CHAPTER 2

MATHEMATICAL MODELING OF COMPONENTS

A detailed mathematical modelling of various sub-systems pertaining to the Circulating Fluidized Bed Combustion Boilers is presented here in this chapter. This involves the modelling of circulation system, furnace, super-heater, re-heater and attemperator. In the circulation system modelling, the associated components like boiler feed pump, high pressure heaters, economiser are also taken into account and is shown in Figure 2.1

2.1 MODELING OF CIRCULATION SYSTEM

(Source : Thenmozhi.G, 2016)

Figure 2.1  Block diagram of circulation system
The Mass balance and Energy balance equations of circulation system is used for modeling the system. The Simulink model of mass and energy balance is given below for reference in Figure 2.2. The mass balance equation for the system consists of three inputs feedwater flow from the economizer \( M_{\text{eco}} \), two phase mixture from waterwall1 \( M_{w1} \) and waterwall2 \( M_{w2} \), three outputs namely mass flow rate of steam \( M_s \), flow rate of water in the downcomer1 \( M_{dc1} \) and downcomer2 \( M_{dc2} \).

\[
\Delta M = M_{\text{eco}} - M_{dc1} - M_{dc2} + M_{w1} + M_{w2} - M_s = \frac{d[(\rho_s V_s + \rho_w V_w)]}{dt} \tag{2.1}
\]

(Source : Thenmozhi.G, 2016)

**Figure 2.2**  Mass and Energy balance simulink model of circulation system
Where $\rho_s$ is the steam density, $\rho_w$ is the water density, $V_s$—volume of steam and $V_w$—volume of water.

Energy balance equation for the circulation system is given by,

$$\Delta E = M_{eco} * H_{eco} - M_{dc1} * H_{wmn} - M_{dc2} * H_w + M_{w1} * H_{xw1} + M_{w2} * H_{xw2} - M_s * H_s + Q_{gww1} + Q_{gww2} = \frac{d[pwVwHw+psVsHs]}{dt}$$

(2.2)

where, $V_{dr} = V_s + V_w$

Initially during start-up $Q_{gww2}$ is zero as no heat is generated by the heat exchangers.

$$H_{xw1} = X_{q1} * H_s + (1-X_{q1}) * H_w$$

(2.3)

$$H_{xw2} = X_{q2} * H_s + (1-X_{q2}) * H_w$$

(2.4)

Here in the boiler drum dynamics, the state variables are chosen to be drum pressure, volume of water in the drum, metal temperature and the steam quality variation. If the heat flux from the combustor is $Q_{gww1}$ and from evaporator is $Q_{gww2}$ and heat flux absorbed by the water walls is $Q_{gww}$, then

$$Q_{gww2} = M_{ww2} C_{pww2} \frac{d[Tmww2]}{dt}$$

(2.5)

The heat required for boiling the water is given by,

$$Q_{gww} = M_s * (H_s - H_{eco})$$

(2.6)

The heat flux from the combustor is obtained by,

$$Q_{gww1} = [Q_{gww} - Q_{gww2}]$$

(2.7)

Kg of Steam produced due to the evaporator

$$M_{s2} = \frac{Q_{gww2}}{[H_s - H_w]}$$

(2.8)
Kg of Steam produced due to the combustor

\[ M_{s1} = M_s - M_{s2} \]  \hspace{1cm} (2.9)

In 1Kg of steam – water mixture, if the steam is of \( X_{q1} \) portion then \((1 - X_{q1})\) is the portion of water. Hence, the volume of steam \( V_s = \frac{X_{q1}}{\rho_s} \) and

volume of water \( V_w = \frac{1-X_{q1}}{\rho_w} \).

Similarly, Quality factor \( X_{q1} \) and \( X_{q2} \) is given by,

\[ X_{q1} = \frac{W_{s1}}{W_{dc1}} \]  \hspace{1cm} (2.10)

\[ X_{q2} = \frac{W_{s2}}{W_{dc2}} \]  \hspace{1cm} (2.11)

where the values of \( X_{q1} \) and \( X_{q2} \) are obtained assuming \( W_{dc1}=600 \) and \( W_{dc2}=165 \).

