Chapter 2

EVANESCENT WAVE FIBRE OPTIC SENSORS: THEORY

Abstract
This chapter gives a theoretical picturization of the evanescent field at the core-cladding interface of dielectric waveguides and deals with the theoretical modelling of evanescent wave based fibre optic sensors. We also discuss some crucial parameters which are very much essential in the design of a variety of intensity modulated evanescent wave fibre optic sensors.

2.1 Introduction:

The exponentially decaying evanescent fields in the lower index region of a waveguide have been offering high potential in the design and fabrication of a variety of sensors. We can find extensive research and development activities in the literature which exploits the evanescent waves in both cylindrical and planar waveguides. As indicated in the previous chapter, this technique offers a lot of advantages. This chapter gives a brief note on the theoretical understanding of the evanescent waves both in planar and cylindrical waveguide structures. It also provides the design aspects of optical fibre based evanescent wave absorption sensors, which form the basis of our investigations. In the literature we can find different designs of these fibre sensors for the enhancement of sensitivity as well as the dynamic range of measurands. B D Gupta et al in their paper discuss the effect of launching condition and geometry of the sensing region on the sensitivity of fibre optic evanescent wave absorption sensors. Another design for the evanescent wave absorption sensors is based on U-shaped fibre sensor probes. Nidhi Nath and Sneh Anand in their recent work on evanescent fibre optic fluorosensor have theoretically analysed signal acquisition from straight fibre and tapered fibre. A fibre optic sensor probe consisting of a uniform core sandwiched between two linear tapered region forms another category.
2.2 Evanescent waves in planar wave guide structure:

The mechanism by which the rays are confined within a waveguide is by total internal reflection and in order to visualise this phenomena closely at the guide cladding interface, it becomes necessary to approach electromagnetic wave theory model at the interface. The simplest way to understand the concept of optical propagation through the waveguide is by considering the case of planar waveguide structure.

![Figure 2.1](image)

**Figure 2.1** Light ray incident on the boundary between two dielectric media with refractive indices $n_1$ and $n_2$. Where $n_1 > n_2$ and $\theta_i$, $\theta_r$, $\theta_t$ are angles of incidence, reflection and transmission respectively.

When an electromagnetic wave is incident on the boundary between two dielectric media whose refractive indices are $n_1$ and $n_2$, then in general, a portion of that wave is reflected and reminder transmitted.

\[ \Delta^2 \mathbf{E} + n^2 k_0^2 \mathbf{E} = 0 \]  

represents the wave equation which is incident from the higher index ($n_1$) region to the lower index ($n_2$) region. The wave is incident on the interface at an angle $\theta_i$ to the normal and the reflected and transmitted waves are at angles $\theta_r$ and $\theta_t$ respectively. These angles are related by the equations

\[ \theta_i = \theta_r + \theta_t \]
\[ \theta_i = \theta_t \]
\[ \frac{\sin \theta_i}{\sin \theta_t} = \frac{n_2}{n_1} \]  

(2)

Across the boundary Maxwell’s equation require that both the tangential components of \( E \) and \( H \) and the normal components of \( D \) and \( B \) are continuous.

\[
\begin{align*}
n \times (E_2 - E_1) &= 0 \\
n \times (H_2 - H_1) &= 0 \\
n \cdot (D_2 - D_1) &= 0 \\
n \cdot (B_2 - B_1) &= 0
\end{align*}
\]  

(2)

Solving the wave equation with suitable approximations, we will get Fresnel's equations and these equations deal with the magnitudes of the transmitted and reflected electric fields relative to the incident field. 

\[
\begin{align*}
E_{\perp}^t &= \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_{\parallel}^i \\
E_{\parallel}^t &= \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_{\parallel}^i \\
E_{\parallel}^t &= \frac{\sin \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_{\parallel}^i \\
E_{\perp}^t &= \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t}
\end{align*}
\]

and

\[
\begin{align*}
E_{\parallel}^r &= \frac{2n_1 \cos \theta_i}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_{\parallel}^i \\
E_{\perp}^r &= \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} E_{\parallel}^i
\end{align*}
\]

(4)

where \( E_{\parallel} \) and \( E_{\perp} \) represent electric field vectors parallel and perpendicular to the plane of incidence.

