CHAPTER 4

MATHEMATICAL APPROACH
FOR OPTIMAL MACHINING FIXTURE LAYOUT
AND CLAMPING FORCES

4.1 INTRODUCTION

The dimensional and form errors induced in the workpiece during machining are mainly influenced by the material, structure and fixturing scheme of a workpiece, clamping preloads and machining forces (Liao and Hu 2001). Some deformation exists in the workpiece when it is clamped to fixture and still more deformation occurs while machining. Few works have been carried out in determining the minimum clamping force required in the fixture system. Particle Swarm Optimization technique (Deng and Melkote 2005) and ε-constraint method (Li and Melkote 2001) have been used for determining the optimum clamping forces. A linear clamp pre-load (LCPL) model has been used to compute the minimum required pre-loads necessary to prevent workpiece slip at the fixture-workpiece joints throughout the machining process (De Meter et al 2001). Most of the studies use either the workpiece rigid-body model or workpiece-elastic contact model, and these studies do not consider the elastic deformation of the workpiece. Precisely for this purpose, the mathematical approach for optimum fixture layout and clamping forces to minimize the overall workpiece elastic deformation is presented in this chapter.
4.2 METHODOLOGY

The mathematical approach for the optimal fixture layout and clamping forces involves three simple steps. The flowchart of methodology is given in Figure 4.1. The formulation of the objective function is the important step as the results wholly depend on it. Here, the fixture is optimized for optimal clamping forces and fixture layout simultaneously. The program developed using Matlab 7.0 software is used to generate layouts and corresponding clamping forces. For each layout generated, the deformation value is calculated along with its respective clamping forces using the FEM software ANSYS 11.0. Finally the set of clamping forces and fixture layout which gives the minimum deformation is selected as the optimal one.

Figure 4.1 Methodology for fixture layout and clamping force optimization
4.3 ILLUSTRATION AND FORMULATION OF THE OBJECTIVE FUNCTION

The workpiece-fixture configuration for the end milling operation described in Figure 3.2 of Chapter 3 is considered to illustrate the mathematical approach for fixture layout and clamping forces optimization.

Here the formulation of the objective function and the constraints that are involved in the problem are discussed. Also the steps in finding the optimal clamping force for a particular layout are discussed. The different layouts and the range of the clamping forces are also listed.

4.3.1 Parameters Involved

In the fixture optimization problem, it is necessary to minimize the deformation in the workpiece. The parameters which influence the deformation of the workpiece are:

i. Material of the workpiece

ii. Clamping forces

iii. Machining forces

iv. Position of locators and clamps

v. Number of locators and clamps

In this work, the position of fixture elements, minimum clamping forces to restrain the workpiece and the machining forces that are required to carry out the end milling operation are considered for analysis.
By optimizing the position of the fixture elements and the magnitude of clamping forces, the deformation of the workpiece can be minimized. So the design variables are three clamping forces, the position of six locators and three clamps as stated in the Section 3.3 of Chapter 3. Thus the design variables are

\[
\{ ZL_1, ZL_2, XL_3, XL_4, XL_5, XL_6, XC_1, ZC_2, XC_3, C^F_1, C^F_2, C^F_3 \}^T
\]

where

\( ZL_1, ZL_2, XL_3, XL_4, XL_5 \) and \( XL_6 \) are the different locator positions along the particular axis

\( XC_1, ZC_2 \) and \( XC_3 \) are the different clamp positions along the particular axis

\( C^F_1, C^F_2 \) and \( C^F_3 \) are the values of clamping forces

### 4.3.2 Condition 1: Equilibrium of Forces

Equilibrium occurs when the resultant of the forces acting in a direction is zero. It states that the body is in equilibrium. All force components acting on the workpiece with relevant directions are shown in Figure 4.2.

Equations for equilibrium of forces are as follows:

\[
\sum F_X = R_1 + R_2 - C^F_2 - F_X = 0 \quad (4.1)
\]

\[
\sum F_Y = R_4 + R_5 + R_6 - C^F_3 - F_Y = 0 \quad (4.2)
\]
\[ \sum F_Z = R_3 - C_{F1} - F_Z = 0 \]  \hspace{1cm} (4.3)

where \(C_{F1}, C_{F2}\) and \(C_{F3}\) are the clamping forces at the corresponding clamping positions.

