Truss optimization can be categorized into three types as topology, shape, and size (TSS) optimization (Kirsch, 1989; Souza et al., 2016). Topological design variables determine a structural initial configuration or layout. Such structural design is challenging and requires higher computational efforts since it is posed to find the best possible structural initial configuration. As a result, the design problem is usually large-scale (Ahrari, Atai, and Deb, 2015; Li, 2015). Moreover, TTO results in mass minimization while searching for the best topology and best possible element cross-sectional areas of the final design (Ali and Dhingra, 2015). With such reasons, many researchers and engineers have investigated TTO problems.

A ground structure method is arguably the most popular method in TTO due to its simplicity and flexibility. Such an approach is accomplished by defining a ground structure composed of user-defined truss elements (ground elements). Those elements are usually connections between nodes based on designer personal preference or construction possibility. The member set off an optimal structure is a subset of members of the selected ground structure. Kaveh and Kalatjari (2003) presented a graph theory to choose a suitable ground structure. A truss with minimum members is known as a simple graph based ground structure, as shown in Figure 3.1(a). A truss with all pairs of nodes being connected by single members is known as a complete graph based ground structure, as shown in Figure 3.1(d). In topology optimization, a
star graph (illustrated in Figure 3.1 (b)) and modified star graph (shown in figure 3.1 (c)) based ground structure is the most popular. In a star graph based ground structure, every node is connected to the neighboring nodes only. However, modified star graph based ground structure adds few members to next neighboring nodes. The use of too many members in the ground structure increases the complexity of the problem and in the process of optimization leads to the non-practical structure. Therefore, a star graph and modified star graph based ground structures are the most suitable forms for TTO problems. TTO process is carried out in such a way to remove or maintain the ground elements until the best element connection is found (Kirsch, 1989). The process of removing or keeping of the ground elements can lead a structural global stiffness matrix to be singular meaning that an optimization run may fail before reaching an optimum. One simple way to avoid such a situation is to assign significantly small values for cross-sectional values of those deleted elements. However, it adds unnecessary computing time when assembling a global stiffness matrix. Moreover, it is always questionable to use this strategy to truss optimization with natural frequency constraints since microelements, to some extent, affect the calculated natural frequency values (Xu et al., 2003). Therefore, this study will not use microelements, but a restructuring approach is used instead. For this method, the finite element model is restructured based on the existence of elements which are controlled by topological design variables. A binary string with 0/1 bits is used for representing removed/existing elements. Another type of design variables is additionally used to determine element cross-sectional areas. This design strategy can help to avoid unnecessary analysis of removed elements. It has been found that most of the study has ignored concentrated masses at nodes; however, such masses should be taken into account since they have an effect on the overall structural mass (Ohsaki, 1995).

A usual constrained optimization problem of trusses is assigned to optimize structural mass and performances whilst fulfilling various design constraints. Some researchers and engineers have investigated truss sizing using metaheuristic methods by considering various constraints such as natural frequency, displacement, stress, and buckling. Nevertheless, size optimization with static loading constraints i.e. stress and displacement has been studied by many researchers. The combined shape and sizing design of trusses with only natural frequency constraints have been proposed by some researchers (Gholizadeh and Barzegar, 2013; Kaveh and Zolghadr, 2014; Pholdee and Bureerat, 2014). Much work related to shape and sizing optimization with natural frequency constraints has been reported, but studies on truss topology optimization with the natural frequency constraints have been somewhat limited (Ohsaki et al., 1999; Xu et al., 2003; Jin and De-yu, 2006; Bai, Klerk, and Pasechnik, 2009; Noilublao and Bureerat, 2011;
Kaveh and Zolghadr, 2013; Gonçalves, Lopez, and Miguel, 2015; Kaveh and Mahdavi, 2015). As a consequence, topological truss design with natural frequency constraints can be considered as a new area to be explored.

