Chapter 5

Creep Analysis of an Isotropic Rotating FGM Disc

Due to rapid growth in technology it is possible to synthesize materials for components that exhibit a graded-variation in their properties. Functionally graded materials (FGMs) are advanced materials that possess continuously graded properties and are characterized by spatially varying microstructures created by both, non-uniform distributions of the reinforcement phase as well as by interchanging the role of reinforcement and matrix materials in a continuous manner. The smooth variation of properties may offer advantages such as reduction of thermal stress and increased strength. A major advantage of FGMs is the possibility of tailoring its gradation to maximize its performance. In case of tailoring material properties, the objective is to determine the material phase volume fraction of each point of the structure.

In recent years, functionally graded materials have gained considerable attention in many engineering applications. FGMs are considered as a potential structural material for future high-speed spacecraft and power generation industries. Metal–ceramic FGMs were first used in the thermal barrier coatings in the fields
of aviation and aerospace. Typically, under severe environments such as high temperature or thermal gradients, the conventional materials, metals or ceramics, alone may not survive. Thus, a new material concept of FGMs emerged and led to the development of superior heat-resistant materials. Such materials withstand severe thermo mechanical loadings. In an FGM, the composition and structure gradually change over volume, resulting in corresponding changes in the properties of the material. Ceramic—particle/whisker—reinforced metal matrix composites have shown superior high—temperature properties and are finding increasing application in the manufacture of components exposed to high temperatures. By applying the many possibilities inherent in the FGM concept, it is anticipated that materials will be improved and new functions for them established. The ceramic in an FGM offers thermal barrier effects and protects the metal from corrosion as well as oxidation and the FGM is toughened and strengthened by the metallic composition. A mixture of ceramic and metal with a continuously varying volume fraction can be manufactured. This eliminates interface problems of composite materials and thus the stress distributions are smooth. Mechanical properties of FGM largely depend on the distribution of volume fraction of metal (or equally ceramic) powder along the thickness of the layer. Extensive studies have been carried out, both theoretically and numerically, on thermal stress distribution and fracture in functionally gradient materials.

Taking into account the mechanical stress in functionally graded rotating disks, Durodola and Attia (2000) presented studies regarding stress distribution of rotating
disks for different gradation of material properties. Tutuncu and Ozturk (2001) provided closed-form solutions for stresses and displacements in FG cylindrical vessels subjected to an internal pressure using the infinitesimal theory of elasticity. They considered Poisson’s ratio as constant with material stiffness varying through the wall thickness as power-law. Mishra and Pandey (1990) have proposed a power law relation between effective strain rate and effective applied stress. It has been reported that steady state creep in aluminium/aluminium alloy based composites can be described in a better way by Sherby’s creep model compared to widely criticized Norton’s creep model. Singh and Ray (2001) estimated steady-state creep response using Norton’s power law in an isotropic FGM rotating disc of aluminum silicon carbide particulate composites without thermal gradient assuming linearly decreasing variation of silicon carbide particles from the inner to the outer radius of the disc. Nieh (1984) has shown that an aluminum-based composite containing silicon carbide whisker has better creep resistance compared to the base aluminum alloy. Pandey et. al. (1992) studied the steady-state creep behavior of Al–SiCp composites under uniaxial loading condition in the temperature range between 623 and 723 K for different combinations of particle size and volume fraction of reinforcement and found that the composite with finer particle size has better creep resistance than that containing coarser ones.

In the study of elastic solutions for axisymmetric rotating disks made of functionally graded material with variable thickness, Bayat et. al. (2008) assumed the
material properties and disk thickness profile to vary according to power–law distributions and found that a functionally graded rotating disk with parabolic or hyperbolic convergent thickness profile has smaller stresses and displacements compared with that of uniform thickness. Noda and Jin (1993, 1994) extended their work by including thermal load. The method they used is to transfer the boundary problems into singular integral equations by Fourier transformation. Shen and Yu (1997) studied the stress in a FGM layer on the surface of a structural component by asymptotic method. Anne et. al. (2006) calculated the strength and residual stresses of functionally graded $\text{Al}_2\text{O}_3/\text{ZrO}_2$ discs by biaxial strength testing and revealed that the strength of such discs, prepared by electrophoretic deposition, was almost doubled from 288 MPa for pure $\text{Al}_2\text{O}_3$ to 513 MPa for the graded $\text{Al}_2\text{O}_3/\text{ZrO}_2$ discs. They observed that the increase in strength was due to the compressive surface residual thermal stresses in the $\text{Al}_2\text{O}_3$ surface layer caused by the graded compositional profile. Singh et. al. (1998, 2002) on rotating aluminum discs containing 20 vol.% silicon carbide undergoing isothermal creep, shown that tangential stress is maximum near the inner radius, and there it results in higher creep strain compared to that at the outer radius. These results indicate that by incorporating a relatively higher amount of particles near the inner radius as compared to that near the outer radius, it may be possible to make the distribution of strain rate more uniform across the disc, which may ultimately result in significantly lower strain rate. Stump et. al. (2005) in their paper presented the application of the Topology optimization Method to tailor the material properties of a Functionally Graded rotating disk with a constraint in the value of the $p$–norm of stress along the structure and in order to avoid the stress
singularity phenomena, a modified von Mises stress evaluation was proposed. Some numerical results were also presented to show the potential of the method. Gupta et. al. (2003) studied the creep behavior of a rotating functionally graded composite disc operating under thermal gradient. Based on their study, concluded that the steady state creep response of the FGM disc was significantly superior compared to that in a disc with the same total particle content distributed uniformly. Rattan et. al. (2009a, 2009b) investigated the effect of stress exponent on creep in an isotropic rotating disc of AlSiC$_p$ using Sherby’s model and further investigated creep in the presence of thermal residual stress.

