CHAPTER II

ASSET PRICING MODELS –

CONCEPTUAL FRAMEWORK

2.1 Introduction

Asset Pricing is one of the oldest topics in finance. Asset pricing asks how the expected rates of return on various assets are determined. The most common answer that has emerged from asset pricing theory is that riskier assets must pay higher returns on average to compensate people for holding them. Asset pricing seeks to determine which risks people care about and demand compensation for, and which risks do not matter to people. In other words, the big issue in asset pricing is measurement of risk and several asset pricing models have evolved over time that find out price of risk in order to determine expected rates of return on assets.

The theoretical asset pricing models have been particularly amenable to empirical testing. The rapid growth of computer technology in the latter part of the twentieth century, taken together with relatively easy access to stock and bond price data, has allowed researchers to examine whether the various asset-pricing models are supported by
rigorous statistical studies. Moreover, in a number of cases the paradoxes revealed in the data have in turn influenced the development of new theoretical models. At the same time, asset-pricing theory has had a direct impact on the world of business and finance. Valuation models have contributed significantly towards the growth of derivatives markets. The large amounts of capital invested using these models have created additional pressures for researchers to explore and examine these theories. The literature on asset pricing has therefore been crucial for our understanding and development of financial markets.

Conventional wisdom has always suggested that investing all the money in assets that may all perform poorly at the same time—that is, whose returns are highly correlated—is not a very prudent investment strategy no matter how small the chance that any one asset will perform poorly. This is because if any one single asset performs poorly, due to its high correlation with the other assets, it is likely that these other assets are also going to perform poorly, leading to the poor performance of the portfolio. In more technical terms, this is addressing the benefits of diversification. Markowitz (1952) quantified the concept of diversification through the statistical notion of covariance between individual securities, and the overall standard deviation of a portfolio, popularly known as mean-variance analysis. He explains that, "the portfolio with maximum expected return is not necessarily the one with..."
minimum variance. There is a rate at which the investor can gain expected return by taking on variance, or reduce variance by giving up expected return”. The basic assumption behind his approach is that an investor’s preferences can be represented by a function (utility function) of the expected return and the variance of a portfolio. The most important contribution made by Markowitz (1952) is his distinction between the variability of returns from an individual security and its contribution to the riskiness of a portfolio. He notes that “in trying to make variance small it is not enough to invest in many securities. It is necessary to avoid investing in securities with high covariances among themselves”. Many asset pricing models are based on the seminal work of Markowitz (1952).

2.2 The Capital Asset Pricing Model

Sharpe (1964) developed capital asset pricing model (CAPM). It builds on the model of portfolio choice developed by Markowitz (1952, 1959) that assumes investors are risk averse and when choosing among portfolios, they care only about the mean and variance of their one-period investment return. Investors may choose mean-variance-efficient portfolios, in the sense that the portfolios: i) minimize the variance of portfolio return, given expected return, and ii) maximize expected return,
given variance. Sharpe’s (1964) CAPM was supplemented by contributions from Lintner (1965) and Mossin (1966).

2.2.1 Assumptions

CAPM is based on certain assumptions. The reservation on the practicality or reasonableness of the assumptions is well answered in the words of Friedman (1953) – ‘the relevant question to ask about the “assumptions” of a theory is not whether they are descriptively “realistic”, for they never are, but whether they are sufficiently good approximations for the purpose in hand. And this question can be answered only by seeing whether the theory works, which means whether it yields sufficiently accurate predictions.’ The assumptions are:

(a) All investors have the same one period horizon.

(b) All assets are marketable and perfectly divisible.

(c) Investors have homogeneous expectations about asset returns.

(d) Investors are risk averse and evaluate portfolios by looking at the expected returns and standard deviation of the portfolios during a one period horizon.

(e) There exists risk free asset such that investors may borrow or lend unlimited amounts at risk free rate.
(f) Markets are frictionless, and information is costless and simultaneously available to investors.

(g) There are no market imperfections such as taxes, regulations or restrictions on short selling.

Under the set of assumptions, the CAPM model can be expressed as \( \tilde{r}_i = r_f + \beta_i (\bar{r}_m - r_f) \) where \( \tilde{r}_i \) is expected return on asset i, \( r_f \) is the risk-free rate of return, \( \bar{r}_m \) is expected return on market portfolio \( (m) \) and \( \beta_i \) is a measure of risk specific to asset i. The market portfolio \( (m) \) is essentially efficient and consists of all securities in which the proportion invested in each security corresponds to its relative market value. \( \beta_i \) is calculated as covariance between asset return and variance of market portfolio.