In the situation of total internal reflections, ie, when \( \theta_i > \theta_c \) where \( \theta_c = \sin^{-1} \frac{n_2}{n_1} \), there will not be a transmitted wave in the second medium. Although all the energy in the beam is reflected when \( \theta_i > \theta_c \), there is still a disturbance in the second medium, whose electric field amplitude decays exponentially with distance away from the boundary.
We can derive an expression for this decay by considering the phase factor $P$ of the transmitted wave. $P$ at a point $r$ may be written as

$$P = \exp i (\omega t - k_t \cdot r)$$  \hspace{1cm} (5)

where, $k_t$ is the wave vector associated with the transmitted wave.

![Figure 2.2](image)

**Figure 2.2** Illustration of the relationship between the rectangular coordinates $y$ and $z$ and the distance $r$ measured from the origin $O$.

$r = z \sin \theta + y \cos \theta$

From figure 2.2, $r$ may be written as, $r = z \sin \theta_i + y \cos \theta_i$

Substituting this in equation 5, we get

$$P = \exp i [\omega t - k_t (z \sin \theta_i + y \cos \theta_i)]$$

$$= \exp i [\omega t - \frac{2\pi n}{\lambda_0} (z \sin \theta_i + y \cos \theta_i)]$$  \hspace{1cm} (6)

where $\lambda_0$ is the wavelength of radiation in vacuum.

We have $\cos \theta_i = (1 - \sin^2 \theta_i)^{1/2}$

From equation 2
\[ \sin^2 \theta_i = \frac{n_1^2}{n_2^2} \sin^2 \theta_i \]

\[ \therefore \cos \theta_i = \left[ 1 - \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_i \right]^{1/2} \]

when \( \theta_i > \theta_c \), \( \sin \theta_i > \frac{n_2}{n_1} \) and \( \cos \theta_i \) then becomes wholly imaginary.

Then \( \cos \theta_i = \pm iB \)

where \( B = \left[ (-1) \sin \theta_i - 1 \right]^{1/2} \)

(7)

(8)

Substituting in equation 6 for \( \sin \theta_i \) and \( \cos \theta_i \).

\[ P = \exp \left[ \frac{2\pi n_2}{\lambda_0} \left( z \frac{n_1}{n_2} \sin \theta_i + (\pm iB)y \right) \right] \]

\[ = \exp \left( \pm B \frac{2\pi n_2}{\lambda_0} y \right) \exp \left( \frac{2\pi n_1 \sin \theta_i}{\lambda_0} z \right) \]

(9)

Thus, in the \( y \) direction the wave either grows or decays exponentially with distance. The former situation is obviously a non-physical solution and we must choose \( \cos \theta_i = -iB \). The decay with distance in the second medium is given by the factor \( F(y) \)

where \( F(y) = \exp \left( -\frac{2\pi n_2}{\lambda_0} By \right) \)

\[ = \exp \left( -\frac{2\pi n_2}{\lambda_0} \left[ \left( \frac{n_1}{n_2} \right)^2 \sin^2 \theta_i - 1 \right]^{1/2} y \right) \]

(10)

Usually \( F(y) \) decays rapidly with \( y \). However, when \( \theta_i \) is very close to \( \theta_c \), then \( \left( \frac{n_2}{n_1} \right)^2 \sin^2 \theta_i - 1 \right]^{1/2} \) will be close to zero and the disturbance may extend an appreciable distance into the second medium. This part of the wave in the second medium is the evanescent wave.
2.3 Evanescent waves in cylindrical waveguide structure:

Consider the electromagnetic waves propagating along a cylindrical fibre as shown in figure 2.3

![Figure 2.3 Cylindrical coordinate system used for analysing wave propagation in an optical fibre](image)

The wave equation in cylindrical co-ordinate system is given by.\(^44,45\)

\[
\begin{align*}
\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} + q^2 E_z &= 0 \\
\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r} \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} + q^2 H_z &= 0
\end{align*}
\]

(11) \hspace{1cm} (12)

