CHAPTER 5
PROPOSED METHODOLOGY FOR MULTI LEVEL THRESHOLD METHOD

This chapter provides overview of the proposed Multi-level threshold technique and Canny method. Multi-level threshold is an improvement of Canny edge-detection technique as discuss in previous chapter. The most important objective of the proposed multi-level OTSU edge-detection scheme is to perform improved and appropriate image edge detection. Multi-level edge-detection technique begin with linear filtering to calculate the value of gradient for an image by the help of intensity distribution function and then thinning process and process of thresholding is completed to get a binary-map of edges. For eliminate texture edge, surround-suppression can be used as general added step between the linear filtering and the thresholding process to obtain mainly including contours and region outlines in the output images.

5.1 INTRODUCTION OF THRESHOLD ALGORITHM

In image segmentation, thresholding technique is play a key role for recognize and pull out a meaningful information from the input image on the basis of the allocation of gray levels in image objects, it means the fundamental idea of thresholding is to choose best possible gray-level threshold value for separating objects of interest in an image from the background based on their gray-level distribution.

A thresholding algorithm will typically classify pixels in two different category or two cluster of objects; first one that have their intensity lower than a certain threshold (generally, the background), and the second one cluster have the features of interest (foreground). For developing the image thresholding algorithms, gray-level histogram of image is taking as an efficient tool[4]. Thresholding process convert gray image into binary image by mark all pixels whose intensity is below a certain value of threshold as zero and all other pixels whose value is equal to or above that threshold value mark as one.
g(x, y) stand for the threshold value of function f(x, y) at some global threshold T, it is also defined as

\[ g(x, y) = 1 \text{ if } f(x, y) \geq T = 0 \]

(5.1)

Otherwise Thresholding process describes as:

\[ T = M[x, y, p(x, y), f(x, y)] \]

(5.2)

where T shows the threshold level; f(x, y) shows gray value of point (x, y). p(x, y) indicate some local property of the point such as the average gray value of adjacent pixels centered on point (x, y) on basis of this idea, thresholding is classified in two methods.

- **Global thresholding:** In global thresholding method, value of threshold T at any point decided by only on gray-level values of that point.

- **Local thresholding:** In this method, value of threshold T based on both f(x, y) and p(x, y). In this technique, an image segmented into several sub regions, and goes for various thresholds T for each sub region reasonably.

Basics of Thresholding calculation for Global Thresholding are:

(i) choose an initial approximation value for Threshold T

(ii) Divide the whole image by that threshold value T. This makes two clusters of pixels. Pixels which have the gray-level values >T, grouped in to G1 and G2 containing of pixels which values is <=T.

(iii) Calculate the average gray-level values. Mean1 is the Average Value for the pixels in regions G1 and Mean2 the Average Value for the pixels in regions G2.

(iv) Calculate a new threshold level by the equation

\[ T = \frac{Mean1 + Mean2}{2} \]

(v) Repeat step 2 to step 4 until distinction in T consecutive iterations is smaller than a predefined parameter T0. Than get the threshold value for the global technique.

### 5.2 OTSU METHOD

OTSU method depends merely on gray-value of the image. In 1979, scholar OTSU was proposed this method. It is a global thresholding selection method and because it is simple and effective therefore OTSU method widely used. This method needs calculation for gray-level histogram before running. Because it used one-dimensional gray-level information for segment the image due to this, it not gives improved
segmentation outcome. Therefore, a method which is based on both gray-level threshold of each pixel and pixels spatial correlation information surrounded by the neighborhood was proposed and this algorithm is called two dimensional OTSU algorithms. OTSU algorithm is able to get suitable segmentation outcomes when it applied to the noisy images[8]. OTSU’s method is one of the improved threshold selection methods for broad real-world images with regard to homogeneity and shape measures. On the other hand, an exhaustive search is used to evaluate the condition for maximizing the between class variance in OTSU method. When the number of classes increases, classes which is based on pixels intensity in image, OTSU’s method gets extra time for multi-level threshold selection[10] but it provides improved results than others method[40]. It is well-known; this technique gives finest value of threshold in the statistical sense along with most appropriate method in the image segmentation. Our work proposed a technique that it confirms a best threshold assessment by OTSU method which is able to gets high threshold value of Canny operator and then extracts the edge of image by using the Canny detector and outcomes will indicate that the method which has proposed is reliable and practicable.

