1.1. Space Plasma

Majority of universe and our solar system comprises of matter that is in the form of plasma. Plasma is the name given to fourth state of matter. The plasma is a gas, containing equal amount of negative charge carriers and positive charge carriers. The gas is so hot that its constituent atoms split into electrons and ions. Same number of charges, with opposite signs in the given volume element, assures that plasma behave quasineutral at the equilibrium state. Also, collective effects play a vital role in plasma physics. In collective behavior, long range electromagnetic forces become more important than interactions between neighboring particles. Since, plasma is made up of electrically charged particles, it is largely affected by electromagnetic and electrostatic fields, which in turn leads to very interesting and complex behavior of plasma waves and instabilities.

When plasma is completely thermalize, the ions and electrons are considered oscillating at their equilibrium position. Any deviation from this equilibrium dislocates the charged particles. To withstand the effect it sets up electric and magnetic field. Due to these restoring forces, electrons execute harmonic motion. A measure of electric field such perturbed plasma exhibits resonance at a specific frequency. This frequency is known as electron plasma frequency. Another important characteristic frequency to be known while studying plasma is electron cyclotron frequency. When the plasma is implanted in quasi-static field, the charged particles get accelerated in perpendicular direction with respect to magnetic field, insisting the particles to move in helix shape around the field lines. This frequency of rotation is known as the electron cyclotron frequency. It is proportional to the strength of magnetic field. To explore the dynamics of space plasma, measurement of these characteristic frequencies is essential. It will enable us to understand basic property of space plasma environment such as its density, magnetic field effect etc. permitting us to study waves in space plasma.

Plasma is bountiful in universe. Approximately 99% of all matter that is known is in the plasma state. Natural plasma is found in solar wind, solar corona, in tails of comets, in the magnetosphere of planets, in the accretion disks of black holes, in the inter-stellar and inter-galactic media. Therefore we use space as a huge plasma laboratory to collect infinite knowledge of our universe and its specific parts. The work in this thesis is focused on plasma in the magnetosphere of planet ‘Uranus’.
Solar wind is the sole source of plasma in Uranian magnetosphere. Solar wind is a stream of charged particles that pervades interplanetary spaces. These ionized charged particles vary in temperature, density and speed over time and distance.

1.2. Waves in Space Plasma

In plasma the charged particles respond very well to electrostatic and electromagnetic fields. The fields could be oscillatory or static. Strong interactions occur between charged particles and plasma waves leading to instabilities. The study of these plasma waves and instabilities is important in understanding and explaining the state of plasma, plasma flux in magnetized plasma, evolution of energy and plenty of more interesting phenomena. When plasma waves propagate, they are strongly influenced by magnetized plasma. Therefore, in thesis, we analyze the behavior of the following plasma waves in the presence of electric and magnetic field of Uranus.

1.2.1. Whistler Mode Waves

Whistlers are low frequency, circularly polarized electromagnetic waves in the audio-frequency range. Wave particle interaction can also create whistler mode radiations in the magnetosphere. Whistler mode emission constitutes electromagnetic waves with frequency below either electron gyro frequency or local electron plasma frequency, whatever is less. Calculation shows that either a loss-cone or a thermal anisotropy in the hot plasma component of the magnetosphere can lead to the generation of incoherent emission of low frequency whistler waves [Pandey and Singh 2011]. Oblique whistler mode elements can provide some scattering mechanism, driving the precipitation. Two types of electron trapping by whistlers can be studied. First one is trapping of electrons at a fixed phase of the whistler in gyrocyclotron resonant condition. In this case electrons are considered moving through the whistler, and then encounter an electric field that rotates in synchronism with the velocity of electron [Omura and Matsumoto, 1982; Matsumoto and Omura, 1981]. The second one, is trapping of whistler mode waves propagating oblique in the electrostatic potential [e.g., Kumagai et al., 1980].