So far, there have been two design approaches employed to tackle truss topology and sizing optimization (Zhong et al., 2015) which are two-stage (Xu et al., 2003; Kaveh and Zolghadr, 2013) and single-stage (simultaneous) (Deb and Gulati, 2001) strategies. For the two-stage approach, ground elements having the same constant cross-sectional area initially created, and topological design is carried out in such a way to delete or maintain some ground elements without varying the truss elements’ cross-sectional areas and/or nodal coordinates. Once an optimum topology is obtained, sizing and/or shape optimization has then activated a search for the optimum element cross-sectional areas. Nevertheless, a global optimum solution may not be achieved by using the two-stage approach; therefore, its single-stage counterpart or an automated design was invented. Such an approach is carried out by simultaneously searching the topology, shape, and size (TSS) variables in one run of optimization. However, the single-stage approach requires more computational efforts because it deals with simultaneous TSS optimization, but it is capable of designing a lightweight structure (Deb and Gulati, 2001; Ahrari, Atai, and Deb, 2014).

![1D bar element](image)

**Fig. 3.2 1D bar element**

The finite element method is currently the most used computational approach in truss analysis to calculate elemental stresses and nodal displacements (Ferreira, 2009). Finite element method was developed initially as matrix method of structural analysis for trusses and frames. Consider a bar element with two nodes, two degrees of freedom, corresponding to two axial displacements \( \delta_1 \) and \( \delta_2 \), as shown in Figure 3.2. The element is subjected to applied loads (or specified displacements) and boundary conditions. Assume an element of length L, cross-sectional area A, modulus of elasticity E, and stiffness k. These equations are written conveniently in matrix form as

\[
\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & k \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix}
\]  

(3.1)

The stress of the bar is represented by

\[
\sigma = \frac{E(\delta_2 - \delta_1)}{L}
\]  

(3.2)
The truss bar can be considered as a line element with simply supported ends, as shown in Figure 3.3. Assume an element of a 2D truss of length L, cross-sectional area A, modulus of elasticity E, and mass density ρ. The bar element is subjected to forces and displacements. These equations are written conveniently in matrix form as

\[
\begin{bmatrix}
F_{x1} \\
F_{y1} \\
F_{x2} \\
F_{y2}
\end{bmatrix} = \frac{AE}{L} \begin{bmatrix}
l^2 & lm & -l^2 & -lm \\
1m & m^2 & -lm & -m^2 \\
-l^2 & -lm & l^2 & lm \\
-lm & -m^2 & lm & m^2
\end{bmatrix} \begin{bmatrix}
\delta_{x1} \\
\delta_{y1} \\
\delta_{x2} \\
\delta_{y2}
\end{bmatrix}
\] (3.3)

in compact form, it is represented by

\[
\{F\} = [K]\{\delta\}
\] (3.4)

Here, K represents the stiffness matrix; δ represents the displacement vector, and F represents the force vector.

The stress of the bar is represented by

\[
\sigma = \frac{E}{L} \begin{bmatrix}
-l & -m & l & m
\end{bmatrix} \begin{bmatrix}
\delta_{x1} \\
\delta_{y1} \\
\delta_{x2} \\
\delta_{y2}
\end{bmatrix}
\] (3.5)

Mass matrix is represented by

\[
M = \frac{\rho AL}{6} \begin{bmatrix}
2 & 0 & 1 & 0 \\
0 & 2 & 0 & 1 \\
1 & 0 & 2 & 0 \\
0 & 1 & 0 & 2
\end{bmatrix}
\] (3.6)

Natural frequencies are represented by
\[ \{f\} = \text{eigen}([M], [K]) \] (3.7)

In this method, the primary unknowns are displacements, obtained by inversion of the stiffness matrix, and the derived unknowns are stresses, strains, and natural frequencies can be obtained (Ferreira, 2009). In this thesis, the single-stage approach design strategy, the ground structure method, and restructuring of a finite element model will be used together.

### 3.1 PROBLEM FORMULATION

The ground structure approach has been widely used for TTO problems and allows the fair comparison of results to benchmark problems. In this method, truss ground elements representing the possible combination of all available truss elements are initially generated (presented in Figure 3.4). Next, the optimization algorithm is applied to finally decide to remove or maintain those elements forming a structural topology (illustrated in Figure 3.4 (a–i)). A typical TTO problem is modeled to find element sizes, nodal positions, and its topology which minimize truss mass subjected to design constraints, e.g. displacement, stress, buckling instability, natural frequencies, and kinematic stability. Trusses are subjected to multiple static load cases. Nodal and element masses are added to the objective function in cases that they exist. The optimization problem can be written as:

Find, \( X = \{A_1, A_2, \ldots, A_m, \xi_1, \xi_2, \ldots, \xi_n\} \) (3.8)

\[ F(X) = \sum_{i=1}^{m} B_i A_i \rho_i L_i + \sum_{j=1}^{n} b_j \]