Multilevel thresholding have more than two threshold level to divide image into different clusters and find the maximum inter-class variance between these cluster and minimum intra-class variance within any cluster. In multi-level OTSU thresholding method, image I segmented into N classes. Multi-level OTSU returns an array (A) containing the cluster indices (from 1 to N) of each point. It denoted as OTSU (I, N).In case if N is not defined then it assume 2 by default. Syntax [A, sep] = OTSU (...) returns the value (sep) of the separability criterion within the range [0 1]. Zero values are assigned to non-finite pixels. Zero is obtained only with data having less than N values, whereas one is obtained only with N-valued arrays. 

It should be noticed that the thresholds generally become less credible as the number of classes (N) to be separated increases. If I image is a RGB image, then it first transform (color to binary/indexed conversion) into three red, green, blue channels. The segmentation is then carried out on the image component that contains most of the energy[44].
5.3 CANNY OPERATOR THEORY

Canny edge-detector is formed by first-order derivative of Gaussian function. Canny operator is symmetry on edge and dissymmetry on vertical edge’s direction \(^7\) because Gaussian function has circular symmetry. First of all, Canny operator processes the image smoothly through Gaussian convolution, then gets gradient of image by differential operation to the image which is processed via Gaussian convolution; subsequently, non-maximum suppression algorithm is applying to find the possible edge points; finally, using the double-threshold process, obtain the edge position in image and pull off the edge with only one pixel wide\(^{11}\). The Gaussian blur is a kind of image-blurring filters to facilitate Gaussian function (which also expresses the normal distribution in statistics) for computing the transformation for applying to every pixel in the image. The formula for Gaussian function in one dimension is given below

\[
G(x) = e^{-\frac{x^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} \]  \hspace{1cm} (5.3)

For two dimensions, it’s the product of two such Gaussians, one in each dimension:

\[
G(x, y) = e^{-\frac{(x^2+y^2)}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} \]  \hspace{1cm} (5.4)

Where \(x\) is the distance from the starting point in the horizontal-axis, \(y\) indicates the distance from starting point in the vertical-axis, and \(\sigma\) is the standard deviation of the Gaussian distribution. This formula creates a surface whose contours are concentric-circles with Gaussian-distribution from the middle point when it used in two dimensions. Latest value of every pixel's is set to a weighted average of that pixel's neighborhood. The original pixel's value gets the extra weight (containing the peak Gaussian value) and neighboring pixels obtain lesser weights as pixels distance increases from the original pixel. This result in a blur that conserves borders and edges improved than other method.

In above equation, \(\sigma\) is a distribution parameter of Gaussian function used for manages the process for smoothing of image. Making use of Gaussian function’s separability, the two filtration convolution templates of \(\nabla G\) can be decomposed to two one-dimension range filters:
\[ \frac{dG}{dx} = k \cdot x \cdot \exp \left( -\frac{x^2}{2\sigma^2} \right) \cdot \exp \left( -\frac{y^2}{2\sigma^2} \right) = h_1(x) \cdot h_2(y) \]  \hspace{1cm} (5.5)

\[ \frac{dG}{dy} = k \cdot y \cdot \exp \left( -\frac{x^2}{2\sigma^2} \right) \cdot \exp \left( -\frac{y^2}{2\sigma^2} \right) = h_1(y) \cdot h_2(x) \]  \hspace{1cm} (5.6)

