Chapter 4

Analysis of Proposed Method for Distortion Reduction

4.1 General Consideration

Proposed method is based on the direct conversion receiver. Therefore analysis of classical direct conversion receiver is required for understanding of the proposed method and its distortion reduction ability. Next section describes the analysis of conventional DCR and then the analysis of proposed method is presented. Later in the chapter, distortion reduction ability of the proposed method is explained. At last the algorithm for computation of calibration coefficients is described.
4.2 Analysis of Classical DCR

A direct conversion receiver is shown in figure 4.1. Antenna receives the RF signal and apply that signal to the preselect filter for selection of the desired band.

![Diagram of Classical Direct Conversion Receiver]

It is important to distinguish between the band and the channel: the former includes the entire spectrum in which the users of a particular standard are allowed to communicate (e.g., the GSM receive band spans 935 MHz to 960 MHz), whereas the latter refers to the signal bandwidth of only one user in the system (e.g., 200 kHz in GSM)[8].

Preselect filter select the desired band signal and apply it to the low noise amplifier (LNA). Major functions of the LNA are (1) to amplify very weak RF signal, (2) as per “Friis formula”, contribute minimum noise to maintain overall noise figure within limit. This amplified RF signal can be represented as $a_{RF}(t)$. This signal applied to both mixer-1 and mixer-2. Local oscillator (LO) generates the signal at the carrier frequency of the desired channel. This LO
signal is first shifted by 90° and then applied to mixer-2, i.e. applied in quadrature phase to mixer-2, while LO signal is applied in-phase to mixer-1, i.e. without any phase change.

Function of mixer-1 is to beat RF signal with in-phase LO signal to generate in-phase component (I(t)) of the desired channel. While, function of mixer-2 is to beat RF signal with quadrature phase LO signal to generate quadrature component (Q(t)) of the desired channel. For this reason, mixer-1 is feed with in-phase while mixer-2 is feed with quadrature phase LO signal. Output of both mixers are filtered through low pass filter (LPF)s to select only the desired channel signal. The bandwidth of pass band of LPF is equal to the channel bandwidth, so that undesired signals can be removed to increase signal to noise ratio (SNR).

Output of LPF-1 and LPF-2 are digitized using analog to digital converter (ADC). These digitized I and Q signals are applied to digital signal processing (DSP) back-end section to extract I and Q data. Next we perform the mathematical analysis of direct conversion receiver.

A direct conversion receiver performs demodulation of a signal $a_{RF}(t)$ with carrier frequency $f_{RF}$, complex envelope $env(t) = I(t) + jQ(t)$, and amplitude $A_{RF}$ using a signal $a_{LO}(t)$ generated by a local oscillator with frequency $f_{LO} = f_{RF}$ and amplitude $A_{LO}$. These signals can be represented by the two complex waves as (1) and (2).

$$a_{RF}(t) = A_{RF}(I(t) + jQ(t))\exp(j2\pi f_{RF}t)$$

(1)
\[ a_{LO}(t) = A_{LO} \exp(j2\pi f_{LO}t). \] (2)

The voltages \( v_{RF}(t) \) and \( v_{LO}(t) \) are obtained by taking the real part of (1) and (2)

\[ v_{RF}(t) = A_{RF}(I(t)\cos(2\pi f_{RF}t) - Q(t)\sin(2\pi f_{RF}t)) \] (3)

\[ v_{LO}(t) = A_{LO}\cos(2\pi f_{LO}t) \] (4)

\( I(t) \) and \( Q(t) \) represent the inphase and quadrature (I/Q) signals. The classical direct conversion receiver is presented in figure 4.1, which performs the demodulation of \( v_{RF}(t) \). As shown in figure 4.1, RF signal input to mixer-1 and mixer-2 is \( v_{RF}(t) \). While, local oscillator (LO) signal to mixer-1 is \( v_{LO1}(t) = v_{LO}(t) \) and to mixer-2 is

\[ v_{LO2}(t) = A_{LO}\cos(2\pi f_{LO}t + \pi/2) \] (5)

Output of low pass filter-1 is \( v_1(t) \) and of low pass filter-2 is \( v_2(t) \).

\[ v_1(t) = \frac{A_{RF}A_{LO}}{2} I(t) \] (6)

\[ v_2(t) = \frac{A_{RF}A_{LO}}{2} Q(t) \] (7)

\( v_1(t) \) and \( v_2(t) \) are converted in digital domain using analog to digital converter (ADC) and then applied to back-end digital signal processing (DSP) section to extract the transmitted data bits.