The present study aims to investigate the steady-state creep response of an isotropic FGM rotating disc of aluminum silicon carbide particulate composites subjected to particle gradient and to investigate the effect of material properties on the distributions of stress and strain through the radial direction of the annular disk. Sherby’s creep law, which is claimed to work better than Norton’s creep law, has been used to describe the creep behavior of the composite.

5.1 Distribution of Dispersoid

Let us consider a functionally graded annular disk of inner radius $a$ and outer radius $b$. The distribution of silicon carbide particles has been assumed to be non-linear (parabolic) from inner to outer radius; therefore, the density and the creep constants will vary with radial distance. The material properties of the annular disk are assumed to be functions of the volume fraction of the constituent materials. The composition
variation in terms of volume percent of silicon carbide, along the radial distance, $V(r)$, is given as:

$$V(r) = A - Br^2, \quad a \leq r \leq b$$  \hspace{1cm} (5.1.1)$$

where

$$A = \frac{b^2V_{\text{max}} - a^2V_{\text{min}}}{b^2 - a^2}$$  \hspace{1cm} (5.1.2)$$

and

$$B = \frac{V_{\text{max}} - V_{\text{min}}}{b^2 - a^2}$$  \hspace{1cm} (5.1.3)$$

Here $V_{\text{max}}$ and $V_{\text{min}}$, are, respectively, the particle contents at the inner and outer radii. Now, using the law of mixtures, one may express the density variation in the composite as:

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \frac{V(r)}{100}$$  \hspace{1cm} (5.1.4)$$

where $\rho_m$ and $\rho_d$ are the densities of the matrix alloy and of the dispersed silicon carbide particles, respectively. Now putting the value of $V(r)$ from equation (5.1.1) into equation (5.1.4), one obtains:

$$\rho(r) = \rho_m + (\rho_d - \rho_m) \frac{A - Br^2}{100}.$$  \hspace{1cm} (5.1.5)$$

If the average particle content in the FGM disc is $V_{\text{avg}}$, and $t$ is the thickness of the disc, then

$$\int_a^b 2\pi rtV(r)dr = V_{\text{avg}}t(b^2 - a^2)\pi.$$  \hspace{1cm} (5.1.6)$$

Putting the expression of $V(r)$ from equation (5.1.1) into equation (5.1.6), one may obtain the relation:

$$V_{\text{avg}} = A - \frac{B(b^2 + a^2)}{2}.$$  \hspace{1cm} (5.1.7)$$
The steady-state creep response of the \( Al - SiC_p \) composite of varying composition has been described in terms of Sherby’s law (1977) of the form:

\[
\dot{\epsilon} = [M(\bar{\sigma} - \sigma_0)]^8
\]  

(5.1.8)

where

\[
M = \frac{1}{E^*} \left[ \frac{ADL^3}{b_r^5} \right]^{1/8}
\]  

(5.1.9)

and the symbols have their usual meanings as given in nomenclature.

In a particle-reinforced composite, the material parameters \( M \) and \( \sigma_0 \) depend on the particle size (\( p \)) and the percentage of dispersed particles (\( V \)) apart from the temperature (\( T \)). The value of \( M \) and \( \sigma_0 \) have been obtained from the creep results reported for \( Al - SiC_p \) composite under uniaxial loading Pandey (1992) and these values have been fitted by the following regression equations:

\[
\ln(M) = 0.2112\ln(p) + 4.89\ln(T) - 0.591\ln(V) - 34.91
\]  

(5.1.10)

\[
\sigma_0 = -0.02050(p) + 0.0378(T) + 1.033(V) - 4.9695
\]  

(5.1.11)

The variation of creep parameters in the rotating FGM disc along the radial distance has been determined in this study from the preceding equations for \( p=1.7 \mu m \) and \( T=623 K \).

5.2 Modeling of Creep Behavior in FGM disc

Consider an aluminium silicon-carbide particulate composite disc of constant thickness \( h \) having inner radius \( a \) and outer radius \( b \), rotating with angular velocity \( \omega \).
From symmetry considerations, principal stresses are in the radial, tangential and axial directions. For the purpose of modeling the following assumptions are made:

(a) Steady state condition of stress is assumed.

(b) Elastic deformations are small for the disc and can be neglected as compared to the creep deformations.

(c) Biaxial state of stress exists at any point of the disc.

