CHAPTER 2

BRAKING FORCE AND ITS CONTROL

2.1 FRICTIONAL FORCE

In pure rolling motion (ideal case), friction is required to start, stop, and change the motion of a wheel. Braking force \((F)\) may develop between the tyre and the road surface as shown in Figure 2.1, to reduce the angular velocity \((\omega)\) of the wheel. In pure rolling motion, friction causes the wheel to catch and stops the sliding and skidding motion.

\[
F \leq \mu N
\]

(Without wheel slide)

Figure 2.1 Braking force for unladen condition
Where

\[ \omega = \text{Angular velocity of wheel}, \]
\[ \mu = \text{Co-efficient of friction between tyre and road surface}, \]
\[ N = \text{Normal reaction on tire under unladen condition} \]
\[ F = \text{Braking force between tyre and road surface, N} \]

Maximum value of braking force is the product of normal reaction on tyre and the coefficient of friction between the tyre and the ground.

Braking force should be less than or equal to the product of the normal reaction on the tyre and the coefficient of friction between the tyre and the ground, to avoid wheel sliding after applying brake.

\[ F \leq \mu N \text{ (without sliding) for unladen condition} \] \hspace{1cm} (2.1)

\[ F \leq \mu N_1 \text{ (without sliding) for laden condition} \] \hspace{1cm} (2.2)

Where, \( N_1 = \text{Normal reaction on tyre under laden condition} \)

From the Equations (2.1) and (2.2), the magnitude of the maximum braking force is not the same for unladen and laden conditions of the motorcycle. Hence, the maximum braking force may be varied depending upon the normal load acting on the wheel for getting the minimum stopping distance without wheel sliding.

2.2 BRAKING FORCE AND BRAKING TORQUE

The normal reaction on the wheel is one of the key factors to consider for varying the braking force because the braking force mainly depends on the coefficient of friction between the tyre and the ground and
normal reaction on the wheel. Consider a free body diagram of the wheel which is shown in Figure 2.2.

![Free Body Diagram](image)

**Figure 2.2 Disc brake- braking force and braking torque**

From Figure 2.2

- $r$ = Effective radius of disc, m
- $R$ = Effective rolling radius of tyre, m
- $F$ = Braking force between tyre and road surface, N
- $F_1$ = Tangential braking force between disc and brake pads, N
- $\mu$ = Co-efficient of friction between tyre and road surface
- $\mu_1$ = Co-efficient of friction between disc and brake pads
- $P_L$ = Hydraulic brake line pressure, N/m$^2$
- $A$ = Caliper cylinder area, m$^2$

\[ F_1 = \mu_1 P_L A \]
$N =$ Normal reaction on wheel under unladen condition, N

At the moment the brake is applied, the torque developed on the brake disc is given by the Equation (2.3),

$$T_c = 2 \mu l P_A r$$ (2.3)

Torque developed on wheel can be written as,

$$T_w = F R$$ (2.4)

$$F = \mu N$$ (2.5)

$$T_w = \mu N R$$ (2.6)

Where

$T_c =$ Torque developed at brake disc

$T_w =$ Torque developed at wheel

Figure 2.3 Load transfer effect during braking
During the deceleration phase, the load on the front wheel increases, while the load on the rear wheel decreases, due to the load transfer effect (LTE). The static equations, applied to the entire vehicle, allow calculating the wheels dynamic load and the load transfer effect. Figure 2.3 shows the forces acting on the motorcycle when the brakes are applied.

From this the static load of the each wheel can be derived;

**Static front axle load**

\[
F_{zf} = mg \frac{L_f}{L} \tag{2.7}
\]

**Static rear axle load**

\[
F_{zr} = mg \frac{L_f}{L} \tag{2.8}
\]

The dynamic load on the front wheel is equal to the sum of the static load and of the load transfer effect.

\[
F_{zf,\text{dyn}} = mg \frac{L_f}{L} + F_{x,\text{total}} \frac{h}{L} \tag{2.9}
\]

While the dynamic load on the rear wheel is equal to the difference between the static load and the load transfer effect:

\[
F_{zr,\text{dyn}} = mg \frac{L_f}{L} - F_{x,\text{total}} \frac{h}{L} \tag{2.10}
\]

