CHAPTER-6

CHOICE OF PAN MATERIAL

6.1 Introduction

When an alternating current flows through a coil placed in close proximity to a conducting surface, the magnetic field of the coil will induce eddy current in that surface. The magnitude and phase of this induced eddy current will determine the loading on the coil and thus its effective impedance. Induction heating is an electromagnetic induction phenomena. To analyse the performance of an induction cooker it is desirable to develop the equivalent circuit representing the induction heating phenomena. Moreover, in case of series resonant circuit, the quality factor (Q) must be properly maintained to keep the maximum value of the stored energy in the inductor or capacitor as high as possible. It has been discussed earlier that if the value of Q be too large, it is harder to generate heat energy. In case of domestic induction cookers the equivalent inductance is very much dependent on the size and material of the cooking pan. Hence the design of the induction coil is one of the most important aspects of all induction cookers. This chapter describes the development of the equivalent circuit model of the proposed induction cooker which will act as a pre-requisite for the design of the induction coil.

6.2 Estimation of power dissipation

To calculate the power dissipation due to the induced eddy current, it is assumed that the field inside the pan is not significantly perturbed for all low frequencies f<<f_{critical}. The power dissipation in the pan due to magnetic induction can be calculated from the following Maxwell’s equations:

\[ \nabla \times \mathbf{H} \approx \mathbf{J} = \sigma \mathbf{E} \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} = -\mu \frac{\partial \mathbf{H}}{\partial t} \]

\[ \nabla \cdot \mathbf{B} = 0 \]

where the linear constitutive laws \( \mathbf{B} = \mu \mathbf{H} \) and \( \mathbf{J} = \sigma \mathbf{E} \) are used.

In case of induction cooker, the cooking pan is considered to be a semi-infinite cylinder placed in an uniform time varying axial magnetic field as indicated in fig.6.1. The cylinder has an electrical conductivity \( \sigma \), electrical resistivity \( \rho \), magnetic permeability \( \mu \), radius \( R \) and length \( l \).
Fig. 6.1: A cooking pan being considered as a semi-infinite cylinder placed in an uniform time varying magnetic field.

In cylindrical coordinates the magnetic field intensity $H$ is $z$-directed. The induced electric field $E$ in the cylindrical pan varies with the radius $r$ and it is $\phi$-directed. Hence the Maxwell equation,

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

can be written as,

$$\frac{1}{r} \frac{\partial}{\partial r} (r E_\phi) a_z = \mu \omega H_0 e^{j \alpha} a_z$$

Solving the above equation, the electric field is given by,

$$E_\phi = \frac{\mu \omega H_0}{2} re^{j \alpha}$$

and the current density,

$$J_\phi = \sigma E_\phi = \frac{\sigma \mu \omega H_0}{2} re^{j \alpha}$$

In the sinusoidal steady state, the time average power dissipation in the cooking pan due to induction from the applied field is given as-

$$\langle P \rangle = \frac{1}{2} \text{Re} \left\{ \left( E \cdot J^* \right) \right\} dV$$

The average power dissipation in the pan can now be expressed as:

$$\langle P_{LF} \rangle = \frac{1}{16} \sigma \mu^2 \omega^2 |H_0|^2 \pi R^4 l$$
The above equation is valid when the operating frequency is below the critical frequency i.e. 

\[ f_{\text{critical}} \leq \frac{1}{\pi R^2 \mu \sigma} \]

However, at frequencies significantly higher than the critical frequency, the current density is concentrated on the surface of the cooking pan and may be approximated as a surface current flowing through a thin layer of skin-depth thickness. The value of the surface current remains constant with \(|K|=|H_0|\). The total current in the pan is equal to \(Kl\).

In the high frequency limit, the average power dissipation in the pan is given as,

\[ \langle P_{HF} \rangle = \frac{\pi R l}{\delta \sigma} |H_0|^2 \]

where the depth of penetration, \(\delta\) is given by,

\[ \delta = \sqrt{\frac{2 \rho}{\omega \mu}} \]

The above two equations are quite useful in calculating the effective resistance of the pan which is seen to be much dependent on the operating frequency, properties of the material and geometry of the pan.

### 6.3 Factors affecting the eddy current in the pan

A large number of factors will affect the eddy current induced in the metallic surface of the pan and these are outlined below.

- **Material conductivity (\(\sigma\))**
  The conductivity of the material of the metal surface has a direct effect on the flow of eddy current. The higher the conductivity of the material the larger will be the flow of eddy currents on the surface.

