CHAPTER 4

DETERMINATION OF THE STRAIN RATE SENSITIVITY INDEX BY THE MULTI DOME ANALYSIS

4.1 STRAIN RATE SENSITIVITY INDEX

The most important mechanical characteristic of a superplastic material is its high strain rate sensitivity of flow stress. The characteristic equation which describes the superplastic behavior is usually written as

\[ \sigma = k \dot{\varepsilon}^m \]  \hspace{1cm} (4.1)

Where \( \sigma \) is the flow stress, \( k \) is a constant, \( \dot{\varepsilon} \) is the strain rate, and ‘m’ is the strain rate sensitivity of the flow stress.

A material is usually considered to be superplastic under conditions, where it displays an m value > 0.3. An important feature of superplastic alloys is that their flow stresses are low compared with those of conventional materials. During tensile deformation, the effect of a high m is to inhibit catastrophic necking. The m values of commercial superplastic alloys lie in the range of 0.4-0.8.

The strain rate sensitivity index is a very important parameter in characterizing structural superplastic deformation, if Equation 4.1 is obeyed

\[ m = \frac{\frac{d}{d (\ln \dot{\varepsilon})} \ln \sigma}{\frac{d}{d (\ln \dot{\varepsilon})} \ln \dot{\varepsilon}} \]  \hspace{1cm} (4.2)
This is the slope of the $\ln \sigma_t$-$\ln \dot{\varepsilon}_t$ curve. There are various methods of measuring $m$ but broadly all lead to a variation of $m$ with $\dot{\varepsilon}_t$ of the form shown in Figure 4.1; $m$ also varies with temperature and grain size (William F. Hosford 2010)

![Variation of the strain rate sensitivity index with $\dot{\varepsilon}_t$](image)

**Figure 4.1 Variation of the strain rate sensitivity index with $\dot{\varepsilon}_t$**

### 4.1.1 Various Methods for Measurement of ‘$m$’ Value

a) Determination of $m$ from the $\sigma_t$-$\dot{\varepsilon}_t$ curve

In this method, $m$ is determined as the slope of the experimental $\ln \sigma_t$-$\ln \dot{\varepsilon}_t$ plots. The latter are obtained using instantaneous values measured in the steady state region of the load elongation curves, or by employing the constant true strain rate deformation. The rate at which the data are collected can be increased, if the incremental increases in the strain rate are carried out in a single test. The slope of the $\ln \sigma_t$-$\ln \dot{\varepsilon}_t$ plot is measured either graphically, or better still, by using a curve-fitting procedure and carrying out differentiation. This method of $m$ evaluation has been used successfully for a
variety of materials. With a constant true strain rate deformation, the measured value of m was found to be dependent on strain or time.

b) Determination of m using change in the strain rate method

This is the most widely used procedure. As shown in Figure 4.2, if the crosshead velocity is suddenly increased from $V_1$ to $V_2$ there is a corresponding increase in the load. If straining is continued for a few percent to eliminate the transient effects, a load comparison can be made. The lower velocity curve is extrapolated to establish a common strain for measurement. Figure 4.2 shows a schematic load-time diagram representing a velocity change from $V_1$ to $V_2$. If $m$ is assumed to be nearly independent of the strain rate in the range covered by the velocity increase, then

$$m = \frac{\ln(P_A/P_B)}{\ln(V_2/V_1)}$$

(4.3)

Figure 4.2 Load-time diagram for velocity change from $V_1$ to $V_2$

(Padmanabhan and Davies 1980)
c) Determination of m by the Stress-relaxation tests

The stress relaxation technique has been widely used to study the time dependent plastic flow of crystalline solids. The procedure involves the plastic deformation of the solid to some chosen stress level, at which stage the crosshead movement is stopped, and a continuous decrease in stress is observed as a fraction of time, t. Strain occurs at this stage as the elastic strain in the machine, and the specimen is relieved.

In analyzing the results of the stress relaxation tests, there are several approaches such as.

(i) The relation similar to equation (4.1) with m independent of the strain rate over the range of interest. It was shown that

\[
\ln \sigma_t = C + \frac{m}{m-1} \ln(t + D)
\]  

where C and D are constants. Thus, a plot of \( \ln \sigma_t \) against \( \ln t \) should yield a straight line of slope \( m/(m-1) \) for \( t >> D \).

