Summary of the Ph.D. thesis in Mathematics entitled

A Study of System Behaviour with Fuzzy and Intuitionistic Fuzzy Sets
by
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Mankind has been attempting to know and predict the future since very early days, The determination of future behaviour was not limited to human beings alone, but was extended to utility items of daily use, machines or systems. With the fast development of science and technology during past few decades, expectations of the society in terms of automation of devices/systems are also increasing. Industries are introducing more complexities into the equipments/systems. User always remains interested in a satisfactory performance of the system for a longer duration. Industries therefore look forward towards mathematicians to develop such models that can provide them solutions of those problems that can predict, estimate or optimize the probability of survival and mean life of components and systems. This has made researches for system behaviour more important in the present day context.

No doubt Mathematics is an exact science, yet the input needed in real life application of mathematical results, is most often not exact. One needs linguistic approach for several variables that involve vagueness and cannot be measured exactly. Under the circumstances, the validity of mathematical application in real life problems sometimes comes under a question mark. The mathematical modeling of fuzzy concepts, given by Zadeh in 1965, is a landmark in this area. In fact fuzzy sets are mathematical objects modeling the vagueness present in our natural language, when we describe phenomena that do not have sharply defined boundaries. Atanassove in 1986 introduced
intuitionistic fuzzy sets that have been found to be very useful to deal with uncertainty of information. The concept of intuitionistic fuzzy set is a generalization of the fuzzy sets.

Present thesis entitled “A study of system behaviour with fuzzy and intuitionistic fuzzy sets” is an effort to study the system behaviour including reliability estimation of several important systems, using approaches based on the fuzzy and intuitionistic fuzzy sets. This thesis is, therefore, divided into two sections. Section A and B. Section A includes the results based on fuzzy set theory while Section B is based on the results on intuitionistic fuzzy set theoretic approach. In all, the thesis consists of six chapters.

**Chapter I : Introduction**

This chapter is introductory in nature that discusses fundamental concepts and definitions of reliability theory, fuzzy set theory and intuitionistic fuzzy set theory, to make the thesis complete in itself.

**Chapter II : A fuzzy set theoretic approach to fault detection and analysis**

Failure causing factors play an important role in the behaviour of any system. Out of several failure modes of a system, only a few play a vital role in governing the failure of the system. These failure modes need to be identified and the rest must be screened out in the analysis of the system. After a careful investigation of the facts, we propose in this chapter, a technique to screen out the failure modes using possibility coefficients.

This chapter thus develops a method of failure mode screening, based on fuzzy set theory. The new approach for failure mode screening provides comprehensive results that can be used to screen out failure modes. This work also provides measurement of failure causing factors using fuzzy sets to get more realistic results. Moreover the weights
of different precipitating factors are fuzzified resulting in the possibility coefficient of the particular failure mode. Further since the threshold value for failure mode is also represented by a fuzzy number, the possibility coefficient obtained in the form a fuzzy number can be compared with the threshold value using ordering of fuzzy numbers. All those failure modes, for which the computed possibility coefficient is less than the threshold value, need not be included in the final analysis of the system. Thus by way of screening out some failure modes, the analysis of the mathematical model becomes less complicated and more realistic. Conclusively, it can favourably be asserted that our approach of failure mode screening provides comprehensive results that can be used to screen out failure modes involving human judgment and vague operating conditions to the area of equipment maintenance management through the development of a fuzzy scheme embedded into a computer expert system.

**Chapter III : Reliability analysis of a system using fuzzy random variables**

Evaluation of system reliability often encounters the problem of subjective judgments, uncertain data and approximate system models. It is well recognized that the classical set theoretic approach in reliability analysis fails to address the uncertainty up to a considerable level. This is due to the fact that a system is composed of large number of components and its functional structure is very complex. This makes fuzzy reliability important.