The critical function of the DCR is to extract the \( I(t) \) and \( Q(t) \) signal without distortion. Faithful reproduction of \( I(t) \) and \( Q(t) \) signals at receiver is greatly affected by distortions.
4.3 Proposed Method

The structure of proposed method for distortion removal is presented in figure 4.2.

\[ E_N = \alpha_1 v_1(N) + \alpha_2 v_2(N) \]  
\[ \phi_N = \beta_1 v_1(N) + \beta_2 v_2(N) \]

The coefficients \( \alpha_i \) and \( \beta_i \) are calculated in such a way that resultant signal \( I(N) \) and \( Q(N) \) are distortion free.

To reduce multiple distortions cascade version of multiplier and adder structure is utilized as shown in figure 4.3.
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Figure 4.3 Proposed method for multiple distortion removal
(cascaded multiplier and adder structure)

The multiplier constants or calibration coefficients $\alpha_i, \beta_i$ are calculated for suppression of particular type of distortion. As an example for the joint mitigation of I/Q mismatch and IMD2, multiplier and adder section-1 of figure 4.3 can be calibrated to remove IMD2 while multiplier and adder section-2 can be calibrated to remove I/Q mismatch or vice-versa. In reality cascaded structure can be replaced by single multiplier and adder structure as shown in figure 4.4 with relation,

$$\alpha_{1,2} = \alpha_1 \alpha_3 + \beta_1 \alpha_4$$  \hspace{1cm} (7_c)

$$\alpha_{2,2} = \alpha_2 \alpha_3 + \beta_2 \alpha_4$$  \hspace{1cm} (7_d)

$$\beta_{1,2} = \alpha_1 \beta_3 + \beta_1 \beta_4$$  \hspace{1cm} (7_e)

$$\beta_{2,2} = \alpha_2 \beta_3 + \beta_2 \beta_4$$  \hspace{1cm} (7_f)
But in practice we implement cascaded structure because, algorithm used for calculation of calibration coefficients required knowledge of input as well intermediate signals. In addition, this structure is going to implement in DSP backend as shown in figure 4.5. Therefore, either we use cascaded structure or its equivalent single structure, resources utilized are same.
4.4 Analysis of Proposed Method in the Presence of I/Q Mismatch

Here, a mathematical analysis is presented to justify that the proposed method is able to detect desired base band signal $I(t)$ and $Q(t)$ from the received RF signal. The proposed method is represented in figure 4.5.

Here, we introduce I/Q mismatch distortion and then demonstrate the ability of proposed method to nullify effect of distortion on the output of the receiver. I/Q mismatch is introduced by taking different gain and phase for the local oscillator path-1 and path-2. In this case the local oscillator signal to mixer-1 and mixer-2, respectively, are

\[
v_{\text{LO1}}(t) = A_{\text{LO1}} \cos(2\pi f_{\text{LO}} t + \phi) \tag{8}
\]

\[
v_{\text{LO2}}(t) = A_{\text{LO2}} \cos(2\pi f_{\text{LO}} t + \pi/2 + \epsilon) \tag{9}
\]

where $\phi$ is the phase shift between received signal and locally generated carrier signal, while $\epsilon$ is phase shift introduced due to non- similarity in design and other factors.

Therefore, the output of low pass filters in the presence of I/Q mismatch are

\[
v_1(t) = A_1.\cos(\phi).I(t) + A_1.\sin(\phi).Q(t) \tag{10}
\]

\[
v_2(t) = A_2.\cos(\epsilon).I(t) + A_2.\sin(\epsilon).Q(t) \tag{11}
\]

where, $A_1$ and $A_2$ are gain of the signals at the output of low pass filters 1 & 2 respectively.