(ii) An alternative approach starts with the same assumptions as above and leads to the relation

\[
\frac{1}{m} = \frac{d(ln-\sigma_t)}{d(ln \sigma_t)}
\]  

Stress – relaxation tests, mostly by using Equation 4.5, have been carried out on a wide range of structurally superplastic materials. Figure 4.3 shows a stress relaxation curve.
4.1.2 Influence and Improvement of the ‘m’ Value in SPF

In an ideal material where the microstructure remains constant, the true flow stresses can be obtained by carrying out tensile tests at a range of constant strain rates and measuring the steady state loads. As indicated above, most engineering materials are microstructured, and the flow stress will continue to increase with increasing strain, due to the effects of grain growth.

The dissimilarities among the various methods as well as their relevance to fundamental studies have been commented upon by a number of authors. The disagreement in the m values can largely be attributed to

(a) Necking, this if significant, can cause strain dependence of m.

(b) The differences in the primitive defect structure arising from the different magnitudes of strain employed in each procedure.
(c) A wide variation in the value of \( m \) over the strain rate range covered by a given method

(d) Grain growth which has a direct relation to strain, and an inverse dependence on the strain rate

(e) The sign and the magnitude of the strain rate change involved, for \( m \) value measurements, when there is negligible grain growth during superplastic deformation (Padmanabhan 2001).

This is because the procedure involves no assumptions or extrapolations, and compares similar initial structures at all strain rates. For best results, however, a new specimen should be used at each strain rate and the method is time-consuming. In this method \( m \) is derived after lengthy calculations, and it cannot be directly obtained by using a simple equation. In single crystals and bicrystals during low stress creep tests, the \( m \)-values of unity have been observed. ‘\( m \)’ increases with decreasing grain size and increasing temperature. In the case of many alloys, the maximum value of \( m \) is reached at a temperature just below the phase boundary, defining the upper limit of the two phase field. The maximum value of \( m \) that can be obtained increases with temperature, and decreasing grain size. Often the maximum value of \( m \) elongation to fracture coincided. The flow stress and grain size are directly related, while an inverse relation is found between the structure and grain size (Jean – Paul Poirier, 1985). When the structure was stable, work hardening was negligible.
4.2 ANALYTICAL EQUATIONS FOR THE STRAIN – RATE SENSITIVITY INDEX

A set of equations was formulated to evaluate the superplastic material characteristics of the sheet metal, and to obtain the relationship between the stress and strain rate. Figure 4.4 shows the illustration of a bulged dome.

![Figure 4.4 Schematic illustration of a bulged dome](image)

The following conditions were assumed to simplify the calculations:

1. The geometry of the formed dome is equivalent to a part of the sphere.
2. The blank material is isotropic and incompressible. The membrane theory is assumed.
3. The blank is rigidly clamped at the periphery.
4. The material obeys the Von Mises effective stress and strain criteria.
5. The initial sheet thickness ($t_0$) is to be small in comparison with the die radius $R$. 
6. The elastic strains are negligible compared to the extensive plastic deformation.

7. Grain growth, cavitations, and strain hardening are not considered in the calculations.

For a spherical surface, the Von Mises theory says, that the principal stresses are equal to the yield stress of the metal (Thomas H. Courtney 2006).

\[ P = \frac{2\sigma t}{r} \]  \hspace{1cm} (4.6)

where

- \( P \) - Forming pressure in MPa
- \( \sigma \) - Yield stress of alloy (for a spherical surface, the principal stresses are equal to the yield stress of a metal/alloy)
- \( t \) - Average thickness of the formed component
- \( r \) - Radius of curvature

\[ r = \left(1 + \frac{R^2}{h^2}\right) \frac{h}{2} \]  \hspace{1cm} (4.7)

\[ t = t_i \left(\frac{R^2}{R^2 + h^2}\right) \]  \hspace{1cm} (4.8)

- \( R \) - Radius of the circular die
- \( h \) - Pole height
- \( t_i \) - Initial thickness
\( \varepsilon = [\text{new surface area} / \text{original area}] \)

**Strain** \( \varepsilon = \ln \left( \frac{\pi (R^2 + h^2)}{\pi R^2} \right) \)  

**Where**

\( h \) = bulge height  

\( R \) = Radius of the die opening

The flow stress varies with respect to the strain rate, and the strain rate sensitivity index. The strain rate is proportional to the grain size of the work material. Two domes with the base diameters of 15 mm and 10 mm were considered. The time taken for forming both domes was the same, since both domes were formed simultaneously from a single specimen material. In this case, the strain rate sensitivity index is the ratio of the change in the flow stress to the change in the strain.