\(R_1, R_2, R_3, R_4, R_5\) and \(R_6\) are the reactions in the corresponding locator positions.

\(F_X, F_Y\) and \(F_Z\) are the machining forces. Thus the Equations (4.1), (4.2) and (4.3) form the first condition.

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**Figure 4.2  Clamping, machining and locator reaction forces**

4.3.3  **Condition 2: Equilibrium of Moments**

The excess amount of imbalanced moment is an important factor influencing workpiece deformation and the moment of force is the tendency of a force to twist or rotate an object. The equilibrium of moments is to equalize the sum of moments at each locator and clamp to zero and is given in Equation (4.4).
\[
\sum M_i = 0 \tag{4.4}
\]

where \(i\) = point about which moment is calculated

The equilibrium of moment is taken at all six locator positions and three clamping positions. The moment about locator \(L_1\) because of machining forces, clamping forces and reaction forces at locator contact points is described in Equation (4.5) by multiplying the force values with the corresponding perpendicular distances with respect to locator position \(L_1\). Similarly Equations (4.6) to (4.13) are developed to calculate moments about other five locators and three clamps.

Those equations are as follows:

\[
\sum ML_1 = 
\begin{bmatrix}
-[(ZL_3 - ZL_2)R_1 - (XL_3 - XL_2)R_3] \\
+(ZL_3 - ZL_4)R_2 + (XL_3 - XL_4)R_3 \\
+(XL_5 - XL_3)R_4 + (XL_6 - XL_3)R_4 \\
+((ZL_4 - ZL_6)R_6 + (XC_1 - XL_4)C_1^\ell) \\
+[(ZC_1 - ZC_2)C_1^\ell] - [(ZL_3 - ZL_3)C_3^\ell] \\
-[(XC_1 - XL_3)C_3^\ell] - [(Z_L - ZL_1)F_y] \\
-[(X_f - XL_1)F_x] - [(Z_f - ZL_3)F_y] \\
+[(X_f - XL_1)F_z]
\end{bmatrix} = 0 \tag{4.5}
\]

\[
\sum ML_2 = 
\begin{bmatrix}
[(ZL_3 - ZL_2)R_1 - (XL_3 - XL_2)R_3] \\
+(XL_4 - XL_2)R_4 + (ZL_4 - ZL_2)R_4 \\
+(XL_5 - XL_2)R_5 + (ZL_5 - ZL_2)R_5 \\
+((ZL_6 - ZL_2)R_6 - (ZL_6 - ZL_2)R_6) \\
+[(X_L - XL_4)C_4^\ell] - [(ZC_2 - ZL_2)C_4^\ell] \\
-[(XC_2 - XL_4)C_4^\ell] - [(Z_L - ZL_3)C_4^\ell] \\
-[(Z_f - ZL_2)F_x] - [(X_f - XL_2)F_y] \\
-[(Z_f - ZL_2)F_z] + [(X_f - XL_2)F_z]
\end{bmatrix} = 0 \tag{4.6}
\]
$$\Sigma_{ML3} = \begin{bmatrix}
[(ZL_1 - ZL_3)R_1] + [(ZL_2 - ZL_3)R_2] \\
+ [(XL_4 - XL_3)R_4] - [(ZL_3 - XL_4)R_4] \\
+ [(XL_5 - XL_3)R_5] - [(ZL_5 - XL_3)R_5] \\
- [(ZC_2 - ZL_3)C_2^f] + [(XL_3 - XC_3)C_3^f] \\
+ [(ZC_3 - ZL_3)C_3^f] + [(Z_F - ZL_3)F_X] \\
- [(X_F - XL_3)F_Y] + [(Z_F - ZL_3)F_Y] \\
+ [(X_F - XL_3)F_Z]
\end{bmatrix} = 0 \quad (4.7)$$