Subjected to:

**Behavior constraints:**

- \( g_1(X):\text{Stress constraints}, \quad |B_i \sigma_i| - \sigma_i^{\max} \leq 0 \)
- \( g_2(X):\text{Displacement constraints}, \quad |\delta_{x_j/y_j/z_j^{\max}} - \delta_{x_j/y_j/z_j}^{\max} \leq 0 \)
- \( g_3(X):\text{Euler buckling constraints}, \quad |B_i \sigma_i^{\text{comp}}| - \sigma_i^{\text{cr}} \leq 0, \text{where } \sigma_i^{\text{cr}} = \frac{k_i A_i E_i L_i^2}{\ell_i^2} \)
- \( g_4(X): f_r - f_r^{\min} \geq 0 \quad \text{for some natural frequencies} \)

**Side constraints:**

- Cross – sectional area constraints, \( A_i^{\text{min}} \leq A_i \leq A_i^{\text{max}} \)
- Shape constraints, \( \xi_{x_j/y_j/z_j}^{\text{min}} \leq \xi_{x_j/y_j/z_j} \leq \xi_{x_j/y_j/z_j}^{\text{max}} \)

Check on kinematic stability

Check on validity of structure
where, \( i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \)

Where, \( B_i \) is a binary element, \( B_i = 0 \) (if \( A_i < \text{Critical area} \)) and \( B_i = 1 \) (if \( A_i \geq \text{Critical area} \)) for deleting and retaining the \( i^{th} \) element respectively. \( A_i, \rho_i, L_i, E_i, \sigma_i, \) and \( \sigma_i^{cr} \) stand for cross sectional-area, mass density, element length, Young modules, stress, and critical buckling stress on the element ‘i’ respectively. \( \delta x_j/y_j/z_j, \delta x_j/y_j/z_j, \) and \( b_j \) are values of nodal displacement, nodal position, and mass values of node ‘j’ respectively, where x, y, and z present x, y, and z axis respectively. \( f_r \) is structural natural frequent obtained from solving structural free vibration analysis of the \( r^{th} \) mode. Superscripts ‘max’, ‘min’, and ‘comp’ denote maximum allowable limit, minimum allowable limit, and compressive stress respectively. \( k_i \) is the Euler buckling coefficient calculated from elements’ cross-sections. The detailed graphical representation of the formulation of the generalize TTO problem is illustrated in Figure 3.4.

It should be noted that the kinematic stability is included in design constraints because it can probably prevent an ill condition in structural analysis (i.e. element number 21 can rotate as shown Figure 3.4(f) and elements 1, 6, 8, and 22 results into a mechanism as shown in Figure 3.4(h)). Two criteria for kinematic instability based on the work by Deb and Gulati (2001) are:

Step (I). Evaluate Grubler's criterion (Ghosh and Mallik, 1994) to check the degree of freedom (DOF) of the truss:

\[
\text{Degree of freedom} = d \times m - n - m_r
\]  

(3.9)

Where, \( d = 2 \) for planar truss and \( d = 3 \) for space truss, \( n \) and \( m \) are numbers of elements and nodes of the truss respectively, and \( m_r \) is restricted number of degrees of freedom at support nodes. The generated truss must be kinematically stable so that it does not turn into a mechanism. If the degree of freedom is non-positive, the truss is not a mechanism (i.e. element number 21 can rotate abut node 8 as shown in Figure 3.4(f)). If the truss is a mechanism, we penalize the solution by assigning a large value. After that, the truss is not sent to Finite element analysis (FEA) model for further analysis.

Step (II). Evaluate positive definiteness of the global stiffness matrix (K) (Rao, 2009) to check singularity of the truss. Grubler's criterion, a necessary, yet not sufficient criterion for the kinematic stability. Therefore, a check of the positive-definiteness of the global stiffness matrix is still needed (Richardson et al., 2012). Analytically, a truss is called kinematically unstable if the determinant of the stiffness matrix is zero; however, FEA model is a numerical method thus the determinant of the stiffness matrix may not be exact zero (Ahrari and Deb, 2016). In this study, Eigenvalue is used to check positive definiteness of the global stiffness matrix. If the
first the value of the Eigenvalue is greater than $10^{-5}$ (a small number near to zero), the truss is assumed to be kinematic stable (positive-definite). If the truss is non-positive-definite, the solution is penalized by assigning a large value. After that, the truss is not sent for further analysis to evaluate stresses, displacements, Euler buckling, and natural frequencies.