- **Permeability of the material (\(\mu\))**
  Permeability is the property of a material describing the ease with which it can be magnetized. For non-ferrous metals such as copper, brass, aluminium etc. and for austenitic stainless steels, the permeability is almost the same as that of ‘free space’, i.e. the relative permeability is very close to unity. For ferrous metals, however, the value of relative permeability will be quite high, of the order of several hundred. The value of \(\mu\) has a very significant influence on the magnitude of the induced eddy current.
- **Operating frequency (ω)**
  The response to eddy current is significantly affected by the frequency chosen. Fortunately, however, this is one property which can be easily controlled.

- **Geometry of the pan**
  Practical heating surface is neither flat nor of infinite size. Besides, geometrical features such as curvature, edges, grooves etc. will exist and they all will affect the eddy current response. Also, if the material thickness be less than the corresponding effective depth of penetration then this will also unduly affect eddy current production.

- **Depth of penetration (δ)**
  The eddy current density and hence the amount of heat produced, is greatest on the surface of the metal being heated and then declines with the depth. It is convenient to define mathematically the standard depth of penetration where the eddy current is 37% of its surface value and may be expressed as:

  \[
  δ = \frac{2\rho}{\sqrt{\omega\mu}}
  \]

  Equation (6.1) reveals that following are the factors that affect the depth of penetration.

  - δ decreases with the increase of frequency
  - δ increases with the increase of resistivity
  - δ decreases with the increase of permeability

6.4 **Equivalent circuit model of heating coil**

The resistance of the coil \( R_{\text{coil}} \) and its inductance \( L_{\text{coil}} \) may be estimated as under.

6.4.1 **Calculation of resistance of the heating coil (R_{coil})**

The coil resistance \( R_{\text{coil}} \) can be calculated from the following formula:

\[
R_{\text{coil}} = \frac{\rho l}{2\pi a \delta} \Omega
\]

where, \( \rho \) = resistivity of the coil material,
\( l \) = total length of the coil conductor,
\( a \) = radius of the coil conductor,
\[ \delta = \text{skin depth of the conductor.} \]

6.4.2 Calculation of self-inductance of the heating coil (\(L_{\text{coil}}\))

The cooking coil is of flat pancake type as shown in fig.6.2. Using Wheeler’s formula, the self inductance of the heating coil can be calculated as follows:

\[
L_{\text{coil}} = 0.0254X \frac{N^2r^2}{8r + 11w} \ \mu\text{H}
\]

Referring to fig.6.2, the average radius as measured from the central axis to the middle of the winding may be written as,

\[ r = a + \frac{b-a}{2} = \frac{a+b}{2} \]

and the width of the coil will be given as,

\[ w = b - a \]

where, \(a\) = inner radius of the coil,

\(b\) = outer radius of the coil,

and \(N\) = number of turns of the coil.

Therefore,

\[
L_{\text{coil}} = \frac{N^2(a+b)^2}{8\left(\frac{a+b}{2}\right)^2 + 11(b-a)}
\]

\[= \frac{N^2(a+b)^2}{4(15b-7a)}\]

6.5 Equivalent circuit model of induction cooker

The heating coil acts as the primary winding of a transformer while the cooking pan acts as its closed secondary winding as shown in fig.6.3. The resistance of the heating coil (\(R_{\text{coil}}\)) changes with frequency due to skin and proximity effects and also due to change in temperature. \(L_{\text{coil}}\) is the self-inductance of the heating coil. \(M\) represents the mutual inductance between heating coil and cooking pan. \(R_{\text{eff}}\) is the effective resistance of the
cooking pan referred to the heating coil side. The pan inductance referred to coil side is 
$L_2 = M$, as there is no physical winding on the cooking pan side. The basic circuit as well as its simplified equivalent configurations is well explained in fig.6.3. The heating coil and the cooking pan (load) can be represented by an equivalent series combination of $R_{eq}$ and $L_{eq}$, where the values of $R_{eq}$ and $L_{eq}$ are given as:

\[ R_{eq} = R_{coil} + \frac{A^2 R_{eff}}{2} \]  \hspace{1cm} (6.2)

\[ L_{eq} = L_{coil} - \frac{A^2 M}{2} \]  \hspace{1cm} (6.3)

where,

\[ A = \frac{\omega M}{\sqrt{R_{eff}^2 + \omega^2 M^2}} \]