Using (10) and (11) a system can be written as (12),
\[
\begin{bmatrix}
  v_1(t) \\
  v_2(t)
\end{bmatrix} = B \begin{bmatrix}
  I(t) \\
  Q(t)
\end{bmatrix}, \quad \text{with} \quad B = \begin{bmatrix}
  A_1 \cos(\phi) & A_1 \sin(\phi) \\
  A_2 \cos(\phi) & A_2 \sin(\phi)
\end{bmatrix}
\] (12)

\[V_{\text{IN}}(N)\]

\[V_{\text{OUT}}(N)\]

Figure 4.5 Proposed Method for distortion removal in Direct Conversion Receiver

If we suppose that the matrix B is nonsingular, then we obtain

\[
\begin{bmatrix}
  I(t) \\
  Q(t)
\end{bmatrix} = B^{-1} \begin{bmatrix}
  v_1(t) \\
  v_2(t)
\end{bmatrix}, \quad \text{with} \quad B^{-1} = \begin{bmatrix}
  \alpha_1 & \alpha_2 \\
  \beta_1 & \beta_2
\end{bmatrix}
\] (13)

The expressions of the \(I(t)\) and \(Q(t)\) signals can be obtained with, (13) and the expression of the inverse of matrix B as,

\[I(t) = \alpha_1(v_1(t)) + \alpha_2(v_2(t))\] (14)

\[Q(t) = \beta_1(v_1(t)) + \beta_2(v_2(t))\] (15)

The relation between \(I(t)\) and \(Q(t)\) signals, the two output voltages \(v_1(t), v_2(t)\) and the four real calibration constants \((\alpha_1, \alpha_2, \beta_1, \beta_2)\) is defined by equations (14) and (15). The calibration of the proposed method gives four real coefficients, which perform faithful \(I/Q\) regeneration from the two output voltages. The four calibration coefficients can be calculated using steps mentioned below.
1) An RF signal with known $I(t)$, $Q(t)$ sequence (length of N symbols) is injected at input of the direct conversion receiver with proposed method. This input generates two output voltages $v_1(t)$ and $v_2(t)$ at the output port of low pass filters. These voltages can be used to write

$$
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix} = \begin{bmatrix}
I(1) \\
I(N)
\end{bmatrix}
$$

(16)

$$
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} = \begin{bmatrix}
Q(1) \\
Q(N)
\end{bmatrix}
$$

(17)

with $C = \begin{bmatrix}
v_1(1) & v_2(1) \\
v_1(N) & v_2(N)
\end{bmatrix}$

2) A deterministic least-square method can be utilized to calculate the four coefficients as

$$
\begin{bmatrix}
\alpha_1 \\
\alpha_2
\end{bmatrix} = (C^T C)^{-1} C^T \begin{bmatrix}
I(1) \\
I(N)
\end{bmatrix}
$$

(18)

$$
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} = (C^T C)^{-1} C^T \begin{bmatrix}
Q(1) \\
Q(N)
\end{bmatrix}
$$

(19)

Thus, after determination of these four real coefficients, faithful $I/Q$ regeneration can be performed. Proposed system can be calibrated using any one of the below mentioned methods.

1) **Pre-calibration method**: In this method, the four calibration coefficients are calculated during manufacturing process. For all the frequencies to be used, a known $I/Q$ sequence is applied at the RF port of DCR and the calibration coefficients are stored in the memory.
2) **Self-calibration method**: Here a self calibration method is presented in figure 4.6. Advantage of this method is that it is standard independent and activated during the power-on process of the device or during the ideal/standby time duration. For explanation purpose the multiplier and adder blocks are represented separately, but usually they are part of the DSP back-end.

![Proposed method with self calibration](image)

**Figure 4.6 Proposed method with self calibration**

When transceiver entered in the self calibration mode, switch S1 becomes open and switch S2 connected to terminal b, to bypass antenna section. DSP back-end section generates and applies $I(t)$ and $Q(t)$ sequences to Transmitter section. Transmitter section modulates the $I(t)$ and $Q(t)$ sequences and generates the RF signal. This RF signal is applied to receiver section. Receiver section down converts the received signal, low pass filters the signal and generates $v_1(t)$ and $v_2(t)$. $v_1(t)$ and $v_2(t)$ are converted in digital domain using analog to digital converter (ADC). These digitized signal $v_1(n)$ and $v_2(n)$, where $n=1,\ldots,N$, are applied to calibration algorithm to calculate calibration coefficients $\alpha_i, \beta_i$. Now, calculated calibration coefficients $\alpha_i, \beta_i$ are applied to
multiplier and adder section to perform error free regeneration of remaining
\( I(t) \) and \( Q(t) \) samples.

