(Centroid of Area aggregated)}$

![Mamdani Inference Model](image)

Fig 4.7. Mamdani Inference Model
4.7.2. Takagi Sugeno Fuzzy Inference System

The Sugeno FIS model is also known by Takagi-Sugeno-Kang wherein the fuzzy inference process is much similar to Mamdani type fuzzy inference process. Both the models have similarity in fuzzifying the inputs and applying the fuzzy operator however the type of outputs they give are different. In Takagi Sugeno model, the output is either constant or a linear (weighted) mathematical expression. The focus is on accuracy and it applies precise fuzzy modelling. The rule base in Fuzzy reasoning is of the form:

If $x$ is $A_1$ and $y$ is $B_1$, then $z_1 = p_1 x + q_1 y + r_1$ and

If $x$ is $A_2$ and $y$ is $B_2$, then $z_2 = p_2 x + q_2 y + r_2$

$$z = \frac{w_1 z_1 + w_2 z_2}{w_1 + w_2}$$

![Fig 4.8. Takagi Sugeno Inference system](image)

Fig 4.8. Takagi Sugeno Inference system
4.8. Design principles while dealing with FIS

So we can make somehow a kind of system which can be given set of inputs and produce some kind of output that may be correct or wrong. If wrong, then we need to perform some kind of actions so that output can be better. The important facts here are:

i. The rules are being provided by the human himself

ii. There is always some kind of division of input between membership functions

iii. And there is always membership curve to be taken care

So if we are trying to make fuzzy robotic controllers or fuzzy machine controllers, and get to know how my controller control the machine as per stated requirements and how it can control the rewarding function so by some means I can judge how well system is performing.

![Output Surface for all applied inputs](image)

**Fig 4.9. Surface view structure**

Above figure shows the surface view structure for the inputs with respective outputs. The two input variables are assigned to the X and Y axis whereas the output variable is assigned to the Z axis.

4.9. Rule Structure and Inference in Fuzzy Logic:

We know we have set of condition that model the system. These conditions are some kind of rules and those rules are fixed whenever conditions are true and based on those firing of rules there are some kind of inference that was made reasoning that was done, some kind of output
produced by the system, we have exactly the same fundamental where we have made some kind of change is that entire modelling can be fuzzy where something can be true to some extend or false to some other extend. We model not only the system but the underlying mathematics behind the system. Only the antecedent part of one rule that we are able to convert into a number and what that number denotes is a degree to which that rule would be fired.

Till now if a system is given where inputs have been broken into MFs along with their curves what we would be able to do is that we will find the various membership values of various inputs to various membership curves and based on all those values, we will analyze the rules that we have been given only the antecedent part and by the application of Or/And/Not we will be able to convert it into some kind of a number and what number denotes is a degree to which that rule is fired. So this antecedent part is done, now we have to do with consequent part. Any rule has two parts—Antecedent and consequent.

The general structure of Rule is:

If \( \text{<Condition (NOT /if any input I belongs to set J AND/OR....> then <consequent>}} \)

**4.9.1. Example of fuzzy rule structure for driving during rain:**

If it’s raining <fuzzy quantifier> And roads are <fuzzy descriptor>, then drive <fuzzy Action>

<Fuzzy quantifier> = low / medium / heavy

<Fuzzy descriptor> = good / bad

![Fig 4.10. Three MF for input rain and Two MF for Input Road.](image-url)
This Fuzzy Action is being represented as membership functions illustrated in the figure below:

\[ <\text{FuzzyAction}> = \text{VeryLessCarefully/LessCarefully/MoreCarefully/VeryCarefully}. \]

Fig 4.11. Illustration of Fuzzy decision making for driving.

Above figure shows the membership functions (triangular) as a linear type of curve.

Here, we first try to divide the inputs and outputs into some kind of logical sets. For every input we get a set of MFs. Further a set of rules are applied to predict the Output that will say how carefully you are supposed to drive based on some numerical value. This is called Inference System

4.9.2. Example for rule base structure for age granulation

Here, Primary terms for Age are: \{Young, Middle-aged, Old\}

Then, Modifiers are: \{not, very, quite, rather…\}

Linguistic terms (F-granulation) are : \{ young, very young, not very young, not very old…\}

Fig 4.12. Graphical Illustration of Fuzzy decision making for age granulation.
Using the input variable ‘age’ and modifiers specified above, it can provide the output consequent category e.g. young. Figure 4.13 below shows some of the rules that are created in this case.

