Starting as early as 1906 the rotating isotropic disc has been studied by Grubler (1906) followed by Donatch (1912). Probably the first recognition given to the industrial importance of creep was by Dickenson in 1922, when he published his researches on the creep-resistance of structural-steel and alloy-steel members in a furnace. One of the first modern dissertations in analyzing the flywheel rotor was the seminal work done by Stodola (1927). In 1947, Manson published a finite difference method to calculate elliptic stresses in gas-turbine disks that could account for the point-to-point variations of disk thickness, temperature, and material properties. Millenson and Manson (1948) extended the method to include plastic flow and creep. These methods have been widely used and extended by industry. Wahl et. al. (1954) conducted the creep test in a rotating disc made of steel and simulated the results theoretically using von Mises and Tresca yield criteria describing creep behavior by power law relation and noticed that the creep deformation based on Mises criterion yielded slightly lower values compared to the experimental values, however, the theoretical results based on maximum shear theory, was found to be in a better agreement with the test values.
The 1960s and 1970s lead to numerous other serious efforts in analyzing the rotor, and introducing different designs for the flywheel, with the onset of composite material development giving added impetus. A detailed review of the rotating disk problem up through the late 1960s is given by Seireg (1970). The past 50 years showed a tremendous amount of work done concerning the rotating disc. Several investigators (Arya (1979), Bhatnagar et. al. (1986), Biner (2002), Bueno et. al. (1992), Boulianne (57th Canadian Geotechnical Conference), Gupta (2008), Sharma (2008), Singh and Ray (2001), You et. al. (2007), Cao and Pollock (2009) etc.) used Norton’s power law to describe creep behavior of metal matrix composites. Arya and Bhatnagar (1979) carried out creep analysis of rotating orthotropic disk using constitutive equations as obtained by Bhatnagar and Gupta (1966) and a time hardening law. Ma (1959) derived some formulae based on the maximum shear theory associated with the Mises flow rule for calculating creep deformations and stress distributions in rotating solid disks of variable thickness and uniform temperature, and used the exponential function creep law at steady-state conditions. Further Ma (1960) derived some formulae based on the theory of the Tresca criterion and its associated flow rule using the exponential function creep law at steady-state conditions for creep strains and stress analysis in rotating solid disks of variable thickness and uniform temperature. Ma (1961) presented the stress analysis of rotating solid disks having variable thickness and variable temperature. The analysis was based on the theory of Tresca criterion and its associated flow rule, and used the exponential function creep law for steady-state conditions. Guven (1997) investigated the deformations of constant thickness rotating annular disks with rigid inclusion in the fully plastic state and
obtained analytical solution using Tresca yield condition assuming linear strain hardening. Elastic—plastic deformations of rotating hyperbolic disks with rigid inclusion were studied by Guven (1998) to obtain exact solution for linearly hardening disks using Tresca yield criterion and its associated flow rule, considering small reductions in disk thickness and small values of the hardening parameter. Gupta and Pankaj (2007) investigated yield condition or incompressibility condition and thus poses and solves a more general problem from which cases pertaining to the assumptions can be worked out. Cnen et al. (2008) investigated the coefficient of thermal expansion (CTE) and accumulated plastic strain of the pure aluminum matrix composite containing 50% SiC particles (Al/SiCp) during thermal cycling (within temperature range 298 K to 573 K) where the composite was produced by infiltrating liquid aluminum into a preform made by SiC particles with an average diameter of 14µm.

Purely elastic deformations of variable thickness rotating annular disks mounted on rigid shafts were studied analytically by Eraslan and Argeso (2002). The stresses and deformations at the elastic limit angular velocity were calculated for constant and variable thickness rotating annular disks. Variable thickness disks were found to yield stress profiles similar to those of constant thickness, with magnitudes smaller than the corresponding constant thickness disks at the same angular velocity. Furthermore, elastic limit angular velocities were found to increase as the thickness reduces at the edge of the disk. Eraslan (2002) also investigated inelastic deformations of constant and variable thickness rotating annular disks with rigid inclusion using von Mises yield criterion. Variable thickness disks, exhibiting linear and nonlinear isotropic strain hardening in the form of Swift’s hardening law, were considered. The stresses
and deformations were studied for two specific thickness variations extending the work of Guven (1997, 1998) to include von Mises yield criterion, nonlinear isotropic hardening, any functional form of thickness variability, small and large values of the hardening parameter.