There are certain structural parameters which partly exhibit random properties. Such parameters can not be modeled by random variable and require fuzzy random variables and fuzzy probability distributions. Therefore the probability theory alone may not be sufficient to deal with the problem of subjectivity. Using fuzzy random variables it
is possible to mathematically describe the uncertainty characterized by so called fuzzy randomness.

In the previous chapter of the thesis we used the concept of possibility coefficients to screen out the failure modes of systems in our effort to study system behaviour. In the present chapter we continue our study of systems in terms of fuzzy reliability evaluation. We deal with representing each quantitative data as a fuzzy real number that gives the value of a fuzzy random variable with certain fuzzy probability distribution. This approach makes it possible to simulate the system reliability by means of fuzzy reliability. Fuzzy failure rates and fuzzy mean time to system failure also have been estimated through this new approach.

**Chapter IV : Fuzzy fault tree analysis for diagnosis of cannula fault in power transformer**

In the earlier two chapters of this thesis we worked on fuzzy schemes for fault detection and reliability evaluation of any system, in general. Present chapter extends our study for the development of a fuzzy scheme for fault tree analysis of any general system. The functioning of the developed fuzzy scheme is demonstrated through the diagnosis of cannula fault in power transformer.

The involvement of a very large number of variables and their multiple interrelations make the design of a power transformer very complicated. This complicity in design of a power system and variations in operating conditions cause uncertain and random occurrence of faults. Fault Tree Analysis (FTA) has been proved to be a very effective tool to predict probability of hazard, caused by a sequence and combinations of faults and failure events. A fault tree is a pictorial representation of various combinations
of faults leading to hazard. In this analysis, first of all the hazard is explored and then the events causing this hazard are located. In conventional FTA the basic events are assigned a crisp number. However, there are various crucial and complex systems of great importance that impart vague characteristics. Due to the complexity of such systems and their vague nature, it is very difficult to obtain an adequate inference about the failure of these systems. Since a very limited statistics is normally available about the power transformer failure, thus it is unrealistic to obtain the probability of basic events up to a required accuracy by using probability distribution. Therefore, in the present chapter probabilistic considerations of basic events are replaced with possibilities.

The failure of power may interrupt various important operations and make a huge damage to the economy of any nation. Power transformer is one of the important electricity equipment used in power networks. Thus the fault diagnosis and its maintenance in power transformer is the utmost priority of power supply enterprises. Accurate failure statistics is crucial requirement for reliability estimation in power transformer failure. In a situation where failure data may not be obtained accurately due to various reasons, it is more practical to employ linguistic terms to express data value for failure of a particular event. Since a power transformer may be installed under different operating conditions, it is impractical to assign a single fuzzy number to the failure possibility of the basic events in fault tree analysis. To overcome this problem, we have categorized the operating conditions of a power transformer as “Worst Case Condition”, “Conducive Environment” and “Highly Conducive Environment” for a power transformer to work. By “Worst Case Condition” we mean a situation that rarely occurs that is the state of emergency. “Conducive Environment” is a normal state where most of
the transformers are installed. Highly Conducive environment is a very special and conducive environment created artificially to keep transformer cool and working for a very long time. Using the linguistic terminology given by different experts for the failure of power transformer working under different operating conditions, each basic event is assigned several fuzzy numbers. In our work in this chapter, we have also proposed a very precise and realistic approach based on PERT method to get a single fuzzy number for each basic event. Our approach uses fuzzy numbers and generalizes the PERT method to evaluate the failure possibility of each basic event to enable us to give more realistic estimates of failure possibility of basic events. The possibilities of basic events were considered to be triangular fuzzy numbers. Unlike previous techniques, here we deliberated over the operating conditions rigorously and assessed the weightage of each of them. The proposed method is observed to be very pragmatic and preclude of failure possibility for basic events.