**Fig 4.13. Inference system for Fuzzy Age Determination**

### 4.10. Fuzzy Relation/ Fuzzy Rule and Fuzzy Operators:

Fuzzy Relation is a fuzzy Rule with If-then statement called Implication [Fakhreddin and Clarence, 2009]. The Fuzzy Relation maps elements of one Universe (set) ‘X’ with other ‘Y’ through Cartesian product (space) represented by (X x Y) where (x, y) represent the tuples having varying membership degrees within the relation e.g. for relation R, the membership degree is shown as $\mu_R(x, y)$. The strength of such relation can be measured with membership function (degrees) that is represented within the interval [0, 1].

The various implications in section 4.10.1, operators in section 4.10.2 and reasoning types in section 4.11 have been discussed below:

#### 4.10.1. Implications in Fuzzy Relation

Several methods of fuzzy Implications have been provided among which some of the Important ones are listed below [Fakhreddin and Clarence, 2009] [Sugeno et al,1992] [Michael 2005] [Kannan, 2008]:

![Fuzzy Relation Diagram](image-url)
i. *Mamdani Implication:* A stronger implication showing Fuzzy Set X coupled with Fuzzy Set Y. Larsen implication is another example of such coupling.

Mamdani Implication \((X \text{ and } Y)\) is represented as:

\[
X \cdot Y : \mu_{X \to Y}(x,y) = \min(\mu_X(x), \mu_Y(y)) \quad ; x \in X \text{ and } y \in Y
\]

ii. *Zadeh Implication:* A weaker implication showing that fuzzy set X entails fuzzy set Y

Zadeh implication \([(X \text{ and } Y) \text{ or (not } X)\] is represented as:

\[
(X \land Y) \lor \neg X : \mu_{X \to Y}(x,y) = \max \{\min \{\mu_X(x), \mu_Y(y)\}, 1 - \mu_X(x)\} \quad ; x \in X \text{ and } y \in Y
\]

This max-min composition is known by Sup-Min Composition also. It is considered to be the standard fuzzy composition operator wherein the basic focus is on the restriction over range membership grade within interval \([0, 1]\).

### 4.10.2. Operators over Fuzzy Relation:

A Fuzzy relation (binary) between two sets \(X\) and \(Y\) is denoted by \(R(X, Y)\) and various operations on Fuzzy relations performed by the fuzzy operators are listed below: [Jang et al., 1997; Kannan.G. 2008]

i. *Intersection Operator:*

For relation ‘\(R\)’ and ‘\(S\)’ on Cartesian space \((X \times Y)\), the Intersection of relations is represented as:

\[
\mu_{R \cap S} = \min(\mu_R(x, y), \mu_S(x, y))
\]

ii. *Union Operator:*

For relation ‘\(R\)’ and ‘\(S\)’ on Cartesian space \((X \times Y)\), the Union of relations is represented as:

\[
\mu_{R \cup S} = \max(\mu_R(x, y), \mu_S(x, y))
\]

iii. *Complement Operator:*

A continuous function ‘\(f(x)\)’ that meets the boundary requirements such that \(f(0) = 1\) and \(f(1) = 0\). For relation ‘\(R\)’ and ‘\(S\)’ on Cartesian space \((X \times Y)\), the complement of a relation is represented as:

\[
\mu_R^c = 1 - \mu_R(x, y)
\]
iv. **Containment Operator:**

For relation ‘R’ and 'S’ on Cartesian space (X x Y), the containment of a relation is represented as:

\[ R \subseteq S = \mu_R(x, y) \leq \mu_S(x, y) \]

**4.11. Decision making procedure in fuzzy logic- Fuzzy Relation Method:**

The three types of Fuzzy reasoning proposed are as follows:

1. *Type 1:* In case of Type 1 the overall output of the fuzzy inference system determined by rules rule’s firing strength which is the weighted average of each rules crisp output.

2. *Type 2:* In case of type 2 the overall output of Fuzzy inference system is determined by applying the max operator between the fuzzy outputs. However fuzzy output is determined by the minimum of the firing strength and the output membership function.

3. *Type 3:* In case of type 3 the overall output of the fuzzy inference system is determined by Takagi and Sugeno fuzzy if then rules.

