CHAPTER VI
MULTIOBJECTIVE MODELS IN WATER RESOURCES
PLANNING AND MANAGEMENT

6.1 GENERAL

Water resource problems are inherently multiobjective which are conflicting with each other and many of them are noncommensurable. National or regional income maximization is only one of many possible and important planning objectives. Others that are not so easily expressed in monetary terms include income redistribution, environmental quality, social wellbeing, national security, self-sufficiency, regional growth and stability and preservation of natural areas. (Loucks et al., 1981). Multiobjective analysis is, thus, a generalization of traditional benefit-cost analysis (Major, 1977).

6.2 APPROACH

Multiobjective planning, which is an iterative process, can be conveniently summarized in the following steps (Major, 1977):

1. Identify the objectives of system design. This step involves selection of objectives in the political process.
2. Translate the objectives of system design into design criteria. This step includes the development of detailed criteria. For example, benefit and cost accounting rules and estimating methods, for reflecting objectives in system design.

3. Use appropriate design criteria to develop system designs that reflect the objectives.

4. Review the results of the design process.

6.3 RISK AND UNCERTAINTY

The important rule in applying multiobjective analysis in risky and uncertain situations is that there should be social attitudes towards risk and uncertainty to guide project and programme design. In multiobjective analysis, risks and uncertainties in many dimensions are to be considered; for example, when a species is endangered, there must be an attitude towards the extreme event of species extinction, and the same is true for the extreme event of regional economic collapse. (Major, 1977).

Situations of risk are those in which outcomes can be characterized by reasonably well known probabilities.

For uncertain events, the underlying process is not well described by a known probability distribution and hence appropriate attitudes can be developed in the political process. Szidarovszky et. al (1976), Wood and
Rodriguez – Iturbe (1975) and Duckstein and Bogardi (1978) have described the following types of uncertainties:

i. **Natural uncertainty**, such as resulting due to rainfall or infiltration, watershed characteristics, heterogeneity of the motion of water through porous media, diffusion process of chemicals.

ii. **Model uncertainty**, which occurs due to the difficulty or impossibility of choosing a proper model to represent the elements of nonhomogeneous and anisotropic surface or groundwater systems.

iii. **Sample or parameter uncertainty**, caused by the finiteness of the sample data that are available to estimate the parameters of various probability density functions such as those of precipitation, storage, transmissivity, recharge or evaporation.

iv. **Economic uncertainty**, due to the inadequacy of knowledge of economic factors such as construction, labour, operation and maintenance costs, shortage and pollution losses.

v. **Technological uncertainty**, due to the difficulty in forecasting technological advances in water cycle control or even in modelling.

vi. **Strategic uncertainty**, caused by not knowing what the institutions and priorities would be at the time a water resource development
plan is implemented: Will the emphasis be on energy, water supply, recreation or ecology? How will decisions be enforced?

All these need to be carefully considered before taking up the development projects.

6.4 METHODS OF SOLUTION

Multiobjective optimization methods can be classified in many ways (Cohon and Marks, 1975; Keeney and Raiffa, 1976; Zeleny, 1982). One classification is as follows:

i) Methods which rely on prior articulation of preferences

In these methods, a single objective optimization is constructed on the basis of the preferences and solved; no further iterations are carried out. The following techniques come under this class:

a) Goal programming
b) Assessment of utility functions
c) Estimation of optimal weights
d) Electre method
e) Surrogate worth trade off method.

ii) Generating Methods

In these methods, through a sequence of properly constructed, single objective optimizations, one generates points on the noninferior
solution set, then one of these points is selected as the best-compromise solution. The following techniques come under this class:

a) Weighting method
b) Constraint method
c) Derivation of a functional relationship for the noninferior set.
d) Adaptive search

iii) **Methods which rely on progressive articulation of preferences**

It is an iterative procedure of stating preferences based on updated information and generation of more solution points to provide additional information. The following techniques come under this class.

a) STEP method
b) Iterative weighting method
c) Sequential multiobjective problem solving (SEMOPES).

Of the above methods, goal programming method is widely used.