### 2.2.2 Derivation of CAPM

The capital market line is an equilibrium relationship that relates the expected rate of return of an efficient portfolio to its standard deviation \( [\tilde{r}_i = r_i + \frac{\bar{r}_m - r_f}{\sigma_m} \sigma_i ] \), but it does not show how the expected rate of return of an individual asset relates to its individual risk. This relation is expressed by the capital asset pricing model. For derivation purpose let us consider a portfolio consisting of a portion \( \alpha \) invested in asset i and a portion \( (1 - \alpha) \) invested in the market portfolio M. (We
allow $\alpha < 0$, which corresponds to borrowing at the risk-free rate.) The expected rate of return of this portfolio is

$$\bar{r}_a = \alpha \bar{r} + (1 - \alpha)\bar{r}_M$$

and the standard deviation of the rate of return is

$$\sigma_a = \left[ \alpha^2 \sigma_i^2 + 2\alpha(1 - \alpha)\sigma_{i,M} + (1 - \alpha)^2 \sigma_M^2 \right]^{1/2}$$

As $\alpha$ varies, these values trace out a curve in the risk ($\sigma$) and return ($\bar{r}$) diagram, as shown in Figure 2.1.

**Figure 2.1 Opportunity Set, Market Portfolio**

In particular, $\alpha = 0$ corresponds to the market portfolio $M$. This curve cannot cross the capital market line. If it did, the portfolio corresponding to a point above the capital market line would violate the very definition of the capital market line as being the efficient boundary of the feasible set. Hence as it passes through zero, the curve
must be tangent to the capital market line at \( M \). This tangency is the condition that is used to derive the formula. The tangency condition can be translated into the condition that the slope of the curve is equal to the slope of the capital market line at the point \( M \). To set up this condition, we first have

\[
\frac{d\bar{r}_a}{da} = \bar{r}_f - \bar{r}_M
\]

Thus,

\[
\frac{d\sigma_a}{da} \bigg|_{a=0} = \frac{\sigma_{i,M} - \sigma_M^2}{\sigma_a}
\]

Using \( \frac{d\bar{r}_a}{d\sigma_a} = \frac{\bar{r}_a}{d\sigma_a} \), we obtain

\[
\frac{d\bar{r}_a}{d\sigma_a} \bigg|_{a=0} = \frac{(\bar{r} - \bar{r}_f)\sigma_M}{\sigma_{i,M} - \sigma_M^2}.
\]

This slope must equal the slope of the capital market line. Hence,

\[
\frac{(\bar{r} - \bar{r}_M)\sigma_M}{\sigma_{i,M} - \sigma_M^2} = \frac{\bar{r}_M - \bar{r}_f}{\sigma_M}.
\]

Solving for \( \bar{r}_f \), we obtain the final equation

\[
\bar{r} = \bar{r}_f + \left( \frac{\bar{r}_M - \bar{r}_f}{\sigma_M^2}\right)\sigma_{i,M} = \bar{r}_f + \beta_i(\bar{r}_M - \bar{r}_f)
\]
The value $\beta_i$ is referred to as the beta of an asset. An asset's beta is all that need be known about the asset's risk characteristics to use the CAPM formula.

The value $r_i - r_f$ is termed the expected excess rate of return of asset $i$, it is the amount by which the rate of return is expected to exceed the risk-free rate. Likewise, $\bar{r}_m - r_f$ is the expected excess rate of return of the market portfolio. In terms of these expected excess rates of return, the CAPM says that the expected excess rate of return of an asset is proportional to the expected excess rate of return of the market portfolio, and the proportionality factor is $\beta$. So with $r_f$ taken as a base point, the expected returns of a particular asset and of the market above that base are proportional.

An alternative interpretation of the CAPM formula is based on the fact that $P$ is a normalized version of the covariance of the asset with the market portfolio. Hence the CAPM formula states that the expected excess rate of return of an asset is directly proportional to its covariance with the market. It is this covariance that determines the expected excess rate of return.
2.2.3 Empirical Test of CAPM

Although we have literature on econometric problems that arises during the empirical testing (Miller and Scholes, 1972; Roll, 1977; Dimson, 1979; and Gibbons, 1982), there have been numerous empirical tests of CAPM. The primary step necessary to empirically test the CAPM is to transform it from ex ante form to ex post form (as expectations cannot be measured) and then run a linear regression like

\[
r_{it} = \alpha_0 + \alpha_i \beta_i + \varepsilon_{it},
\]

where

\[
r_{it} = r_{it} - r_f = \text{Excess return on } i^{th} \text{ asset over risk free rate } r_f
\]

\[
\beta_i = \text{Ratio of covariance between asset return and market portfolio return to variance of market portfolio.}
\]

\[
\varepsilon_{it} \text{ is the error term with } E(\varepsilon_{it}) = 0
\]

\[
\alpha_0 \text{ and } \alpha_i \text{ are the parameters to be estimated.}
\]

The following predictions made by CAPM are generally tested using the above regression:

1. The intercept term \( \alpha_0 \) should not be significantly different from zero. If it is different from zero there may be something left out of the CAPM that is captured in the empirically estimated intercept term.