Where,

\[
F_{zf} = \text{Static front axle load, N}
\]

\[
\frac{F_{x,\text{total}} h}{L} = \text{Load transfer effect, N}
\]

\[
m = \text{Total mass of the motorcycle, kg}
\]


\( g = \text{Acceleration due to gravity } 9.81 \, \text{m/s}^2 \)

\( W = \text{Total motorcycle weight, N} \)

\( F_{jR} = \text{Static rear axle load, N} \)

\( F_{df, \text{dyn}} = \text{Dynamic front axle normal force, N} \)

\( F_{dR, \text{dyn}} = \text{Dynamic rear axle normal force, N} \)

\( h = \text{Vertical center of gravity of the total mass of the motorcycle from the ground, m} \)

\( L = \text{Length of wheelbase while motorcycle is level, m} \)

\( L_f = \text{Horizontal distance from centre of gravity to front axle, m} \)

\( L_r = \text{Horizontal distance from centre of gravity to rear axle, m} \)

\( a = \text{Motorcycle deceleration in ‘g’ units.} \)

\( F_{xF} = \text{Dynamic braking force for front axle, N} \)

\( F_{xR} = \text{Dynamic braking force for rear axle, N} \)

\( F_{x, \text{total}} = \text{Total braking force} \)

\( F_{x, \text{total}} = F_{xF} + F_{xR} \, \text{N} \) \hfill (2.11)

### 2.3 METHODS TO CONTROL WHEEL LOCKING

In the present scenario the braking force may be controlled by two methods.

- ABS
- EBD
2.3.1 Anti Lock Braking System (ABS)

An anti lock braking system (ABS) is a safety system on motor vehicles which prevents the wheels from locking while braking. The Anti-lock Braking System is designed to maintain vehicle control, directional stability, and optimum deceleration under severe braking conditions on most road surfaces. Anti lock braking system is shown in Figure 2.4.

![Figure 2.4 ABS system (Courtesy: 2carpros.com)](image)

2.3.2 ABS Operation

It operates effectively by monitoring the rotational speed of each wheel and controlling the brake line pressure to each wheel during braking. This prevents the wheels from locking up. In general, wheel locks at 100% wheel slip. It is normal to have 15-20% wheel slip which is indicated in Figure 2.5 for effective traction (maximum coefficient of friction value). ABS does no work on snow and loose gravel because there is no peak. Encoders detect excessive wheel slippage and not lock-up. The sensors - one at each wheel, send a variable voltage signal to the control unit, which monitors these
signals, compares them to its program information, and determines whether a wheel is about to lock up.

**Figure 2.5 Coefficient of friction Vs slip on various surfaces**

When a wheel is about to lock up, the control unit signals the hydraulic unit to reduce hydraulic pressure (or not increase it further) at that wheel’s brake caliper that helps preserve vehicle stability and steering control as shown in Figure 2.6 in emergency braking condition. Pressure modulation is handled by electrically-operated solenoid valves.

**Figure 2.6 ABS effect (courtesy: Nissan-global.com)**
2.3.3 **Electronic Brake Force Distribution (EBD)**

An application of anti-lock braking system (ABS) technology, EBD controls rear-wheel braking based on detection of loss of traction at the front and the rear wheels when the brakes are applied. By the controlling distribution of braking force to the front and the rear wheels according to the passenger and payload, EBD minimizes differences in braking performance whether the vehicle load is light or heavy as LTE is greater with laden vehicle. Figure 2.7 shows the EBD effect.

![Figure 2.7 EBD effect (courtesy: Nissan-global.com)](image)

Brake force distribution is now performed under electrical control of the skid control electronic control unit (ECU). The skid control ECU precisely controls the braking force in accordance with the vehicle’s driving conditions. Generally, when the brakes are applied the load transfer effect reduces the load on the rear wheels. When the skid control ECU senses this condition (based on speed sensor output), it signals the brake actuator to regulate rear brake force so that the vehicle will remain under control during the stop. The amount of applied force to the rear wheels varies based on the
amount of deceleration. The amount of brake force that is applied to the rear wheels also varies based on whether or not the vehicle is carrying a load.