(d) The composite shows a steady state creep behavior, which may be described by Sherby’s constitutive model as given by equation (5.1.8).

The different material combinations in the composite are conceptually replaced by an equivalent monolithic material that has the yielding and creep behavior similar to that displayed by the composite. Taking reference frame along the directions $r$, $\theta$ and $z$, the generalized constitutive equations for creep in an isotropic rotating disc takes the form:

$$
\dot{\epsilon}_r = \frac{\dot{\epsilon}}{2\sigma} \left[ 2\sigma_r - (\sigma_\theta + \sigma_z) \right]
$$

(5.2.1)

$$
\dot{\epsilon}_\theta = \frac{\dot{\epsilon}}{2\sigma} \left[ 2\sigma_\theta - (\sigma_z + \sigma_r) \right]
$$

(5.2.2)

$$
\dot{\epsilon}_z = \frac{\dot{\epsilon}}{2\sigma} \left[ 2\sigma_z - (\sigma_r + \sigma_\theta) \right]
$$

(5.2.3)

where the effective stress, $\sigma$, is given by,

$$
\sigma = \frac{1}{\sqrt{2}} \left[ (\sigma_\theta - \sigma_r)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2 \right]^{1/2}
$$

(5.2.4)

and $\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$, $\dot{\epsilon}_z$ and $\sigma_r$, $\sigma_\theta$, $\sigma_z$ are the strain rates and stresses respectively in the directions indicated by the subscripts and $\dot{\epsilon}$ is the effective strain rate. For biaxial
state of stress \((\sigma_r, \sigma_\theta)\),

\[
\bar{\sigma} = \frac{1}{\sqrt{2}} [\sigma_\theta^2 + \sigma_r^2 + (\sigma_r - \sigma_\theta)^2]^{1/2}
\]

(5.2.5)

and the constitutive equations are,

\[
\dot{\epsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{[M(r)(\bar{\sigma} - \sigma_0)]^8(2x - 1)}{2(x^2 - x + 1)^{1/2}}
\]

(5.2.6)

\[
\dot{\epsilon}_\theta = \frac{d\dot{u}_r}{dr} = \frac{[M(r)(\bar{\sigma} - \sigma_0)]^8(2 - x)}{2(x^2 - x + 1)^{1/2}}
\]

(5.2.7)

where \(x = \sigma_r / \sigma_\theta\) is the ratio of radial and tangential stress at any radius \(r\). Equations (5.2.6) and (5.2.7) can be solved to obtain \(\sigma_\theta(r)\) as given:

\[
\sigma_\theta(r) = \frac{(\dot{u}_a)^{1/8}}{M(r)} \psi_1(r) + \psi_2(r)
\]

(5.2.8)

where

\[
(\dot{u}_a)^{1/8} = \frac{\int_a^b M(r)\sigma_\theta dr - \int_a^a M(r)\psi_2(\sigma_\theta)dr}{\int_a^b \psi_1(r)dr},
\]

(5.2.9)

\[
\psi_1(r) = \frac{\psi(r)}{(x^2 - x + 1)^{1/2}},
\]

(5.2.10)

\[
\psi_2(r) = \frac{\sigma_\theta(r)}{(x^2 - x + 1)^{1/2}},
\]

(5.2.11)

\[
\psi(r) = \left[\frac{2(x^2 - x + 1)^{1/2}}{r(2 - x)} \exp \int_a^r \frac{\phi(r)}{r} dr \right]^{1/8},
\]

(5.2.12)
and

\[ \phi(r) = \frac{(2x - 1)}{(2 - x)}. \]  \hspace{1cm} (5.2.13)

The equation of motion for a rotating disc of constant thickness \( h \) may be obtained by considering the equilibrium of an element in the composite disc confined between radial distances \( r \) and \( r + dr \) and an interval of angle between \( \theta \) and \( \theta + d\theta \). The equilibrium of forces in the radial direction of the element implies that

\[
\frac{d}{dr}[r\sigma_r(r)] - \sigma_\theta(r) + \rho(r)\omega^2r^2 = 0 \tag{5.2.14}
\]

where \( \rho \) is the density of the composite. Knowing the tangential stress distribution \( \sigma_\theta(r) \), values of \( \sigma_r(r) \) can be as obtained from above relation as follows:

\[
\sigma_r(r) = \frac{1}{r} \int_a^r \sigma_\theta(r)dr - \frac{\omega^2}{r} \left[ \left\{ \rho_m + \frac{(\rho_d - \rho_m)}{100} A \right\} \frac{(r^3 - a^3)}{3} - \frac{B(\rho_d - \rho_m)}{100} \frac{(r^5 - a^5)}{5} \right]. \tag{5.2.15}
\]

As the tangential stress, \( \sigma_\theta \), and the radial stress, \( \sigma_r \), are determined by equations (5.2.8) and (5.2.15) at any point within the composite disc. Then the strain rates \( \dot{\epsilon}_r \), \( \dot{\epsilon}_\theta \) and \( \dot{\epsilon}_r \) are calculated from equations (5.2.6), (5.2.7) and (5.2.3) respectively.