Any truss topology is said to be unusable if none of the truss elements are connected to nodes with applied loads (i.e. node 3 is free as shown in Figure 3.4(b)), nodes with boundary conditions (i.e. support node 8 is having no connection with the truss as shown in Figure 3.4(d)), and unchangeable nodes pre-defined by a user (Li and Liu, 2011).
To handle those constraints, a penalty function technique is used. For no violation of the constraints, the penalty becomes zero; otherwise, the penalty is intended by following criteria (Deb and Gulati, 2001; Kaveh and Zolghadr, 2013):

$$\text{Penalized } F(X) = \begin{cases} 
10^9 & \text{if invalid structure} \\
10^8 & \text{if violation in DOF} \\
10^7 & \text{if violation in positive definiteness} \\
F(X) \times F_{\text{penalty}} & \text{otherwise}
\end{cases} \quad (3.10)$$
Where, \( F_{\text{penalty}} = (1 + \varepsilon_1 \ast C)^{\varepsilon_2}, \quad C = \sum_{i=1}^{q} C_i, \quad C_i = \left| 1 - \frac{p_i^*}{p_i} \right| \) (3.11)

Where, \( p_i \) is the level of constraint violation having the bound as \( p_i^* \). The parameter \( q \) is a number of active constraints. The variables \( \varepsilon_1 \) and \( \varepsilon_2 \) are pre-determined by a user. In this study, the values of both \( \varepsilon_1 \) and \( \varepsilon_2 \) are set as 1.5, which were obtained from experimenting their effect on the balance of the exploitation-exploration balance. The detailed graphical representation of the formulation of the TTO problem is illustrated in Figure 3.5.

### 3.2 PROPOSED METHODOLOGY

The brief stepwise discussion of proposed methodology is as below:

Step 1: Define the ground structure of the truss. In this method, nodes are first generated over the predefined design domain and then all possible element connections are assigned. Also, assign material property, loading, and boundary conditions.

Step 2: Go to optimization algorithm: Define an objective function, population size, design variables, bounds, algorithm controlling parameters, and termination criterion.

Step 3: Initialize the randomly generated set of trusses (i.e. population) within its upper and lower bounds.

Step 4: Go to truss configurations: Generate trusses as per the ground structure.

Step 5: Find invalid trusses, if none of the truss element is connected to unchangeable nodes (i.e. a node subjected to load or support) is called invalid truss (discussed in section 3.1). Also, evaluate degree of freedom of valid trusses as per Equation 3.9. If a truss is invalid or its degree of freedom is a positive number, assign a large penalty to the objective function. Go to Step 12.

Step 6: If truss is a valid structure, go to finite element analysis.

Step 7: Compute global stiffness matrix of each of trusses.

Step 8: Evaluate positive-definiteness of the global stiffness matrix to check singularity of trusses. If a truss is nonpositive-definiteness, assign a large penalty to the objective function. Go to Step 12.

Step 9: Compute mass matrix, force vector, and displacement vector.

Step 10: Compute natural frequencies, element stresses, nodal displacements, and Euler buckling of the truss.

Step 11: Go to penalty function: Check constraint violation. If constraint violation, assign penalty as per Equation 3.10 otherwise compute total mass of the truss.

Step 12: Go to optimization algorithm: Assign functional values.

Step 13: Check termination criteria. If not satisfied generate new trusses (i.e. solutions) as per algorithm. Go to step 4.
Fig. 3.6 Schematic representation of proposed methodology
Step 14: Output: The best solution with natural frequencies, natural frequencies, element stresses, nodal displacements, and Euler buckling values.

Graphical representation of the proposed methodology is presented in Figure 3.6.

