CHAPTER 2

RELATED LITERATURE

2.1 Major Directions of Research

In this chapter we present a detail survey of existing literature on the methodology used and the various option pricing models to search for a viable alternative model using an alternative to the Geometric Brownian Motion assumption for the underlying. Before that we would like to point out the major directions of research in empirical study of financial time series as was evident from our discussions in the first chapter. As we have pointed out already, the traditional analysis of financial time series has been a part of standard econometric analysis. The research in this direction has centred on application of a mix of statistical tools and assumptions and Fourier analysis. There has been extensive work in this direction and a volume of literature exists in this field. However, since our work concentrates on the other younger school which has evolved from physics and non-linear dynamics and has been extended to various disciplines including economics, our survey concentrates on research works based on application of these newer methods.

Historically there are two views of nonlinear dynamics: the Fourier and the Poincaré. The Fourier system consists of reducing the nonlinear equation to a system of linear ones. The final solution becomes a sum of these linear equations. This has one major problem. It is easily seen that the properties of a nonlinear equation should be different from a collection of linear ones. Only
recently this problem has been addressed to and a new method has evolved based on an approach that differs from Fourier or Fourier based wavelet analysis. This method is known as the Empirical Mode Decomposition method.

Poincare’s system defines the mapping of the phase space onto itself, enabling a graphical presentation of the dynamics. The major problem with phase space representation is multi-dimensionality. However, Recurrence Plot is a method that enables representation of phase space characteristics with the help of single time series on a two dimensional graph. Our work uses both these methods to empirically test the financial time series under study. Our survey also presents the existing literature on these two methods in detail.

2.2 Existing Literature on Option Pricing Models

The option valuation model proposed by (Black and Scholes 1973) and (R. Merton 1973) represent the most used development in finance. At a practical level, the valuation of options and the hedging strategies based on derivatives too, are governed by the Black and Scholes model, given that the price is obtained with the use of the implied volatility parameters derived from the model. We can assert, without any doubt, that this model is the first theoretical model for option valuation, used in an intensive manner by practitioners for valuation, speculation and hedging purposes. Empirical studies demonstrated that the

---

61 Black, F. and M. Scholes, 1972, op cit
volatility of the underlying asset is not constant, a fact that contradicts the Black and Scholes hypothesis related to a non-variant volatility. This represents the starting point for numerous studies focused on extensions of the original model. An alternative theory to the Black and Scholes model considers that the volatility describes a diffusion process, governed by a second Brownian process. This new generation of valuation models is labelled as the generation of the 'valuation models with stochastic volatility'.

Econometric studies stressed that the valuation models with stochastic volatility would help to explain the 'smile' or 'smirk' effects and would also lead to a more realistic theory of the term structure of the implied volatility. The fact that the variations of the volatility of an asset's return can only partially be explained by the movements in its price lead many researchers, like (Wiggins 1987)\(^63\), Hull and White(1987\(^64\)), Stein and Stein (1991)\(^65\), Heston (1993)\(^66\), Bates (1996a\(^67\), 1996b\(^68\)), Bakshi, Cao and Chen (1997\(^69\), 2000\(^70\), to conclude that the volatility


of an asset's return could be itself a random variable describing a specific process. These authors assumed that the price of an option is a function of the underlying asset price and of the standard deviation of the returns of this asset. In this context, the non-expected variations of a call option prices are explained by the random variables representing the underlying asset price and variations in volatility. Hull and White (1987)\textsuperscript{71} suggested an analytical formula in order to calculate the price of a European call option, which uses a Taylor expansion about the point where the volatility is non-stochastic. Stein and Stein (1991)\textsuperscript{72} used the Fourier transform in order to understand the distribution of the underlying prices, while Heston (1993)\textsuperscript{73} obtained an analytical formula for the option price starting from the characteristic function of the risk neutral probability and the inverse of the Fourier transform. The formulas obtained until now have the complex number $i$ (where $i^2 = -1$) in their expressions, a fact which makes their practical use be difficult.