### 5.3 Result and Discussion

Figure 5.1 shows similar variation of radial stress along the radial distance for three different types of rotating discs. It is observed in all these discs that the radial stresses increases as one move from inner radius to center radial distance, reaches maximum
Figure 5.1: Variation of radial stress along radial distance in FGM/non-FGM disc.

Figure 5.2: Variation of tangential stress along radial distance in FGM/non-FGM disc.
Figure 5.3: Variation of radial strain along radial distance in FGM/non-FGM disc.

Figure 5.4: Variation of tangential strain along radial distance in FGM/non-FGM disc.
nearly at the center and then decreases on going towards outer radius of the disc. The radial stresses are lesser for the non FGM disc in comparison to FGM disc with linear or parabolic distribution of particle content along the radial distance.

Figure 5.2 shows the variation of tangential stress along the radial distance of the rotating discs. It is observed from the figure that as one moves from inner to outer radial distance the tangential stress increases in all these discs, reaches maximum nearly at one-fifth of the radius and then decreases. The tangential stress changes drastically for discs with non linear or linear distribution of particle content as one moves from inner to outer radius of the disc whereas it shows small variation for the disc with uniform distribution of particle content.

Figure 5.3 shows the variation of radial strain rate along the radial distance for these rotating discs. It can be seen from the figure that compressive radial strain rate is largest for the disc with uniform distribution of particle content and least for non linear FGM disc. Also radial strain rate becomes tensile at the middle radial distances for the discs with functionally graded distribution of particle contents whereas it remains compressive throughout the entire radius of non FGM disc.

Figure 5.4 shows the variation of tangential strain rate along the radius of the rotating discs. The figure shows that the tangential strain rate is maximum for the uniformly distributed disc particularly at the inner radius and minimum for the disc with parabolic distribution of particle content. It is also observed that for FGM disc the tangential strain rate remains almost same throughout the disc, whereas it changes drastically for non FGM disc.
5.4 Conclusion

Based on the results and discussion presented in this paper it can be concluded that the creep behavior in the disc can be controlled by the suitable distribution of particle contents as the strain rates are minimum when particle content is distributed parabolically along the radial distance of the disc.
Chapter 6

Creep Behavior of an Anisotropic Rotating Disc

Rotating disc is a very useful component in many engineering applications such as turbines, rotors, compressors, flywheels and computer’s disc drive. Many of these applications require the disc to operate at very high temperatures where the problem of creep becomes important. Composites often exhibit anisotropy in yielding depending on the reinforcement geometry and their orientation in the matrix. A composite containing aligned fibers or whiskers would start yielding at different stresses in the direction of alignment and in the transverse direction. Processing of composites containing whiskers and short fibers often result in anisotropy due to preferential alignment of reinforcing element during flow as in extrusion. A great deal of work has been dedicated to the area of steady state creep behavior, since the majority of component’s creep life is spent in this stage. Tsai and Wu (1971) investigated general theory of strength for anisotropic materials. Axisymmetric creep analysis has also been of interest. Wahl (1954) was the first to analyse axisymmetric secondary creep problems. He considered different loading of a rotating disc. Wahl et. al. (1954) theoretically studied steady state creep deformation in a rotating turbine disc made
of 12%–chromium steel using von Mises and Tresca yield criteria and compared the results with experimental values. They described creep behavior by a power function relation of the form $\dot{\epsilon} = k\sigma^n f(t)$. Pandey et. al. (1992) studied the roles of volume fraction and particle size on the steady state creep behavior of aluminium–silicon carbide particulate composites in the temperature range 623-723 K. The authors reported that steady state creep in aluminium/aluminium alloy based composites can be described in a better way by substructure invariant model of Sherby et. al. (1977) as compared to widely criticized Norton’s creep model. Damage anisotropy is experimentally evident mainly in the case of brittle damage response where the appropriate modification of the fourth rank constitutive tensors, stiffness or compliance, is used (Litewka (1985), Murakami and Kamiya (1997)). Under high temperature creep conditions it is usually assumed that the damage response is of isotropic nature (Kachanov(1958), Hayhurst (1972), Lemaitre and Chaboche (1985)). Nevertheless, even if creep damage is concerned, for some materials it is necessary to account for the effect of damage anisotropy. Ganczarski and Skrzypek (2001) demonstrated a modification of the creep-damage equations proposed by Murakami, Kawai and Rong. Singh and Ray (2002, 2003) have studied steady state creep in the rotating disc of Al–SiC composites using von Mises and Hill criteria. Bhadauria et.al. (2009) discusses the effects of stress triaxiality on yielding behavior of anisotropic materials using Hill-von Mises criteria for anisotropic material with triaxiality factor (TF) and formulated mathematical model that combines the yield stress and anisotropic ratio R (ratio of width strain to thickness strain) along with triaxiality. Singh (2008) studied the creep
behavior of a whisker reinforced anisotropic rotating disc made of $Al - SiC_w$ composite. He described creep behavior by Norton’s power law. Singh and Ray (2003) proposed a new yield criterion for residual stress, which at appropriate limits reduces to Hill anisotropic and Hoffman isotropic yield criterion and carried out analysis of steady state creep in a rotating disc made of $Al - SiC_w$ composite using this criterion and compared the results obtained using Hill anisotropic yield criterion ignoring difference in yield stresses.