**Strain rate sensitivity index** \( m = \frac{\ln \sigma_1 - \ln \sigma_2}{\ln \varepsilon_1 - \ln \varepsilon_2} \)  

where \( \sigma_1 \) and \( \sigma_2 \) are the flow stresses of the component projected through the 15 mm and 10 mm diameter holes respectively. Similarly, \( \varepsilon_1 \) and \( \varepsilon_2 \) are the surface strains of the component projected through the 15 mm and 10 mm diameter holes; the ‘m’ values were calculated.
The Von Mises effective stress ($\sigma_{\text{eff}}$) and effective strain ($\varepsilon_{\text{eff}}$) are defined as follows:

$$\sigma_{\text{eff}} = \frac{[(\sigma_\theta - \sigma_m)^2 + (\sigma_m - \sigma_s)^2 + (\sigma_s - \sigma_\theta)^2]^{0.5}}{1.414}$$  \hfill (4.12)

$$\varepsilon_{\text{eff}} = \frac{1.414[(\varepsilon_\theta - \varepsilon_m)^2 + (\varepsilon_m - \varepsilon_s)^2 + (\varepsilon_s - \varepsilon_\theta)^2]^{0.5}}{3}$$  \hfill (4.13)

Where $\sigma_\theta$, $\sigma_m$ and $\sigma_s$ are the hoop, meridional and thickness stresses, and $\varepsilon_\theta$, $\varepsilon_m$ and $\varepsilon_s$ are the hoop, meridional and thickness strains.

The volume of the deforming material remains constant, which implies that,

$$\varepsilon_\theta + \varepsilon_m + \varepsilon_s = 0$$  \hfill (4.14)

The flow stress in the thickness direction is ignored.

$$\sigma_s = 0$$  \hfill (4.15)

4.3 EXPERIMENTAL SETUP

4.3.1 Multi Dome Forming Die Assembly and Accessories

The geometry of the blanks used as specimens was, 70 mm diameter, and an initial thickness of 1.5 mm. The template plate has five holes of diameters, 1, 2, 5, 10, and 15 mm as shown in Figure 4.5. Figure 4.6 shows the experimental setup for the multi-dome forming test. In the experiment as shown, each test piece was clamped between the blank holder and the die holder. The electric furnace was used to heat the test piece, and the forming temperature was maintained within $\pm 2^\circ$C by using a controller, and compressed air was used to deform the specimen to various heights within the mold. The experiments were conducted under different forming pressures, temperatures, and annealing time modes.
4.4 EXPERIMENTAL PROCEDURE

4.4.1 Formation of a Multi Dome Under Various Pressures

The experimental work was divided into three segments; in the first segment, three samples, 1, 2 and 3 were considered. Sample 1 was formed under a constant forming pressure of 0.4 MPa; sample 2 under 0.5 MPa, and sample 3 under 0.6 MPa. The forming process of samples 1, 2 and 3 was performed at 530°C, and the forming time was 60 minutes. The formed samples were taken out from the die setup, and the dome height and apex thickness were measured, using a digital micrometer. For all the samples, the initial sheet thickness was 1.5 mm.
4.4.2 Formation of a Multi Dome Under Various Temperatures

In the second segment, four samples, 4, 5, 6 and 7 were considered. Sample 4 was formed under a constant forming temperature of 500°C, sample 5 under 510°C, sample 6 under 520°C and sample 7 under 540°C. The forming process of samples 4, 5, 6 and 7 was performed at a constant forming pressure of 0.5 MPa and a forming time of 60 minutes. The formed samples were taken out from the die setup, and the dome height and apex thickness were measured, using a digital micrometer.

4.4.3 Formation of a Multi Dome Under Various Annealing Times

In the third segment, four samples, 8, 9, 10 and 11 were considered. Sample 8 was formed under 60 minutes’ annealing time, sample 9 under 90 minutes, sample 10 under 120 minutes, and sample 11 under 150 minutes’ annealing time. The forming process of the samples 8, 9, 10 and 11 was performed at a constant forming pressure of 0.5 MPa, the forming time was 60 minutes and a temperature of 530°C. The formed samples were taken out from the die setup, and the dome height and apex thickness were measured using a digital micrometer.

