CHAPTER - 2

2. LITERATURE REVIEW

2.1 Surveys / Reviews:

Numerical and analytical methods for shape optimization of structures are reviewed by Ding\textsuperscript{16}. Several steps in the shape optimization process such as model description, selection of the shape variables, objective function, representation of boundary shape, finite element mesh generation, sensitivity analysis and solution methods are reviewed in detail.

Haftaka and Grandhi\textsuperscript{22} presented a survey of structural shape optimization on techniques dealing with shape optimization of the boundaries of two and three dimensional bodies. They studied some special problems on structural shape optimization.

Shape optimization process integrates geometrical modeling, structural analysis, and optimization into one complete automated computer aided design process. Hsu\textsuperscript{33} presents a general review of structural shape optimization with emphasis on the recent developments. Different approaches for shape optimization which consists of geometrical representation, structural analysis, sensitivity analysis and optimization algorithms are reviewed.

A state-of-the-art review of recent developments in the area of structural optimization is presented by Levy and Lev\textsuperscript{36}. Recent
reviews, techniques and civil engineering design applications are covered, citing list of papers, books and computer programs.

The various structural shape optimization techniques are reviewed by Pathak et al. A literature survey on gradient based and gradient less finite element methods of shape optimization has been presented. Some guidelines are suggested for selection of method of shape optimization for a particular problem.

Qatu et al. presents the research carried out in the years 2000-2009 on the dynamic behavior of composite shells. This review is conducted with emphasis on the type of testing, type of analysis (free vibration, transient, impact, shock, etc.), complicating effects in material (damping, piezoelectric etc.), structure (stiffened shells etc.) and the various shell geometries (cylindrical, spherical, conical etc.) that are subjected to dynamic loads. A brief description of the various theories (classical, shear deformation, non-linear etc.) is presented.

Sennah and Kennedy highlights the references related to straight and curved box girder bridges in the form of single cell and multi cell cross sections. This paper deals with elastic analysis and experimental studies on the elastic response of box girder bridges.

Box girder bridges are increasingly used in modern highways and many of them are complex in plan geometry. The finite element method has been proved to be the most general and can be used for the analysis of skew and curved bridges of constant or variable cross section. Sisodiya et al. presents recent developments on box girder bridges investigated by several authors.
A review of the recent developments on the finite element analysis for laminated composite plates from 1990 is presented by Zhang and Yang. This paper discusses the various laminated plate theories for the free vibration and dynamic analysis, geometric nonlinearity and large deformation analysis, buckling and post buckling analysis, and failure and damage analysis of composite laminated plates.

2.2 Static Analysis and Optimization:

Optimization techniques are effective in finding out alternative geometries of shell structures to improve their mechanical behavior particularly reducing or avoiding the bending moments. Antonio Tomas and Pascual Marti show how the field of computers is widened using these techniques in the design of concrete shells allowing the user to obtain optimum designs subjected to the constraints. Some optimum geometric designs of an actual concrete shell were found, similar to that initially planned by the designer. The results show that significant improvements in the structural behavior can be achieved with only slight changes in geometry.

Bletzinger et al. presents numerical methods to simulate the physical experiments and how they can be merged among each other. Two different lines of research have been developed which deal with the generation of structural shapes i.e. Form finding and Structural optimization respectively. The methods of form finding are usually limited to tensile structures (cables and membranes) whereas the
methods of structural optimization can be applied to any kind of structures. This paper will give an overview on both the approaches and shows strategies about how to combine them effectively.

Chapelle and Bathe\textsuperscript{11} describes the fundamental considerations of the finite element analysis of shell structures. They presented theoretical formulations and proposed appropriate test cases on shell analysis for numerical evaluations.

Cheung\textsuperscript{13} used the finite strip method to analyze the curved box girder bridges supported along radial lines and having constant width. Many authors analyzed the curved box girders as curved beams of prismatic thin-walled closed cross-section. But, such an assumption is not appropriate for many of the wider box girder sections commonly used in bridge design, in which considerable transverse bending occurs in the flanges. The finite strip method used in this paper incorporates all the bending and membrane actions.