The Approximate reasoning starts with the knowledge base represented by a set of if-then rules comprising of two sections of variables: Antecedent variables and Consequent variables. These variables are known as fuzzy descriptors. The data is pre-processed for fuzzification i.e.

\[ \text{Fuzzy\_Descriptor} = \text{Fuzz\_process}(\text{Data}) \]

The fuzzy inference is obtained using composition operator (°) as a fuzzy matching operator between the fuzzy knowledge base and fuzzy descriptor i.e.:

\[ \text{Fuzzy\_Inference} = \text{Fuzzy\_Descriptor} \circ \text{Fuzzy\_Knowlege\_Base} \]

This can be represented as:

\[ \text{MF}_{\text{FIS}} = \text{SupMin}_x (\text{MF}_{\text{FD}}, \text{MF}_{\text{KB}}) \]

Where,

\( \text{MF}_{\text{FI}} \) is the Membership function of Fuzzy Inference (FI)

‘x’ is the set of context variables used in knowledge base matching.
MF\textsubscript{KB} is the Membership function of Rule base or Knowledgebase (KB) or fuzzy relation obtained from fuzzy reasoning rule. (Knowledge base contains several fuzzy rules).

MF\textsubscript{FD} is the Membership function of Fuzzified data (FD)

### 4.12. Formulation of knowledge using Fuzzy Systems:

Generally, the two basic paths for constructing fuzzy systems and knowledge extraction in those systems are [Timothy, 2010]:

1. **Knowledge formulation from Conscious Path:**

   Under this mechanism, the rules and membership functions are derived or adjusted intuitively by external (human) intelligence or supervisor that is based on expertise, experience, understanding, observation etc. so as to improve overall performance and restricting to system to specified domain [Tamani et al., 2006]

2. **Knowledge formulation from Sub-Conscious Path:**

   Under this mechanism, the rules and membership functions are developed using an automated techniques/adaptive methods such as Least Square method, Gradient method, learning techniques, clustering methods etc [Passino and Yurkovich, 1988].

![Pathway to construct Fuzzy System](image-url)
STRUCTURE IDENTIFICATION IN FUZZY INFERENCE SYSTEM

4.13. Structure Identification in FIS

The Cartesian Product of Fuzzy sets results in the fuzzy relation. Before determining the relation, the major concern is to partition the Input and Output Spaces (Universe of discourse) into fuzzy sets i.e. partitioning the input space into ‘n’ simple partitions over a specified interval (i.e. ‘n’ partitions over the input variable) and partitioning the output space into ‘m’ simple partitions over a specified interval (i.e., ‘m’ partitions over the output variable).

Actually, the concept of fuzzy logic started with the problem of the pattern recognition and classification [Bellman et al., 1996] [Timothy, 2010] that is integral to human perception (fuzzy perception). A simple idea of clustering is used in the area of classification whose basic objective is to classify or partition the dataset into homogeneous clusters wherein the points within the cluster share similar attributes. The idea of data clustering in the area of fuzzy partitions was among the first introduction to clustering. In Fuzzy partitions, it’s the membership degree that measures the similarity between points of cluster [Yang, 1993]. The advantage of Fuzzy clustering is to classify the data points (outliers) isolated between the clusters i.e. those data points would have low membership values in the clusters from which they are isolated [Ruspini, 1973]. As far as crisp data classification is concerned, isolated data points need to be grouped under any one of the cluster wherein their membership is represented as a value 1. So, the extent to which they are isolated from a cluster (distance) cannot be determined by the membership degree measure.

In general, by Clustering we mean classifying or grouping whole dataset into subsets or clusters in such a way that data within the clusters are similar enough, is known as clustering. i.e., Data within the subset or cluster share some common features that is usually evaluated by distance measure (Euclidean and Manhattan). The different types of similarity measures that control how the groups/ clusters are being formed include distance etc.

Data Clustering is viewed as a data (Input / Output Space) partitioning problem wherein the data objects are relocated between clusters and the initial assumption is made that each cluster has at least one object and each object belongs to only one cluster [Forgy, 1965] [MacQueen, 1967] [Anderberg, 1973] [Hartigan, 1975] [Chandon et al , 1980] [Fisher, 1987] [Jain and Dubes, 1988] [Fukunaga, 1990] [Bandefield et al, 1993] [Lozano et al., 1999] [Kaufman et. al, 2009] [Gupta, 2006] [Petrović,2011].
partitioning was to minimize the number of rules of FIS. Here, the number of rules is taken as a
design parameter and is determined by grouping the input output pairs into clusters, each cluster
having one rule i.e. No. of rules equals the no. of clusters [Li-Xin, 1997]

The various methods of clustering for Input / Output Space Partitioning are listed below:

4.13.1. Grid Partitioning:

Under such partitioning the multidimensional input space is divided into partitions (rather than
the data) on which adaptations over the partitions can be applied that may involve adaptations in
the antecedent of the fuzzy rules. For (n) inputs with (m) number of membership functions
associated with each input, the number of fuzzy rules generated using fixed grid partitioning is
(m^n) fuzzy rules. However, this technique may lead to exponential increase in the size of the
fuzzy rules (“Curse of dimensionality”) as the number of input variables increases although
fuzzy rules can be generated using small number of linguistic variables [Jang et al., 1997]
[Gerasimos, 2011].