1.2.2. Ion-Cyclotron Waves

Ion-cyclotron waves occur very often and are most intense while magnetic storms [Erlandson and Ukhorskiy, 2001; Braysy et al. 1998], although the waves are
present while comparatively quiet geomagnetic conditions too. Studies have suggested that localized ion cyclotron waves may also be excited by solar wind density enhancements [Usanova et al., 2008]. Ion-cyclotron waves could also be generated when ion temperature anisotropy exceeds specific positive threshold [Cornwall, 1964; Kennel and Petschek 1966]. The propagation of left hand polarized waves are supported by the presence of ions in plasma for frequencies lower than ion-gyro frequency. The ion-cyclotron waves while propagating through plasma, exchange energy by interacting with energetic particles. For the case of isotropic Maxwellian velocity distribution function of ions, the wave amplitude attenuates as energy exchange takes place from waves to the ions [Ahirwar et al. 2010, Singh et al. 2003]. Since resonant absorption of ion cyclotron waves is believed to be one of the important mechanism that accelerates and heats the high-speed solar wind, it is considered universally that ion-cyclotron waves play the most crucial role in accelerating and heating of ions [Hollweg 1986; Hollweg and Isenberg 2002; Cramer 2002; Lu and Wang 2005; Lu and Chen 2009]. Therefore, ion-cyclotron waves have been a subject of deep interest due to their tendency to heat heavy ions and links in ion diffusion in magnetospheres of planets.

1.2.3. Electron-Cyclotron Waves

Electron-cyclotron waves, in the frequency range of electron cyclotron frequency, have been observed in space [Kennel and Scarf 1968; Wu and Davidson 1972] and in surroundings of the outer planets [Barbosa and Kurth 1980; Gurnett et al. 1986; Gurnett 2005; Kurth and Gurnett 1991]. The particles whose Doppler shifted wave frequency lie in the neighborhood of gyrofrequency, interact strongly with these right-hand circularly polarized electron-cyclotron waves [Scharer and Trivelpiece, 1967]. Such cyclotron wave-particle interactions serves as a pathway via which waves can scatter, accelerate or heat space plasma [Summers et al. 2007; Kennel and Petschek, 1966]. The resonant wave-particle interactions supervise the established dynamics in space plasma and anisotropic electrons excite space plasma instabilities [Xiao 2006; Lu and Wang 2006], one of them being electron-cyclotron waves. These wave-particle interactions are the major source of energy diffusion and pitch angle diffusion in planetary magnetosphere. Due to pitch angle diffusion, the diffused auroras are produced, which are observed by spacecraft, thus providing us the information of plasma oscillations in the magnetospheric environment. Literature
presents electron-cyclotron harmonics (ECH) wave-particle interaction as the major source of energy diffusion and pitch-angle diffusion in magnetospheres of planets. These diffusions, because of growth of waves, leading to electron precipitation are enough to generate diffuse auroral emissions detected by spacecrafts in planetary magnetospheres [Belmont, Fontaine & Canu 1983; Roeder & Koons 1989; Kurth & Gurnett 1991; Summers, Tang & Thorne 2009; Meredith et al. 2009].

1.3. Exploration of Uranus

1.3.1. Voyager 2 and Uranus

On 20 August 1977, NASA launched a space probe from Florid, named Voyager 2, to study solar system and interstellar space. Voyager 2 has returned knowledge about the outer planets that had not existed in all of the preceding history of planetary science and astronomy. The primary mission of Voyager 2 ended on December 31, 1989 after encountering the Jupiter’s system in 1979, Saturn’s system in 1980, Uranus’s system in 1986 and Neptune’s system in 1989. It sent its last picture 13 years ago, just before shutting down its camera to conserve power. Since that final picture, Voyager has been sending back pure data-measurements of plasma waves, magnetic fields and cosmic particles. After 38 years of launch, the space probe is now in interstellar space, still receiving and transmitting data via the Deep Sea Network. It were the vibrations in the plasma surrounding the spacecraft that told the scientists of plasma-wave team at the University of Iowa that Voyager had entered the cold region interstellar space, where there are no particles from sun but the remnants of ancient exploded stars begin.
The Voyager 2 flyby of Uranus announced that the planet has a giant and remarkable magnetosphere. The magnetic field experiment carried on Voyager 2 consisted of dual low field (LFM) and high field magnetometer (HFM) systems. The LFM permitted to distinguish between of spacecraft’s magnetic fields and the ambient fields with great accuracy of dual systems [Behannon et al. 1977].The encounter of Voyager 2 spacecraft with the Uranian planetary system led to a series of out bound crossing of Uranian bow shock between Jan. 27 and Jan 30 of 1986. The Plasma Wave Instrument carried by Voyager 2 detected important processes regarding local wave-particle interactions, electrostatic Bernstein waves, radio emissions and plasma waves [Scarf et al. 1987; Kurth et al.1987; Kurth et al. 1989; Russell, Song & Lepping 1989; Kurth & Gurnett 1991; Coroniti et al. 1987; Kurth, Gurnett & Scarf 1986; Russell, Lepping & Smith 1990]. The Cosmic Ray System on Voyager 2 detected the noteworthy fluxes of energetic protons and electrons in those regions of magnetospheres of planets where these particles were expected to be stably trapped [Selesnick & Stone 1991; Stone et al. 1986].