$$\Sigma_{ML4} = \begin{bmatrix}
[(ZL_1 - ZL_4)R_1] + [(XL_4 - XL_3)R_2] \\
- [(ZL_5 - ZL_4)R_5] - [(XL_4 - XL_6)R_6] \\
- [(ZL_6 - ZL_4)R_6] - [(XL_4 - XC_1)C_4^f] \\
- [(ZC_2 - ZL_4)C_2^f] + [(XL_4 - XC_3)C_3^f] \\
+ [(ZC_3 - ZL_4)C_3^f] - [(Z_F - ZL_4)F_X] \\
+ [(Z_F - ZL_4)F_Y] + [(X_F - XL_4)F_Z]
\end{bmatrix} = 0 \quad (4.8)$$

$$\Sigma_{ML5} = \begin{bmatrix}
- [(ZL_5 - ZL_3)R_2] + [(XL_5 - XL_3)R_3] \\
- [(ZL_5 - ZL_6)R_6] - [(XL_5 - XC_1)C_4^f] \\
+ [(ZL_5 - ZC_2)C_2^f] + [(XL_5 - XC_3)C_3^f] \\
- [(ZL_5 - ZC_3)C_3^f] - [(Z_F - ZL_5)F_X] \\
+ [(Z_F - ZL_5)F_Y] + [(X_F - XL_5)F_Z]
\end{bmatrix} = 0 \quad (4.9)$$

$$\Sigma_{ML6} = \begin{bmatrix}
[(ZL_1 - ZL_6)R_1] - [(ZL_6 - ZL_2)R_2] \\
- (XL_3 - XL_6)R_3] + [(XL_4 - XL_6)R_4] \\
+ [(ZL_6 - ZL_4)R_4] + [(XL_5 - XL_6)R_5] \\
- [(ZL_6 - ZL_5)R_5] + [(XC_1 - XL_6)C_7^f] \\
- [(ZC_3 - ZL_6)C_7^f] - [(Z_F - ZL_6)F_X] \\
+ [(X_F - XL_6)F_Y] + [(Z_F - ZL_6)F_Y] \\
+ [(X_F - XL_6)F_Z]
\end{bmatrix} = 0 \quad (4.10)$$
\[ \sum MC_1 = 1 \begin{bmatrix} -[(ZC_1 - ZL_1)R_1] - [(ZC_1 - ZL_2)R_2] \\
+ [(XL_4 - XC_4)R_4] + [(ZC_1 - ZL_4)R_4] \\
+ [(ZC_1 - ZL_5)R_5] - [(XC_1 - XL_5)R_6] \\
+ [(ZC_1 - ZL_6)R_6] + [(ZC_1 - ZC_2)C^f_2] \\
+ [(XC_1 - XC_3)C^f_3] - [(ZC_1 - ZC_3)C^f_3] \\
- [(X_F - XC_1)F_Y] - [(ZC_1 - Z_F)F_Y] \\
+ [(XC_1 - X_F)F_Z] \end{bmatrix} = 0 \] (4.11)

\[ \sum MC_2 = 1 \begin{bmatrix} -[(ZL_1 - ZC_2)R_1] - [(ZC_2 - ZL_2)R_2] \\
+ [(XC_2 - XL_3)R_3] - [(XC_2 - XL_4)R_4] \\
+ [(ZC_2 - ZL_4)R_4] - [(XC_2 - XL_5)R_5] \\
- [(ZC_2 - ZC_4)C^f_4] - [(Z_F - ZC_2)F_X] \\
+ [(XC_2 - XC_3)C^f_3] - [(Z_F - ZC_2)F_Y] \\
- [(X_F - XC_2)F_Z] \end{bmatrix} = 0 \] (4.12)

\[ \sum MC_3 = 1 \begin{bmatrix} -[(ZL_1 - ZC_3)R_1] - [(ZC_3 - ZL_2)R_2] \\
+ [(ZC_3 - ZL_4)R_4] + [(XL_5 - XC_3)R_5] \\
+ [(Z_F - ZC_3)F_Z] \end{bmatrix} = 0 \] (4.13)

where

(XL_i, YL_i, ZL_i) refers to the position of the respective locators along the particular axis (i = 1 to 6).

(XC_i, YC_i, ZC_i) refers to the position of the clamps along the particular axis (i=1 to 3).
(X_f, Y_f, Z_f) refers to the position of the machining force acting on the workpiece.