A great deal of work has been dedicated to the area of steady-state creep behavior. Efforts are usually conducted to study the creep mechanisms associated with this stage or they involve the development and use of a life-predicting time-temperature parameter. This makes sense, since the majority of a component’s creep life is spent in this secondary stage. In the present study the creep behavior of an anisotropic rotating disc made of $Al - SiC_p$ composite has been investigated using Hill’s yield criteria and the creep behavior in this case is assumed to follow Sherby’s constitutive model. The creep parameters used in the analysis have been determined using the regression equations developed on the basis of the experimental results of Pandey et. al. (1992). The values of anisotropic constants have been taken from Kulkarni et. al. (1985).

6.1 Modeling of Creep for anisotropic disc

Consider a particle reinforced composite disc of inner and outer radii $a$ and $b$, respectively and with a constant thickness $h$, rotating with angular velocity $\omega$ (rad/s).
From symmetry considerations, principal stresses are in radial, tangential and axial
directions. For the purpose of analysis the following assumptions are made:

(a) Steady state condition of stress is assumed.

(b) Elastic deformations are small for the disc and can be neglected as compared to
the creep deformations.

(c) Biaxial state of stress exists at any point of the disc.

(d) The composite shows a steady state creep behavior, which may be described by
Sherby’s constitutive model of the form:

\[
\dot{\varepsilon} = [M(\bar{\sigma} - \sigma_0)]^8
\] (6.1.1)

where,

\[
M = \frac{1}{E} \left[ \frac{AD_L \lambda^3}{|b_r|^5} \right]^{1/8}.
\] (6.1.2)

The values of creep parameters M and \(\sigma_0\) are described by the regression equations
(3.3.3) and (3.3.4) as a function of particle size (p), temperature (T) and volume
percent (V), which are obtained by using the experimental results of Pandey et. al.

The generalized constitutive equations for creep in anisotropic composite take the
following form when reference frame is taken along the principal directions \(r, \theta\) and
\(z\):

\[
\dot{\varepsilon}_r = \frac{\dot{\varepsilon}}{2\bar{\sigma}}[(G + H)\sigma_r - H\sigma_\theta - G\sigma_z]
\] (6.1.3)

\[
\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}}{2\bar{\sigma}}[(H + F)\sigma_\theta - F\sigma_z - H\sigma_r]
\] (6.1.4)
\[ \dot{\epsilon}_z = \frac{\dot{\epsilon}}{2\bar{\sigma}}[(F + G)\sigma_z - G\sigma_r - F\sigma_\theta] \] (6.1.5)

where \( F, G, H \) are anisotropic constants and \( \bar{\sigma} \) is the effective stress based on the Hill’s yield criterion for anisotropic material as given by,

\[ \bar{\sigma} = \frac{1}{\sqrt{G + H}}[H(\sigma_\theta - \sigma_r)^2 + G(\sigma_r - \sigma_z)^2 + F(\sigma_z - \sigma_\theta)^2]^{1/2}. \] (6.1.6)

For biaxial state of stress (\( \sigma_z = 0 \)), solving constitutive equations along with equations (4.1.1) and (6.1.6) the following relations for strain rates \( \dot{\epsilon}_r, \dot{\epsilon}_\theta, \dot{\epsilon}_z \) may be obtained as

\[ \dot{\epsilon}_r = \frac{d\dot{u}_r}{dr} = [M(\bar{\sigma} - \sigma_0)]^8 \sqrt{F(G + H)}[(\frac{G}{F})(\bar{\sigma} + H) - \frac{H}{F}x] \sqrt{2\left[\frac{G}{F}(x^2 - 2\frac{H}{F}x + (1 + \frac{H}{F}))\right]^{1/2}} \] (6.1.7)

\[ \dot{\epsilon}_\theta = \frac{\dot{u}_r}{r} = [M(\bar{\sigma} - \sigma_0)]^8 \sqrt{F(G + H)}[(1 + \frac{H}{F}) - \frac{H}{F}x] \sqrt{2\left[\frac{G}{F}(x^2 - 2\frac{H}{F}x + (1 + \frac{H}{F}))\right]^{1/2}} \] (6.1.8)

where \( x = \sigma_r/\sigma_\theta \) is the ratio of radial and tangential stress at any radius \( r \).