The Voyager 2 encounter of Uranus gave us the golden chance to investigate one more enriched planetary magnetosphere and compare physical plasma phenomena taking place there, with those happening in magnetosphere of Saturn, Jupiter and terrestrial planet [Gurnett et al. 1986]. The presence of magnetic field at Uranus was not revealed until the Voyager’s arrival. This offered an enriched surroundings for the study of magnetospheric phenomena.
The Voyager 2 flyby of Uranus revealed that the planet has huge magnetosphere. The magnetic field experiment carried on Voyager 2 [Behannon et al. 1977] reported the maximum magnitude of magnetic field as 413 nT, observed at 4.19 \( R_U \), just before the closest approach to Uranus [Ness et al. 1986]. The unique feature of Voyager 2 encounter was the fact that the angle between Uranus’s angular momentum vector and dipole moment vector has the large value of 60 degrees. So the spin axis of Uranus is aligned nearly along the planet-sun axis. This leads to the situation that the flow system rotational electric field is oriented in such a way that instead of shielding the middle magnetosphere from the solar wind, it allows solar wind effects to deeply penetrate into the magnetosphere of Uranus [Gurnett et al. 1986]. As the consequences of this, Uranian system permits us to study magnetospheric instabilities and strong dynamics including Earth like injection phenomena. Therefore, in present study we have also conducted the theoretical investigation to find the behavior of various parameters when cold/hot plasma beam is injected in the magnetosphere of Uranus.

The resonant interaction occurs when Doppler shifted wave frequency is equal to multiples of electron’s cyclotron frequency [Stix 1992]. Such interactions are energies relativistic electrons after geomagnetic storm in outer radiation belt [Horne et al. 2005, Shprits et al. 2006]. After those of Earth and Mercury, magnetosphere of Uranus is the third planetary magnetosphere for which substorm activity has been cited [Cheng et al. 1987]. And Meredith et al. [2001] have shown in their work that chorus are effective in accelerating electrons to relativistic energies following/during geomagnetic storms. Whistler mode waves are competent of trapping and instantaneous scattering into loss-cone of radiation belt particles. [Kellogg et al. 2010; Kersten et al. 2011]. Test particle simulations results conclude that large amplitude
waves may energise electrons while simultaneously scattering them into the loss cone [Bortnik et al., 2008; Cattell et al., 2008].

1.3.2. Observations in Magnetosphere of Uranus

Uranus is one of the four giant planets of the solar system that possess planetary magnetic field namely Jupiter, Saturn, Uranus and Neptune. The magnetosphere of Uranus, shown by means of instrumentation on Voyager 2 spacecraft, has proved to be a rich environment for study of magnetospheric interactions. In addition to the standard array of magnetospheric phenomena ranging from upstream ion events to the bow shock and magnetopause boundary structures from an intense inner region of hard particle radiation to a soft magnetotail plasma sheet population in the vicinity of magnetotail neutral sheet, the magnetosphere of Uranus presents unusual characteristics. The most exceptional plasma waves were found inside nearly $23R_U$, by the plasma wave receiver on board. The frequency of these waves lie between 10-100 Hz. In every case, the upper frequency limit of the emissions is found to be less than localgyrocyclotron frequency [Kurth et al.1987; Kurth et al. 1989].

The Voyager 2 plasma wave observations during the encounter at Uranus [Gurnett et al. 1986; Kurth et al. 1986] revealed the presence of electrostatic and electromagnetic plasma turbulences, low frequency radio emissions and also non-Maxwellian high-energy tail distribution. The examination of data of Uranus plasma waves from the inner magnetosphere was based on wide-band observations and 16-channel measurements covering the range 10 Hz to 56 kHz, which gave details on low frequency radio emissions, electron gyrofrequency harmonics and strong wave activity. Coroniti et al. [1987] used the observed energetic ($E > 22$ keV) electron distribution and the cold plasma density profile deduced by Kurth et al. [1987] to calculate the ray path integrated spatial amplification of whistlers which arrive at Voyager 2 from the magnetic equator. Hudson et al. [1898] and Clark et al. [1987] investigated the process of generation of electric field in Uranian magnetosphere and reported its magnitude as used by Pandey et al. [2001] in their theoretical magnetospheric studies. In magnetosphere and in shock regions, AC electric field observations have been reported perpendicular and along the magnetic field [Wygant et al. 1987]. Voyager 2 encounter of Uranus on 24 January 1986 revealed a strong planetary magnetic field, an associated magnetosphere and fully bipolar magnetic tail of Uranus. The results from Voyager2 plasma wave instrument show radio emissions
from Uranus about 5 days before the closest approach at frequencies 31.1 and 56.2 kHz and then about 10 hours before closest approach the bow shock was identified [Gurnett et al. 1986]. The instrument also detected that inner magnetosphere is characterized by both the types of strong whistler mode emissions, chorus and hiss. On the basis of the frequency range, these detected waves were identified as whistler mode waves ($f_c = 28$ B Hz, where B is in units of nanotesla).