2. Beta should be the only factor that explains the rate of return of the risky asset.
3. The relationship should be linear in beta.

4. The coefficient of beta i.e., $\alpha_1$ should be equal to $\bar{r}_M - r_f$.

5. The coefficient of beta i.e., $\alpha_1$ should be greater than zero.

2.2.4 CAPM Anomalies

An anomaly is usually a disorder, a deviation from the norm. In natural science, it has induced researchers to formulate new theories. In finance however, what could not be explained by traditional asset pricing theories\(^1\) was easily arbitrated, and later labeled as anomaly.

The research on anomalies began with researchers recognizing the existence of asset mispricing that surpassed available economic theories' ability to explain them. It is always easier to determine the causes of the occurrences with the benefit of hindsight. But when anomalies are actually taking place, it is not easy to identify them, let alone incorporate them into pricing models. This is the benefit market speculators get for their efforts in identifying anomalies. When an anomaly gets detected, and enough arbitrageurs have made money, the trend disappears. This is when the anomaly is ripe for public introduction and the race begins for providing extensive analysis in financial journals. Amongst the reasons

\(^1\) Traditional asset pricing theories cover the CAPM and the factor model (APT)
for anomalies are: tax evasion, window-dressing of portfolio fund managers, behavioural biases, etc.

The phenomenon of anomalies is best explained by an amalgam of available financial literature. The following section presents a sample of the more important contributions in this area that collectively stand as a challenge for alternative asset pricing models.

(a) The Value Effect

The value effect refers to the positive relation between security returns and the ratio of accounting based measures of cash flow or value to the market price of the security. Examples of the accounting-based measures are earnings per share and book value of common equity per share. Investment strategies based on the value effect have a long tradition in finance and can be traced at least to Graham and Dodd (1940). Ball (1978) argues that variables like the earnings-to-price ratio (E/P) are proxies for expected returns.

Basu (1977) was the first to test the notion that value-related variables might explain violations of the CAPM. He found a significant positive relation between E/P ratios and average returns for U.S. stocks that could not be explained by the CAPM. Reinganum (1981) confirmed and extended Basu's findings. Rosenberg et al. (1985), DeBondt and
Thaler (1987) and many others have documented a significant positive relation between returns and the book-to-price ratio.

Dividend yield, the ratio of cash dividend to price, has also been shown to have cross-sectional return predictability. Although similar in construction to the value ratios, the explanatory power of dividend yields is most often attributed to the differential taxation of capital gains and ordinary income as described in the after-tax asset pricing models developed by Brennan (1970) and Litzenberger and Ramaswamy (1979).

(b) The Size Effect

The size effect refers to the negative relation between security returns and the market value of the common equity of a firm. Banz (1981) was the first to document this phenomenon for U.S. stocks. The size effect has been reproduced for numerous sample periods and for most major securities markets around the world (Hawawini and Keim, 2000).

Others have argued that the size effect is actually a liquidity effect in which small-cap stocks are less liquid than large-cap stocks and therefore provide correspondingly higher returns to offset the higher transaction costs (e.g., Brennan et al. 1998). Still others have suggested that the size and B/P results may be due to survivor biases in the databases used by researchers (e.g., Kothari et al. 1995).
(c) The Prior Return or Momentum Effect

Prior stock returns have been shown to have explanatory power in the cross section of common stock returns. Stocks with prices on an upward (downward) trajectory over a prior period of 3 to 12 months have a higher than expected probability of continuing on that upward (downward) trajectory over the subsequent 3 to 12 months. This temporal pattern in prices is referred to as momentum. Jegadeesh and Titman (1993) show that a strategy that simultaneously buys past winners and sells past losers generates significant abnormal returns over holding periods of 3 to 12 months. The abnormal profits generated by such offsetting long and short positions appear to be independent of market, size or value factors and has persisted in the data for many years.