The equilibrium equation for the disc of constant thickness is given by (3.1.23). Equations (6.1.7) and (6.1.8) along with equation (3.1.23) can be solved to obtain \( \sigma_\theta(r) \), as given below:

\[ \sigma_\theta(r) = \sqrt{\frac{G + H}{F}} \left[ \frac{\sqrt{\frac{F}{G + H}}(b - a)\sigma_{\theta a g} - \int_a^b \psi_2(r)dr}{\int_a^b \psi_1(r)dr} \right] \psi_1(r) + \psi_2(r), \] (6.1.9)

where,

\[ \psi_1(r) = \frac{\psi(r)}{\left[\left(\frac{H}{F} + \frac{G}{F}\right)x^2 - 2\frac{H}{F}x + (1 + \frac{H}{F})\right]^{1/2}}, \] (6.1.10)

\[ \psi_2(r) = \frac{\sigma_0}{\left[\left(\frac{H}{F} + \frac{G}{F}\right)x^2 - 2\frac{H}{F}x + (1 + \frac{H}{F})\right]^{1/2}}. \] (6.1.11)
\[ \psi(r) = \left[ \frac{2\left(\frac{H}{F} + \frac{G}{F}\right)x^2 - 2\frac{H}{F}x + (1 + \frac{H}{F})}{r \sqrt{F(G + h)}[(\frac{G}{F} + \frac{H}{F})x - \frac{H}{F}]} \right] \exp \int_a^r \frac{\phi(r)}{r} dr \right]^{1/8}, \]  
(6.1.12)

and

\[ \phi(r) = \left[ \frac{\left(\frac{H}{F} + \frac{G}{F}\right)x - 2\frac{H}{F}}{[(\frac{G}{F} + \frac{H}{F})x - \frac{H}{F}]} \right]. \]  
(6.1.13)

Also, equation of equilibrium (3.1.23) can be solved to obtain the radial stress \( \sigma_r(r) \) as eq. (3.1.28). Knowing the value of \( \sigma_\theta \) and \( \sigma_r \) strain rates \( \dot{\epsilon}_r, \dot{\epsilon}_\theta \) and \( \dot{\epsilon}_z \) are calculated using equations (6.1.7), (6.1.8) and (6.1.5), respectively.

### 6.2 Modeling of Creep for anisotropic FGM disc

The equilibrium equation for the disc of constant thickness is given by (3.1.23). Equations (6.1.7) and (6.1.8) along with equation (3.1.23) can be solved to obtain \( \sigma_\theta(r) \), as given below:

\[ \sigma_\theta(r) = \left( \frac{\dot{u}_a}{M(r)} \right) \frac{x}{\psi_1(r) + \psi_2(r)} \]  
(6.2.1)

where,

\[ \left( \frac{\dot{u}_a}{M(r)} \right)^\frac{1}{2} = \frac{\int_a^b M(r)\sigma_\theta dr - \int_a^b M(r)\psi_2 dr}{\int_a^b \psi_1 dr} \]  
(6.2.2)

Knowing the radial distribution of \( \sigma_\theta(r) \), values of \( \sigma_r(r) \) can be as obtained from above relation as follows:

\[ \sigma_r(r) = \frac{1}{r} \int_a^r \sigma_\theta(r) dr - \frac{\omega^2}{r} \left[ \frac{\left( \rho_d - \rho_m \right) A}{100} \right] \frac{(r^3 - a^3)}{3} - \frac{B(\rho_d - \rho_m)}{100} \frac{(r^5 - a^5)}{5}. \]  
(6.2.3)
As the tangential stress, $\sigma_\theta$, and the radial stress, $\sigma_r$, are determined by equations (6.2.1) and (6.2.3) at any point within the composite disc. Then the strain rates $\dot{\epsilon}_r$, $\dot{\epsilon}_\theta$ and $\dot{\epsilon}_r$ are calculated from equations (6.1.3), (6.1.4) and (6.1.5) respectively.

### 6.3 Result and Discussion

A computer program based on the analysis developed to calculate stress and strain rate distributions. To study the effect of anisotropy on the stress and strain rates, numerical values of anisotropic constants shown in Table 6.1 are taken from Kulkarni et al., (1985).

Figure 6.1 shows the variation of the radial stress in a rotating disc along the radius of the disc for 5 different cases of anisotropy. It can be seen clearly from this figure that the radial stress is not much affected by the anisotropy of the material as the values of radial stress are very close for the different cases of anisotropy. Figure 6.2 shows the variation of tangential stress in a rotating disc along the radius of the disc. It is clear that near the inner region of the disc, the tangential stress is highest for the case I and lowest for the case V. As the radius increases the tangential stress also increases and attains a maximum value near the middle of the disc and then decreases towards the outer region of the disc. The variation of tangential stress is same for the cases I and II and also the values are same for the cases IV and V.
Figure 6.3 shows the variation of the radial strain rate along with the radius of the disc. It can be seen from the figure that the value of compressive radial strain rate is lowest for the case I and highest for the case V at the inner radius of the disc while for all other cases it lies in between. The radial strain rate goes on increasing as radius increases for all the cases before decreasing near the outer radius of the disc but the fall is sharp for the case I. Figure 6.4 shows the variation of the tangential strain rate along the radial distance of the disc. The tangential strain rates are highest at the inner radius of the disc and then decreases towards the outer radius of the disc. The tangential strain rate is maximum for the case I and minimum for the case V.