Equations (4.5) to (4.13) form the second condition. These equations are solved using the program developed by means of MATLAB 7.0. The result of these equations includes the three clamping forces and the reactions at the six locators. The boundary values presented in Table 3.2 of Chapter 3 are taken as coordinate values of the locators and clamps.

4.3.4 Coulomb’s Static Friction Law

Thus Coulomb’s static friction law states

\[ \text{Friction force} = \mu \times F_N \]

where, \( \mu \) is the coefficient of static friction and \( F_N \) is the normal force exerted. Most of the dry materials in combination have friction coefficient values between 0.2 and 0.6.

4.3.5 Feasible Clamping Forces by Stick/Slip Conditions

However, clamping and locator reactions have to satisfy the stick/slip condition to ensure that the workpiece remains stable while machining without any slip. The stick slip conditions for the workpiece-fixture system are formulated by the following equations.

\[ \text{Frictional force} \geq \text{external force (machining force)} \]

Only if the frictional force is greater than the machining forces along with the clamping forces, the workpiece will not slip out of the clamps. This frictional force can be ensured by the amount of clamping
forces. Those equations which form the checking condition are presented in Equations (4.14) to (4.16). The Co-efficient of friction (μ) between aluminium (workpiece material) and steel (locator material) is 0.6 under dry conditions. The μ value is considered as 0.25 for safety condition.

In the X-direction

\[ (C_1^F + C_3^F + R_3 + R_4 + R_5 + R_6)(0.25) > F_X \]  

(4.14)

In the Y-direction

\[ (C_1^F + C_2^F + R_1 + R_2 + R_3)(0.25) > F_Y \]  

(4.15)

In the Z-direction

\[ (C_2^F + C_3^F + R_1 + R_2 + R_4 + R_5 + R_6)(0.25) > F_Z \]  

(4.16)

The layouts which provide the results that satisfy the above conditions are said to be in the selected feasible range. Only those clamping forces which satisfy the above equations are fit to be counted to find the deformation using its corresponding layout in ANSYS software.

**4.3.6 Generation of Layouts and Respective Clamping Forces**

The objective functions formulated are put in a program to generate different sets of layouts with their corresponding clamping forces and reaction forces at the supports. Then the deformation values of those layouts generated are found by using ANSYS software.

MATLAB 7.0 software is used to develop the required program. The program is developed in such a way that the clamping forces resulting
by solving the equations 1 to 12 are put to check with the stick/slip conditions and only those forces which satisfy this condition are taken into account. Such a set of results that are obtained as a result of the program are shown in Table 4.1.

Table 4.1 shows the different values for the variable sets of positions of fixture elements and the clamping forces to the corresponding layout. About 100 intervals are selected between the chosen boundary values. The variable sets are gradually and simultaneously increased. The increment value is based on the number of intervals selected (100 intervals for this case). Thus the program solves each set of layouts, its corresponding clamping forces and the respective reaction forces at the positions of fixture elements. Then these forces are checked for the stick/slip condition and only then they are selected to find the deformation value by using FEM software ANSYS 11.0.

4.3.7 Optimum Fixture Layout and Clamping Forces

The different boundary conditions are given to the workpiece-fixture system model and the respective deformation values are obtained. Among all the deformation values, 0.059829 mm is found as the minimum. Thus the fixture layout providing this minimum deformation is said to be the optimal one. Table 4.2 shows the optimal fixture layout and its corresponding clamping forces. The respective workpiece deformations obtained from ANSYS 11.0 is shown in Figure 4.3. The optimal clamping forces for the optimal layout are 186.34 N, 412.75 N and 26.76 N for clamps 1, 2 and 3 respectively.
Table 4.1  A set results of the program using MATLAB 7.0