### 4.4.1 Properties of Calibration Constants

For the precise understanding of the regeneration process of \( I(t) \), \( Q(t) \),
the analysis of properties of calibration constants is required. Define the five
following vectors.

\[ V_o = \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix} \]  
(20)

\[ G_{\cos} = \begin{bmatrix} A_1 \cos(\phi) \\ A_2 \cos(\varepsilon) \end{bmatrix}, \quad G_{\sin} = \begin{bmatrix} A_1 \sin(\phi) \\ A_2 \sin(\varepsilon) \end{bmatrix} \]  
(21)

\[ \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \]  
(22)

The system defined by (10) and (11) becomes the vectorial relation

\[ V_o = I(t)G_{\cos} + Q(t)G_{\sin} \]  
(23)

Similarly, system defined by (14) and (15) becomes

\[ I(t) = \alpha \cdot V_o \]  
(24)

\[ Q(t) = \beta \cdot V_o \]  
(25)

By putting the expression of the vector \( V_o \) defined by (23) in (24),
following relation for the \( I \) channel can be derived:

\[ I(t) = I(t)\alpha \cdot G_{\cos} + Q(t)\alpha \cdot G_{\sin} \]  
(26)

Therefore, the relations for the \( I \) channel can be expressed as:

\[ \alpha \cdot G_{\sin} = 0 \]  
(27)

\[ \alpha \cdot G_{\cos} = 1. \]  
(28)
Similarly, the relations for the $Q$ channel can be expressed as:

\begin{align*}
\beta \cdot G_{\cos} &= 0 \\
\beta \cdot G_{\sin} &= 1.
\end{align*}

From (27)-(30) following relations for the calibration procedure can be deduced:

1) The vectors $a$ and $\beta$ are, respectively, perpendicular to the vectors $G_{\sin}$ and $G_{\cos}$, as per (27) and (29).

2) Equations (28) and (30) indicate that if $A_i$ is low, the norms of vector $a$ and $\beta$ are high and vice versa.

### 4.5 Multiple Distortions Reduction Ability of Proposed Method

In a direct conversion receiver, pre-select filter (figure 4.5) suppresses the outband signals and allow the desired signal at frequency $f_{LO}$ to be applied at the RF port of the direct conversion $I/Q$ demodulator. In this situation, inband adjacent channel can pass through the preselect filter and makes its way to demodulator. In the direct conversion receiver, low-pass filters at output of the $I/Q$ demodulator perform the task of channel selection. But, if the direct conversion receiver is suffering from second order intermodulation distortion (IMD2), then $I$ and $Q$ signals get distorted by a spurious baseband term produced by the adjacent channel signals [84]. We will study the joint effect of IMD2 and $I/Q$ mismatch on the demodulation with the proposed method. Here the DC-offset due to self-mixing is removed by AC coupling...
between mixer and low pass filter (LPF). This is possible, due to utilization of modulation scheme having very low power near to 0Hz i.e at DC. The proposed method for joint mitigation of IMD2 and I/Q mismatch, implemented in DSP back-end, is presented in figure 4.7. Here $v_1(N)$ and $v_2(N)$ are the signals respectively from ADC of LPF-1 and ADC of LPF-2. First multiplier and adder structure is utilized for IMD2 removal. The calibration algorithm for IMD2 removal take $v_1(N)$ and $v_2(N)$ signals as input and vary the coefficients $\alpha_1$, $\alpha_2$, $\beta_1$, $\beta_2$ in such a way that resultant signals $v_3(N)$ and $v_4(N)$ are IMD2 free. These $v_3(N)$ and $v_4(N)$ are feed as input signals to Calibration algorithm for I/Q mismatch removal. This calibration algorithm vary the coefficients $\alpha_3$, $\alpha_4$, $\beta_3$, and $\beta_4$ in such a way that resultant signals $v_3(N)$ and $v_4(N)$ are IMD2 and I/Q mismatch free. The algorithm for calculation of coefficients for I/Q mismatch removal is presented in section 4.4 and algorithm for calculation of coefficients for IMD2 removal is presented here.

**Figure 4.7 Proposed method for multiple distortion removal**
With an adjacent channel present in the receiver reception frequency band, the input signal to the receiver is

\[ a_{in}(t) = a_{RF}(t) + a_{adj}(t) \]  \hspace{1cm} (31)

with \( a_{adj}(t) = A_{adj}(I_{adj}(t) + jQ_{adj}(t)) \exp(j2\pi f_{adj}t). \)

The term \( a_{adj}(t) \) represents the adjacent channel signal at frequency \( f_{adj} \).