Table 6.1: Values of anisotropic constants

<table>
<thead>
<tr>
<th>Case</th>
<th>Case II</th>
<th>Case III</th>
<th>Case IV</th>
<th>Case V</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G/F$</td>
<td>0.8159</td>
<td>1.2200</td>
<td>1.0000</td>
<td>0.7452</td>
</tr>
<tr>
<td>$H/F$</td>
<td>0.6081</td>
<td>0.7452</td>
<td>1.0000</td>
<td>1.2200</td>
</tr>
</tbody>
</table>

Figure 6.5 shows similar variation of radial stress along the radial distance for anisotropic and anisotropic FGM rotating discs. It is observed that the radial stresses increases as one move from inner radius to center radial distance, reaches maximum nearly at the center and then decreases on going towards outer radius of the disc. The
Figure 6.1: Variation of radial stress along radial distance in anisotropic disc.

Figure 6.2: Variation of tangential stress along radial distance in anisotropic disc.
Figure 6.3: Variation of radial strain along radial distance in anisotropic disc.

Figure 6.4: Variation of tangential strain along radial distance in anisotropic disc.
radial stresses are lesser for the anisotropic disc in comparison to anisotropic FGM disc with parabolic distribution of particle content along the radial distance.

Figure 6.6 shows the variation of tangential stress along the radial distance of the rotating discs. It is observed from this figure that as one moves from inner to outer radial distance the tangential stress increases in these discs, reaches maximum nearly at one-fifth of the radius and then decreases. The tangential stress changes drastically for anisotropic FGM discs with parabolic distribution of particle content as one moves from inner to outer radius of the disc whereas it shows small variation for the anisotropic disc.

Figure 6.7 shows the variation of radial strain rate along the radial distance for these rotating discs. It can be seen from the figure that compressive radial strain rate is largest for the anisotropic disc and least for anisotropic FGM disc. Also radial strain rate becomes tensile at the middle radial distances for the anisotropic disc with functionally graded distribution of particle contents whereas it remains compressive throughout the entire radius of anisotropic disc.

Figure 6.8 shows the variation of tangential strain rate along the radius of the rotating discs. The figure shows that the tangential strain rate is maximum for the anisotropic distributed disc particularly at the inner radius and minimum for the disc with parabolic distribution of particle content. It is also observed that for FGM disc the tangential strain rate remains almost same throughout the disc, whereas it changes drastically for non FGM disc.
**Figure 6.5:** Variation of radial stress along radial distance in FGM/non-FGM anisotropic disc.

**Figure 6.6:** Variation of tangential stress along radial distance in FGM/non-FGM anisotropic disc.
6.4 Conclusion

It is concluded from the result and discussion that the anisotropy of the material has a significant effect on the creep of a rotating disc. The stresses and strain rates are dependent upon the material anisotropy. Material with anisotropy of the type case $V$ exhibits lower strain rate as compared to the isotropic case while the material with anisotropy of the case $I$ exhibits higher strain rates so it is suggested to use the material with anisotropy of the type $V$ to have a longer life. Based on the results and discussion presented in this chapter it can be concluded that the creep behavior in the anisotropic disc can be controlled by the suitable distribution of particle contents as the strain rate are minimum when particle content is distributed parabolically along the radial distance of the disc.
Figure 6.7: Variation of radial strain along radial distance in FGM/non-FGM anisotropic disc.

Figure 6.8: Variation of tangential strain along radial distance in FGM/non-FGM anisotropic disc.
Chapter 7

Conclusion, Summary and Future Prospects

This chapter summarizes all conclusions drawn from different models and calculations performed as a part of this thesis and also addresses the possibilities of further work in this direction. The detailed chapter wise conclusions are given at the end of each chapter.

The research work presented in this thesis deals with the study of “Mathematical Modeling of Creep in Rotating Discs of Composites and Functionally Gradient Materials”. In this thesis, models have been developed to find stress and strain rate distributions in a rotating disc made of aluminum/aluminum alloy matrix reinforced with silicon carbide in the form of particulates. For this study constitutive equations for composite have been developed using different yield criteria as described in the chapters. To compute stress and strain rate distributions in a rotating disc, the equilibrium equation of the continuum mechanics and the constitutive equations have been solved.
7.1 Conclusion and Summary

The present thesis can broadly be divided into four phases.

In the first phase of the thesis, analysis has been carried out for an isotropic rotating disc made of particulate reinforced composites where the creep behavior has been described by the threshold stress based creep law. The values of stress exponent \( n \) have been taken as 3, 5 or 8. The constitutive equations for an isotropic composite have been developed using von Mises criterion of yielding and the impact of stress exponent on the stress and strain rate distributions in a rotating disc have been investigated. On the basis of results obtained in this phase the choice of exponent 8 have been made for the material of the disc. At various temperatures, the creep response in lightweight aluminium base particle reinforced composite rotating disc can be brought to the level required for a given application by controlling both particle content and particle size. The work done in this phase has been published as Effect of Stress Exponent on Creep in an Isotropic Rotating Disc of Al-SiCp in Bulletin of the Calcutta Mathematical Society, Vol. 101(6), 2009.

