CHAPTER - 3

STRUCTURAL OPTIMIZATION

3.1 General Perspective:

In structural engineering design the ultimate goal frequently is to develop the “best” possible structural system that meets the requirements and the client’s demands. “Best” could mean, for instance, cheapest, lightest, stiffest, or any compromise design balancing a number of design criteria. Structural optimization can help to achieve an overall goal by replacing intuitive decision making with a systematic procedure based on a rigorous mathematical formulation of the design problem.

In structural design, it is necessary to obtain an appropriate geometric shape for the structure so that it can carry the imposed loads safely and economically. This may be achieved by the use of structural shape optimization (SSO) procedures, in which the shape or the thickness of the components of the structure is varied to achieve a specific objective satisfying certain constraints. Such procedures are iterative and involve several re-analyses before an optimum solution can be achieved. SSO tools can be developed by the efficient integration of structural shape definition, automatic mesh generation, structural analysis, sensitivity and mathematical programming methods. The growing interest in shape optimization problems can be attributed to the effectiveness of shape changes in improving
structural performance coupled with intensive developments in analysis and optimization techniques, as well as in the enormous growth in computer speed and power.

Structural optimization can be seen as a rational method of finding a structural design that is the best of all possible designs for a chosen objective function and a given set of geometrical and behavioural constraints. This global optimization might not always be achievable for technical or cost reasons. Structural optimization is a systematic way of improvement of structural designs. Conceptually, structural optimization involves:

- The selection of a set of design variables to describe design alternatives.
- The selection of a mathematically formulated objective function expressed in terms of the design variables that is to be minimized or to be maximized.
- The determination of a set of constraints, expressed in terms of the design variables that must be satisfied by any acceptable design.
- The determination of a set of values for the design variables that minimize (or maximize) the objective function while satisfying all the constraints.

3.2 Definition of Structural Optimization:

A typical optimization problem can be written as

Minimize (or Maximize) : f(x)
\[ g_i(x) \leq 0 \quad i = 1 \ldots N_1 \]  
\[ h_i(x) = 0 \quad i = N_1 + 1 \ldots N_2 \]  
\[ k_i(x) \geq 0 \quad i = N_2 + 1 \ldots N_{bc} \]

Where \( x_{lk} \leq x_k \leq x_{uk} \quad k = 1 \ldots N_{dv} \)

Here \( f(x) \) is the objective functions

\( g, h \) and \( k \) are \(< = >\) type constraints

\( x \) is the design variable vector with lower \( x_{lk} \) and upper \( x_{uk} \) bounds for \( k^{th} \) design variable

\( N_{bc} \) is the no. of boundary conditions,

\( N_{dv} \) is the no. of design variables

The following table summarizes the list of commonly used design variables, objective functions and constraints in SSO.

**Design variables s**

- Coordinates of key points \( s_k \)
- Thickness at key points \( t_k \)

**Objective functions \( F(s) \)**

- Weight \( W = \int_{\Omega} \rho \, \omega \, d\Omega \)
- Strain energy \( \| W \|^2 \)
- Stress leveling \( S = \int_{A} (\sigma - \sigma_{avg})^2 \, dA \)
- Fundamental frequency \( \omega \)
- Critical buckling load \( \lambda \)
Constraint functions $g(s)$

- **Stress constraint**
  \[ g^\sigma = 1 - \frac{\sigma}{\sigma_{all}} \]  
  (3.7)

- **Displacement constraint**
  \[ g^d = 1 - \frac{d}{d_{all}} \]  
  (3.8)

- **Volume constraint**
  \[ g^v = 1 - \frac{v}{v_{all}} \]  
  (3.9)

- **Frequency constraint**
  \[ g^\omega = 1 - \frac{\omega}{\omega_{all}} \]  
  (3.10)

- **Buckling load constraint**
  \[ g^\lambda = 1 - \frac{\lambda}{\lambda_{all}} \]  
  (3.11)

### 3.3 Optimization Techniques

Structural optimization problems can be divided into two main categories based on the type of design variables i.e. Sizing optimization and shape optimization. During optimization of structures with sizing design variables, the shape of the structures remains unchanged. Sizing design variables can be c/s area and M.I., plate thickness, laminate angle in case of composite structures etc. Shape optimization is more complex than the pure sizing minimization in the sense that the shapes are continuously changing during the optimization process. Careful consideration has to be paid to describe the changing boundary shape, to maintain an adequate finite element mesh, to impose proper constraints etc.

In general, structural optimization techniques are mixture of engineering concepts and mathematical principles. The various optimization techniques that have been developed and used
extensively in engineering design optimization are briefly reviewed in this section.