In induction heating as the effective resistance ($R_{eff}$) of the cooking pan and its magnetizing reactance ($\omega M$) are same (refer eqns. 6.4 & 6.5), eqns. 6.2 & 6.3 can be written as:

\[ R_{eq} = R_{coil} + \frac{1}{2} R_{eff} \]

\[ L_{eq} = L_{coil} - \frac{1}{2} M \]

Fig. 6.3 : Equivalent circuit model of induction cooker
6.5.1 Determination of the equivalent resistance and equivalent reactance of the cooking pan

As induction heating is an electromagnetic phenomena, the effective values of resistance and reactance of the cooking pan referred to the heating coil side can be derived from the fundamental electromagnetic field theory. This derivation is made in three steps as mentioned below:

- Calculation of magnetic field due to a flat pancake coil.
- Calculation of induced emf in the pan.
- Calculation of the equivalent parameters of the pan.

6.5.2 Calculation of magnetic field due to a flat pancake coil

The magnetic field due to one circular Loop \((N=1)\) is given by,

\[
H = \frac{I}{2r} \frac{AT}{m}
\]

Fig.6.4 : Elemental coil

For the flat pancake coil of ‘n’ number of turns per unit width, inner radius ‘a’ and outer radius ‘b’, as shown in fig.6.4, one can write:

\[
dH = \frac{nI}{2r} \frac{dr}{b-a} \times \frac{L}{2r}
\]

Therefore,

\[
H = \int_a^b dH = \frac{N}{2(b-a)} \ln \frac{b}{a}
\]

6.5.3 Calculation of induced e.m.f. in the pan

In calculating the induced emf in the pan the following assumptions are made.

- Coil is in x-y plane and the field is z-directed.
Cooking pan is on the flat pancake coil.
Cooking pan can be considered as cylindrical.

Assuming the current through the coil to be a sinusoid of angular frequency $\omega$ rad/sec, the magnetic field created by the coil may be written as:

$H = H_0 \cos \omega ta_z$

or,

$B = \mu_0 H_0 \cos \omega ta_z$

$= B_0 \cos \omega ta_z$

Due to skin effect the surface value of the induced flux in the pan gets modified both in amplitude and in phase as it passes through the pan material and it may be written as:

$B = B_0 e^{-\gamma/\delta} \cos(\omega t - z/\delta)$

where,

$\delta = \text{skin depth} = \sqrt{\frac{2\rho}{\mu \omega}}$

$\rho = \text{resistivity of the pan material (}\Omega/\text{m})$

$\mu = \text{permeability of the pan material (H/m)}$

$\omega = \text{angular frequency in rad/sec.}$

The total flux in the pan can be approximately estimated by integrating the flux density over the entire depth of the pan as:

$\Phi = \int_0^\infty B_0 e^{-\gamma/\delta} \, dz$

$= \frac{B_0 \delta}{\sqrt{2}} \angle - 45^\circ$

$\Phi(t) = H_0 \frac{\delta \mu}{\sqrt{2}} \cos(\omega t - \pi/4)$

The induced emf in the pan is given by,

$E = N \frac{d\Phi}{dt}$

Now for the pan $N=1$. Therefore,
\[ E = \frac{d\Phi}{dt} \]

Referring back this voltage to the side of the heating coil of ‘N’ number of turns, the reflected value of the induced emf due to the flux in the pan may be written as:

\[ E = N\omega \frac{\delta lH_o}{\sqrt{2}} \cos\left(\omega t + \frac{\pi}{4}\right) \text{ V/m} \]

As,
\[ H_o = \frac{1}{2} \times \frac{N}{(b-a)} I \times \ln\left(\frac{b}{a}\right) \text{ AT/m} \]

Therefore,
\[ E = \frac{1}{2\sqrt{2}} \times N^2 \omega \delta \mu l \times \frac{\ln\left(\frac{b}{a}\right)}{(b-a)} \cos\left(\omega t + \frac{\pi}{4}\right) \text{ V/m} \]

### 6.5.4 Calculation of equivalent parameters of the pan

The equivalent impedance of the pan,
\[ Z = \frac{E}{I} \Omega/m \]

\[ = \frac{1}{2\sqrt{2}} \times N^2 \omega \delta \mu \frac{\ln\left(\frac{b}{a}\right)}{(b-a)} \angle 45^\circ \Omega/m \]

\[ = R_{\text{pan}}' + j X_{\text{pan}}' \]