(d) Predicting Returns with Past Returns I: Individual Security Autocorrelations

Much research finds that autocorrelations of higher-frequency (daily, weekly) individual stock returns are negative and that the autocorrelations are inversely related to the market capitalization of the stock. The exception is that the largest market cap stocks have positive autocorrelations for daily returns. The inverse relation between individual return autocorrelations and market capitalization is due to the influence of a bid-ask bounce in high frequency stock prices that may
induce "artificial" serial dependencies into returns. Niederhoffer and Osborne (1966) find that successive trades tend to occur alternately at the bid and then the ask price, resulting in negative serial correlation in returns.

(e) Predicting Returns with Past Returns II: Aggregate Return Autocorrelations

Because of variance reduction obtained from diversification, aggregated or portfolio returns provide more powerful tests of return predictability using past returns. However, this increased power may be offset by upward-biased autocorrelations caused by the infrequent trading of securities in the portfolios (Fisher, 1966). This bias is more serious for portfolios of smaller-cap stocks that contain less-frequently traded stocks. In the U.S. and other global equity markets positive autocorrelations for high frequency portfolio returns range from 0.4 for small-cap stocks to 0.1 for large-cap stocks. Research has shown, however, that positive portfolio autocorrelations are not due to infrequent trading of the securities in the portfolio. Lo and MacKinlay (1990) reconcile the paradox of positive portfolio autocorrelations and negative individual stock autocorrelations: because the autocorrelation of portfolio returns is the sum of individual security autocovariances and cross-autocovariances, if the cross-autocovariances are sufficiently large.
relative to the autocovariances, then the cross-autocovariances will overshadow the contribution of the autocovariances.

(f) Calendar / Seasonal Effect

French (1980) first identified the weekend effect while studying daily stock returns from 1953 to 1977. French finds a “weekend effect” where Monday’s mean return is significantly less than zero, while the other weekday returns are significantly greater than zero. Keim (1983) and others document that fifty percent of the annual size premium in the U.S. is concentrated in the month of January, particularly in the first week of the year. This finding has been reproduced on many equity markets throughout the world. Blume and Stambaugh (1983) subsequently demonstrated that, after correcting for an upward bias in average returns for small stocks (related to the magnitude of bid-ask spreads), the size premium is evident only in January.

The empirical evidence on some of the important and significant asset pricing anomalies (relative to the CAPM) found in financial studies during the last few decades are summarized below in Table 2.1. The anomaly and the authors credited for validating such anomalies are represented in first column. The second column represents the factor or the variable causing such anomalies and the third column shows the effect of such factors on equity returns, i.e., impact on share premium.
Table 2.1

Asset Pricing Anomalies

<table>
<thead>
<tr>
<th>Authors</th>
<th>Variable/factor</th>
<th>Impact on the premium</th>
</tr>
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<tbody>
<tr>
<td><strong>Seasonal:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reinganum (1983)</td>
<td>January dummy</td>
<td>+</td>
</tr>
<tr>
<td>French (1980)</td>
<td>Monday dummy</td>
<td>-</td>
</tr>
<tr>
<td><strong>Price-related:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basu (1977, 1983)</td>
<td>Earning/Price ratio</td>
<td>+</td>
</tr>
<tr>
<td>Stattman (1980)</td>
<td>Book-to-market ratio</td>
<td>+</td>
</tr>
<tr>
<td>Banz (1981)</td>
<td>Size: Market Equity</td>
<td>-</td>
</tr>
<tr>
<td>Bhandari (1988)</td>
<td>Leverage: Debt/Equity</td>
<td>+</td>
</tr>
<tr>
<td><strong>Past performance:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jegadeesh and Titman (1993)</td>
<td>Momentum</td>
<td>+</td>
</tr>
<tr>
<td>De Bondt and Thaler (1985)</td>
<td>Asymmetric reaction to news/events</td>
<td>-</td>
</tr>
<tr>
<td><strong>Market microstructure:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brennan and Subrahmanyam (1996)</td>
<td>Liquidity: Trading Volume</td>
<td>-</td>
</tr>
</tbody>
</table>

These anomalies have been empirically explored for quite some time both in developed as well as in developing countries. However, the evidences of these factors as causes of anomalies universally are weak and inconclusive especially in emerging markets and it is still an interesting area of research.
2.2.5 Extensions of CAPM