A finite element based shape optimization program is developed for three dimensional shell structures by Habib Uysal et al.\textsuperscript{21} The shape optimization program is implemented by a job control language and a reliable finite element based program ANSYS. The objective is to minimize the weight of the shell structure under the constraint that the maximum von Mises stress in each element should not exceed a certain limit. The design sensitivities are calculated by using the finite difference method. The search for the final optimum shape of the structure is performed using linear programming technique.
The linear elastic analysis of prismatic shell structures supported on diaphragms at two opposite edges, other two edges arbitrarily restrained are investigated by Hinton and Ramana Rao\textsuperscript{24,25}. The analysis is carried out using variable thickness, curved finite strips. They studied box girder bridges and cylindrical shells, including the transverse shear deformation effects. In the second part, they developed the computational tools for structural shape optimization of shells in which the weight of the structure or strain energy is minimized subjected to certain constraints. Both shape and thickness variables defining the cross-section of the structure are considered. Optimal shapes are presented for various kinds of shells and folded plates of variable thickness. It is shown that the finite strip method offers an accurate and inexpensive tool for the optimization of a wide variety of structures having regular prismatic geometry with diaphragm ends.

The structural shape optimization of shells is investigated by Hinton and Ramana Rao\textsuperscript{26} using two nodded finite strips. Both shape and thickness variables defining the cross-section of the structure are considered. The objective is to minimize the strain energy stored in a structure with a constraint that the total material volume remains constant. It is shown that minimization of strain energy leads to optimum structures in which the stress resultants and deflections in the members are considerably reduced.

Hinton et al.\textsuperscript{29,32} developed a program ADOPT (Adaptivity and shape optimization) integrating the automatic mesh generation, finite
element analysis, shape optimization and adaptive mesh refinement (AMR) procedures for linear elastic structures. The methodology of the program and the selection of suitable design variables are presented. Several examples excluding the use of adaptivity are solved to illustrate the benefits of this approach. In Part II, anomalies arising in predicting the optimum shapes of structures due to discretization error are discussed. The effect of the number of design variables on the final optimum shape of the structure is investigated when two different types of curves are used to describe the boundary of the structure, i.e. quadratic Bezier and cubic B–spline curves.

Hinton and Ramana Rao\textsuperscript{31} developed an efficient computational tool for the structural optimization of variable thickness prismatic and axisymmetric shells using computer aided analysis and design procedures. In static conditions, the composition of the strain energy is monitored during the optimization process to get an insight into the energy distribution for the optimum structures. The mode shapes of the initial and optimum solutions are presented for the vibrating structures.

An efficient optimization methodology combining a finite element based analytical model, a sequential quadratic programming algorithm and a direct method of design sensitivity analysis is developed by Mahmoud\textsuperscript{37}. The methodology involves the solution of a sequence of high quality explicit approximate problems subject to specified move limits in the design space. A new technique for developing approximation function to the original function is proposed.
with a high quality adaptive capability by using the state variables (stresses and/or displacements) and their derivatives.

A finite triangular facet element is presented by Mohr\textsuperscript{40} for the analysis of doubly curved thin shells. A simple design procedure iterative in nature is developed. The optimality criteria are the minimization of surface integral of the membrane stresses and elimination of bending stresses. The procedure is used to predict the optimal shapes for constant thickness shells and arches.

Finite element models of a wide variety of problems are optimized by Mohr\textsuperscript{41,42} using the steepest descent method and merit functions. Part I of the paper deals with typical structural problems in which the merit functions are developed based on the stresses caused by loading of the structure. In part II, he analyzed the structures loaded by a fluid flow and alter the shape of the structure to enhance its hydrodynamic efficiency.

Ozakca and Taysi\textsuperscript{46} developed computational tools to analyze and find optimum shapes of box girder bridges in curved plan form in which the weight of the structure or the strain energy is minimized subjected to certain constraints. The finite strip method is used to determine the displacements and stresses. It is shown that the finite strip method offers an accurate tool for the optimization of box girder bridges having regular prismatic geometry with diaphragm ends and in curved plan form.