These two equations contain either \(E_z\) or \(H_z\). This implies that the longitudinal components of \(E\) and \(H\) are uncoupled and can be chosen arbitrarily provided that they satisfy equations 1 and 2. Mode solutions can be obtained in which either \(E_z\) or \(H_z\) = 0. When \(E_z = 0\) modes are called transverse electric or TE modes and when \(H_z = 0\), transverse magnetic or TM modes result. Hybrid modes exist if both \(E_z\) and \(H_z\) are non-zeros. They are designated as HE or EH modes. We can solve the above two equations by variable separable method. The solution of the equation is of the form

\[
E_z = A F_1(r) F_2(\phi) F_3(z) F_4(t)
\]

where time and \(z\) - dependant factors are given by \(F_3(z)F_4(t) = e^{(\omega t - \beta z)}\)
Because of the circular symmetry of the waveguide, each field component must not change when the co-ordinate $\phi$ is increased by $2\pi$. Thus we assume $F_2(\phi) = e^{iv\phi}$, where the constant ‘$v$’ can be positive or negative. Substituting these in wave equation for $E_z$ we will get

$$\frac{\partial^2 F_1}{\partial r^2} + \frac{1}{r} \frac{\partial F_1}{\partial r} + \left( q^2 - \frac{\nu^2}{r^2} \right) F_1 = 0$$

(14)

This is a differential equation for Bessel function. An exactly identical equation can be derived for $H_z$ as well. Equation 14 is solved for the regions inside the core and outside the core. For the inside region the solutions for the guided modes must remain finite as $r \to 0$, whereas in outside, the solutions must decay to zero as $r \to \infty$. Thus for $r < a$, the solutions are Bessel functions of first kind of order $v$.

$$E_z(r < a) = AJ_v(\alpha r) e^{j(\alpha r - \beta r)}$$

(15)

$$H_z(r < a) = BJ_v(\alpha r) e^{j(\alpha r - \beta r)}$$

(16)

where $\alpha^2 = k_1^2 - \beta^2$, $k_1 = 2\pi n_1/\lambda$ and $A$ and $B$ are arbitrary constants.

Outside the core, the solutions are given by modified Bessel functions of the second kind, $K_v(\nu r)$, where $\nu^2 = \beta^2 - k_2^2$ and $k_2 = 2\pi n_2/\lambda$. The expression for $E_z$ and $H_z$ outside the core are given by

$$E_z(r > a) = CK_v(\nu r) e^{j(\alpha r - \beta r)}$$

(17)

$$H_z(r > a) = DK_v(\nu r) e^{j(\alpha r - \beta r)}$$

(18)

where $C$ and $D$ are arbitrary constants.

From the definition of modified Bessel function, it is seen that $K_v(\nu r) \to e^{-\nu r}$ as $\nu r \to \infty$. The modified Bessel function decays exponentially with respect to $r$. Hence $K_v(\nu r)$ must go to zero as $r \to \infty$.

The field distribution in the core and cladding regions have the same form and the electric field pattern corresponds to a non-uniform wave travelling in the $z$-direction.
Moreover, specifically, it is a standing-wave pattern in the fibre core and a decaying or evanescent wave in the cladding region as illustrated in figure 2.4.\textsuperscript{42}

![Evanescent field illustration](image)

**Figure 2.4** Illustration of standing wave pattern and exponentially decaying evanescent wave

### 2.4 Fibre optic evanescent wave absorption sensors:

Exponentially decaying evanescent fields were utilised for developing different types of intensity modulated fibre optic sensors. This includes optical fibre based refractometers to measure refractive indices to a high degree of accuracy.\textsuperscript{31,46} Variation in the refractive index of the uncladded region of the cylindrical waveguides were used for making different sensors to detect or measure different physical or chemical variables. Lisa et al developed evanescent wave immunosensor. Here, the toxin from clostridium botulinum and pseudexin has been detected using the above principle.\textsuperscript{47} Another type of evanescent wave fibre sensor was based on excitation and detection of fluorescence using evanescent waves.\textsuperscript{29} However, most commonly used fibre optic evanescent wave sensors are based on evanescent wave absorption phenomena.\textsuperscript{5,21-23,25,27,28,38,39,48}