Optical microscopy was used to study the microstructures. The specimens were cut so as to obtain a flat surface for metallographic examination, mechanically polished, and then etched with Keller’s reagent, for 15 seconds. From the digitized images, taken with a CCD camera through an optical microscope, the grain size was measured and calculated, using the Biovis material plus software.
4.5 RESULTS AND DISCUSSION

Sample 1 was formed under 0.4 MPa after 30 minutes annealing at 530°C; the calculated ‘m’ value was 0.325, because of less pressure. Sample 2 was formed under 0.5 MPa after 30 minutes annealing at 530°C; the calculated ‘m’ value was 0.494. Sample 3 was formed under 0.6 MPa after 30 minutes annealing at 530°C; the calculated ‘m’ value was 0.616, but the thickness at the apex of the dome shows higher thinning. Normally the aluminium alloys have high grain boundary strength and due to that high resistance to slide exist at high pressure at initial stage. But sliding takes place due to that same high pressure encountered after large thinning. From the first segment, sample 2 gives a better strain rate sensitivity index with the apex thickness. Figure 4.7 shows the formed component, Table 4.1 shows the strain rate sensitivity of samples 1, 2 and 3 after the multi dome test, and Figure 4.8 shows the variation of the bulge height with respect to the forming pressures of samples 1, 2 and 3. The bulge height increases with an increase in the pressure, but the apex thickness was reduced; it is clear that the forming pressure influences the forming capability of the component (ASM Handbook 1988).

![Figure 4.7 Formed component in the multi dome test](image-url)
Table 4.1 The strain rate sensitivity of samples 1, 2 and 3

<table>
<thead>
<tr>
<th>sample</th>
<th>Forming pressure (MPa)</th>
<th>Diameter of opening (mm)</th>
<th>Bulge height (mm)</th>
<th>Pole thickness (mm)</th>
<th>Effective flow stress (MPa)</th>
<th>Strain</th>
<th>$M$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4</td>
<td>15</td>
<td>2.88</td>
<td>1.36</td>
<td>1.72</td>
<td>0.137</td>
<td>0.325</td>
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<td>1.39</td>
<td>1.47</td>
<td>1.97</td>
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<tr>
<td>2</td>
<td>0.5</td>
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<td>1.33</td>
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<td>0.191</td>
<td>0.494</td>
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<td>0.251</td>
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<td>1.39</td>
<td>1.68</td>
<td>0.149</td>
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</tr>
</tbody>
</table>

Figure 4.8 Variation of the bulge height with respect to the forming pressures of samples 1, 2 and 3

In the second segment, sample 4 was formed under 0.5 MPa after 30 minutes annealing at 500°C; the calculated ‘$m$’ value is quite less 0.0162 due to the very low forming temperature. Sample 5 was formed under 0.5 MPa after 30 minutes annealing at 510°C; the calculated ‘$m$’ value is less 0.349 due the low forming temperature. Sample 6 was formed under 0.5 MPa after 30 minutes annealing at 520°C; the calculated ‘$m$’ value is 0.469 due to the forming temperature. Sample 7 was formed under 0.5 MPa after 30
minutes annealing at 540°C; the calculated ‘m’ value is 0.49 but the apex thickness of the dome is (1.13mm) less; due to the high forming temperature, the grains lost their stability. Table 4.2 shows the strain rate sensitivity of samples 4, 5, 6 and 7 after the multi dome test. Figure 4.9 shows the variation of the bulge height with respect to the forming temperature of samples 4, 5, 6 and 7. The bulge height increases with an increase in the temperature, but the apex thickness was reduced; it is clear that the forming temperature influences the forming capability of the component.