Numerous empirical studies have found that the Black–Scholes (BS) model results in systematic biases across moneyness and maturity (for example, Rubinstein, 1985\textsuperscript{74}, Gemmill, 1996\textsuperscript{75}). The BS pricing biases usually produce an implied volatility smirk for equity options indicating the underlying distribution

\begin{footnotesize}
\begin{itemize}
\item \textsuperscript{71} Hull, J. and A. White, 1987 \textit{op cit}
\item \textsuperscript{72} Stein, E.M. and J.C. Stein, 1991 \textit{op cit}
\item \textsuperscript{73} Heston, S.L. 1993 \textit{op cit}
\item \textsuperscript{75} Gemmill, G. "Did Option Traders Anticipate the Crash—Evidence from Volatility Smiles in the UK with US Comparisons." \textit{Journal of Futures Markets} 16 (1996): 881-897.
\end{itemize}
\end{footnotesize}
implicit in option prices is skewed and fat-tailed. Studies of time-series returns also find evidence of substantial skewness and leptokurtosis in underlying returns (for example, Kon, 198476, Taylor, 198677). These moment properties of the underlying distribution depart from those of the lognormal assumed by Black and Scholes (1973)78. One possible explanation for the documented leptokurtosis in stock returns is the presence of stochastic volatility. Option pricing models incorporating stochastic volatility start with the bivariate diffusion processes of Hull and White (1987)79, Scott (1987)80, and Wiggins (1987)81.

Assuming stock return variances are uncorrelated with aggregate wealth and stock returns, Hull and White (1987)82 use a Taylor series expansion to derive a formula for European call options on a stock. Similar to Hull and White's work, Scott's (1987)83 solution is an integral of the Black-Scholes model over the distribution of average variance over time to maturity given a mean-reverting Ornstein-Uhlenbeck process for volatility. Wiggins (1987) incorporates investor utility into his option pricing model. To justify the assumption that the market price of volatility risk is approximately zero, Wiggins restricts his model to the pricing of options on a market portfolio and to stocks where all the correlation between the

78 Black and Scholes 1973, op cit
79 Hull, J. and A. White, 1987 op cit
81 Wiggins, J.B. 1987, op cit
82 Hull, J. and A. White, 1987 op cit
83 Ibid
stock's volatility and the market return comes through the stock return. He then shows that under log utility, volatility risk premiums are identically zero, and produces a partial differential equation that can be solved numerically for any given correlation between volatility and returns. Chesney and Scott (1989)\textsuperscript{84} choose to work with currency options, adopting the equilibrium asset-pricing model of Cox, Ingersoll and Ross (1985)\textsuperscript{85} (CIR) to derive an option pricing function. Bailey and Stulz (1989) analyse the pricing of stock index options in a simple equilibrium model in which the index volatility follows a square root process and is negatively correlated with stochastic interest rates. Their specification explains the phenomenon that an increase in the volatility of the index has an ambiguous effect on the value of the option. Melino and Turnbull (1990\textsuperscript{86}, 1991\textsuperscript{87}) use numerical techniques to solve a two-dimensional Partial Differential Equation and their approach are successful in explaining the prices of currency options. Stein and Stein (1991)\textsuperscript{88} (henceforth S&S) adopt Hull and White's (1987)\textsuperscript{89} assumption of $\rho_{SV} = 0$ but take into account systematic volatility risk. In the light of the potential convergence problems associated with a power-series solution, S&S rely on properties of characteristic functions. S&S give an analytic representation for the stock price distribution evaluated by an inverse

\textsuperscript{88} Stein, E.M. and J.C. Stein, 1991 \textit{op cit}
\textsuperscript{89} Hull, J. and A. White, 1987 \textit{op cit}
Fourier series transformation, and provide a closed-form formula for European options that involves a single numerical integration. Heston’s (1993) model is mathematically similar to S&S’s procedure, both involving largely analytic approaches relying on characteristic functions. However, Heston relaxes the zero correlation restriction and specifies a mean-reverting square root process for volatility. Heston’s model is an appealing choice for modelling option pricing under stochastic volatility, most importantly because it is sufficiently flexible to allow for systematic volatility risk and arbitrary correlation between volatility and underlying returns. It is also justified by empirical appeal to the observed time-varying and mean-reverting historical volatility process.