Static, vibration and buckling analysis of axisymmetric circular plates using the finite element method is carried out by Pardoen\textsuperscript{51}. 
The static analysis of circular and annular orthotropic plates is performed. He presented the exact displacement functions for circular and annular plates made of isotropic and orthotropic materials.

Pourazady and Fu\textsuperscript{54} developed an integrated approach to structural shape optimization. The proposed approach combines the finite element method and the optimization process together by implementing design sensitivity analysis. The design sensitivities are obtained by directly differentiating the finite element equations. The geometry of a structure is represented by design element concept and an isoparametric mapping technique is used to automatically generate and regenerate the finite element mesh. Move limits, B-spline functions and constraint equations are introduced to obtain a realistic optimum shape that meets the manufacturing requirements. Using a linear programming technique, the search for the final shape of a structure is performed based on the sensitivity information.

An integrated shape optimal design system is developed by Rajan et al.\textsuperscript{56} consisting of a geometric modeler, mesh generator, design sensitivity calculator, finite element analyzer and an NLP optimizer that operates on a common database.

Linear elastic analysis of prismatic shell structures with curved plan form is investigated by Ramana Rao and Hinton\textsuperscript{58,59}. The analysis is carried out using finite strips of curved cross-section and having variable thickness. All the features of the curved finite strip formulations are tested using benchmark solutions for structures with curved plan form and comparisons are provided with known solutions.
In next part of the paper, these finite strips are used for carrying out structural shape optimization of prismatic type structures with curved plan form.

Ramana Rao and Hinton\textsuperscript{60} developed the computational tools for the structural optimization of free form shells and variable thickness plates. Some benchmark examples are considered showing optimal forms under different boundary and loading conditions.

Shape optimization with prescribed movement directions of design variables has been investigated by Rupesh Kumar and Ramana Rao \textsuperscript{63,64,65} with stress leveling index and strain energy as objective functions. They studied the effect of movement directions of shape design variables on the optimum shapes of various structures such as plate with a square hole, plate with a circular hole, connecting rod and deep beam problem.

Sisodiya and Ghali\textsuperscript{70} deals with finite element analysis of single box girder skew bridges curved in any shape. The bridge may have any type of support conditions and having varying width. Results obtained from the analysis of curved bridges are compared with the results of experiments conducted on a model.

A cylindrical shell having varying thickness subjected to axisymmetric deformation is analyzed by Sundarasivarao and Ganesan\textsuperscript{72} using the finite element displacement method. Numerical results are presented for clamped–free, clamped–clamped and simply supported–simply supported end conditions. It is shown that the
deflections and stresses can be reduced considerably by properly selecting the thickness variation of the shell.

A method based on energy criteria for optimum design of structures subjected to static loading is presented by Venkayya\textsuperscript{76}. The method can efficiently handles the multiple loading conditions, constraints on displacements, stresses and sizes of the elements. Examples of beam and bar type structures are presented to illustrate the effectiveness of the method. The method is very efficient in obtaining the minimum weight of structures using linear and nonlinear programming methods.

The optimal shape of a mechanical component is investigated by Zhixue Wu\textsuperscript{79} with the aim of developing a simple and efficient numerical approach for minimizing the stress concentration factor. The proposed approach is based on the concept of finite element method in conjunction with the widely used fully stressed design criterion. An essential requirement for optimality in the structural shape optimization is the achievement of constant stresses along a section of the boundary to be optimized. This is achieved iteratively by adjusting the design boundary shape based on simple logic and algorithm.

### 2.3 Free Vibration Analysis and Optimization:

Afonso and Hinton\textsuperscript{1,2} studied the free vibration analysis of plate and shell structures. The theoretical formulations are presented for shells with arbitrary thickness variation and various support
conditions. The convergence and accuracy of the formulation is checked with several benchmark problems. In paper II, sensitivity analysis has been carried out to obtain optimum shapes for shells and plates in which the natural frequencies are maximized. Both the shape and thickness design variables are considered. The optimal solution is found by the use of a structural optimization algorithm that integrates the sensitivity analysis, finite element analysis and a mathematical programming method such as SQP.