Further since, all basic events do not contribute equally in failure of a system that is, in the occurrence of top event, so it is important to assess the importance of each basic event. We have, in our work, employed a very effective and computationally easy technique to obtain fuzzy important index. The implementation of proposed methods is demonstrated through the diagnosis of cannula fault in power transformer. We classified eighteen basic events, which lead to the occurrence of top event. We finally reached to the conclusion that the reliability of cannula and hence of Power Transformer may be improved by preventing occurrence of one of the basic events.
Chapter V : Reliability analysis of network systems using intuitionistic fuzzy sets

In section ‘A’ of this thesis we have studied several characteristics of system behaviour using fuzzy sets. The utility of the application of fuzzy sets depends on the capability of the user to construct appropriate membership functions, which are often very precise. The hesitation involved in the appropriate choice for the degree of belongingness in fuzzy sets lead to the introduction of intuitionistic fuzzy sets. Starting from the present chapter, in this part of the thesis, that is, section B, we confine our study of system behaviour using Intutionistic Fuzzy Sets. Intutionistic fuzzy sets consider the concept of membership functions for favourable as well as unfavourable cases separately to provide a more effective tool than the fuzzy sets for handling imprecision and uncertainty.

This chapter studies reliability of series and parallel network systems using the concept of intuitionistic fuzzy sets. Results of the network system reliability have been applied to illustrate the interrelation among the HTTP (Hyper Text Transfer Protocol) basic events through fault tree analysis. A fault tree is a graphical model of pathways that interconnect contributory events and conditions using series and parallel networks. In a fault tree, the symbol “+” and “.” means that the systems are connected in parallel and series respectively. The vague and imprecise data, like LOG data, web tools results etc are the integral part of Web Server reliability analysis. A LOG file consists of information regarding the service request from a system, the upcoming response and the genesis of information etc. that usually appear with imprecise and vague information. To address this vague and unreliable resource of data, we have assigned intuitionistic fuzzy sets to the basic events. We have obtained system reliability expressions for series and
parallel networks, when the reliabilities of each of the system components are prescribed either as a triangular intuitionistic fuzzy sets or the trapezoidal intuitionistic fuzzy sets. Application of these results has been done in fault tree analysis of Real Web Server Log data wherein the failure of a server is modeled as a combination of different basic and intermediate events. Operations on intuitionistic fuzzy sets have been used to evaluate the possibility of series and parallel system which ultimately lead to the determination of failure possibility of top event. Being based on intuitionistic fuzzy sets, our results clearly indicate the percentage of support associated with the result. It also throws light as to how many are against the result and how many hesitate to take any side. This analysis is very useful in those cases where the hesitating part is substantially great. In such cases the hesitation function is reduced by involving more evidences before any decision is taken for the result.

**Chapter VI: A study of water system using intuitionistic fuzzy correlation coefficient**

The present and the last chapter of this thesis continues our study of system behaviour based on intuitionistic fuzzy sets. It is devoted to the study of physico-chemical characteristics of water system of lakes, using a new concept of intuitionistic fuzzy correlation coefficient. It is quite important in statistical analysis of data to find the correlation coefficient between the variables or attributes. Although, the correlation coefficient defined on crisp sets has been studied in many works of conventional statistics yet it is useful and interesting to extend the notion of correlation to fuzzy and intuitionistic fuzzy sets.

In the study of different kind of systems one most often comes across vague data or values. To handle such kind of situations in the study of system behaviour, we in this
chapter, introduce the concept of correlation coefficient of intuitionistic fuzzy sets. The intuitionistic fuzzy correlation is a better tool than the fuzzy correlation coefficient, because in intuitionistic fuzzy sets we take account of membership as well as non-membership functions, instead of only membership function in fuzzy sets. This chapter also deals with the application of the intuitionistic fuzzy correlation to determine physico-chemical characteristics of water system of lakes around Jaipur city in Rajasthan. This study is significant because fishes from these lakes are consumed by people in the city; apparently healthy fishes may, in fact, be carrier of pollutants which biomagnifies in human bodies. For the analysis of water system, the source data has been taken from Neera et al. [78]. In this work we not only converted the source data into fuzzy data to force the data for all parameters to lie in the interval [0, 1] but also proposed the concept of proportion of variability.

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