The classical CAPM as developed by Sharpe (1964) was extended by several researchers to improve the predictive ability of the model. The most frequently cited modification is by Black (1972), who shows how the model needs to be adapted when riskless borrowing is not available; his version is known as the zero-beta CAPM. Another important variant is by Brennan (1970), who finds that the structure of the original CAPM is retained when taxes are introduced into the equilibrium. Mayers (1972) shows that when the market portfolio includes non-traded assets, the model also remains identical in structure to the original CAPM. The model can also be extended to encompass international investing, as in Solnik (1974) and Black (1974). The theoretical validity of the CAPM has even been shown to be relatively robust if the assumption of homogenous return expectations is relaxed, as in Williams (1977). Finally, there are extensions from the classical one-period setting to a continuous time environment. Extensions of CAPM have resulted in prominent asset pricing models that include the multifactor models as well. Some of these important asset pricing models are like:

(a) **Intertemporal Capital Asset Pricing Model** (ICAPM) by Merton (1973),

(b) **Arbitrage Pricing Theory** (APT) by Ross (1976),
(c) **Consumption-Based Capital Asset Pricing Model** (CCAPM) as first developed by Breeden (1979),

(d) **Production-Based Asset Pricing Model** made popular by Cochrane (1991),

(e) **Conditional Capital Asset Pricing Model** by Jagannathan and Wang (1996) and

(f) **Stochastic Difference Equations** that considers changing volatility of asset returns like GARCH models made popular by Bollerslev (1986).

### 2.3 The Intertemporal Capital Asset Pricing Model

In order to construct a framework that is both more realistic and at the same time, more tractable than the discrete time model, Merton (1973) developed an Intertemporal Capital Asset Pricing Model (ICAPM) by assuming that time flow is continuous. The framework of continuous time turns out to be one of the major developments of modern finance, in both equilibrium asset pricing and derivative valuation.

CAPM is a static single-period model. The amount invested into assets was fixed for a given period of time and the amounts invested into each asset could not be changed. At the end of a time period it was assumed that the investors consume their wealth and therefore the CAPM...
ignores the multi-period nature of participation in the capital markets. Merton's (1973) intertemporal capital asset pricing model (ICAPM) was developed to capture this multi-period aspect of financial market equilibrium. The ICAPM framework recognizes that the investment opportunity set might shift over time, and investors would like to hedge themselves against unfavorable shifts in the set of available investments. If a particular security tends to have high returns when bad things happen to the investment opportunity set, investors would want to hold this security as a hedge. This increased demand would result in a higher equilibrium price for the security (all else constant). One of the main insights of the ICAPM is the need to reflect this hedging demand in the asset pricing equation. The resulting model is:

\[
E(R_j) = R_f + \beta_{jM}\lambda_M + \beta_{j2}\lambda_2 + \ldots + \beta_{jn}\lambda_n
\]

where \(E(R_j)\) is expected return on asset \(j\), \(R_f\) is return on a risk free asset, \(\beta_{ji}\) is the reaction coefficient describing the change in asset \(j\)’s return for a unit change in factor \(i\), and \(\lambda_i\) is the premium for risk associated with factor \(i\). The equation form of ICAPM is very similar to that of the APT. There are subtle differences, however. The first factor of the ICAPM is explicitly identified as being related to the market portfolio. Further, while the APT gives little guidance as to the number and nature of
factors, the factors that appear in the ICAPM are those that satisfy the following conditions:

(a) They describe the evolution of the investment opportunity set over time.

(b) Investors care enough about them to hedge their effects.

For example, there might be a priced factor for unexpected changes in the real interest rate. Such a change would certainly shift the investment opportunity set and the effect would be pervasive enough that investors would want to protect themselves from the negative consequences. We still don’t know exactly how many factors there are, but the ICAPM at least gives us some guidance.

In particular, Merton (1973) demonstrates that an agent’s welfare at any point in time is not only a function of his own wealth, but also the state of the economy. If the economy is doing well then the agent’s welfare will be greater than if it is doing badly, even if the level of wealth is the same. Thus the demand for risky assets will be made up not only of the mean-variance component, as in the static portfolio optimisation problem of Markowitz (1952), but also of a demand to hedge adverse shocks to the investment opportunity set.