The undamped free vibration analysis of a shell of general shape is formulated by Aksu with a curved isoparametric trapezoidal finite element including shear deformations, rotatory inertia effects with consistent mass matrix. The shell element with 8 nodes and 40 d.o.f. is used both for thin and moderately thick shell analysis. The accuracy and efficiency of this proposed finite element is checked by benchmark examples and the results are compared with those obtained by other numerical and analytical methods.

The classical optimization problems of shells and plates are studied by Awrejcewicz and Krysko. Orthotropic plates and shells with low transverse stiffness and variable thickness are analyzed. The problems related to free vibrations of shells with transverse deformation and rotary inertia effects are discussed.

Barbosa et al. carried out the sensitivity analysis for the optimization of axisymmetric shells subject to static and dynamic constraints. Thickness and shape design variables are considered. The objective of the design is the minimization of the volume of the
shell material or the minimization of the maximum stress or the maximization of the fundamental natural frequency. The constraint functions are the stresses, displacements, volume of the structure, or the natural frequency of a specified mode shape.

A general finite element formulation is presented for the free vibration analysis of cylindrical shell structures by Chakravorty and Bandyopadhyay. The natural frequencies for clamped short, intermediate and long shells are obtained and the effect of release of boundary constraints on the frequencies are investigated.

A finite element method for free vibration analysis of generalized shallow shells is described by Chakravorty and Bandyopadhyay. Results for natural frequencies of corner supported cylindrical, spherical and hyperbolic paraboloidal shells are presented. Natural frequencies of reinforced concrete clamped cylindrical, elliptic paraboloid and conoidal shells of the same thickness and material properties are presented.

Chakravorty et al. used the finite element method to solve free and forced vibration problems of laminated composite isotropic shells with and without cutouts. Free vibration problems of cylindrical, spherical, hypar (hyperbolic paraboloid bounded by straight lines) saddle (hyperbolic paraboloid bounded by parabolas) and conoidal shells with cutouts and forced vibration of composite conoidal and hypar shells without cutouts are examined.

Free vibration analysis of curved and straight beam-slab or box girder bridges is investigated by Cheung. He used the finite
strip method to study the curved box-girder bridges supported along radial lines and having constant width.

A general mathematical approach is developed by Hamed and Frostig\textsuperscript{23} for the free vibration analysis of multi cell box bridges with a single/multi span including the effects of transverse deformations of the bridge cross section. The model developed is valid for any boundary and continuity conditions and is applicable for multi girder bridges with longitudinal and cross beams. Closed form solutions of the governing equations are derived and the eigen frequencies are determined using Newton-Raphson method.

The structural shape optimization of vibrating folded plates and prismatic shells is investigated by Hinton et al.\textsuperscript{27} The natural frequencies and mode shapes are determined by using the finite strip method. The objective is to maximize the fundamental frequency of the structure by changing the shape and thickness design variables defining the cross section of the structure with a constraint that the volume of the structure remains constant.

Hinton et al.\textsuperscript{28} investigated the free vibration analysis of prismatic shell structures supported on diaphragms at two opposite edges and the other two edges arbitrarily restrained. The analysis is carried out by using variable thickness, curved finite strips which allows for rotatory inertia effects and transverse shear deformation.

An automated optimization approach is adopted by Hinton et al.\textsuperscript{30} to generate structural shapes for axisymmetric shells and plates in which some vibration characteristics are optimized. The free
vibration analysis is carried out by using finite elements having variable thickness which allow for rotatory inertia effects and transverse shear deformation. Semi analytical sensitivity calculations are used in conjunction with a sequential quadratic programming optimization algorithm to obtain the optimum shapes.

A unified formulation and efficient numerical techniques are presented by Jean Win Hou\textsuperscript{34} to perform shape optimum design. The numerical scheme uses finite element method and mathematical programming technique which consists of boundary parametrization.

The finite strip method is used by Morris and Dawe\textsuperscript{43} for the prediction of the natural frequencies of vibration of rigidly connected assemblages, longitudinally invariant curved and flat strips having diaphragm end supports. Two types of refined and transversely curved finite strips are presented. Results are shown for a limited range of problems including circular cylinder.