In above equation

\[ h_1(x) = \sqrt{k} \cdot x \cdot \exp \left( -\frac{x^2}{2\sigma^2} \right) \] \hspace{1cm} (5.7)

\[ h_1(y) = \sqrt{k} \cdot y \cdot \exp \left( -\frac{y^2}{2\sigma^2} \right) \] \hspace{1cm} (5.8)

\[ h_2(x) = \sqrt{k} \cdot \exp \left( -\frac{x^2}{2\sigma^2} \right) \] \hspace{1cm} (5.9)

\[ h_2(y) = \sqrt{k} \cdot \exp \left( -\frac{y^2}{2\sigma^2} \right) \] \hspace{1cm} (5.10)

\[ h_1(x) = x \cdot h_2(x) \] \hspace{1cm} (5.11)

\[ h_1(y) = y \cdot h_2(y) \] \hspace{1cm} (5.12)

and \( k \) is constant

Then, after convoluting the image \( f(x, y) \) and the Eq.(5.5) and Eq.(5.6) respectively, it can obtain

\[ M_x = \frac{dG}{dx} \cdot f(x, y) \] \hspace{1cm} (5.13)

and \[ M_y = \frac{dG}{dy} \cdot f(x, y) \] \hspace{1cm} (5.14)

Commands: \( T(i, j) = \sqrt{M_x^2(i, j) + M_y^2(i, j)} \) \hspace{1cm} (5.15)

\[ \theta(i, j) = \arctan \left[ \frac{M_y(i, j)}{M_x(i, j)} \right] \] \hspace{1cm} (5.16)

To pursue, the \( T(i, j) \) reproduces edge at the point \( (i, j) \) and \( \theta(i, j) \) reproduces the normal vector at the point \( (i, j) \) in a image.

In summing up, the thorough algorithm step of Canny operator as follows:

(i) Gaussian filter is applied to make the image noise-free (wipe off noise);

(ii) By using First-order differential coefficient method, image’s gradient intensity and directions of filtrate the image are received;
(iii) Non-maximum suppression, for this process quantized an image into pixels value.

All the pixels are surveyed, the algorithm will scan the image again for checking and comparing the value of pixel with thresholding value and all the marked pixels value will be replaced by gray value zero and pixel value more than threshold value replaced with one. Then mark the edge by suppression of weak edges. Edge association or linking by scan the image until meet the first non-zero pixel then follow the counter line from begins to destination point\(^{[1]}\). When it complete the connection of contour line which include the origin point S, algorithm marks it accessed and afterward come again on the primary step to search another contour line and algorithm will repeat steps while find the new contour line in image. By Above mentions all steps, detection by Canny edge method has been finished.

5.4 OTSU METHOD PRESENTATION

It is well known that the OTSU method is a very advantageous method of choosing threshold value automatically which was proposed in 1979\(^{[4]}\). The fundamental principle is divided the image’s pixels in to two classes and confirms the finest threshold value through the variance maximum value from that classes.\(^{[8][9][10]}\)

In OTSU’s method, best possible threshold value is select through minimizing the within-class variance of the two group of pixels or in other words optimal threshold is selected by maximizing the between class variance of two group. It is histogram based thresholding method such that a threshold value is selected from gray level histogram which results in reduction to binary image from gray-level image. It supposes a bimodal distribution of gray-level values such as foreground and background.\(^{[44]}\). This method iterate through all the possible threshold values and calculates a spread for the pixel levels on each side of the threshold, i.e. the pixels that either belongs to foreground or background. The key thought is to obtain the threshold value where the addition of background and foreground spreads is at its minimum.\(^{[2]}\)

An image is a two dimension gray scale intensity function, and includes N pixels by gray levels from 1 to L. The number of pixels along with gray level I is indicated by \(f_i\), giving a probability of gray level I in an image of

\[ P_i = \frac{f_i}{N} \]
Let $M*N$ is an image histogram with $L$ intensity level, i.e. $[0….L-1]$.