Merton’s (1973) analysis was at the same time disconcerting because it runs counter to the basic intuition of the CAPM, that an asset has greater value if its marginal contribution to wealth is greater.
2.4 Arbitrage Pricing Theory

A substitute and concurrent theory to the CAPM is one that incorporates multiple factors in explaining the movement of asset prices. Ross (1976) proposed the Arbitrage Pricing Theory (APT) that offers a testable alternative to the CAPM. The central insight of APT is that if investors understand the generating processes of security returns they can use this information to design portfolios in which net wealth is zero (because short and long positions are equal) and in which all risk has been completely diversified away. The familiar arbitrage condition suggests that the expected return on such portfolios should be zero for otherwise infinite wealth could be arbitrarily accumulated without any net investment or exposure to risk. The CAPM predicts that security returns will be linearly related to a single common factor – the rate of return on the market portfolio. The alternative model for asset pricing APT assumes that security returns are generated by a factor model but does not identify the factors. It implies that securities or portfolios with equal factor sensitivities should offer the same expected returns. If not, investors will take advantage of arbitrage opportunities, causing their elimination. The equilibrium expected return on a security is a linear function of its sensitivities to the factors. Unlike CAPM, which requires strong restrictions on return distributions or preferences, the APT gives a
characterization of expected returns on assets based only on the weak assumptions that there are no arbitrage opportunities and that the return follows a factor structure. The intuition for APT is drawn from the special case in which there are no idiosyncratic shocks so that return on each security depends solely on its exposure to risk factors. With idiosyncratic shocks ruled out the assumption that there are no arbitrage implies that there exist expected returns associated with each factor such that the expected return of any security is expressible as a linear function of its exposure to the factor risks, with coefficients being just the expected returns on the factors. The result follows directly from the duality theorem of linear programming (e.g., Dybvig and Ross, 1987; Ingersoll, 1984) if there are a finite number of securities. APT can be described by the following equation:

$$\tilde{r}_i = r_f + \gamma_1 \beta_{i1} + \gamma_2 \beta_{i2} + \gamma_3 \beta_{i3} + \ldots + \gamma_j \beta_{ij}$$

where $\tilde{r}_i$ is expected return on asset $i$, $r_f$ is return on a risk free asset, $\beta_{ij}$ is the reaction coefficient describing the change in asset $i$'s return for a unit change in factor $j$, and $\gamma_j$ is the premium for risk associated with factor $j$. If there are portfolios with identical risk but different returns then investors will push up the prices of undervalued portfolios and vice versa for overvalued portfolios till risk and return are equated. The sensitivity to factors (like beta in CAPM) are estimated as follows:
where $r_{i,t}$ is the return on asset $i$ in period $t$, $\beta_{i,0}$ is the estimated return on asset $i$ when all $\delta_{j,t}$ values are zero, $\delta_{j,t}$ is the value at time $t$ of factor $j$ common to the returns of all assets, $\beta_{i,j}$ is the estimated sensitivity of asset $i$ to factor $j$, and $u_{i,t}$ represents residual risk.

The APT provides theoretical support for an asset pricing model where there is more than one risk factor. Consequently, models of this type are referred to as multifactor risk models. Three different types of multifactor risk models used in equity portfolio management: statistical factor models, macroeconomic factor models, and fundamental factor models.

2.4.1 Statistical Factor Model

In a statistical factor model, historical and cross-sectional data on stock returns are tossed into a statistical model. The goal of the statistical model is to best explain the observed stock returns with “factors” that are linear return combinations and uncorrelated with each other. This is typically accomplished by principal component analysis (PCA).
2.4.2 Macroeconomic Factor Model

In a macroeconomic factor model, the inputs to the model are historical stock returns and observable macroeconomic variables. The goal is to determine which macroeconomic variables are persistent in explaining historical stock returns. Those variables that consistently explain the returns then become the factors and are included in the model.

2.4.3 Fundamental Factor Model

Fundamental factor models use company and industry attributes and market data as raw descriptors. The inputs into a fundamental factor model are stock returns and the raw descriptors about a company. Those fundamental variables about a company that are pervasive in explaining stock returns are then the raw descriptors retained in the model.

2.5 Consumption-Based Capital Asset Pricing Model

Breeden (1979) reconciled Merton’s (1973) ICAPM with the classical CAPM by highlighting the dichotomy between wealth and consumption. In an intertemporal setting, Breeden showed that agents’ preferences must be defined over consumption and thus “always, when the value of an additional dollar payoff in a state is high, consumption is low in that state, and when the value of additional investment is low,
optimal consumption is high. This is not always true for wealth, when investment opportunities are uncertain. The implication is that assets are valued by their marginal contribution to future consumption and not wealth. Breeden’s (1979) model which became known as the Consumption CAPM (CCAPM) allows assets to be priced with a single beta as in the traditional CAPM. In contrast to the latter, the CCAPM’s beta is measured not with respect to aggregate market wealth, but with respect to an aggregate consumption flow and, as Breeden states, “the higher that an asset’s beta with respect to consumption is, the higher its equilibrium expected rate of return”. However, one important insight from Merton’s (1973) ICAPM is that multiple risk factors are needed to explain asset prices.