Free vibration analysis of shells of revolution using the finite element method is investigated by Ozakca and Hinton\textsuperscript{44,45}. A family of curved, variable thickness finite elements is presented which include rotatory inertia effects and shear deformation. The accuracy and convergence of these elements are explored through a series of free vibration analyses of axisymmetric shell structures. Paper II considers the structural optimization of axisymmetric shells undergoing free vibrations. The sensitivities of the objective function and constraints to changes in the design variables are determined and some examples are illustrated.
A structural optimization algorithm is developed by Sedaghati\textsuperscript{68} for beams and trusses under stress, displacement or frequency constraints. The algorithm combines the finite element technique based on the Integrated Force Method and the mathematical programming based on the Sequential Quadratic Programming (SQP) technique. It is shown that the force method requires very less computational effort compared to that of displacement method. In structural optimization problems consisting of multiple frequency constraints, the analysis procedure (displacement or force method) significantly affects the final optimum design. The structural optimization using the force method may result in a lighter design.

A simple finite element method is presented for free vibration analysis of axisymmetric and conical shells with uniform/varying wall thickness by Tambiratnam and Yan Zhuge\textsuperscript{74}. This is obtained by considering the analogy between the theories of bending of a conical shell and a beam on elastic foundation.

\section*{2.4 Dynamic Analysis and Optimization :}

A method to optimize a composite laminate with layers oriented according to a limited set of angles is described by Conti et al.\textsuperscript{15} The laminate must be balanced, symmetric and loaded in plane. The optimization process is used in conjunction with a finite element program. It provides the designer with a list of all the possible
orientation combinations ranked as per their safety factor and the optimal overall engineering characteristics.

Falco et al.\textsuperscript{17,18} developed computational tools for the analysis of variable thickness free form shells and plates under dynamic loads. The direct integration algorithm developed by Newmark is used for the time discretization of the dynamic equilibrium equations. In paper II, structural shape and sizing optimization procedure is used to obtain optimum designs for shells and plates subjected to dynamic loads. The optimization problem is solved by sequential quadratic programming algorithm. In the optimal design of symmetrical shells, special focus is given to eliminate the clusters of multiple eigen values.

An optimum design of laminated composite structures subjected to in-plane loading is presented by Fukunaga and Sekine\textsuperscript{19}. A simplified design approach is developed to optimize layer thickness distributions and layer angle. Based on the minimum strain energy criterion, an optimal relation is derived explicitly for layer thickness ratios and layer angles in a single element laminate. With this optimal relation in the single element laminate, an optimization approach is developed for determining layer thickness distributions and layer angles in multi element composite structures with strength constraints. A rough structural shape is determined in addition to layer thickness distributions and layer angle.

Fukunaga and Vanderplaats\textsuperscript{20} investigated laminated composites under in-plane loading for the strength optimization based upon the mathematical programming method. Layer thickness and
layer orientation angles are taken as the design variables. A technique is also developed to delete the strength constraints of nearly zero thickness layers. This new technique gives an efficient optimization approach for the laminated composites.

A multilevel approach to the optimal design of laminated composite plates is presented by Kam and Chang\textsuperscript{35} by considering deflection, damping and natural frequency. Ply orientation and thickness are designed to minimize the volumes of the plates subjected to constraints on the first natural frequencies, deflections induced by the applied loads and damping of the first vibration modes. The optimization process is carried out in two steps. Optimal ply orientations are determined using quasi-Newton method in the first step and an optimal ply thickness is determined using the optimality criteria method in the second step of optimization process.

A simple isoparametric finite element formulation for dynamic analysis of multi layer symmetric composite plates based on a higher order displacement model is presented by Mallikarjuna and Kant\textsuperscript{38} with an explicit time marching scheme. The proposed higher order theory which is more accurate than the Mindlin theory is used to evaluate the plate response to different types of dynamic loads. The parametric effects of the finite element mesh, time step, orthotropy and lamination scheme on the transient response are investigated.