Gupta et al. (2004 a) investigated steady state creep in a rotating disc made of isotropic aluminium—silicon carbide particulate composite using Sherby’s creep law. The study revealed that for given operating conditions, the strain rates in the disc can be controlled by selecting optimum particle content and/or particle size of the reinforcement. Park et al. (1990, 1994), Li and Langdon (1999) showed the measured experimental creep rates passing through a distinct minimum in a plot of the shear strain rate against the shear strain for an Al − 6061 composite reinforced with 30 vol.% of SiC particulates tested at a temperature of 648 K at two different levels of the applied shear stress and stated that a quasi—steady—state or minimum condition may be defined in these experiments, although it was recognized that the secondary stage was very short by comparison with most pure metals and solid solution alloys. By contrast, some MMCs exhibit creep curves where the secondary stage was more clearly delineated, and was shown in a plot of the normal strain rate vs the strain for an Al − 6092 composite reinforced with 25 vol.% of SiC particulates. It was evident that the steady-state condition was reasonably well established in this material, especially at a testing temperature of 723 K at the higher stress of 16.4 MPa, thereby demonstrating that an abrupt and possibly poorly defined minimum creep rate is not an intrinsic property of all MMCs. It is important to note that the extent of the steady—state stage is also often of a very short duration in the creep of unreinforced
powder metallurgy materials. They showed creep data for an unreinforced powder metallurgy Al − 6061 alloy tested at 648 K at two different levels of shear stress and observed curves similar in appearance to those documented in Al − 6061 composite reinforced with 30 vol.% of SiC particulates and Al − 6092 composite reinforced with 25 vol.% of SiC particulates, thereby demonstrating that the relatively short duration of the steady—state stage in the creep of MMCs is not necessarily related to the advent of damage mechanisms within the matrix such as the occurrence of debonding at the interfaces between the matrix and the reinforcement. The close similarities in the creep curves of unreinforced powder metallurgy materials and MMCs provides support for analytical or theoretical treatments of the creep results for MMCs in terms of the minimum or steady—state creep rates.

Mohamed (1998) pointed out that for Al alloys and their composites the true stress exponent $n$ is usually selected as 3, 5 and 8. These values of $n$ correspond to three well—documented creep cases for metals and alloys: $n=3$ for creep controlled by viscous glide processes of dislocation, $n=5$ for creep controlled by high temperature dislocation climb (lattice diffusion), and $n=8$ for lattice diffusion—controlled creep with a constant structure. Pandey et al. (1992, 1994) and Gonzalez—Doncel and Sherby (1993) have used a true stress exponent of 8 to SiC/Al and TiB$_2$P/Al composites. Mishra and Pandey (1990) reanalyzed the experimental creep data of Nieh (1984), Nieh et. al. (1988) and Morimoto et. al. (1988) for 6061 Al − SiC$_{p,w}$ composites (subscript $p$ for particle and $w$ for whisker) and found a good fit with the substructure invariant model (Sherby, 1977). However, other research groups ( Park et. al. (1990), Mohamed et. al. (1992), Park and Mohamed (1995), Cadek et. al.
(1995), Li and Mohamed (1997), Cadek et. al. (1988), Li and Langdon (1999)) have observed that a stress exponent of either 5 or 3, rather than 8, provides a better description for the creep data of discontinuous SiC/Al. Li and Langdon (1999) analyzed the creep behavior of metal matrix composites using different stress exponents and observed that a stress exponent of either 3 or 5 provides better description for the creep data of discontinuous Al – SiC. They indicated that true stress exponent of Al₂O₃/6061Al composite is close to 3. Z. Y. Ma and S. C. Tjong (2001) reviewed the creep behavior of discontinuously reinforced Aluminium matrix composites (DRAMCs) at elevated temperature, like the shapes of creep curves, origin and characteristics of the threshold stress, the creep strengthening of the DRAMCs, nature of rate controlling process, the effect of cyclic stress and creep rupture. They observed that DRAMCs exhibits high values of apparent stress exponent and apparent activation energy for creep. But incorporation of the threshold stress in the analysis reduces these values to those anticipated from the creep of pure metals and solid solution alloys. Ma and Tjong (2001) reported that a stress exponent of 8 rather than 3 or 5 can be used to rationalize the creep data of Al₂O₃ – TiB₂/Al composite.

Gustafson et. al. (1997) showed the relationship between the inter–particle spacing, λ, particle size, d and the volume fraction of the reinforcements, \( f_v \) by equation

\[ \lambda = d(f_v^{\frac{1}{3}} - 1). \]  

(2.0.1)

Earlier work of Kouzeli and Mortensenon (2002) on aluminum–based composites was primarily based on dispersion of ceramic particles of large size (typically a few microns) as reinforcements with high volume fractions as shown in area 1 of figure
2.1. It was observed that larger ceramic particles (above $1.5\mu m$) tend to increase the inter–particle spacing in the composite and eventually lead to lower strengths. In addition to this they act as micro–concentrators of stress and give rise to cleavage in the particles. The medium size particles ($0.2–1.5\mu m$) lead to the formation of cavities and pits through loss of interphase cohesion. This prompted some investigators to add smaller size particles (below $200\ nm$) into the matrix as these particles bond well to the matrix and do not initiate cavities in the particles. However, such composites were produced with low volume fraction of the ceramic phase as shown in area 2 of figure 2.1. To bridge this gap, Katiyar (2004) used the dispersion of sufficiently high volume fraction of nanometer–sized ceramic phase, shown in area 3 of figure 2.1.