Table 4.2 The strain rate sensitivity of samples 4, 5, 6 and 7

<table>
<thead>
<tr>
<th>Sample</th>
<th>Forming temperature (°C)</th>
<th>Diameter of opening (mm)</th>
<th>Bulge height (mm)</th>
<th>Pole thickness (mm)</th>
<th>Effective flow stress (MPa)</th>
<th>Strain</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>500</td>
<td>15</td>
<td>2.41</td>
<td>1.43</td>
<td>2.37</td>
<td>0.098</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.962</td>
<td>1.48</td>
<td>2.33</td>
<td>0.036</td>
<td></td>
<td></td>
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<tr>
<td>5</td>
<td>510</td>
<td>15</td>
<td>2.88</td>
<td>1.40</td>
<td>2.14</td>
<td>0.137</td>
<td>0.349</td>
</tr>
<tr>
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<td>10</td>
<td>1.39</td>
<td>1.45</td>
<td>1.74</td>
<td>0.075</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>520</td>
<td>15</td>
<td>3.194</td>
<td>1.36</td>
<td>2.05</td>
<td>0.167</td>
<td>0.469</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>1.59</td>
<td>1.41</td>
<td>1.57</td>
<td>0.097</td>
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</tr>
<tr>
<td>7</td>
<td>540</td>
<td>15</td>
<td>4.12</td>
<td>1.13</td>
<td>1.93</td>
<td>0.264</td>
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</tr>
<tr>
<td></td>
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<td>1.95</td>
<td>1.30</td>
<td>1.42</td>
<td>0.142</td>
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<td></td>
</tr>
</tbody>
</table>

Figure 4.9 Variation of the bulge height with respect to the forming temperature of samples 4, 5, 6 and 7
In the third segment, sample 8 was formed under 0.5 MPa after 60 minutes annealing at 530°C; the calculated ‘m’ value is 0.426. Sample 9 was formed under 0.5 MPa after 90 minutes annealing at 530°C; the calculated ‘m’ value is 0.415. Sample 10 was formed under 0.5 MPa after 120 minutes annealing at 530°C; the calculated ‘m’ value is 0.415. Sample 11 was formed under 0.5 MPa after 150 minutes annealing at 530°C; the calculated ‘m’ value is 0.395. The annealing time increases, while the ‘m’ value is reduced, due to a marginal increase in the grain size during annealing. Table 4.3 shows the strain rate sensitivity of samples 8, 9, 10 and 11 after the multi dome test. Figure 4.10 shows the variation of the bulge height with respect to the annealing times of samples 8, 9, 10 and 11. The bulge height does not change with an increase in the annealing time.

Table 4.3 The strain rate sensitivity of samples 8, 9, 10 and 11

<table>
<thead>
<tr>
<th>Sample</th>
<th>Annealing time (minutes)</th>
<th>Diameter of opening (mm)</th>
<th>Bulge height (mm)</th>
<th>Pole thickness (mm)</th>
<th>Effective flow stress (MPa)</th>
<th>Strain</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>60</td>
<td>15</td>
<td>2.39</td>
<td>1.38</td>
<td>2.38</td>
<td>0.097</td>
<td>0.4258</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>1.22</td>
<td>1.39</td>
<td>1.19</td>
<td>0.058</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>90</td>
<td>15</td>
<td>2.23</td>
<td>1.32</td>
<td>2.49</td>
<td>0.085</td>
<td>0.4146</td>
</tr>
<tr>
<td></td>
<td></td>
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<td>1.37</td>
<td>2.02</td>
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<tr>
<td>10</td>
<td>120</td>
<td>15</td>
<td>2.23</td>
<td>1.32</td>
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<td>1.14</td>
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<tr>
<td>11</td>
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<td>15</td>
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<td>1.32</td>
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<td>1.14</td>
<td>1.37</td>
<td>2.11</td>
<td>0.046</td>
<td></td>
</tr>
</tbody>
</table>
4.6 SUMMARY

It was found that the material after 30 minutes’ annealing has obtained a higher strain rate sensitivity of 0.49, with the forming parameters, of the pressure of 0.5 MPa, forming time of 60 minutes and temperature of 530°C.

In the second segment, sample 7 gives a higher strain rate sensitivity of 0.49 with the forming parameters of the pressure of 0.5 MPa, forming time of 60 minutes and temperature of 540°C, but the apex thickness was the minimum of 1.136mm, because of the high forming temperature. Hence, when compared with sample 2, sample 7 has obtained a lesser thinning factor. It affects the integrity of the components.

When the annealing time is increased from 60 to 150 minutes, the strain rate sensitivity index value was reduced, due to the grain instability during the high annealing time. Also, there is no change in the bulge height; it is evident that there is a marginal increase in the grain size during annealing.

Figure 4.10 Variation of the bulge height with respect to the annealing times of samples 8, 9, 10 and 11