The consumption-based model of Breeden (1979) provides a logical extension of the previous work in asset pricing. Breeden’s model is based on the intuition that an extra rupee of consumption is worth more to a consumer when the level of aggregate consumption is low. When things are going really well and many people can afford a comfortable standard of living, another dollar of consumption doesn’t make us feel very much better off. But when times are hard, a few extra dollars to spend on consumption goods is very welcome. Based on this “diminishing marginal utility of consumption”, securities that have high returns when aggregate consumption is low will be demanded by
investors, bidding up their prices (and lowering their expected returns). In contrast, stocks that co-vary positively with aggregate consumption will require higher expected returns, since they provide high returns during states of the economy where the high returns do the least good.

Based on this line of reasoning, Breeden (1979) derives a consumption-based capital asset pricing model (CCAPM) of the form:

$$E(R_j) = R_f + \beta_{jc} [E(R_M) - R_f]$$

In this model, $\beta_{jc}$ measures the sensitivity of the return of asset $j$ to changes in aggregate consumption. $\beta_{jc}$ is referred to as the consumption beta of asset $j$. $[E(R_M) - R_f]$ is the total risk premium and the CCAPM’s main result is that expected returns should be a linear function of consumption betas.

Despite the intuitive appeal of the consumption-based model, empirical tests have not supported its predictions (Breeden et al. 1989). Accordingly, consumption-based asset pricing has not received as much attention in practice as the other models discussed earlier.

2.6 Production-Based Asset Pricing Model

A complementary asset pricing approach looks on the production side instead of the consumption side. It includes some quite different contributions. Based on the asset pricing models of Brock (1982) and
Lucas (1978), respectively, Balvers et al. (1990) and Cecchetti et al. (1990) argue that aggregate output is equal or proportionate to aggregate consumption and that one could evaluate the marginal utility of consumption at the observed level of output so that aggregate output growth becomes the key asset pricing factor. The advantage is that output growth is likely measured more accurately than consumption growth. Within this approach, a different production-based perspective is provided by Cochrane (1991, 1996) who explicitly derives an expression for investment returns and assumes that these can serve as a pricing kernel for asset returns. By taking the demand side (consumption) as given and modeling the supply side (production) of the economy Cochrane (1991) developed the Production-based Asset Pricing Model (PAPM). Cochrane (1996) finds that investment returns (using two factors – residential and non-residential investment growth – as proxies) are significantly priced.

2.7 Conditional Capital Asset Pricing Model

The performance of the traditional CAPM has been debatable. It has been proposed that the reason may be the static nature of the model and the therewith following assumptions of a fixed beta and fixed risk premium. Allowing the beta to vary over time can be justified by the
reasonable assumption that the relative risk and the expected excess returns of an asset may vary with the business cycle. It therefore is reasonable to use all available information on the business cycle and other relevant variables to form expectations, i.e. to use conditional moments. This gave rise to the Conditional Capital Asset Pricing Model. Jagannathan and Wang (1996) developed this model that can be expressed as:

For each asset \( i \) and in each period \( t \),

\[
E(R_{it} / I_{t-1}) = \gamma_{0t-1} + \gamma_{1t-1} \beta_{it-1},
\]

Where \( \beta_{it-1} \) is the conditional beta of asset \( i \) defined as:

\[
\beta_{it-1} = \frac{\text{Cov}(R_{it}, R_{Mt} / I_{t-1})}{\text{Var}(R_{Mt} / I_{t-1})}
\]

\( \gamma_{0t-1} \) is the conditional expected return on a “zero-beta” portfolio, and

\( \gamma_{1t-1} \) is the conditional market risk premium.

\( R_{Mt} \) is the market return.

\( I_{(t-1)} \) denotes the common information set of the investors at the end of period \( t - 1 \).