A preliminary analysis of Voyager 2 observations [Gurnett et al. 1986] revealed planetary magnetic field of Uranus. Examination of the magnetic field data recorded in close proximity to the shock, unfolded the series of whistler wave events that appeared to be generated from processes associated with the shock. The reports discussing the observations made by Voyager 2, show enhanced wave activity at frequency almost 0.1-0.5 times of electron cyclotron frequency. These were interpreted as whistler hiss and chorus by Scarf et al. [1987]. Whistlers are generated by cyclotron resonance interaction with energetic electrons. Due to loss cone in distribution of electrons, anisotropy is produced which gives rise to free energy source [Gurnett et al. 1986]. Therefore, the study of resonant interactions between whistler mode waves and electrons are extremely important for deep understanding of planetary magnetospheres. It performs controlling role for terrestrial radiation belts [Horne et al., 2005; Summers et al. 1998]. The plasma wave spectrum at Uranus is dominated largely by whistler mode emissions [Gurnett et al. 1986]. Whistler mode waves propagating in the magnetosphere of Uranus provide a variety of interesting effects. Whistler waves are also important for the survey of wave phenomena. For the detailed study of such mode of waves, it can be assumed that plasma consist of two species of electron; hot and cold electrons. Along with high frequency radio emission, whistler radiations, which exist as a steady hiss and at closest approach in the form of rising chorus, were also identified by Voyager-2. Scarf et al. [1987] discussed in detail the characteristics of Uranus waves with frequency ($f$) less than gyrofrequency ($f_c$). Since the model used by them yielded a high value for electron plasma frequency, it was interpreted as whistler hiss and chorus. The entire closest approach to Uranus is characterized by the band of emission below about $f_c/2$. The band is very intense especially on the outbound leg of the encounter and could be more intense at the magnetic equator. Also the wave instrument carried by Voyager 2 spacecraft detected a continuous nad of signals with $f < 0.5 f_c$ during the inner magnetosphere transversal.
So at the radial distance of less than 8 $R_U$ intense whistler mode waves, chorus and hiss, were reported along with the strong fluxes of energetic electrons.

Electron cyclotron waves were also reported in the same region of inner magnetosphere (less than 8 $R_U$) of Uranus [Gurnett et al. 1986]. They are believed to be generated by cyclotron resonance interaction with energetic electrons. From the anisotropy generated due to the loss cone distribution of electrons that are trapped, free source of energy comes. Gurnett and his team also reported the presence of electron cyclotron waves in the region of inner magnetosphere of Uranus [Gurnett et al. 1986]. The electromagnetic electron cyclotron instability can be considered as one of the most important topic to study in space physics. Another detailed analysis of the measurements done by Voyager 2 were reported by Bridge et al. [1986]. It shows that for $L > 5$, Uranian magnetospheric plasma comprises of hot components of few kiloelectron volts. And cold components were observed both inside and outside of $L=5$ with 4-50 electron volts temperature [Bridge et al. 1986]. Therefore we chose to study electromagnetic electron cyclotron waves driven by energetic electrons of thermal energy 100 eV and cold ions of 10 eV [Pandey and Kaur, 2012; Pandey et al. 2001].