The input voltages at the input of the demodulator is

\[ v_{in}(t) = A_{RF}(I(t) \cos(2\pi f_{RF}t) - Q(t) \sin(2\pi f_{RF}t)) + A_{adj}(I_{adj}(t) \cos(2\pi f_{adj}t) - Q_{adj}(t) \sin(2\pi f_{adj}t)) \]  \hspace{1cm} (32)

Using the down-conversion behaviour model given by [55], output of mixer-1 and mixer-2 can be expressed as

\[ v_{m1}(t) = v_{in}(t) v_{LO1}(t) + \gamma_1 [v_{in}(t) v_{LO1}(t)]^2 \]  \hspace{1cm} (33)

\[ v_{m2}(t) = v_{in}(t) v_{LO2}(t) + \gamma_2 [v_{in}(t) v_{LO2}(t)]^2 \]  \hspace{1cm} (34)

where, \( \gamma_i \) represents the second order transconductance of mixers. From (33) and (34) output of low pass filters are,

\[ v_1(t) = A_1 \cos(\phi) I(t) + A_1 \sin(\phi) Q(t) + \gamma_1 [x_{RF}(t) + x_{adj}(t)] \]  \hspace{1cm} (35)

\[ v_2(t) = A_2 \cos(\epsilon) I(t) + A_2 \sin(\epsilon) Q(t) + \gamma_2 [x_{RF}(t) + x_{adj}(t)] \]  \hspace{1cm} (36)

where, \( x_{RF}(t) \) represents component of desired signal and \( x_{adj}(t) \) represents component of adjacent channel.

Interference signal only affects the receiver when its power is much larger than that of the desired signal [92], i.e. \( v_{RF}(t) << v_{adj}(t) \), this implies that

\[ x_{RF}(t) << x_{adj}(t) \]  \hspace{1cm} (37)

Using this hypothesis, we can write (35) and (36) as

\[ v_1(t) = A_1 \cos(\phi) I(t) + A_1 \sin(\phi) Q(t) + \gamma_1 x_{adj}(t) \]  \hspace{1cm} (38)
Using (14) and (15) with the output voltages defined by (38) and (39), the in-phase demodulated signal $I$ and the quadrature demodulated signal $Q$ can be expressed as:

$$I = a_1v_1(t) + a_2v_2(t)$$

$$Q = \beta_1v_1(t) + \beta_2v_2(t)$$

Assuming the system to be calibrated without the adjacent channel signal and by using the properties of calibration coefficients described in section 4.3, we obtain the two signals $I$ and $Q$:

$$I = I(t) + \left(\sum_{i=1}^{2} \alpha_i y_i \right)x_{adj}(t)$$

$$Q = Q(t) + \left(\sum_{i=1}^{2} \beta_i y_i \right)x_{adj}(t)$$

The calibration procedure allows the regeneration of I/Q signals, but does not remove the adjacent channel signal $v_{adj}(t)$. The response of the direct conversion receiver with proposed method at $f_{RF}$ and $f_{adj}$ are different, therefore the factors $\gamma_i$ do not verify the relations (14) and (15). The desired signals $[I(t)$ and $Q(t)]$ are corrupted by the adjacent channel. Next we describe a method to reject the effect of IMD2 and I/Q mismatch jointly.

### 4.5.1 Method for Joint Reduction of Distortions

Here, a method has been proposed that calibrates the system and rejects the effect of IMD2 and I/Q mismatch distortion jointly. A new vector
can be defined that consists of second order transconductance of mixers, as follow:

\[ \gamma = \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \]  \hspace{1cm} (44)

This vector corresponds to the adjacent channel. Using (21) and (44) in (38) and (39) we obtain the relation as follow:

\[ V_o = x_{adj}(t)\gamma + I(t)G_{\cos} + Q(t)G_{\sin} \]  \hspace{1cm} (45)

In order to reject the adjacent channel signal represented by \( x_{adj}(t) \) and the vector \( \gamma \), we propose the method of the projection of the vector \( V_o \) onto a plan \( H \) that is perpendicular to the vector \( \gamma \). This concept is explained in the figure 4.8.