4.13.2. Subtractive Clustering Partitioning:

The density based method of SCP, also known as modified mountain clustering
[Chiu, S.L., 1994] [Yager et al, 1994] is used when there is no clear view of the cluster number
that should be pre-specified for data organization. This is considered to be the fast method for
searching the number of clusters along with centers. The aim is to estimate both number of
clusters and position of the cluster centers. The method starts by assuming each data point to be a
cluster center and the point having maximum neighbor points is reassumed as a new cluster
center and other points with fewer neighbors will be ignored. The search process is repeated until
all the points are analyzed. The most important parameter in subtractive clustering is the radii
which on changing can provide different number of clusters and the algorithm is investigated
using different settings. Specifying the smaller cluster radii produces many small clusters of data
thus resulting in many rules and larger radii results in few large clusters of data thus produces
fewer rules. So, the smaller the radius, the greater is the number of clusters and vice versa. Or we
say, the number of clusters or rules depends on distribution of input data points and the radius r.
[Chiu,1994] [Delmirli and Muthukumaran, 2000] [MATLAB User’s Guide, 2006].
The technique starts with calculating the initial potential of data object or data point as follows:

\[ P_i = \sum_{j=1}^{N} \exp(-\alpha \|X_i - X_j\|^2) = \frac{4}{r_a^2} \]

Where \( r_a \) is the radius that defines the neighborhood.

The data point with highest potential is taken as the first Cluster Centre.

And the revision in potential of each data point is revised as follows:

\[ P_i = P_i - P_i^* \exp\left(-\beta \exp\left\|X_i - X_j\right\|^2\right); \quad \beta = \frac{4}{r_b^2} \]

Where \( r_b \) is the radius larger than \( r_a \) that is able to avoid cluster centers with high density.

The data point with highest potential is taken as second cluster center. The process of finding new clusters and revising the potentials is continued until the potential of all the data points is below the potential of first cluster center. After identifying the ‘n’ number of clusters on the basis of radii specified, this method characterizes each input and output data by ‘n’ number of membership functions and the no. of rules is also same as no. of Clusters [Matlab 2006].

4.13.3. Fuzzy C-Means (FCM) Partitioning:

This kind of partitioning involves the partition of set of data objects into distinct subsets or clusters wherein the objects in a cluster are “similar enough” and each cluster has a selected center. In case of hard computing, that deals with crisp data, each data object is assigned to exactly single crisp cluster e.g. K-means algorithm [Dave et al.,1997]. However, in soft computing, dealing with fuzzy data, each object is associated to more than one cluster by a degree of membership. One of the important fuzzy clustering technique is the FCM (Fuzzy C-means) technique and was introduced by [Dunn, 1973]. The algorithm partitions or divides a finite set ‘X’ into a collection of ‘C’ fuzzy clusters taking into consideration some criteria. When it comes to FCM, the parameters like cluster number and their initial locations are specified before the algorithm starts and hence the quality of the solution depends on the choice of such initial values [Bezdek, 2013].

Fuzzy C means focuses on the centers of the clusters as a classical C means technique does. The most important parameter is the number of clusters on the basis of which the algorithm is
investigated. The algorithm starts with a generated cluster and calculates the validity of the algorithm. If the Validity index is satisfactory then further model processing is performed else the ‘cluster number’ parameter is changed and the algorithm runs with the new specified parameter. So, the aim of this algorithm is to minimize the fuzzy objective function by choosing an efficient number of groups or clusters [Sugeno and Yasukawa, 1993] [Bataineh, 2011] [Petrović, 2011] [Li et al, 2008].

After randomly initializing the cluster membership values, it calculates the cluster centres as follows which are updated accordingly:

\[
C_i = \frac{\sum_k (\mu_{ik})^m x_k}{\sum_k (\mu_{ik})^m}
\]

Where \( m \geq 1 \), \( k \) is a data point or data object in \( x_i \) and \( \mu_{ik} \) is the membership of the \( k \)th object.