2.8 Linear Stochastic Difference Equations

A difference equation expresses a variable as a function of its own lagged value, time and other variables. Time series econometrics is
concerned with the estimation of difference equations containing stochastic components. Autoregressive (AR) models, Moving Average (MA) models, Autoregressive Moving Average (ARMA) models and Integrated Autoregressive Moving Average (ARIMA) models are often successfully used to forecast asset returns. Along with this, many financial time series was found to exhibit diverse form of non-linear dynamics, the crucial one being the strong dependence of variability of the series on its own past. This spawned research to specify how the information could be used to forecast the mean and variance of the return, conditional on the past information. Engle (1982) and Bollerslev (1986) gave rise to a different class of econometric models that considers changing volatility of asset returns. Their models of conditional heteroskedasticity, together with a large number of variations developed on their basic work, have become widely applied to model the returns of assets. Some of these models are discussed below:

2.8.1 Autoregressive (AR) Models

AR is autoregressive process where the outcome is modeled as a weighted average of past observations plus an error term. A $p^{th}$ order autoregressive model takes the form $y_t = \alpha_0 + \sum_{i=1}^p \alpha_i y_{t-i} + \epsilon_t$ where $y_t$ is the response (dependent) variable at time $t$, $y_{t-1}, \ldots, y_{t-p}$ are response
variable at time lags t-1, t-2, ... t-p respectively, \( \alpha_0, \alpha_1, \alpha_2, ..., \alpha_p \) are the coefficients to be estimated and \( \epsilon_t \) is the error term at time t that represents the effects of variables not explained by the model.

2.8.2 Moving Average (MA) Models

MA is moving average processes where outcome is expressed as a moving average of the past values of the error term. The \( q^{th} \) order MA process may be expressed as \( y_t = \sum_{i=0}^{q} \beta_i \epsilon_{t-i} \) where \( \beta_i \) are the coefficients to be estimated and \( \epsilon_t \) is the error term.

2.8.3 Autoregressive Moving Average (ARMA) Models

In ARMA models the value of the series at any point is written as a linear function of its lagged values and past values of some noise processes. It may be represented as \( y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-i} + \sum_{j=0}^{q} \beta_j \epsilon_{t-j} \) where \( \alpha_i \) and \( \beta_j \) are the parameters to be estimated.
2.8.4 Integrated Autoregressive Moving Average (ARIMA) Models

An ARIMA model consists of a unit root non stationary time series which can be rendered stationary by a difference equation. By introducing the difference operator $\nabla$, it may be said that a time series $X_t$ follows a ARIMA $(p,d,q)$ model if $X_t$ differenced $d$ times ($\nabla^d X_t$) is an ARMA$(p,q)$ process.

2.8.5 Autoregressive Conditional Heteroskedasticity (ARCH) Models

Autoregressive Conditional Heteroskedasticity (ARCH) model was proposed by Engle (1982) that aims to model variance $\{\sigma_i^2\}$ and can be expressed as $\sigma_i^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2$ with mean equation similar to that of an ARMA model like $y_t = \beta_0 + \sum_{i=1}^{p} \beta_i y_{t-i} + \sum_{j=0}^{q} \beta_j \varepsilon_{t-j}$. Here, the "autoregressive" property in principle means that old events leave waves behind a certain time after the actual time of the action. The process depends on its past. The terms "conditional heteroskedasticity" means that the variance (conditional on the available information) varies and depends on old values of the process. One can resemble this with the process having a short-term memory and that the behaviour of the process
is influenced by this memory. However, empirical evidence shows that high ARCH order has to be selected in order to catch the dynamic of the conditional variance. Some areas of empirical application of the ARCH specification are (a) effects of shocks on the variance of stock market returns, (b) effects of increases in the variance of excess returns of bonds on risk premiums. Gourieroux (1997), Das and Bhattacharya (2011a) among others, have shown application of ARCH models to asset pricing.

2.8.6 Generalised Autoregressive Conditional Heteroskedasticity (GARCH) Models

Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model of Bollerslev (1986) is often used where the expected return equation (often called mean equation) is an ARMA series with the conditional variance as a linear function of past squared innovations and past conditional variances. Such a model is represented as:

$$
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
$$

where $\varepsilon_t$ denote a real-valued discrete-time stochastic process and the $\alpha_i$’s and the $\beta_j$’s are non-negative parameters, and $p > 0, q \geq 0, \alpha_0 > 0, \alpha_i \geq 0$.

There are many variants GARCH model (Das and Bhattacharya, 2011a). The GARCH model as discussed above is a symmetric volatility
model characterized by a symmetric response of current volatility to positive and negative lagged return. This is because, the GARCH expression for conditional variance shows that it considers square of return terms. The empirical literature on returns of risky assets shows that, future volatility of stock returns is much more affected by negative news compared with positive news (Black, 1976). In order to consider this leverage effect, asymmetric GARCH models like GJR GARCH models are often considered. The variance equation for GJR GARCH model can be expressed as

\[ \sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \varepsilon_{t-1}^2 I_{t-1} \]

where \( I_{t-1} = 1 \) if \( \varepsilon_{t-1} < 0 \) and = 0 otherwise.