![Figure 4.8 Vector Diagram explaining removal of IMD2](image)

The plane \( H \) is defined by the orthogonal base \( p = (p_x, p_y) \) with the following properties:

\[ p_x \perp p_y, \quad p_x \perp \gamma, \quad p_y \perp \gamma. \]  \hspace{1cm} (46)
The projection of the vector $V_o$ onto the plane $H$ can be defined using the following two scalar product:

$$h_x = V_o \cdot p_x$$  \hspace{1cm} (47)$$

$$h_y = V_o \cdot p_y$$  \hspace{1cm} (48)$$

Using (45) in (47) and (48) with the properties defined in (46), we get the following expressions:

$$h_x(t) = I(t)(p_x \cdot G_{\cos}) + Q(t)(p_x \cdot G_{\sin})$$  \hspace{1cm} (49)$$

$$h_y(t) = I(t)(p_y \cdot G_{\cos}) + Q(t)(p_y \cdot G_{\sin})$$  \hspace{1cm} (50)$$

We can see that the two projections $h_x$ and $h_y$ are independent of the adjacent channel signal. To calculate these projections, first we need to determine the base $P$. The vectors $p_x$ and $p_y$ can be determined as follows.

(1) Determine the coordinates of the vector $\gamma$.

(2) Construct a base $P$ that verifies the properties defined by (46).

If the power of the adjacent channel signal is higher than the desired signal then and then, adjacent channel signal severely affect the performance of DCR. In this situation, the two output voltages are almost proportional to the term $x_{adj}(t)$, so we have the following approximation:

$$V_o \approx x_{adj}(t)\gamma.$$  \hspace{1cm} (51)$$

In order to cancel the term $x_{adj}(t)$, we need to calculate the two standard deviations of the two output voltages using following expression

$$STD(v_i(t)) = \sqrt{\frac{1}{N} \sum_{l=1}^{N} (v_i(l) - \langle v_i(l) \rangle)^2}.$$  \hspace{1cm} (52)$$
We obtain the two standard deviation values using N samples of output voltages \((v_1, v_2)\), represented as below

\[
V_{STD} = \begin{bmatrix} \bar{D}_1 \\ \bar{D}_2 \end{bmatrix} = \begin{bmatrix} \text{STD}(v_1(t)) \\ \text{STD}(v_2(t)) \end{bmatrix}
\] (53)

With, \(D_1\) and \(D_2\) representing the two elements of the vector \(V_{STD}\). The vector \(V_{STD}\) is constant and proportional to the vector \(\gamma\). Using (51), vector \(V_{STD}\) can be calculated as

\[
V_{STD} = \begin{bmatrix} \bar{D}_1 \\ \bar{D}_2 \end{bmatrix} = \text{STD}([v_{adj}(t)]^{\gamma_1} [\gamma_2])
\] (54)

With known elements of the vector \(V_{STD}\), the base \(P\) can be constructed as

\[
p_x = \frac{1}{D_1}, \quad p_y = \frac{D_1/(D_1^2 + D_2^2)}{D_2/(D_1^2 + D_2^2)}
\] (55)

The vectors \(p_x\) and \(p_y\) shown above satisfy the properties defined by (46).

The system of two equations (49) and (50) becomes the matrix relation

\[
\begin{bmatrix} h_x(t) \\ h_y(t) \end{bmatrix} = E \begin{bmatrix} I(t) \\ Q(t) \end{bmatrix}
\] (56)

with \(E = \begin{bmatrix} p_x \cdot G_{cos} & p_x \cdot G_{sin} \\ p_y \cdot G_{cos} & p_y \cdot G_{sin} \end{bmatrix}\)

Suppose that the matrix \(E\) is nonsingular, then

\[
\begin{bmatrix} I(t) \\ Q(t) \end{bmatrix} = E^{-1} \begin{bmatrix} h_x(t) \\ h_y(t) \end{bmatrix}
\] (57)

where \(E^{-1} = \begin{bmatrix} s_1 & s_2 \\ u_1 & u_2 \end{bmatrix}\).

Using (57), the expressions of the \(I(t)\) and \(Q(t)\) signals are

\[
I(t) = s_1 h_x(t) + s_2 h_y(t)
\] (58)
Equations (58) and (59) define the relation between $I(t)$ and $Q(t)$ signals, the two projections $h_x$ and $h_y$, and the four real calibration constants ($s_1$, $s_2$, $u_1$, $u_2$). The calibration of the proposed method gives four real constants, which allow faithful $I/Q$ regeneration from the two projections $h_x$ and $h_y$ voltages in the presence of second order intermodulation distortion. The method for the calculation of four calibration constants is as follows.