Recent research has examined the empirical applications of models based on Heston’s closed form option pricing solution. On currencies, Bates (1996b) estimates the parameters of a diffusion-jump model with stochastic volatility for deutsche mark options. He finds that a stochastic volatility sub-model explains implicit excess kurtosis only by assigning implausible values to stochastic volatility parameters. Guo (1998) uses Heston’s model to examine volatility risk premiums implicit in currency option prices and finds that the compensation for volatility risk is a significant factor in currency market risk premiums. On stock index options, Bates (2000) examines post-1987 data for S&P 500 futures

---

90 Bates 1996b, op cit
options. He finds that a stochastic volatility model captures the strong negative skewness implicit in S&P 500 futures options through a negative correlation between index and volatility shocks and an implausibly high volatility of volatility. With memories of the 1987 Crash being recent, a stochastic volatility–jump diffusion model provides more plausible parameter values, but as Bates points out, this model is inconsistent with the absence of large moves in the S&P 500 index over 1988–93. Bakshi et al. (1997) 93 evaluate the performance of alternative models for the SPX contract and examine the contribution of stochastic volatility to hedging performance. While their results are consistent with Bates(2000) 94 , they find that stochastic volatility provides a first-order improvement over the BS model. Their results also show that the stochastic volatility model achieves the best hedging results. Nandi (1998) 95 documents the importance of allowing for correlation between S&P 500 index returns and volatility in a stochastic volatility option-pricing model. He finds that allowing for non-zero correlation provides a significant improvement in mispricing of out-of-the-money options and overall pricing performance.

On the other hand a host of research work has grown around the fact that despite the successes of the Black-Scholes model based on Brownian motion and normal distribution, two puzzles, emerging from many empirical

---

93 Bakshi, G., C. Cao, and Z. Chen, 1997, op cit
94 Bates 2000, op cit
investigations, have got much attention recently. (1) The asymmetric leptokurtic features. In other words, the return distribution is skewed to the left, and has a higher peak and two heavier tails than those of the normal distribution. (2) The volatility smile. More precisely, if the Black-Scholes-Merton model is correct, then the implied volatility should be constant; but it is widely recognized that the implied volatility curve resembles a “smile,” meaning it is a convex curve of the strike price. Many researches have been conducted to modify the Black-Scholes model to explain the two puzzles. An account of many models incorporating the second property has already been discussed. To incorporate the asymmetric leptokurtic features in asset pricing, a variety of models have been proposed, including, among others: (a) chaos theory, fractal Brownian motion, and stable processes; see, for example, Mandelbrot (1963\textsuperscript{96}, 1967\textsuperscript{97}), Fama (1963\textsuperscript{98}, 1965\textsuperscript{99}), Rogers (1997)\textsuperscript{100}, Samorodnitsky and Taqqu (1994)\textsuperscript{101}; (b) generalized hyperbolic models, including log t model and log hyperbolic model; see, for example, Barndorff-Nielsen and Shephard (2001)\textsuperscript{102}, Praetz (1972)\textsuperscript{103}, Blattberg

and Gonedes (1974); (c) time changed Brownian motions, including log variance gamma model; for example, Clark (1973), Madan and Seneta (1990), Madan, Carr, and Chang (1998), Andersen (1996), Hurst, Platen and Rachev (1997), Geman, Madan, and Yor (1998), and Heyde (2000).