3.3 DESIGN PROBLEMS

Three benchmark trusses of simultaneous topology and size optimization with continuous cross-sectional areas from Xu et al. (2003), Kaveh and Zolghadr (2013) and seven new benchmark problems (Hajela and Lee, 1995; Deb and Gulati, 2001; Richardson et al., 2012; Miguel et al., 2013) are modified for simultaneous TSS optimization to investigate the performance of proposed algorithms. Trusses have continuous and discrete design variables. The Euler buckling coefficient \( k_i, i = 1,2, \ldots, m \) and mass of nodes \( b_j, j = 1,2,\ldots,n \) are set as 4.0 and 5 kg respectively for all the problems. The lower and upper limits of design variables are \( X_{lower} = -A_{max} \) and \( X_{upper} = A_{max} \) respectively. The critical area is \( A_{min} \), which is applied to remove elements from the ground structure. The ten truss optimization problems are detailed in Appendix A. Truss constraints consist of displacement, stress, natural frequency, and buckling while trusses are acted upon by multiple load cases. The search space is approximately converted into two times of the design variable limits to deal with the topology optimization.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Objective function</th>
<th>Stress constraints</th>
<th>Displacement constraints</th>
<th>Buckling constraints</th>
<th>Frequency constraints</th>
<th>Total No. of constraints</th>
<th>Number of design variables</th>
<th>Feasible space (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L and NL signify linear and non-linear functions respectively.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-bar truss</td>
<td>NL</td>
<td>4</td>
<td>16</td>
<td>4</td>
<td>4</td>
<td>16</td>
<td>1</td>
<td>45</td>
</tr>
<tr>
<td>14-bar truss</td>
<td>NL</td>
<td>6</td>
<td>22</td>
<td>4</td>
<td>6</td>
<td>22</td>
<td>1</td>
<td>61</td>
</tr>
<tr>
<td>15-bar truss</td>
<td>NL</td>
<td>0</td>
<td>30</td>
<td>4</td>
<td>0</td>
<td>30</td>
<td>1</td>
<td>65</td>
</tr>
<tr>
<td>24-bar truss</td>
<td>NL</td>
<td>28</td>
<td>20</td>
<td>4</td>
<td>28</td>
<td>20</td>
<td>1</td>
<td>101</td>
</tr>
<tr>
<td>20-bar truss</td>
<td>NL</td>
<td>4</td>
<td>36</td>
<td>4</td>
<td>4</td>
<td>36</td>
<td>2</td>
<td>86</td>
</tr>
<tr>
<td>72-bar 3D truss</td>
<td>NL</td>
<td>0</td>
<td>144</td>
<td>16</td>
<td>0</td>
<td>144</td>
<td>2</td>
<td>306</td>
</tr>
<tr>
<td>39-bar 3D truss</td>
<td>NL</td>
<td>8</td>
<td>70</td>
<td>12</td>
<td>8</td>
<td>70</td>
<td>1</td>
<td>169</td>
</tr>
<tr>
<td>45-bar 3D truss</td>
<td>NL</td>
<td>56</td>
<td>34</td>
<td>12</td>
<td>56</td>
<td>34</td>
<td>1</td>
<td>193</td>
</tr>
<tr>
<td>25-bar 3D truss</td>
<td>NL</td>
<td>2</td>
<td>48</td>
<td>36</td>
<td>2</td>
<td>48</td>
<td>1</td>
<td>137</td>
</tr>
<tr>
<td>39-bar 3D truss</td>
<td>NL</td>
<td>2</td>
<td>76</td>
<td>36</td>
<td>2</td>
<td>76</td>
<td>1</td>
<td>193</td>
</tr>
</tbody>
</table>

Table 3.1 Problem complexity

Table 3.1 presents complexity-measuring parameters such as nature of the objective function, constraints’ nature, design variables details, constraints’ details, and feasible space of proposed problems. This study also measures the influence of feasible and infeasible search space for the
set of 100000 random points for 10 independent runs. Sampling points are selected by means of a random numbers technique. The objective functions of proposed problems are of a non-linear type, whereas stress and buckling constraints are a combination of linear and non-linear type and displacement constraints are linear in nature. Therefore, the proposed problems are of mixed linear and non-linear type. Moreover, feasible space of the problems is too narrow. Therefore, proposed problems are challenging for optimization methods.