1) An RF signal with known $I(t)$, $Q(t)$ sequence (length of $N$ symbols) is applied at input of the direct conversion receiver, which generates two output voltages $v_1(t)$, $v_2(t)$ that can be used to find $D_1$ and $D_2$ using (52) and (53). $D_1$ and $D_2$ are utilized to calculate $p_x$ and $p_y$ using (55). $p_x$ and $p_y$ with $v_1(t)$, $v_2(t)$ are utilized to calculate $h_x$ and $h_y$ using (47) and (48). Using (58) and (59) these projection signals can be used to write

$$L \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} I(1) \\ I(N) \end{bmatrix}$$

(60)

$$L \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} Q(1) \\ Q(N) \end{bmatrix}$$

(61)

with

$$L = \begin{bmatrix} h_x(1) & h_y(1) \\ h_x(N) & h_y(N) \end{bmatrix}$$

2) The four constants can be calculated using the deterministic least-square approach as follows:
\[
\begin{align*}
\begin{bmatrix}
 s_1 \\
 s_2
\end{bmatrix} &= (I^T L) \cdot I^T \begin{bmatrix}
 I(0) \\
 I(N)
\end{bmatrix} \\
\begin{bmatrix}
 u_1 \\
 u_2
\end{bmatrix} &= (L^T L) \cdot I^T \begin{bmatrix}
 Q(0) \\
 Q(N)
\end{bmatrix}
\end{align*}
\] (62) (63)

Once these four real coefficients are determined, IMD2 and I/Q mismatch distortion free I/Q regeneration takes place using the two projection signals. The relation between \( s_1, s_2, u_1, u_2 \) and \( \alpha_1, \alpha_2, \beta_1, \beta_2 \) can be established as follows. Using (47), (48) and (55) the expressions of the \( h_s(t) \) and \( h_q(t) \) signals are

\[
h_s(t) = \frac{v_1(t)}{D_1} - \frac{v_2(t)}{D_2}
\] (64)

\[
h_q(t) = \frac{D_1 v_1(t)}{(D_1^2 + D_2^2)} + \frac{D_2 v_2(t)}{(D_1^2 + D_2^2)}
\] (65)

Therefore, the expressions of \( I(t) \) and \( Q(t) \) signals using (58), (59), (64) and (65) are

\[
I(t) = \left[ \frac{s_1 + s_2 D_1}{D_1} \right] v_1(t) + \left[ \frac{s_2 D_2}{(D_1^2 + D_2^2)} - \frac{s_1}{D_2} \right] v_2(t)
\] (66)

\[
Q(t) = \left[ \frac{u_1}{D_1} + \frac{u_2 D_1}{(D_1^2 + D_2^2)} \right] v_1(t) + \left[ \frac{u_2 D_2}{(D_1^2 + D_2^2)} - \frac{u_1}{D_2} \right] v_2(t)
\] (67)

By comparing (14) and (15) with (66) and (67) following relations can be established

\[
\begin{align*}
\alpha_1 &= \frac{s_1}{D_1} + \frac{s_2 D_1}{(D_1^2 + D_2^2)} , \\
\alpha_2 &= \frac{s_2 D_2}{(D_1^2 + D_2^2)} - \frac{s_1}{D_2} , \\
\beta_1 &= \frac{u_1}{D_1} + \frac{u_2 D_1}{(D_1^2 + D_2^2)} , \\
\beta_2 &= \frac{u_2 D_2}{(D_1^2 + D_2^2)} - \frac{u_1}{D_2}
\end{align*}
\] (68)

Equation (68) is only utilized when (51) is fulfilled. In the case when, (51) is not satisfied, (18) and (19) are utilized for calculation of calibration coefficients. Calibration algorithm decides which equations are to be utilized
on the basis of norm of $h_x$ and norm of $h_y$. If condition (51) is satisfied the norm of $h_x$ and norm of $h_y$ are very low, otherwise norm of $h_x$ and norm of $h_y$ are high. A threshold is calculated as a part of calibration process to decide whether the norm of $h_x$ and norm of $h_y$ are low or high.

This method can also deal with multiple interferences. The detail mathematical analysis of the proposed method for multiple interference scenario is presented in appendix-A.