The numeral of pixels along with gray level $i$ is showed $n_i$, such that

$$M = \sum n_i$$

Lower limit is $I = 0$, higher limit $L-1$ so probability of gray level $I$ in an image is:

$$P_i = \frac{n_i}{MN} \quad \text{.................................................................}(5.17)$$

$$\sum_{i=0}^{L-1} P_i = 1 \quad \text{.................................................................}(5.18)$$

where $P_i \geq 0$

Using a threshold say $k$ pixels of image are partitioned into two classes say $C_1$ and $C_2$ (e.g., object and background) such that $0<k<L-1$, as threshold, $T=k$. the gray level probability distributions for two classes $C_1$ (pixels in $[0, k]$) and $C_2$ (pixels in $[k+1, L-1]$) are:

$$P_1 = P(C_1) = \sum_{i=0}^{k} P_i$$

$$P_2 = P(C_2) = (x+\alpha)^2 = \sum_{i=k+1}^{L-1} P_i = 1 - P_1$$

Where $P_1$ is Probability of class $C_1$ and $P_2$ is Probability of class $C_2$. Means $m_1$ and $m_2$ for class $C_1$ and $C_2$ respectively are given by following equations:

$$m_1 = \sum_{i=0}^{k} i.P(i/C_1)$$

$$m_1 = \frac{1}{P_1} \sum_{i=0}^{k} i.P_i$$

Where $P(C1/i)=1$, $P(i) = P_i$ e $P(C1) = P_1$

$$m_2 = \frac{1}{P_2} \sum_{i=k+1}^{L-1} i.P_i$$

Mean global intensity, $m_G$:

$$m_G = \sum_{i=0}^{L-1} i.P_i \quad \text{.................................................................} (5.19)$$

While the mean intensity up to the $k$ level, $m$:

$$m = \sum_{i=0}^{k} i.P_i$$

Hence

$$P_1m_1+P_2m_2 = m_G \quad \text{.................................................................} (5.20)$$

$$P_1+P_2 = 1$$
The global variance, $\sigma^2$ is given by:

$$\sigma^2 = \sum_{i=0}^{L-1} (i - mg)^2 . P_i \quad \text{………………………………………….. (5.21)}$$

Hence between classe variance of the threshold image is defined by OTSU as:

$$\sigma^2 = (MGp_1 - m)^2 / p_1(1 - p_1) \quad \text{………………………………………….. (5.22)}$$

### 5.5 GET HIGH AND LOW THRESHOLD WITH OTSU ALGORITHM

In image processing and computer vision OTSU method is used to execute clustering based image thresholding or the diminution into a binary image from a gray-level image. The algorithm assume that the image allocate threshold includes two classes/clusters of pixels or bi-model histogram (foreground and background) then computes the best possible threshold dividing those two classes so that their joined spread is smallest $^{[29]}$. The enhanced form of the original method is taking as the multi-level threshold OTSU method.

**Method:** - In OTSU method, exhaustively look for the threshold to diminish the intra-class variance, defined as a weighted sum of variances of two classes-

$$\sigma_{w}^2(t) = w_1(t) \sigma_1^2(t) + w_2(t) \sigma_2^2(t) \quad \text{………………………………………….. (5.23)}$$

Weight $w_i$ is the probability of two classes divided by a threshold $t$ and $\sigma_i^2$ variances of these classes. OTSU indicates that reducing the intra-class variance is the similar as maximizing inter-class variance:

$$\sigma_b^2(t) = \sigma^2(t) - \sigma_{w}^2(t) = w_1(t) w_2(t) [\mu_1(t) + \mu_2(t)]^2 \quad \text{………………………………………….. (5.24)}$$

Which is showed in conditions of class probabilities $w_i$ and class means $\mu_i$.

The class probability $w_1(t)$ is calculated from the histogram as $t$:

$$W_1(t) = \sum_0^i p(i)$$

even as the class mean $\mu_1(t)$ is:

$$\mu_1(t) = \frac{\sum_0^i p(i) x(i)}{W_1(t)} \quad \text{………………………………………….. (5.25)}$$

Where $x(i)$ is the value at the middle of the $i^{th}$ histogram bin. Likewise, $w_2(t)$ can compute easily and $\mu_2$ on the right-hand side of the histogram for bins greater than $t$. The class means and class probabilities can be computed iteratively. This proposal yields an effectual algorithm.
5.6 CONNECT EDGES
In graph theory, a graph is k-edge-connected if it remains connected whenever fewer than k edges are removed. The edge-connectivity graph is largest k for which the graph is k-edge-connected.