Evanescent waves in the cladding region were exploited for developing evanescent wave absorption sensors. This is achieved by removing a certain region of
the cladding of the fibre and allowing interaction with the external medium. Evanescent field absorption occurs when the medium, which forms the cladding of the waveguide absorbs the light at the wavelength being transmitted through the fibre. Compared to other sensing methods these evanescent wave fibre optic sensors have a lot of advantages, which have motivated different groups to work in this field. In evanescent wave fibre optic sensors, the interrogating light remains guided in the sensor and no coupling optics are required at the sensing region. Also, these sensors offer the possibility to achieve a considerable miniaturisation. The technique provides enhanced sensitivity over conventional bulk optics attenuated total internal reflection (ATR) crystals. Also, in bulk optics method, it is often difficult to perform accurate absorption measurements on highly absorbing or scattering media. But fibre optic evanescent wave sensors are suitable for such samples because the effective path length in this case is small. Also, this technique is much less sensitive to scattering. Fibre optic sensors offer high potentiality for remote operation and for on-line measurements of different physical and chemical parameters.

There are two different types of evanescent wave absorption sensors. The first one is direct spectroscopic evanescent wave sensor. In this type, evanescent field at the lower index region can interact directly with the analyte, if the wavelength of guided waves coincides with the absorption band of the measurand. The second type is called reagent mediated evanescent wave sensors. Here, an intermediate reagent, which responds optically to the analyte, may be attached to the waveguide. This type of sensor provides greater sensitivity than direct spectroscopic devices.

Design of evanescent field absorption based sensor devices requires a knowledge about certain parameters. These design parameters play a crucial role in determining the sensitivity, dynamic range etc. of fibre optic sensors. The degree of penetration of evanescent field into the low refractive index medium is very important. This quantity is called penetration depth of the evanescent field, $d_p$. This is defined as the perpendicular distance from the core-cladding interface at which the electric field
amplitude has become $1/e$ of its value at the waveguide interface. If $E_0$ represents the electric field amplitude at the interface, after a distance $d_p$ it falls to

$$E = E_0 \exp(-zd_p)$$

The magnitude of the penetration depth is

$$d_p = \frac{\lambda}{2\pi n_1 \left[ \sin^2 \theta - \left( \frac{n_2}{n_1} \right)^2 \right]^{1/2}}$$

$\lambda$ is the wavelength, $\theta$ is the angle of incidence to the normal at the interface, $n_1$ and $n_2$ are the refractive indices of denser and rarer media respectively.49

Another critical parameter, which has a prominent role in the design is the evanescent power that resides in the cladding. This power fraction of the total guided power is also very important both in fibre refractometers and in evanescent field absorption sensors. This quantity is given by33

$$r = \frac{P_{\text{clad}}}{P_{\text{total}}}$$

This fraction is determined by the fibre V-parameter. The fibre V parameter is given by

$$V = \frac{2 \pi \rho}{\lambda} NA$$

where $\rho$ is the fibre core radius and NA is the numerical aperture $= \left( n_1^2 - n_2^2 \right)^{1/2}$ and $\lambda$ is the propagation wavelength.

Analysing the evanescent power for different fibres we can see that substantial values of ‘r’ can be achieved for single mode fibres. But for fibres with high V-parameter, the average fractional power in the cladding is very low. The value of ‘r’ is maximum for modes close to cut-off and for higher order modes.

Even though single mode fibres have higher evanescent wave power fraction and higher sensitivity, multimode fibres can also be used for developing evanescent wave sensors as multimode fibres have greater power throughput and easiness in handling.29
For weakly guiding \( (n_1 \approx n_2) \) multimode fibres, the average value of \('r'\) is

\[ r = \frac{4\sqrt{2}}{3V} \]

Since the amount of absorption depends both on the amplitude of evanescent field in the sample medium and the number of reflections within the sensing region, the amplitude of evanescent field increases dramatically for incident angles approaching waveguide–sample critical angle. The number of reflections is inversely proportional to the waveguide thickness. Removal of cladding results in V-number mismatch between the cladded portion and the sensing portion of the fibre. \(^{40}\) V-number, which determines the mode capacity of the fibre is lower for the cladded region than that for the sensing region. The numerical aperture of the fibre is considerably less than that at the core-sample region. There are different techniques adopted by different groups to enhance the evanescent power. Also, there are strong theoretical models suggested by different groups to tackle this problem and to enhance the transmission of higher order modes through the fibre by launching selected modes in the multimode fibre. \(^{23,25,26}\)

The power transmitted by an optical fibre where cladding has been locally replaced by an absorbing medium is given by

\[ P(z) = P(0) \exp(-\gamma z) \]

\( P(0) \) is the power transmitted through the fibre in the absence of an absorbing medium and \( \gamma \) is the evanescent wave absorption coefficient, \( z \) is the distance along the uncladed length.