Heat transfer co-efficient for loop1 (HTC 1) and loop2 (HTC 2) are given by,

\[ \text{HTC 1} = \frac{Q_{gww1}}{Q} \]  \hspace{1cm} (2.12)

\[ \text{and HTC 2} = \frac{Q_{gww2}}{Q} \]  \hspace{1cm} (2.13)

where

\[ Q = (T_{mww1} - T_{dr})^{3.57} \cdot P_{dr}^{0.86} \cdot 100 \]  \hspace{1cm} (2.14)

From equations (1) and (2), the state variables \( \frac{dP_{dr}}{dt}, \frac{dV_{w}}{dt}, \frac{dX_{q1}}{dt}, \frac{dX_{q2}}{dt}, \frac{dT_{mww1}}{dt}, \frac{dT_{mww2}}{dt} \) are obtained.
\[
\frac{dP_{dr}}{dt} = \frac{\Delta M(\rho_w H_w - \rho_s H_s) - \Delta E(\rho_w - \rho_s)}{[(\rho_{w2} - \rho_{w1})V_w + (\rho_{s2} - \rho_{s1})V_s] - (\rho_w H_w - \rho_s H_s) - (\rho_w - \rho_s)V_w} \\
[[(\rho_{w2} - \rho_{w1})H_w + (H_{w2} - H_{w1})\rho_w + V_s((\rho_{s2} - \rho_{s1})H_s + \rho_s(H_{s2} - H_{s1}))]]
\]

(2.15)

2.2 MODELING OF FURNACE

A schematic diagram of a 210MW CFBC boiler as shown in Figure 2.3 has been considered for modelling the furnace. The coal, primary air, secondary air and sorbent are sent to the furnace where it is being burnt and heat is produced. The combustion of boiler produces heat and flue gases. Further solid particles which are not burnt in the furnace are recycled back to the furnace through cyclone separators and heat exchangers.
Most of the combustion occurs only in the lower part of the furnace and the heat transfer due to convection and radiation takes place in the upper half of the furnace. The bed temperature control plays an important role in the combustion control and hence this is achieved by the control of primary air and coal flow so that the bed temperature is maintained a constant.

Here, coal flow is introduced as the disturbance. Increase in coal flow increases the bed temperature. Hence different controllers are used and their performance is studied. Among the different control variables like excess primary air, limestone injection and fuel inventory Kortela. U et al. (1991) the proposed method makes use of fuel flow to control the furnace bed temperature. Performance and development of fuzzy logic controller has been studied earlier by B. Lixia et. al (2003) and Liptak et. al (1999) and optimization of boiler performance in K. Åström and T. Hagglund (1995).

Under steady state conditions, the inlet mass flow and the outlet mass flow are equal. The energy balance is shown below and the simulink diagram is shown in Figure 2.4.

The inlet energy is given by,

\[ Q_{in} = Q_{fuel} + Q_{pa} + Q_{sa} + Q_{sorbent} + Q_{sp} \]  \hspace{1cm} (2.16)

where \( Q_{fuel} \) is fuel inlet energy, \( Q_{pa} \) is the energy due to primary air, \( Q_{sa} \) is the energy due to secondary air, \( Q_{sp} \) - energy due to solid particles.

Here the energy due to solid particles \( Q_{sp} \) is the combination of all the four heat exchangers namely the FBHE SH –I, FBHE SH –II, FBHE RH-II, FBHE – EVP. This is mainly based on the mass flow rate of solid particles.
to the above said heat exchangers which depends on the opening of the spiess valve.

\[ y = m^*x + c \]  

(2.17)

where  \( y = \) solid particle flow in kg/sec,  \( x = \) percentage opening of the spiess valve,  \( c = \) constant.  \( OV \) represents the valve opening of super-heater I, super-heater II, re-heater II and evaporator

\[
M_{spsh1} = 1.615^* OV_{sh1} + 18.1
\]
\[
M_{spsh2} = 1.615^* OV_{sh2} + 18.1
\]
\[
M_{sprh1} = 1.615^* OV_{rh1} + 18.1
\]
\[
M_{spevp} = 1.615^* OV_{evp} + 18.1
\]

\[
M_{sp} = SPCR - ( M_{spsh1} + M_{spsh2} + M_{sprh1} + M_{spevp})
\]

(2.18)

Solid particles flow from the cyclone separator is associated with the super heater1, super heater 2, reheater1 and evaporator and it flows back to the combustor. Hence, Input Port1 specifies the mass flow of solid particles from the super heater1, Port2 the mass of solid particles from super heater 2, port3 and port4 specifies the mass flow of solid particles from reheater1 and evaporator respectively. The total mass flow rate of solid particles is the difference between solid particles circulation rate and the mass flow due to all the above said heat exchangers as specified in the equation (2.18)

The simulation values of \( M_{spsh1} = 47 \text{Kg/sec}, M_{spsh2} = 31.7 \text{Kg/sec}, M_{sprh1}=48.8 \text{Kg/sec}, M_{spevp}= 38 \text{Kg/sec} \)

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\[ Q_{sp} = M_{sp} \times CP_{sp} \times T_{cy} \]  \hspace{1cm} (2.19)

The solid particles and flue gases leave the furnace. Hence the circulation rate of solid particles and flue gas is taken into account by Sivakumar, L (2010).