<table>
<thead>
<tr>
<th>Layout No.</th>
<th>Coordinate Values of Layout (mm)</th>
<th>Clamping Forces (N)</th>
<th>Maximum deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZL₁</td>
<td>ZL₂</td>
<td>XL₃</td>
</tr>
<tr>
<td>1</td>
<td>107.0</td>
<td>43.5</td>
<td>92.4</td>
</tr>
<tr>
<td>2</td>
<td>107.4</td>
<td>43.9</td>
<td>93.5</td>
</tr>
<tr>
<td>3</td>
<td>107.9</td>
<td>44.4</td>
<td>94.5</td>
</tr>
<tr>
<td>4</td>
<td>108.3</td>
<td>44.8</td>
<td>95.6</td>
</tr>
<tr>
<td>5</td>
<td>108.7</td>
<td>45.2</td>
<td>96.7</td>
</tr>
<tr>
<td>6</td>
<td>109.2</td>
<td>45.7</td>
<td>97.7</td>
</tr>
<tr>
<td>7</td>
<td>109.6</td>
<td>46.1</td>
<td>98.8</td>
</tr>
<tr>
<td>8</td>
<td>110.0</td>
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<td>99.9</td>
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</tr>
<tr>
<td>12</td>
<td>111.8</td>
<td>48.3</td>
<td>104.2</td>
</tr>
<tr>
<td>13</td>
<td>112.2</td>
<td>48.7</td>
<td>105.2</td>
</tr>
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Table 4.1 (Continued..)

<table>
<thead>
<tr>
<th>Layout No.</th>
<th>Coordinate Values of Layout (mm)</th>
<th>Clamping Forces (N)</th>
<th>Maximum deformation (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ZL₁</td>
<td>ZL₂</td>
<td>XL₃</td>
</tr>
<tr>
<td>14</td>
<td>112.7</td>
<td>49.2</td>
<td>106.3</td>
</tr>
<tr>
<td>15</td>
<td>113.1</td>
<td>49.6</td>
<td>107.4</td>
</tr>
<tr>
<td>16</td>
<td>113.5</td>
<td>50.0</td>
<td>108.4</td>
</tr>
<tr>
<td>17</td>
<td>114.0</td>
<td>50.5</td>
<td>109.5</td>
</tr>
<tr>
<td>18</td>
<td>114.4</td>
<td>50.9</td>
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</tr>
<tr>
<td>19</td>
<td>114.8</td>
<td>51.3</td>
<td>111.7</td>
</tr>
<tr>
<td>20</td>
<td>115.3</td>
<td>51.8</td>
<td>112.7</td>
</tr>
</tbody>
</table>
Table 4.2  Optimal fixture layout and clamping forces

<table>
<thead>
<tr>
<th>Position of fixture elements</th>
<th>X (mm)</th>
<th>Y (mm)</th>
<th>Z (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Locator 1 (L₁)</td>
<td>0</td>
<td>19.05</td>
<td>114.39</td>
</tr>
<tr>
<td>Locator 2 (L₂)</td>
<td>0</td>
<td>19.05</td>
<td>50.89</td>
</tr>
<tr>
<td>Locator 3 (L₃)</td>
<td>110.58</td>
<td>19.05</td>
<td>0</td>
</tr>
<tr>
<td>Locator 4 (L₄)</td>
<td>119.09</td>
<td>0</td>
<td>14.94</td>
</tr>
<tr>
<td>Locator 5 (L₅)</td>
<td>119.09</td>
<td>0</td>
<td>112.06</td>
</tr>
<tr>
<td>Locator 6 (L₆)</td>
<td>50.89</td>
<td>0</td>
<td>63.5</td>
</tr>
<tr>
<td>Clamp 1 (C₁)</td>
<td>110.58</td>
<td>19.05</td>
<td>127</td>
</tr>
<tr>
<td>Clamp 2 (C₂)</td>
<td>127</td>
<td>19.05</td>
<td>110.58</td>
</tr>
<tr>
<td>Clamp 3 (C₃)</td>
<td>74.76</td>
<td>38.1</td>
<td>63.50</td>
</tr>
</tbody>
</table>

\[ C^F_1 = 186.34 \text{ N} \quad C^F_2 = 412.75 \text{ N} \quad C^F_3 = 26.76 \text{ N} \]

Figure 4.3  Workpiece deformations for the mathematically optimized fixture layout
4.4 FINETUNING OF MATHEMATICALLY OPTIMIZED LAYOUT