Kennel and Engelmann [1966] considered velocity space diffusion for waves propagating at specific angle with respect to planet’s magnetic field. In a similar way, theory of resonant interaction between energetic charged particle and whistler cyclotron waves propagating obliqueto non-uniform geomagnetic field in inhomogenous plasma was studied [Shklyar & Matsumoto 2009]. The work considered Hamiltonian approach to analyze particle equation of motion. In contrast to this methodology, we have adapted kinetic approach to investigate particle equation of motion, in present study. A detailed review on the kinetic electromagnetic instabilities and importance of this approach, provided by Cuperman [1981], is referred. It dealt with effect of cold plasma addition, variation of growth rates, acceleration of heavier ions and results of computer simulation experiments explaining the transformation of distribution function of particles and also the wave activity. The studies show that ion cyclotron noise couples to ions and similarly whistler mode noise give rise to pitch angle diffusion of electrons. This diffusion advances to particle precipitation and thus producing a distribution of trapped particles which does not support wave growth further [Kennel & Petschek 1966].
Low-Energy Charged-Particle (LECP) instrument on Voyager 2 revealed that magnetospheric particle population principally consists of electrons and protons. And electron intensities substantially exceed proton intensities at a given energy [Krimigis et al. 1986]. The evidences given by Plasma Science Experiment on Voyager 2 suggested that distribution functions at Uranus must be characterized by a “warm” (i.e., subsonic) core and a non-Maxwellian tail that varied significantly along the spacecraft trajectory. The non-Maxwellian tail carried most of the plasma energy density in the observed energy range (<6 keV), which is small compared to the energy density of the planetary magnetic field. [McNutt Jr. Selesnick & Richardson 1987]. Therefore, on the basis of above mentioned observation, we concluded that natural space plasmas in the magnetosphere of Uranus owns a non-Maxwellian tail distribution which can be studied by kappa distributions. So, kappa distribution is employed instead of usual Maxwellian distribution for particles [Kurth 1992; Vin„as, Mace & Benson, 2005].

A substantial increase in the energetic electron precipitation was found by the injection of very modest amount of cold plasma into the radiation belt of magnetosphere [Brice et al. 1971]. Since then, because of the possibility of excitation of specific modes in magnetosphere, more investigations have been done in whistler mode instability. Parallel propagating plasma waves in the proximity of magnetosphere at very low frequencies have been studied by many workers. Using a series of 1-d simulations, the growth of whistler wave was studied by Zhang et al. [1993] from an anisotropic electron beam of various electrostatic and electromagnetic wave modes at various propagation angles.

1.4. Methodology

A surprising feature of near-Uranus space plasma is that it exhibits traits of an open-system. Its dynamics are strongly controlled by solar, geomagnetic activity and by ambient geomagnetic field. Thus according to plasma physics as subject, the dynamics of the fields and charged particles trapped in magnetosphere of Uranus, may be treated as an issue of wave-particle interactions occurring in a magnetized plasma. The results of these interactions conclude itself as an enhancement of particle distribution and wave fields.
1.4.1. Wave Particle Interactions

The acceleration and scattering of particles, the damping or the growth of waves and the emission of radiations are the consequences of wave-particle interaction. The interactions in which wave exchanges energy with the particles, play crucial role in several phenomena occurring in space plasma [Gary 1992] and in laboratory [Gill 1981]. Resonant wave-particle interaction is an elemental agenda, particularly in magnetospheric physics. Since low frequency waves interact with charged particles over long scale lengths within magnetospheres and can transfer energy from one region to another, we chose to study these low frequency waves in an inhomogenous magnetosphere of Uranus. Also, as compared to characteristic time scale of the system, the collision time between the particles is very long. Therefore the space plasma is treated as collisionless, which rules out the possibility of particle-particle collisions.

Magnetized space plasma supports a variety of plasma waves. The investigations done in planet’s orbit have showed two general classes of wave-particle interactions, controlling the important facet of plasma dynamics in magnetospheres, namely, electromagnetic and electrostatic plasma instabilities. The resonant interactions between particle and electromagnetic waves have been dealt by researchers [Kennel and Petschek 1966, Lyons and Williams 1984]. Such narrow banded spontaneous emissions scatter particles in the loss cone, that leads to modified pitch-angle distributions, diffusion of energy etc.

1.4.2. Kinetic Theory

Since plasma consists of enormous number of particles and waves interacting with each other, it is suitable to adopt a statistical approach to present a macroscopic description of magnetospheric plasma phenomena. In plasma kinetic theory, it is necessary to describe plasma in terms of particle distribution function. The particle distribution function presents the particle density in six-dimensional phase space for particular species. This function is a continuous function of its argument, always positive and finite at any point of time. Also, it tends to zero for infinitely large velocity.

The kinetic approach was developed by Lee and Kaw [1972] and Kaw [1990]. Inhester [1990] used a drift-kinetic methodology to show that maximum growth rate obtained by fluid treatment is reduced by thermal effects. It emphasized the need for a
full kinetic theory or treatment of the problem. To characterize the conditions for growth rate and properties of plasma instabilities, we use kinetic dispersion theory [Brinca and Tsurutani 1989a, 1989b; Gary 1993]. The linear theory of electromagnetic waves for uniform cold plasma is given in regular texts [Stix 1992].