When the brakes are applied while the vehicle is cornering, the load applied to the inner wheels decreases while the load applied to the outer wheel increases. When the skid control ECU senses this condition (based on speed sensor output), it signals the brake actuator to regulate brake force between the left and right wheels to prevent skid. Figure 2.8 shows the vehicle response due to EBD. It proportions brake force between left and right because if any difference in the magnitude of the braking force between the left and the right wheels may lead the vehicle being subjected to yawing moment which affects the stability of the vehicle.

![Figure 2.8 Vehicle response due to EBD (Courtesy :toyota-mideast.com)](image)

### 2.3.4 New Concept of Braking Force Control

\[
T_c = 2 \ast \mu_1 \ast P_L \ast A \ast r
\]

(2.12)

Where,

- \(\mu_1\) = Co-efficient of friction between disc and brake pads
- \(P_L\) = Hydraulic brake line pressure, N/m²
From the above braking torque Equation (2.12) braking torque may be varied by changing brake fluid pressure or effective radius of disc. Effective disc radius is measured from the disc center of rotation to the center of pressure of caliper pistons. Figure 2.9 shows the disc and caliper assembly. In this work, braking torque developed at the brake disc is varied by changing effective radius of brake disc. If the brake torque is varied, the braking force developed between the tyre and the ground also varies for the constant wheel radius.

Figure 2.9 Disc and caliper assembly (Courtesy: autorepair.about.com)

The disc is mechanically coupled to the hub and wheel assembly, and the tyre is assumed to be rigidly attached to the wheel, the torque will be constant in both brake disc and wheel. The torque can be written as the product of force and radius.

It can be understood by considering the torques:
\[ T_c = T_w \] (2.13)

Where

\[ T_c = \text{Torque developed at brake disc} \]
\[ T_w = \text{Torque developed at wheel} \]

\[ F l^* r = F^* R \] (2.14)

Where

\[ F = \text{Braking force between tyre and road surface} \]
\[ F l = \text{Tangential braking force between disc and brake pads} \]
\[ r = \text{Effective radius of disc} \]
\[ R = \text{Effective rolling radius of tyre} \]

\[ F l = \mu_1^* \text{Normal force on disc} \] (2.15)

Where

\[ \mu_1 = \text{Co-efficient of friction between disc and brake pads} \]

Braking force is equal to the product of the coefficient of friction between the tyre and the ground and normal reaction on wheel.

\[ F = \mu_1 N \] (2.16)

Where

\[ \mu_1 = \text{co-efficient of friction between tyre and road surface} \]
\[ N = \text{Normal reaction on tire under unladen condition} \]
From the general expression, normal force on disc can be written as the product of caliper brake fluid pressure and cross section area of the caliper.

\[
\text{Normal force on disc} = P_L \times A
\]  

(2.17)

Where, \(P_L\) = Hydraulic brake line pressure

\(A\) = Caliper cylinder area.

The relation between braking force at brake disc and normal reaction is coefficient of friction. Substituting Equation (2.17) in (2.15),

We have \(F_1 = \mu_1 \times P_L \times A\)  

(2.18)

With this support, the basic caliper torque and wheel torque can be written as

\[\mu_1 \times P_L \times A \times r = \mu \times N \times R\]

(2.19)

By re-arranging the above Equation (2.19), the effective radius of disc is given as

\[r = \frac{\mu \times N \times R}{\mu_1 \times P_L \times A}\]

(2.20)

From the above Equation (2.20), the effective radius of disc is directly proportional to the normal reaction on the tyre of the motorcycle, when the other parameters in the Equation (2.20) are constant. This is the effective radius (Calculated from Equation 2.20) of disc at which the brake pads are to be positioned for a normal load on the rear tire, coefficient of friction between tyre and road surface, coefficient of friction between disc and
brake pads, effective rolling radius of tyre, hydraulic brake line pressure and the caliper cylinder area.

\[ r = \frac{r_1 + r_2}{2} \]  

(2.21)

Where, \( r_1 \) = Outer radius of the brake pad when it is squeezed against the disc.

\( r_2 \) = Inner radius of the brake pad when it is squeezed against the disc.

2.4 SUMMARY

In this chapter, the frictional force, braking force, and braking torque developed during braking condition are discussed. The load transfer of the two-wheeler during braking is presented. The methods to control wheel locking are discussed. Also the new concept for controlling braking force is discussed.