5.7 EDGE THINNING
It is a method applied to eliminate the unwanted spurious points on the edges in an image. This technique is employed after the image has been filtered for noise (using median, Gaussian filter etc.), the edge operator has been applied to detect the edges and after the edges have been smoothed using an appropriate threshold value. This removes all the unwanted points and if applied carefully, results in one pixel thick edge elements.

5.8 ALGORITHM
The steps are in this algorithm are as below:
Step 1: Read Image and change image in to gray level and apply Linear filtering for removing noise from the image
Step 2: Calculate Lx Intensity distribution function.
Step 3: Calculate Ly Intensity distribution function
Step 4: Canny edges detection.
Step 5: Calculation of norm of gradient.
Step 6: Thresholding which is depends on OTSU method.
Step 7: Thinning for sharp edges.
Step 8: After 5 levels thresholding get edge.
Step 9: After 4 levels thresholding get edge.

5.9 PERFORMANCE PARAMETER
In thesis work the multi-level threshold method with following parameters are analyses. Here some brief details of this parameter are given below.
5.9.1 VARIANCE

Variance show how far a set of numbers is spread out. Zero variance indicates that all the values are identical. It is always non-negative: a small variance shows that data point is very close to the mean. While a high variance indicates that the data points are very spread out around the mean and from each other. Let $X$ is second central moment of the variance, the expected value of the squared deviation from the mean

$$\mu = E[X]:$$

$$\text{Var}(X) = E[(X - \mu)^2] \quad \cdots \cdots \cdots \cdots \cdots \cdots (5.26)$$

This definition encompasses random variables that are discrete, continuous or mixed. The variance can also be thought of as the covariance of a random variable with itself:

$$\text{Var}(X) = \text{Cov}(X,X) \quad \cdots \cdots \cdots \cdots \cdots \cdots (5.27)$$

5.9.2 MEAN

For a data set, the terms arithmetic mean, mathematical expectation, and sometimes average are applied to impart to a central value of a distinct group of numbers. Specifically sum of the values divided by the number of values. The mathematics mean of a group of number $x_1, x_2, ..., x_n$ is typically denoted by $\bar{x}$, pronounced bar. If the data element was pedestal on a sequences of annotations achieved by sampling from a statistical population, the mathematics mean is termed the sample mean (denoted $\bar{x}$) to distinguish it from the inhabitants mean\textsuperscript{12}. Supportive of a fixed population, the population mean of a property is equal to the arithmetic mean of the given property while considering every member of the population. For example, the population means height is equivalent to the summation of the heights of every individual divided by the total number of individuals \textsuperscript{30}. The sample mean possibly will quit differ from the population mean, mainly for small samples. The law of enormous numbers dictates that larger the size of sample, the additional probable it’s that the samples mean will be close to the population mean\textsuperscript{24}. Outside of probability and statistics, a large variety of other notions of mean are frequently used in geometry and analysis; examples are given below.

The arithmetic mean (or simply mean) of a sample $x_1, x_2, x_3, \ldots, x_n$ is the sum of the sampled value divided by the number of objects in the sample:
\[ x = (x_1 + x_2 + x_3 + \ldots + x_n)/n \]  \hspace{1cm} \text{-------------------------------------- (5.28)}

5.9.3 SIGNAL TO NOISE RATIO (SNR)

The SNR in decibels, it is straignt index to evaluate the fused image to the reference one \(^[7]\). For multiband images, it can be computed band-by-band and also globally averaged, SNR-

\[ SNR(Z, Z') = 10 \log_{10} \frac{\sum z^2}{\sum (z - z')^2} \]  \hspace{1cm} \text{-------------------------------------- (5.29)}

Signal-to-noise ratio (often abbreviated SNR or S/N) is generally used in science and engineering that measure up to the level of a desired signal to the level of background noise. It is described as the ratio of signal power to the noise power, commonly showed in decibels unit.