Table 3.2 Proportion of infeasible trusses in the search space (Continuous sections)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Invalid trusses (%)</th>
<th>Kinematic instable trusses in DOF (%)</th>
<th>Kinematic instable trusses in the global stiffness matrix (%)</th>
<th>Constraint violation (%)</th>
<th>Feasible solutions (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-bar truss</td>
<td>52.82</td>
<td>41.44</td>
<td>1.48</td>
<td>4.06</td>
<td>0.19</td>
</tr>
<tr>
<td>14-bar truss</td>
<td>17.42</td>
<td>43.85</td>
<td>9.34</td>
<td>28.37</td>
<td>1.02</td>
</tr>
<tr>
<td>15-bar truss</td>
<td>56.26</td>
<td>41.58</td>
<td>0.79</td>
<td>1.25</td>
<td>0.12</td>
</tr>
<tr>
<td>24-bar truss</td>
<td>12.32</td>
<td>35.47</td>
<td>16.59</td>
<td>35.32</td>
<td>0.30</td>
</tr>
<tr>
<td>20-bar truss</td>
<td>26.93</td>
<td>66.10</td>
<td>4.39</td>
<td>2.54</td>
<td>0.04</td>
</tr>
<tr>
<td>72-bar 3D truss</td>
<td>34.14</td>
<td>53.45</td>
<td>10.67</td>
<td>1.37</td>
<td>0.36</td>
</tr>
<tr>
<td>39-bar truss</td>
<td>11.38</td>
<td>51.11</td>
<td>12.71</td>
<td>24.69</td>
<td>0.11</td>
</tr>
<tr>
<td>45-bar truss</td>
<td>2.57</td>
<td>4.87</td>
<td>29.71</td>
<td>58.51</td>
<td>4.34</td>
</tr>
<tr>
<td>25-bar 3D truss</td>
<td>24.79</td>
<td>58.66</td>
<td>12.26</td>
<td>3.79</td>
<td>0.51</td>
</tr>
<tr>
<td>39-bar 3D truss</td>
<td>4.92</td>
<td>30.53</td>
<td>25.84</td>
<td>34.78</td>
<td>3.93</td>
</tr>
</tbody>
</table>

Table 3.2 and Table 3.3 present detail analysis of the infeasible and feasible solutions to proposed problems of structurally constrained optimization. The truss is called invalid if none of truss elements are connected to nodes with applied loads (see Figure 3.4(b)), nodes with boundary conditions (see Figure 3.4(d)), and unchangeable nodes pre-defined by a user (see Figure 3.4(b)). Therefore, these solutions are rejected. As shown in the Table 3.2, the 15-bar truss generates 56.26 % and 41.58 % invalid and Kinematic instable trusses respectively while the 45-bar truss generates only 2.57 % and 4.87 % invalid truss and Kinematic instable trusses for continues sections. Thus, the 15-bar truss is the most difficult problem and the 45-bar truss is the least difficult problem and this observation is identical for discrete sections. Invalid and kinematic instable trusses add further difficulties in optimization problems. The feasible space is ranging from 0.11 % to 4.34 % for continuous section and 0.08 % to 3.99 % for discrete sections (see Table 3.2 and Table 3.3). Thus, it can be seen from the results that feasible region
is too small for all the problems. Moreover, it is also observed from result tables that feasible space of discrete problems is smaller than that of continuous sections. Therefore, it can be concluded that discrete problems are more challenging problems than continuous problems.

Table 3.3 Proportion of infeasible trusses in the search space (Discrete sections)

<table>
<thead>
<tr>
<th>Problem</th>
<th>Infeasible solutions</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Invalid trusses (%)</td>
<td>Kinematic instable trusses in DOF (%)</td>
<td>Kinematic instable trusses in the global stiffness matrix (%)</td>
<td>Constraint violation (%)</td>
<td>Feasible solutions (%)</td>
</tr>
<tr>
<td>10-bar truss</td>
<td>52.39</td>
<td>41.76</td>
<td>1.50</td>
<td>4.18</td>
<td>0.18</td>
</tr>
<tr>
<td>14-bar truss</td>
<td>17.09</td>
<td>43.49</td>
<td>9.41</td>
<td>29.03</td>
<td>0.98</td>
</tr>
<tr>
<td>15-bar truss</td>
<td>55.38</td>
<td>42.91</td>
<td>0.88</td>
<td>0.71</td>
<td>0.12</td>
</tr>
<tr>
<td>24-bar truss</td>
<td>11.79</td>
<td>33.83</td>
<td>16.71</td>
<td>37.38</td>
<td>0.29</td>
</tr>
<tr>
<td>20-bar truss</td>
<td>26.46</td>
<td>66.39</td>
<td>4.52</td>
<td>2.59</td>
<td>0.04</td>
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