![Figure 2.1](image)

**Figure 2.1:** Relationship between the inter-particle spacing, $\lambda$, particle size, $d$ and the volume fraction of the reinforcements, $f_v$.

Singh and Ray (2004) have studied the effect of thermal residual stress on the
steady-state creep behavior of a rotating disc made of 6061Al – 20vol% SiC\textsubscript{w} composite using isotropic Hoffman yield criterion while describing the creep by Norton’s power law. The tensile residual stress significantly affected the strain rates in the disc when compared with the strain rate in the disc without residual stress. The high values of apparent stress exponent and apparent activation energy observed for aluminium based MMCs disqualify the selection of Norton’s power law to describe creep behavior in these composites. In order to rationalize the strong stress and temperature dependency of creep rate reported for discontinuously reinforced aluminium/aluminium alloy matrix composites, the concept of an effective stress has been used. Durodola and Attia (2000) investigated the potential benefits of using several forms of fiber gradation in FGM rotating discs using finite element method and direct numerical integration. It was observed that the different forms of property gradation modify the stress and displacement fields in FGM discs compared with uniformly reinforced discs. Singh et. al. (2001) estimated steady state creep response using Norton’s power law in an isotropic FGM rotating disc of aluminium—silicon carbide particulate composites at uniform elevated temperature. They concluded that the steady state creep response of the FGM disc is significantly superior compared to that in a disc with the same total particle content distributed uniformly. Gupta et. al. (2004b) investigated creep in a rotating FGM disc made of Al – SiC\textsubscript{p} using Sherby’s law. The disc was considered having a thermal gradient in radial direction. The study indicated that the steady state strain rates in FGM disc are significantly lower compared to those in an isotropic disc with uniform particle distribution. Gupta et. al. (2005) investigated the creep behavior of a rotating disc made of isotropic composite
containing varying amounts of silicon carbide in the radial direction in the presence of a thermal gradient, also in the radial direction. The creep behavior of the disc under stresses developing due to rotation had been determined following Sherby’s law and compared with that of a similar disc following Norton’s law. The difference in the distribution of stresses and strain rates in the discs did not follow any definite trend but the values were somewhat different. The presence of thermal gradient and a linear particle gradient separately or their simultaneous presence result in a significant decrease in steady-state creep rates as compared to that in a composite disc with the same average particle content distributed uniformly and operating under isothermal condition. Further, the study revealed that the creep behavior of a FGM disc could be significantly improved by increasing the gradient of particle distribution while keeping the same average particle content of silicon carbide in the disc.

Work by Reddy and Srinath (1974), Chang (1975), Gurushankar (1975), Christensen and Wu (1977) and Genta and Gola (1981) dealt with determining stresses via an elasticity approach in orthotropic single-ply circular plates with the outer boundary free of any constraints. Khalfallah (2002) presented inverse identification strategy of constitutive laws for elastoplastic behavior proposing inverse algorithm composed on an appropriate finite element calculation combined with an optimisation procedure. It was applied to identify material anisotropic coefficients using a set up of easy performed laboratory tests. Tutuncu (1995) examined the effect of anisotropy on the stress distribution in laminated plates for various boundary conditions. Singh et. al. (1998) performed creep analysis in an anisotropic 6061Al – SiCw disc rotating at 15,000 rpm and undergoing steady state creep described by Norton’s power law at
The study revealed that the presence of anisotropy leads to significant change in the strain rates.

In summary, the previous work conveys the complexity of the problem, in that a very large number of conditions and factors must be considered. In particular, each of the listed investigations have necessarily focused on some specified conditions, particular material behavior and related conclusions. As a first step toward this end, an annotative study is conducted herein, with the major restriction being that of disks with constant thickness. In particular, emphasis is on rotation, volume, particle content, temperature and the effect of material isotropy/anisotropy. In addition, our second primary objective is to present a number of new results with regard to limiting conditions of stress. These will then enable us to reach a number of important conclusions in parallel under both service conditions (stress analysis). The literature reveals that the creep model based on the effective stress seems to describe well the creep behavior of aluminium/aluminium based composites but still the controversy exists over the value of stress exponent. Keeping in view the controversies over the value of true stress exponent $n$ (3, 5 or 8) there is a need to carry out further investigations on creep in a rotating disc made of aluminium/aluminium alloy based composites on creep laws based on true stress exponent of 3, 5 and 8. Further, the study pertaining to creep deformation of a rotating FGM disc having linear distribution of reinforcement is available in the literature. But in order to explore the potential application of FGMs, there is a strong need to investigate the effect of using other kind of reinforcement gradients on the creep behavior of rotating disc. In this work models are developed to find stress and strain rate distributions in a rotating
disc made of aluminum/aluminum alloy matrix reinforced with silicon carbide in the form of particles. For this constitutive equations for composite are developed using different yield criterion and the equilibrium equation of the continuum mechanics and the constitutive equations are solved as described in following chapters.