An immediate problem with these models is that it may be difficult to obtain analytical solutions for option pricing; more precisely, they might give some analytical formulae for regular call and put options, but certainly not for interest rate derivatives and path-dependent options, such as perpetual American options, barrier and look back options. In a parallel development, different models are also proposed to incorporate the "volatility smile" in option pricing some of which have already been discussed elsewhere in this article. Popular ones are: (a) stochastic volatility and ARCH models; see, for example, Hull and White (1987), Engle (1995), Gourieroux (1997), Fouque, Papanicolaou,

---

112 Hull, J. and A. White, 1987 op cit
and Sircar (2000)\textsuperscript{115}, (b) constant elasticity model (CEV) model proposed by Cox and Ross (1976)\textsuperscript{116}; (c) normal jump models proposed by Merton (1976)\textsuperscript{117}; (d) affine stochastic-volatility and affine jump diffusion models; see, for example, Heston (1993)\textsuperscript{118} and Duffie, Pan, and Singleton 2000\textsuperscript{119}; (e) Models based on Levy processes; see, for example, Geman, Madan, and Yor (1999)\textsuperscript{120} and reference therein; (f) a numerical procedure called "implied binomial trees"; see, for example, Derman and Kani (1994)\textsuperscript{121}, Dupire (1994)\textsuperscript{122}, Rubinstein (1994)\textsuperscript{123}. Aside from the problem that it might not be easy to find analytical solutions for option pricing, specially for path-dependent options (such as perpetual American options, barrier and look back options), some of these models may not produce the asymmetric leptokurtic feature.

The empirical deficiencies of the Black-Scholes-Merton (BSM) model have prompted research along three dimensions. The univariate diffusion models maintained the diffusion and no-arbitrage foundations of BSM, but relaxed the assumption of geometric Brownian motion. Examples include the constant

\textsuperscript{118} Heston, S.L. 1993, \textit{op cit}
\textsuperscript{119} Dupire, B., Jan 1994, \textit{op cit}
\textsuperscript{120} Geman, Helyette G, Dilip B. Madan, and Marc Yor. "Time Changes for Levy Processes." \textit{Working paper} (University of Paris IX), 1999
\textsuperscript{122} \textit{Ibid}
elasticity of variance model of Cox and Ross (1976)\textsuperscript{124} and Cox and Rubinstein (1985)\textsuperscript{125}; the leverage models of Geske (1979)\textsuperscript{126} and Rubinstein (1983)\textsuperscript{127}; and the more recent implied binomial and trinomial trees models. Stochastic volatility models allow the instantaneous volatility of asset returns to evolve stochastically over time -- typically as a diffusion, but possibly as a regime-switching (Naik, 1993)\textsuperscript{128} or jump-diffusion process (Duffie, Pan, and Singleton, 2000)\textsuperscript{129}. Jump models such as Merton (1976)\textsuperscript{130} relax the diffusion assumption for asset prices. Recent approaches have combined features of these three approaches. Stochastic generalisations of binomial tree models are surveyed in Skiadopoulos (2000)\textsuperscript{131}, while the currently popular affine class of distributional models creates a structure that nests particular specifications of the stochastic volatility and jump approaches.

While alternatives to the BSM model began appearing within a few years of the original Black-Scholes (1973)\textsuperscript{132} paper, and continued in the 1980’s with the stochastic volatility models, actually testing those alternatives using options data was handicapped by several factors. First, options data weren’t readily available

\textsuperscript{124} Cox, J.C. and S.A. Ross, 1976, \textit{op cit}
\textsuperscript{125} \textit{Ibid}
\textsuperscript{130} Merton, \textit{op cit}
\textsuperscript{132} Black and Scholes 1973, \textit{op cit}
until after the advent of option trading on centralised exchanges. Second, the new models were perforce more complicated. Pricing options was a substantial computational burden for the (mainframe) computers available at the time, while employing these option pricing models in empirical work was a major computational and programming challenge. Third, the extra risks introduced by jumps and stochastic volatility raised the issue of how to price those risks. The early literature tended to assume zero risk premia; e.g., Merton (1976)\textsuperscript{133} for jump risk, and the stochastic volatility models of Hull and White (1987)\textsuperscript{134}, Johnson and Shanno (1987)\textsuperscript{135}, Scott (1987)\textsuperscript{136}, and Wiggins (1987)\textsuperscript{137}.