1.4.3. Distribution Functions

Research requires the knowledge of plasma properties for various distribution functions. Recently, many researchers have explored various plasma regimes for instabilities using different distribution functions with different techniques. In this thesis we have dealt whistler mode waves with generalized distribution function. And according to data reported by Voyager 2, we have applied Lorentzian kappa distribution to analyze electron cyclotron and ion cyclotron waves in magnetoplasma of Uranus. These distribution functions have dependence on velocity in phase space.

1.4.3.1. Generalized Distribution Function

For Maxwellian velocity distribution to be applied, the plasma should be in thermal equilibrium with no exchange of energy taking place between the particles of plasma. But space plasma is not as simple as isotropic Maxwellian. The presence of magnetic field in planet’s magnetosphere results in different particle velocities in direction perpendicular ($v_\perp$) and parallel ($v_\parallel$) to magnetic field, thus, introduces an anisotropy. Since these two velocity ($v_\perp$ and $v_\parallel$) components are independent, they are modelled as product of two Maxwellians [Baumjohann and Treumann 1997]. Therefore, generalized distribution function is written as:

$$f_0(v) = \left( \frac{n_0 v^{2j}}{\pi^{3/2} \alpha_\perp^{2j+1} \alpha_\parallel^j} \right) \exp \left[ -\left( \frac{v_\perp}{\alpha_\perp} \right)^2 - \left( \frac{v_\parallel}{\alpha_\parallel} \right)^2 \right]$$

Here $\alpha_\perp$ and $\alpha_\parallel$ are the thermal energies, $\alpha_\perp = \sqrt{2K_B T_\perp/m}$ and $\alpha_\parallel = \sqrt{2K_B T_\parallel/m}$. For spectral index $j=0$, the distribution scales down to bi-Maxwellian distribution function and for $j=1$, it becomes loss-cone distribution function. In inner magnetospheric plasma, mirroring particles possessing high velocities will lose inside the loss cone, thus giving rise to loss-cone distribution. These particles actually had Maxwellian distribution and may be lost to ionosphere due to more velocity parallel to magnetic field [Baumjohann and Treumann 1997]. Pandey et al. [2001] and Ahirwar et al.

1.4.3.2. Kappa Distribution Function

Solar wind in near-planet space plasma, exhibits non-thermal particle distributions [Feldman et al. 1975; Montgomery et al. 1968; Maksimovic et al. 1997; Pilipp et al. 1987; Zouganelis 2008]. Solar wind particles show enhanced suprathermal deviations from Maxwellian equilibrium, reduced as a power law of velocity. They are expected to prevail in plasma with low density found in Universe, where collision of charges are limited [Pierrard and Lazar, 2010]. So these plasma particle velocity distribution is well explained by the family of kappa or generalized Lorentzian velocity-distribution functions (VDFs):

$$f_i^k(r, v) = \frac{n_i}{2\pi(\kappa \alpha_{si}^2)^{3/2}} \frac{\Gamma(\kappa + 1)}{\Gamma(\kappa - 1/2)\Gamma(3/2)} \left(1 + \frac{v^2}{\kappa \alpha_{si}^2}\right)^{-(\kappa+1)}$$

and $\alpha_{si} = \sqrt{(2\kappa - 3)kT_i/km_i}$

Where $\alpha_{si}$ is thermal velocity, $m_i$ is the mass of particle of species $i$, $T_i$ equivalent temperature, $n_i$ its number density, $v$ its velocity and $\Gamma(x)$ is the gamma function. Following the previous work done [Xiao et al. 2007] the instability threshold condition for this distribution reduces as $\kappa$ increases and leads to the lowest values of bi-Maxwellian distributions $\kappa \rightarrow \infty$.

Voyager 2 encounter revealed that natural space plasmas in the magnetosphere of Uranus contains a non-Maxwellian distribution which is better investigated by kappa distributions or VDFs [Kurth, 1992; Vinas et al. 2005]. The work in recent past has showed that highly energetic electrons are found to be better fitted by kappa-type distributions [Xiao 2006, Xiao et al. 2008a]. This kappa distribution function fits very well for solar particles of high energy [Xiao et al. 2008b], and also well with the geostationary orbit electrons [Xiao et al., 2008c]. Therefore, in this study we consider energetic electrons shaped with a kappa distribution, driving the whistler mode wave instability.