Assuming the problem of classifying ‘\( N \)’ samples into ‘\( k \)’ clusters, FCM is based on minimization of the objective function as represented by the formula:

\[
j_m = \sum_{j=1}^{k} \sum_{i=1}^{N} (\mu_{ij})^m ||x_i - C_j||^2
\]

Where,

\{\( X_1, X_2, ..., X_N \)\} given dataset (\( X_1 \in R \))

\( \mu_{ij} \in [0, 1] \) is the degree of membership or association of \( X_i \) to cluster center ‘\( C_j \)’.

\( ||x_i - C_j|| \) is the Euclidean distance between sample \( X_i \) and center \( C_j \).

‘\( m \)’ is “fuzziness index” (\( m > 1 \)) used in controlling membership fuzziness of each data point. The small the ‘\( m \)’, the less will be the fuzziness and vice versa.

when \( m = 1 \), it works much similar to similar to the working of original C-Means clustering. However, value of 2.0 has been mostly used in fuzzy clustering [Naqvi, 2009].

The sum of fuzzy membership values to each cluster is equal to 1.

\[
\sum_{i=1}^{k} \mu_{ij} = 1
\]
The calculation of objective function is done iteratively, and the process stops when threshold value is reached or the max number of iterations is met.

The partitioning methods are applied over the dataset to recognize the membership functions. GP helps in doing that to some extend but huge dataset needs to be categorize that could help in recognizing the system behavior. Initially the number of clusters is unknown we can use the subtractive clustering partitioning (SCP) technique wherein the number of clusters and their centers can be determined using radii parameter. Further, the FCM technique is an improved version of SCP wherein the nonlinear data is grouped on the basis of their degree of membership. However, both the techniques are used in finding the number of clusters and their centers. The Input Output Fuzzy models, to model the original input data, are built using the best clustering results obtained. SCP results are directly used to build the system while as FCM results are further applied to any training routine used to obtain local minimum solution i.e. Objects relocated between clusters as the clusters are being refined (e.g. using Sugeno type) [Gupta, 2006] [Fakhreddine, 2009] [Bataineh, 2011]

4.14. Working with FIS on Matlab:

The Fuzzy Logic controller (FLC) can be built using the GUI toolbox of Fuzzy Logic. Some of the important GUI tools for this available on Matlab are listed below [Bansal et al., 2009]:

4.14.1. Rule Editor:

Matlab provides a Rule editor that is used to modify the rules of FIS structure and displays them accordingly. The rule editor is invoked using ruleedit function using the input (antecedent) and output (Consequent) options in the list box provided in the editor. Figure below shows the fuzzy rule editor displaying some of the rules of FIS structure containing one input variable in the antecedent and one output in the consequent. Next Figure represents the Rule editor for two input FIS structure:
Fig 4.15. Fuzzy Rule Editor for one input variable

Fig 4.16. Fuzzy Rule Editor for two input variable
4.14.2. Rule Viewer:

Rule viewer is used to view the implication process in the rule base system of FIS structure. It provides the line indices corresponding to the inputs that can be moved around to readjust and compute the new output value. The rule viewer can be invoked using ‘ruleview’ function. Figure below shows the rule viewer for one input ad one output FIs structure.

![Rule Viewer for one input FIS structure](image)

Fig 4.17. Rule viewer for one input FIS structure

In the above figure, the input i.e. distance variable would help in knowing the consequent breaking intensity value.

Next figure below represents the rule viewer for two input one output FIS structure i.e. the adjustments in values of two inputs i.e. distance and velocity would give the consequent output of breaking intensity accordingly.
4.14.3. Surface Viewer:

Again you can test the system by choosing different values of inputs. If the result is not Satisfactory, then refine the system. E.g. it seems that the area of short distance and high speed does not give strong enough braking response. Finally, the surface view of the rules is shown below in figure . Surface viewer is invoked using ‘surfview’ function and is used to examine the output surface of an FIS structure. The two input variables are assigned to the X and Y axis whereas the output variable is assigned to the Z axis.
4.15. Conclusion:

Fuzzy inference system provides a better way to extract the hidden knowledge out of data using fuzzy If-then Rules. However, Human Expertization is important in FIS to provide membership values properly. Another issue in this method is the cure of dimensionality where number of rules increase with the number of input variables. So, we moved towards the better option. Instead of using fixed input cell partition we used the concept of clustering. All we have to do is to provide the cluster radii on the basis of which the FIS will partition the data and determine the number of membership functions and rules. But the issue is the radii specification. The more the radii the smaller the big clusters and smaller will be the rules. To avoid the confusion in the radii specification we allowed the FIS to search for the best number of clusters. This was done using the best machine learning algorithm called Fuzzy C means.