The equivalent resistance of the pan per unit length,
\[ R_{\text{pan}}' = \frac{1}{4} \times N^2 \omega \delta \mu \frac{\ln\left(\frac{b}{a}\right)}{(b-a)} \Omega/m \]

and the equivalent reactance of the pan per unit length,
\[ X_{\text{pan}}' = \frac{1}{4} \times N^2 \omega \delta \mu \frac{\ln\left(\frac{b}{a}\right)}{(b-a)} \Omega/m \]

The total equivalent resistance of the pan,
\[
R_{\text{eff}} = \frac{1}{4} \times N^2 \omega \delta \mu \times \frac{\ln \left( \frac{b}{a} \right)}{(b-a)} \times 2\pi \Omega
\]

\[
= \frac{1}{2} \times a\pi N^2 \omega \delta \mu \times \frac{\ln \left( \frac{b}{a} \right)}{(b-a)} \Omega \quad \text{------------------------(6.4)}
\]

and the total equivalent reactance of the pan,

\[
X_{\text{eff}} = \frac{1}{2} \times a\pi N^2 \omega \delta \mu \times \frac{\ln \left( \frac{b}{a} \right)}{(b-a)} \Omega \quad \text{------------------------(6.5)}
\]

6.6 Developed induction coil assembly

It is well known that due to skin-effect, pushing high-frequency alternating current through an induction coil is always a very difficult proposition especially for higher diameter enameled copper wire as the current prefers to flow only through the outer layer of the conductor. As a result, the primary current through the induction coil will be quite low. This in turn will cause a very low induced eddy current in the metal pans or in the metallic packages. The corresponding copper loss will be too small for any useful generation of heat in the metallic packages. To avoid this difficulty, a bundled conductor (Litz wire) of enameled copper and forced air cooled has been chosen for the induction coil as the high frequency current penetration capacity can always be improved by using a number of thin wires of higher gauge number but bundled together to form a single conductor of equivalent higher cross-section. In the present scheme 16 number of 24-SWG enameled copper wires were twisted together to form a single conductor instead of a single lower gauge wire of equivalent cross-section. The cross-sectional view of this bundled conductor is shown in fig.6.5. By using this construction, the high frequency current can effectively penetrate the bundle conductor assembly although the current will pass only through the outer skin of the individual 24-SWG enameled copper wires. Thus, with the use of bundle conductor assembly the current is forced to pass more or less throughout the entire cross sectional area of the equivalent conductor of lower gauge.
Fig. 6.5: Cross-section of twisted bundled conductor of induction coil

number. Now, as the current can penetrate through the primary winding of the induction coil, the secondary eddy current, which is flowing in pan or vessel, will be quite high resulting fruitful induction heating. The overall geometrical shape of the induction coil, for better linking of magnetic flux with pan or vessel of home appliances, is of spiral type as indicated in fig. 6.2.

6.7 Material for heating element

The material for heating element should have the following properties:

- **High specific resistance**
  It should have high specific resistance so that a short piece will be required for a particular resistance i.e. for the same piece and same current, the heat produced will be more.

- **High melting point**
  It should have high melting point so that higher temperature can be safely attained.

- **Free from oxidation**
  It should not oxidize at higher temperature; otherwise its life span will be shortened.

- **Low temperature coefficient of resistance**
  The material should have a low temperature coefficient of resistance so that its resistance does not change much during its operation at different temperature ranges. Also, the current drawn by the element at cold (start) will not be very much different
from that when it is hot.
The materials normally used as heating elements are either alloys of Nickel-chromium, Nickel-chromium-iron, nickel-chromium-aluminium or nickel-copper. The use of iron into the alloy even though cheapens the final product but reduces the life of the alloy as it gets oxidized soon. However, it has been possible to make alloys containing iron, chromium, cobalt and aluminium which can withstand temperatures as high as 1300°C.

6.8 Structure of load
The radio-frequency mirror inverter can be used in low, medium and high power applications by using high power switches like thyristors, IGBTs etc. A flat coil, as shown in fig.6.2, is used for induction cooking. A thermal insulator is placed in between the cooking vessel and the induction coil to protect the coil from over heating and also to support the cooking vessel. A ferrite disc is often used to enhance the coupling but with an additional in cost. In order to obtain maximum coupling, the space between the vessel and the coil should be kept as small as possible. But at the same time this gap should be large enough for sufficient strength of support, insulation and air flow. The vessel must be made from a material for which the product of resistivity and relative permeability is high enough to yield an acceptable efficiency.