A ratio higher than 1:1 (greater than 0 dB) indicates more signal than noise. While SNR is commonly quoted for electrical signals, it can be applied to any form of signal (such as isotope levels in an ice core or biochemical signaling between cells).

5.9.4 MEAN SQUARE ERROR (MSE)

In statistics, the mean squared error (MSE) of an estimator measures the average of the squares of the "errors", that is, the dissimilarity between the estimator and what is estimated. MSE is a risk function, analogous to the expected value of the squared-error loss or quadratic loss \(^[29]\). The dissimilarity arises due to randomness or because the estimator doesn't account for information that possibly will create a more precise estimate.

The MSE therefore evaluates the superiority of an estimator or set of predictions in forms of its variation and degree of bias. Because MSE is an expectation, it’s not technically a random variable, but it will be subject to estimation error when calculated for a particular estimator of with unknown true value. Thus, any estimation of the MSE on the basis of an estimated parameter is in fact random variable\(^[23]\). The objective of a signal reliability compute is to match up to two signals by provided that a quantitative score that explain the degree of likeness/reliability or, conversely, the level of error or distortion between them. Normally, it is supposed that one of the signals is a immaculate original means
estimator, even as the other is damaged or contaminated by errors, it can call estimated signal.

The MSE value gives the average difference of the pixels throughout the original image with edge detected image. The higher MSE indicates a greater difference between the input image and resultant image. The MSE value is calculated using this equation:

\[
MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|f(i, j) - g(i, j)\|^2 \tag{5.30}
\]

Where \( f \) shows the matrix-data of original image, 
\( g \) stand for the matrix data of degraded image in question, 
\( m \) stand for the numbers of rows of pixels of the images, 
\( i \) shows the index of that row \( n \) represents the number of columns of pixels of the image and \( j \) shows the index of that column.

### 5.9.5 UNIVERSAL IMAGE QUALITY INDEX (UIQI):

For image resemblance assessments, UIQI parameter is mostly used. UIQI is simple to compute and easy to employ on different image processing applications. Image distortion is collection of three factors: loss of correlation, luminance distortion and contrast distortion. UIQI of two images (A and B) is defined as

\[
Q = \frac{4\sigma_{AB} \mu_A \mu_B}{(\sigma_A^2 + \sigma_B^2)(\mu_A^2 + \mu_B^2)} \tag{5.31}
\]

Where \( \mu_{AB} = \text{Mean of image AB} \); \( \mu_A = \text{Mean of image A} \)
\( \mu_B = \text{Mean of image B} \); \( \sigma_{AB} = \text{Variance of image AB} \)
\( \sigma_A = \text{Variance of image A} \); \( \sigma_B = \text{Variance of image B} \)

The vibrant assortment of \( Q \) is \([-1, 1]\) and the best value 1 is obtained if \( A = B \). When use this index to a multi-band image, it is applied band-by-band and averaged over all bands \([7]\).

A novel universal objective image quality-index is proposed for analysis of image, which is effortless to compute and appropriate to a range of image-processing applications. Instead of using conventional error summing up methods, the proposed index is designed by modeling any image distortion as a combination of three factors- loss of correlation, luminance distortion and contrast distortion.
5.9.6 PEAK SIGNAL TO NOISE RATIO (PSNR)

Peak Signal-to-Noise Ratio, is ratio of highest possible power of a signal to the power of degrading noise that affects the faithfulness of its demonstration. The PSNR generally showed in terms of the decibel scale (dB). Peak Signal to Noise Ratio is used to appraise image edge detection quality. PSNR is an irregular assessment to human acuity of restoration quality. Even though high values PSNR usually point out that the restoration is of higher quality in image processing techniques\textsuperscript{[4]}. The PSNR calculated based on the MSE by,

\[ PSNR = 10 \log_{10} \left( \frac{R^2}{MSE} \right) \] (5.32)

R is the maximal variation in the input image data. If it have an 8-bit unsigned integer data type, \( R^2 \) is 255.