The processing of composites often involves cooling from high temperature resulting in the residual stress. So the analysis of the steady state creep in a rotating disc made of composites containing SiC particulates have been carried out in the presence of residual stress using Hoffman yield criterion in the second phase. The creep behavior of the disc has been described by Sherby law and the value of stress exponent 8 was chosen on the basis of result of work done in the first phase. In this phase it is concluded that the presence of the residual stress does not significantly
affect the stress distributions but it definitely effect the strain distributions in the disc rotating at high temperatures. This work has been published as *Creep Analysis of an Isotropic Rotating Al-SiC Composite Disc Taking into Account the Phase Specific Thermal Residual Stress* in the *Journal of Thermoplastic Composite Materials*. Online First published on September 9, 2009 as doi: 10.1177/0892705709345938, Sage Publisher, U.K.

In the third phase of the thesis, analysis of creep in a rotating disc made of FGMs having non-linear distributions of silicon carbide from inner to outer radius have been carried out. The results have been compared with the available results for linearly decreasing reinforcements having the same average particulate content and also with disc having uniform particulate distribution. The strain rate remains almost same throughout the functionally graded disc having linear or parabolic distribution of particle content, whereas it changes drastically for non FGM disc. Thus steady-state creep responses of FGM disc is better than that of uniform disc having the same average particle content. But this work shows that the steady state creep response for FGM disc having parabolic distribution of particle content is even better than that of linear distribution. This work has been published as *Creep Analysis Of An Isotropic Functionally Graded Rotating Disc* in *International Journal of Contemporary Mathematical Sciences* Vol. 5, no. 9, pp. 419-431, 2010.

In the fourth phase of the thesis, analysis of creep in a rotating disc made of anisotropic material have been carried out. Further the analysis has also been carried out for FGM disc made of anisotropic material having non-linear distributions of silicon carbide from inner to outer radius using Hill yield criterion. It is concluded
that the stresses and strain rates depend upon the material anisotropy and the strain rates are minimum when particle content is distributed parabolically. Thus steady state creep rates can be controlled well by the suitable distribution of particle content. This work has been published as *Creep Behavior of an Anisotropic Rotating Disc of Composites* in *International Journal of Contemporary Mathematical Sciences* Vol. 5, no. 11, pp. 509 - 516, 2010.

Overall it is concluded that while designing rotating disc, material isotropy, anisotropy and the nature of the distribution of reinforcement plays a crucial role and therefore it should be taken care of.

### 7.2 Future Scope of work

The work done in this thesis allows the computation of strain rate and stresses at different points of a discs rotating at high speeds and working at elevated temperatures and hence life-time of variety of machine parts can be improved using these concept. For example, life-time of discs rotating at elevated temperatures in space crafts, rotors, turbines etc. can be improved taking care of material isotropy/anisotropy and the distribution of reinforcement.

This work can be extended to investigate the steady state creep behavior of rotating discs made of other composite material. Analysis of creep behavior on discs having variable thickness and having different profiles may be performed in continuation of this work. Further this can be extended to study the impact of residual stresses having different distributions of reinforcements in rotating discs. The analysis
may be extended for creep behavior of rotating cylinder subjected to internal pressure as well as external pressure. At this point it can only be concluded that extensive research work is needed in this direction both in theoretical and experimental areas. One can use softwares like ABAQUAS and ANSYS to find stresses and strain rate distributions at different points in the rotating disc made of composites. Primary creep can also be taken with steady state creep.

As a last word, this thesis sets up a stage to perform several rigorous experiments and studies to establish the controlled strained behavior which is a primary requirement for the design of machines parts. The theoretical models and conclusions that have been developed in this work are helpful to pursue such studies. This work actually lays a base for the constructive work in the area of Creep research in general and the Creep of rotating discs of composites at elevated temperatures in particular.
Appendix A

Equation of Equilibrium

Equilibrium equation for a disc, shown in figure A.1, of constant thickness \( h \), density \( \rho \) and rotating with angular velocity \( \omega \) may be obtained by considering the equilibrium of an element in the composite disc confined between radial distances \( r \) and \( r + dr \) and an interval of angle \( \theta \) and \( \theta + d\theta \) as shown in figure A.2. The equilibrium of forces in the radial direction of the element of disc implies that

\[
2(h dr)\sigma_r sin\frac{d\theta}{2} - \sigma_r (r + dr)hdr\theta + (\sigma_r - d\sigma_r)rhdr\theta = \frac{hrd\theta dr\rho \omega^2 r^2}{r}
\]  

\[\Rightarrow\ \sigma_\theta dr - \sigma_r dr - r d\sigma_r = \rho \omega^2 r^2 dr \]  

\[\Rightarrow\ \sigma_\theta - (\sigma_r + r \frac{d\sigma_r}{dr} = \rho \omega^2 r^2)\]  

\[\Rightarrow\ \frac{d}{dr} (r \sigma_r) - \sigma_\theta + \rho \omega^2 r^2 = 0\]
Figure A.1: Free body diagram of a rotating disc.

Figure A.2: Free body diagram of an element of a rotating disc.
Appendix B

Numerical Scheme for Computation
Bibliography