Energy out is given by

\[ Q_{out} = [(SPCR + M_{fg}) \times C_{sp} \times T_{bed}] + Q_{gww1} \]  \hspace{1cm} (2.20)
where $M_{fg}$ is the mass flow rate of flue gas, $C_{sp}$ is the specific heat capacity of solid particles, $T_{\text{bed}}$ is the bed temperature, and SPCR is the solid particles circulation rate and is given by

$$SPCR = 785.0 \times (U_{fl} - U_{mf}) \times e^{-6630 \times \text{Chardia} \times D_{\text{combustor}} \times W_{\text{combustor}}} \quad (2.21)$$

where the fluidization velocity $U_{fl} = 4.5$ m/s and minimum fluidization velocity $U_{mf} = 0.7736$ m/s and is shown below in the simulink diagram Figure 2.5

![Simulink diagram of SPCR in the furnace](image)

**Figure 2.5** Simulink diagram of SPCR in the furnace

But total stored energy in the furnace is given by,

$$Q_{\text{total}} = Q_{\text{in}} - Q_{\text{out}} \quad (2.22)$$

The bed temperature is obtained by

$$T_{\text{bed}} = \frac{Q_{\text{total}}}{M_{\text{bed}} \times C_{sp}} \quad (2.23)$$
Change in Bed pressure $\Delta P_{\text{furnace}}$ is obtained by the equation given below and the simulink diagram is shown in Figure 2.6

$$\Delta P_{\text{furnace}} = \left[ H_{\text{bed}} \times (1-\varepsilon_f) \times (\rho_{\text{sp}} - \rho_{\text{air}}) \times g_{\text{const}} \right] / 100$$ \hspace{1cm} (2.24)

where $\varepsilon_f$, the fixed bed voidage is given by the expression,

$$\varepsilon_f = 1 - (1-\varepsilon) \times \frac{\rho_{\text{bed}}}{\rho_{\text{bedo}}}$$ \hspace{1cm} (2.25)

where

$$\rho_{\text{bed}} = \frac{M_{\text{bed}}}{V_{\text{vol,combustor}}}$$ \hspace{1cm} (2.26)

![Simulink diagram](image)

Figure 2.6 Bed pressure calculation in the furnace

### 2.3 MODELING OF SUPER-HEATERS AND RE-HEATERS

Heat Exchangers are of three types. They are

1. Water inside the tube and gas outside the tube. (eg.) Economiser
2. Water inside the tube and steam outside the tube. (eg.) Low pressure heaters (LPH) and high pressure heaters (HPH)
3. Steam inside the tube and gas outside the tube. (eg.) Super-heater and Re-heater
Mathematical modeling of Super-heaters and Re-heaters are based on mass balance and energy balance.

(Source : Sivakumar, 2010)

Figure 2.7  Layout of Super-heater /Re-heater with inlet and outlet variables

The Figure 2.7 represents the diagram of Super-heater / Re-heater with steam within the tube and gas outside the tube.
Steam Side:

Mass balance of Super-heater is given by,

\[ \Delta M = M_{si} - M_{so} \] (2.27)

\[ M_{si} - M_{so} = \frac{b(V \rho_s)}{bt} \] (2.28)

\[ = V \frac{b(\rho_s)}{bP} \frac{dp}{dt} + \rho_s \frac{b(V)}{bt} \] (2.29)

\[ \Delta M = VC_2 P + \rho_s V \] (2.30)

At steady state energy balance is obtained by,

\[ \Delta E = M_{si} H_{si} - M_{so} H_{so} + Q_{ms} \] (2.32)

where \( Q_{ms} \) is the heat transferred from metal to steam.

\[ \Delta E = \frac{b(VH_s \rho_s)}{bt} \] (2.33)

\[ \Delta E = \rho_s \frac{b(VH_s)}{bt} + H_s V \frac{b(\rho_s)}{bP} \] (2.34)

\[ \Delta E = \rho_s H_s V + H_s V \frac{b(\rho_s)}{bP} \frac{dp}{dt} + \rho_s V \frac{b(H_s)}{bP} \frac{dp}{dt} \]

\[ \Delta E = \rho_s H_s V + (H_s V \frac{b(\rho_s)}{bP} + \rho_s V \frac{b(H_s)}{bP})P \]

\[ \Delta E = \rho_s H_s V + (H_s VC_2 + \rho_s VC_4 )P \] (2.35)

where \( C_2 = \frac{b(\rho_s)}{bP} \); \( C_4 = \frac{b(H_s)}{bP} \)
By Cramer’s Rule,

\[
\begin{bmatrix}
\Delta M \\
\Delta E
\end{bmatrix} = \begin{bmatrix}
V C_2 \\
(H_s V C_2 + \rho_s V C_2) \\
H_s \rho_s
\end{bmatrix} \begin{bmatrix}
\rho \\
\rho
\end{bmatrix} * p.
\]