The buckling analysis of circular and annular plates is investigated by Ozakca et al.\textsuperscript{48,49} using the finite element method. A family of curved, variable thickness axisymmetric finite elements is
developed which include rotary inertia effects and shear deformation. The accuracy and convergence of these elements is tested using a series of buckling analyses of plates and the results are compared with those obtained by other numerical and analytical methods. In paper II, these efficient and accurate finite elements are used for the structural shape optimization. An automated finite element analysis and optimization procedure is adopted which integrates parametric cubic spline thickness definition, automatic mesh generation, finite element analysis, sensitivity analysis and mathematical programming.

The structural shape optimization of prismatic folded plates is investigated by Ozakca et al.\textsuperscript{50} under buckling load consideration. Buckling loads are determined using cubic, quadratic and linear variable thickness finite strips. The objective is either the minimization of the c/s of the prismatic folded plate or the maximization of the critical buckling load with constraints on the buckling load and volume.

Pathak et al.\textsuperscript{53} studied the transient dynamic analysis of three dimensional composite laminated plates. Simply supported plate made up of five layers, two orthotropic materials stacked in alternate layers are investigated. Number of plates are analyzed by varying the fiber orientation, thickness and type of loading.

Optimum vibrating shapes of circular plates and beams are investigated by Thambiratnam and Thevendran\textsuperscript{73}. They studied the fundamental mode of axisymmetric vibration of circular plates and fundamental mode of lateral vibration of beams. They concluded that
a considerable saving (>50%) in the material for frequency constraint and a significant increase (>100%) in the fundamental frequency for volume constraint are possible by merely altering the shape.

Several simultaneous formulations are described, analyzed and compared by Wang and Arora for transient dynamic response optimization. A key feature of the formulations is that the state variables are treated as independent variables in addition to the real design variables during the optimization process. The state variables include different combinations of generalized accelerations, velocities and displacements. The performance features of the formulations and numerical results for some example problems are compared.

2.5 SUMMARY:

The shells are broadly classified as prismatic shells having rectangular planform, prismatic shells having curved planform and axisymmetric shells. The work done by various researchers on static analysis and optimization of above said three classes of shell structures has been presented in section 2.2. Many researchers dealt the optimization of shell structures with different objective functions and geometric constraints. However, in many of the cases, the optimum shapes of structures are obtained by prescribing the movement directions of the shape design variables in one direction (mostly radial) only. To the author’s knowledge, no work has been reported on the effect of prescribing the move directions of the shape design variables both in horizontal and vertical directions simultaneously on the optimum shapes of shell structures. A
thorough investigation has been made in this research work to arrive at the better optimum shapes of structures as detailed in chapter - 4.

In the design of axisymmetric and prismatic structures, the knowledge of dynamic characteristics such as natural frequencies and mode shapes is vital for sound design. The work done by various researchers on free vibration analysis and optimization of shell structures has been presented in section 2.3. Many researchers dealt the structural optimization of vibrating axisymmetric and prismatic shell structures using shape or thickness design variables. In many cases, the shape design variables are allowed to move in one direction (mostly radial) only. A thorough investigation has been carried in this research work to enhance the fundamental frequency of prismatic and axisymmetric shell structures using shape & thickness design variables simultaneously and by allowing the design variables to move in horizontal and vertical directions simultaneously as detailed in chapter - 5.

The prediction of dynamic characteristics of plates and shell structures is very essential for the design of structures. It is necessary to study the dynamic behavior of such structures to guard against failure due to resonance. Laminated composite plates which are used in aerospace, automobile and construction industries are often subjected to dynamic loads. The work done by various researchers on the dynamic analysis and optimization of plates and shell structures has been presented in section 2.4. The behavior of plates and shell structures is affected by excessive vibrations, higher displacements
and accelerations which may deteriorate the structural performance. Hence, many researchers tried to reduce the dynamic response parameters such as displacement, velocity and acceleration but the percentage reduction is less than 50%. Hence, a thorough investigation has been made in this research work to enhance the percentage reduction in response parameters by changing the fiber orientation in laminated composite plates as detailed in chapter – 6.