The evanescent wave absorption coefficient \( \gamma \) is related to the bulk absorption coefficient \( \alpha \),

\[ \gamma = r \alpha \]

where \( r \) is the fraction of the power outside the fibre core when all bound modes are launched in the multimode fibre.

Then output power through the fibre is given by the equation

\[ P(z) = P(0) \exp(-r \alpha z) \]
The evanescent wave absorbance is given by \( A = \log_{10} \frac{P(0)}{P(z)} \)

\[ \therefore A = \frac{\gamma L}{2.303} = \frac{r \alpha L}{2.303} \]

where \( L \) is the length of the uncladded region of the fibre.

V. Ruddy et al suggested a model for the enhancement of transmission of higher order modes through the fibre by launching only selected modes through the fibre. Spatial filtering was used to restrict modes that were having substantial power in the evanescent field in the uncladded region.  

Evanescent absorption coefficient is given by

\[ \gamma = NT \]

\( N \) is the number of reflections per unit length and \( T \) is the Fresnel transmission coefficient at the interface of loss-less core and lossy cladding. The refractive index of lossy cladding is given by \( n_2 = nk \). The bulk absorption coefficient of the cladding material is given by

\[ \alpha = \frac{4 \pi k}{\lambda}, \text{ where } k \text{ is extinction index.} \]

In a weakly guiding fibre \((n_1 \geq n_2)\) the relationship between evanescent absorption coefficient \( \gamma \) and bulk absorption coefficient \( \alpha \) of the lossy cladding is given by

\[ \frac{\gamma}{\alpha} = \frac{1}{V} \left( \frac{\theta_z}{\theta_c} \right)^2 \left[ 1 - \left( \frac{\theta_z}{\theta_c} \right)^2 \right] \]

\( V \) = normalised frequency parameter of the waveguide, \( \theta_z \) is the angle the ray makes with the core axis, \( \theta_c \) is the complementary critical angle \( (\cos^{-1} n_2/n_1) \)

In the case of evanescent field sensors in aqueous solutions, the difference in refractive index between core and cladding at the launch end is very much smaller than
the refractive index difference at the core-sample region (at the sensing region). In that case the weakly guiding approximation is not valid. In such a condition the value of $T$ can be obtained as

$$T = \frac{\alpha n_2 \sin \theta}{\pi n_1^2 \sin^2 \theta_c \sqrt{\cos^2 \theta_c - \cos^2 \theta_c'}},$$

and $N = \frac{\tan \theta_c'}{2 \rho}$, $\rho$ is the fibre core radius

The ratio of evanescent absorption coefficient to bulk absorption coefficient becomes

$$\frac{\gamma}{\alpha} = \frac{\lambda n_2 \cos \theta \cot \theta}{2 \pi \rho n_1^2 \cos^2 \theta_c \sqrt{\sin^2 \theta_c - \sin^2 \theta_c'}},$$

where $\theta$ is the angle between the ray and the normal to the interface $\theta = \frac{\pi}{2} - \theta_c'$ and $\theta_c'$ is the critical angle for the two media. $\theta_c = \sin^{-1} n_2/n_1$

We have discussed two different theoretical aspects in the design of evanescent wave fibre optic sensors, viz. uniform core with all bound modes launched and uniform core with only selected modes launched into the fibre. But, we have carried out our investigations based on the former theoretical framework only. The following chapters discuss our experimental studies based on this framework.

2.5 Conclusion:

Theoretical outline for the evanescent waves in planar waveguides and cylindrical waveguides has been given. Some aspects of the theoretical background in the case of fibre optic evanescent wave absorption sensors have also been discussed.