### 3.3.1 Gradientless FE Methods

In gradientless methods of structural shape optimization, stress derivatives are not used to determine optimal geometries. In these methods of structural shape optimization, boundary stresses are directly used for shape modifications in various iterations. The boundary shape is adjusted iteratively such that there is a nearly constant stress distribution around the boundary with the aim of minimizing the peak stress. These methods are very simple to handle and takes very less time in comparison to gradient based methods. In these methods, mathematical programming techniques are not used. These methods can be divided into the following categories.

#### 3.3.1.1 Direct Methods:

These methods are based on the fact that stress at a boundary point depends upon the amount of the material under this point. The most intuitive way to achieve a fully stressed boundary is to add material if the stress is high and removes material if stress is low.

#### 3.3.1.2 Curvature Methods:

The stress value at a boundary is determined by normal stress and stress concentration effect. Curvature of a boundary curve can be used for this purpose since it is closely related to stress concentration effect. The stress concentration factor at a notch or fillet increases as the radius of the notch or fillet decreases and vice versa.
3.3.1.3 Pattern Transformation Methods:

This is a technique of transforming the shape of a boundary based on the stress ratio in the boundary finite elements. Initially, stress ratio in the boundary finite elements is calculated and then the size of the boundary elements is scaled up or down based on their ratio.

3.3.2 Gradient-based FE Methods

Shape optimization problems using gradient based methods are generally solved using numerical optimization techniques. In these methods, objective function and constraints are expanded using Taylor series. The order of expansion of objective functions, constraints and selection of proper bounds on the design variables play very crucial role in obtaining the optimum shape. Some of the most important and generally used numerical methods are briefly explained as follows.

3.3.2.1 Sequential Linear Programming (SLP):

This method involves linearizing the non linear objective function and constraints about a design point using Taylor series expansion up to first order and neglecting higher order terms. With these approximations, the optimization problem is reduced to a sequence of linear programming problems. The SLP method approximates the objective function and constraint function with appropriate linear functions. SLP has been used in many engineering problems and it has been proved to be a useful method for finding an optimum solution, even though it has many drawbacks.
3.3.2.2 Sequential Quadratic Programming (SQP):

In SQP methods, the non-linear objective function is expanded by Taylor’s series about a given point up to second order. Here, the objective function is a quadratic function and constraints are linear. The techniques of nonlinear programming are especially suited for size and shape optimization problems in combination with finite elements and finite strips.

3.4 Integrated approach to shape optimization

The basic algorithm for structural shape optimization is shown in Fig. 3.1. It consists of the following modules.

(a) Problem definition  (b) Geometric modeling  (c) Structural analysis  
(d) Mathematical programming  (e) Optimization module.

A Structural Shape Optimization (SSO) problem can be stated mathematically as one of minimizing/maximizing a specific objective function subjected to certain constraints which might include equality/inequality constraints and bounds on the design variables. Having defined the problem, the initial structural shape is defined in some convenient form. Typically, the geometry of the box girder bridge is described with the coordinates specified at certain key points. The thickness distribution may also be defined with thickness values specified at the key points. These coordinates and thickness values are taken as the design variables. This module reads the data concerning the initial geometry of the structure to be analyzed, mesh parameters, material properties, boundary conditions, loads etc. The mesh generator requires only the definition of the boundaries with a
desired mesh density to generate meshes of good quality with a facility for automatically updating the loading and boundary conditions. The structural analysis module uses FEM to evaluate the stiffness and the loads & then to assemble and solve the governing equations incorporating the boundary conditions. This module gives the nodal displacements, reactions and evaluates the stress resultants.

**Fig 3.1 : Basic algorithm for structural shape optimization**
The results of the finite element (FE) analysis are processed to provide an error estimate. If the error is acceptable, the solution proceeds to the next module, otherwise remeshing is performed using the automatic mesh generator. Further, finite element analysis are carried out based on revised meshes until the error is deemed to be insignificant. By using a suitable mathematical programming algorithm, a new structural shape is generated with an improved value of the objective function. The optimization module consists of variety of algorithms depending on the type of problem. If the convergence criterion for the optimization algorithm is satisfied, the optimum solution has been found and the process is terminated. Otherwise, new values of the design variables are sent to the mesh generator and the whole sequence of operations is repeated until an optimum solution is obtained.

For the present research work, ANSYS Release 9.0 which is robust, reliable, compatible to many fields of engineering, consisting many types of elements and having all the above mentioned modules is used as solver and optimizer.