6.5 GOAL PROGRAMMING APPROACH

6.5.1 GENERAL

Goal Programming is a mathematical programming approach for dealing with decision problems having multiple, conflicting and incommensurable objectives. Goal Programming model does not attempt to maximize or minimize the objective function as does the linear
programming model. It seeks to minimize the deviations among the desired goals and the actual results according to the priorities assigned. Goal Programming may be linear or nonlinear, deterministic or probabilistic.

The general multiobjective optimization problem with 'n' decision variables, 'm' constraints and 'p' objectives can be stated as:

Maximize \( z (x_1, x_2, \ldots, x_n) \)

\[
= \{ z_1 (x_1, x_2, \ldots, x_n), z_2 (x_1, x_2, \ldots, x_n), \ldots, z_p (x_1, x_2, \ldots, x_n) \}
\]

subject to \( g_i (x_1, x_2, \ldots, x_m) \leq 0 \quad (i = 1, 2, \ldots, m) \)

\( x_i \geq 0, (j = 1, 2, \ldots, m) \)

where \( z (x_1, x_2, \ldots, x_n) \) is the multiobjective objective function and \( z_1 ( ), z_2 ( ), \ldots, z_p ( ) \) are the "p" individual objective functions.

6.5.2 GOAL LINEAR PROGRAMMING

The general goal deterministic linear programming is expressed mathematically as follows (Moskowitz, et al., 1979):

Minimize \( z = \sum_{i=1}^{m} w_i (d_i^+ + d_i^-) \)

Subject to \( \sum_{j=1}^{n} a_{ij} x_j + d_i^- - d_i^+ = b_i \)

for all "i"

\( x_j, d_i^-, d_i^+ \geq 0 \) for all \( i, j \)
where \( w_i \) represents the weights attached to each goal

\[ d_i' \] represents underachievement of \( i^{th} \) goal

\[ d_i^+ \] represents overachievement of \( i^{th} \) goal

\( b_i \) represents target level for \( i^{th} \) goal

It is required that one or both of the deviational variables \((d_i' \text{ or } d_i^+)\) be zero in the solution, since it is not possible for both underachievement and overachievement to occur at the same time.

In case of multiple goals, goals are achieved according to their priorities. To do this, goals are classified into 'k' ranks and deviational variables associated with the goals are assigned a priority number \( P_i (j = 1, 2, \ldots, k) \)

The priority factors have the relationship, \( P_1 \gg P_2 \gg \cdots \gg P_k \)

The lower priority goals are considered only after higher priority goals are achieved. With 'm' goals, the Goal Linear Programming model may be formulated mathematically as requiring the minimization of the linear weighted ranking function (Fabrycky W.J et al., 1984), as follows:

\[
\sum_i \sum_k P_i (w_{i,k} d_i' + w_{i,k}^+ d_i^+) 
\]

where

\( P_j \) : Priority no. of deviational variable \((j=1,2,\ldots,k)\)
$W_{i,k}$: Relative weight of the $d_i$ in the $k$th rank

$W_{i,k}^*$: Relative weight of the $d_i^*$ in the $k$th rank

subject to the linear constraints:

$$\sum_{j=1}^{n} a_{ij} x_j + d_i - d_i^* = b_i$$

where $i = 1,2, \ldots, m$

and $x_j, d_i^-, d_i^* \geq 0$

### 6.5.2 NONLINEAR GOAL PROGRAMMING

The nonlinear (quadratic) goal programming problem can be expressed mathematically as follows (Moskowitz et al, 1979):

Minimize $z = \sum_{i=1}^{l} \sum_{j=1}^{m} w_i \left( d_i^- d_j^- + d_i^+ d_j^+ + d_i^- d_i^* \right)$

subject to $\sum_{j=1}^{n} a_{ij} x_j + d_i - d_i^* = b_i$ for all $i$

$x_i, d_i^- d_i^* \geq 0$ for all $i, j$

The objective function must be a convex function to guarantee a global optimal solution. The objective function is composed of squared terms (e.g. $d_i^- d_i^-$) and cross product terms (e.g. $d_i^* d_j^*$).
6.6 CONCLUSION

In the present study, it is proposed to adopt linear goal programming approach to achieve the targetted goals in order of their priorities. The problem formulation, prioritization of goals and the solution process is described in detail in chapter VII.