The Simulink diagram of Metal temperature is shown in Figure 2.8 and the mathematical equation is obtained from the below said equation and is given by

\[
Q_{gm} - Q_{ms} = M_m C_{pm} \frac{dT_m}{dt}
\]

(2.36)

where \( Q_{gm} \) is the heat transferred from gas to metal and \( Q_{ms} \) is the heat transferred from metal to steam, \( M_m \) is the metal mass and \( C_{pm} \) is the specific heat capacity.

Figure 2.8  Simulink diagram of metal temperature of Super-heater

Heat transfer from metal to steam is given by

\[
Q_{ms} = \alpha_{ms} A_1 (T_m - T_{sm})
\]

(2.37)
where \( \alpha_{ms} \) - heat transfer co-efficient from metal to steam, \( A_i \) - inlet surface area, \( T_{sm} \) - Steam to metal temperature

Heat transfer from gas to metal is given by

\[
Q_{gm} = (\alpha_c + \alpha_r)A_o(T_{gm} - T_m) \tag{2.38}
\]

where \( \alpha_c \) - convective heat transfer co-efficient from gas to metal, \( \alpha_r \) - radiative heat transfer coefficient, \( A_o \) - outlet surface area, \( T_{gm} \) - gas to metal

Gas Side:

\[
M_{gl}C_{pg}\frac{dT_{go}}{dt} = M_{gl}C_{pg}T_{gi} - Q_{gm} \tag{2.39}
\]

Heat transfer co-efficient depends on the coal flow, air flow, flue gas flow and also the temperature as described by Sivakumar et al. (1979)

In the case of Super-heater I, Super-heater II, Re-heater II, and evaporator the solid particles are taken into account while calculating the inlet energy whereas for the other Super-heaters like LTSH, FSH and RH-I, the solid particles doesn’t flow. Hence they are ignored.

### 2.4 MODELING OF ATTEMPERATOR

Attemperator or de-super heater is used to reduce the temperature of super heater and re-heater of the secondary Super-heater. The modelling of attemperator is based on the mass balance and energy balance equations with the input and output variables. The mass balance equation of the attemperator is shown in Figure 2.9.
At steady state mass balance equation is given by,

\[ M_{ai} + M_{spray} = M_{ao} \]  \hspace{1cm} (2.40)

For a change in the spray water flow, the inlet remains the same while the outlet changes.

\[ M_{ai} + M_{spray} + \Delta M_{spray} = M_{ao} + \Delta M_{ao} \]  \hspace{1cm} (2.41)

Hence \[ \Delta M_{spray} = \Delta M_{ao} \]  \hspace{1cm} (2.42)

Energy balance is given by,

\[ M_{ai} H_{ai} + M_{spray} H_{spray} = M_{ao} H_{ao} \]  \hspace{1cm} (2.43)

For a change in the spray water flow,

\[ M_{ai} H_{ai} + (M_{spray} + \Delta M_{spray})H_{spray} = (M_{ao} + \Delta M_{ao})(H_{ao} + \Delta H_{ao}) \]

\[ \Delta M_{spray}(H_{spray} - H_{ao}) = (M_{ao} * \Delta H_{ao}) \]  \hspace{1cm} (2.44)

But \[ \Delta H_s = \frac{bH_s}{bP} * (\Delta P) + \frac{bH_s}{bT} * (\Delta T) = \frac{bH_s}{bT} * (\Delta T) = C_p * \Delta T \]
Substituting this in the above said equation,

\[ \Delta M_{\text{spray}}(H_{\text{spray}} - \Delta H_{ao}) = (M_{ao} \cdot C_{pao} \cdot \Delta T_{ao}) \text{ where } \Delta P = 0 \]

Gain \( K_u = \frac{\Delta T_{ao}}{\Delta M_{\text{spray}}} = \frac{(H_{\text{spray}} - \Delta H_{ao})}{MaoCpao} \) \hspace{1cm} (2.45)

### 2.5 SUMMARY

Thus the mathematical and simulink models of circulation system, super-heater, Re-heater, de-superheater and furnace are obtained in this chapter and from this the system dynamics are studied in the chapters later.