This section deals with the variation of deformation values for various positions of locators and clamps for further finetuning of mathematically optimized layout. Workpiece deformations for the mathematically optimized layout are found by using FEM. To check the possibility of near optimum values the position of fixture elements are refined based on workpiece deformations given by FEM. First the position of $L_1$ is varied within the variable range and other locators and clamping positions are kept as same given by the mathematically optimized layout. Within the boundary range, for the various positions of $L_1$ the corresponding workpiece deformation values are found out by using ANSYS. Then a new position for $L_1$ is taken for minimum deformation of the workpiece. Later, the same procedure is repeated for all locators and clamps and a new refined fixture layout is found for minimum deformation of the workpiece. Figures from 4.4(a) to 4.4(i) show the variation of workpiece deformation for various positions of locators and clamps.

![Figure 4.4(a) Position of locator $L_1$ vs workpiece deformation](image)
Figure 4.4(b) Position of locator L₂ vs workpiece deformation

Figure 4.4(a) and 4.4(b) show that workpiece deformation gradually decreases with the movement of L₁ and L₂ along Z axis. A minimum deformation of 0.0598 mm occurs when the position of L₁ is at 112.06 mm and L₂ is at 50.894 mm along Z axis.

The following graphs in Figures 4.4(c), 4.4(d) and 4.4(e) shows that the deformation of workpiece initially increases with the movement of locators L₃, L₄ and L₅ along X axis and finally decreases to the minimum values. A minimum deformation of 0.0598 mm is reported when the position of L₃ is at 110.67 mm. Also the same amount of deformation is reported when the L₄ is at 119.09 mm and L₅ is at 120.65 mm along X axis. Figure 4.4(f) shows variations of workpiece deformation for different positions of locator L₆ and reports the same minimum deformation of 0.0598 mm at 50.89 mm along X axis.
Figure 4.4(c)  Position of locator $L_3$ vs workpiece deformation

Figure 4.4(d)  Position of locator $L_4$ vs workpiece deformation
Workpiece deformation changes with respect to different clamping positions are shown in Figures 4.4(g), 4.4(h) and 4.4(i).
They indicate that the minimum deformations occur when the clamping elements are placed near the machining area.

![Figure 4.4(g)](image), **Position of clamp C₁ vs workpiece deformation**

![Figure 4.4(h)](image), **Position of clamp C₂ vs workpiece deformation**
Based on the graphs from Figure 4.4(a) to 4.4(i) the minimum deformation achieved in all the cases is the same and is equal to 0.0598 mm. No changes are observed in optimal layout during finetuning. So it is evident that the mathematically optimized layout yields the same result before and after the finetuning process and thus finetuning reports no improvements.

![Graph showing deformation vs position of C3](image)

**Figure 4.4(i) Position of clamp C3 vs workpiece deformation**

4.5 CONCLUSION

In this chapter, a mathematical approach for optimum fixture layout and clamping forces based on the equilibrium condition of forces and moments to minimize the overall workpiece deformation has been proposed. The proposed approach is illustrated with a prismatic workpiece-fixture model and the optimal machining fixture layout and clamping forces have been determined. By using stick slip condition and Coulomb’s friction law the obtained clamping forces are checked and ensured that those forces are good enough to hold the workpiece without slip during
machining. The final optimal model has the maximum deformation of 0.059829 mm. This is found to be the minimum within the boundary ranges considered. The optimal fixture layout which is found by using the above approach provides the near equilibrium state for the workpiece-fixture system. It makes the resultant of forces and moments to approximately zero value. It further minimizes the dimensional and form errors of the workpiece system. The optimum clamping forces for the corresponding layout are the minimum forces, which are required to hold the workpiece without slip. Though many intelligent algorithms have been used in fixture layout optimization, the proposed approach is mathematically simple and easily adaptable for fixture design problems. Thus the proposed approach can be applied to relevant fixture design problems and the overall workpiece deformation can be minimized. It leads to the minimization of dimensional and form errors in the components.

4.6 LIMITATIONS

In the mathematical approach, simultaneous increase of the values of all the layout variables is only possible. So the consideration of all possible combinations of design variables is not possible and finetuning of the optimal layout given by the approach does not yield better results. To overcome this drawback, some nontraditional optimization tool can be used to find the optimal location of the locators and clamps.