The 1990’s witnessed several important developments that have facilitated greater empirical scrutiny of alternative option pricing models. The most important has been the Fourier inversion approach to option pricing of Stein and Stein (1991)\textsuperscript{138} and Heston (1993)\textsuperscript{139}. While other methods already existed for pricing options under general stochastic volatility processes -- e.g., the Monte Carlo approach of Scott (1987)\textsuperscript{140}, or the higher-dimensional finite-difference approach of Wiggins (1987)\textsuperscript{141}-- Fourier inversion is substantially more efficient when applicable. Furthermore, as discussed by various authors, ( e.g., Bates

\textsuperscript{133} Merton, R.C. 1976, \textit{op cit}
\textsuperscript{134} Hull, J. and A. White, 1987 \textit{op cit}
\textsuperscript{136} Scott, L.O. 1987, \textit{op cit}
\textsuperscript{137} Wiggins, J.B. 1987, \textit{op cit}
\textsuperscript{138} Stein, E.M. and J.C. Stein, 1991, \textit{op cit}
\textsuperscript{139} Heston, S.L. 1993, \textit{op cit}
\textsuperscript{140} Scott, L.O. 1987, \textit{op cit}
\textsuperscript{141} Wiggins 1987, \textit{op cit}
(1996a\textsuperscript{142}, 2000\textsuperscript{143}), Bakshi, Cao and Chen (1997)\textsuperscript{144}, Scott (1997)\textsuperscript{145}, Bakshi and Madan (2000)\textsuperscript{146}, Duffie, Pan and Singleton (2000)\textsuperscript{147} and Dai and Singleton (2000)\textsuperscript{148},) it can be applied to multiple types of risk (stochastic volatility, jumps, stochastic interest rates) and more general structures (multiple additive or concatenated latent variables; time-varying jump risk) without sacrificing option pricing tractability. Finally, the affine pricing kernels first developed by Cox, Ingersoll, and Ross (1985)\textsuperscript{149} imply risk premia on untraded risks are linear functions of the underlying state variables, creating a method for going between objective and risk-neutral probability measures while retaining convenient affine properties for both measures.

Affine models have two major advantages for econometric work. First, European call (and put) option prices $c(S_t, Y_t, T; \theta, \phi)$ with maturity $T$ can be computed rapidly conditional upon the observed underlying asset price $S_t$, the values of relevant underlying state variables $Y_t$, and the various model parameters $\theta$ and risk premia $\phi$ that determine the risk-neutral probability measure.

Second, the joint characteristic function associated with the joint conditional transition density \( p(S_{t+T}, Y_{t+T} \mid S_t, Y_t, \theta) \) has an analytic solution, implying objective transition densities can also be evaluated via Fourier inversion or other methods. Consequently, it becomes relatively straightforward to infer \( Y_t \) values from observed option prices, and to test whether the observed time series properties of asset and/or option prices conditional on those values are consistent with the predicted properties.

Reverse tests based upon the time series properties of asset and/or option prices and their option pricing implications have been handicapped by the difficulty of estimating continuous-time models with latent state variables such as stochastic volatility. However, the time series literature has advanced sufficiently over the last decade that this approach is now feasible. Examples include the Bayesian Monte Carlo Markov Chain approach of Jaquier, Polson and Rossi (1994)\(^{150}\), and Gallant and Tauchen’s EMM/SNP approach. The two approaches are surveyed in Johannes and Polson (2003)\(^ {151}\) and Gallant and Tauchen (2001)\(^ {152}\), respectively. The latter approach derives moment conditions from a particular auxiliary time series model that captures discrete-time data properties, uses Simulated Method of Moments to estimate the parameters of a continuous-time

---


model consistent with those moment conditions, and uses “re-projection” to
generate filtered and smoothed estimates from times series data. As illustrated in
the Ahn et al (2002)\textsuperscript{153} paper in this issue, the methodology can be used to
estimate model parameters and latent variable values from the joint time series
properties of the underlying and of derivatives prices.

Many more attempts have been made to modify or extend the Black-Scholes
model in order to accommodate for the skew observed on option markets. A
large class of those models is based on modelling the skew surface itself. For
example, so-called local volatility models (B. Dupire, 1994\textsuperscript{154}, E. Derman and I.
Kani, 1994\textsuperscript{155}) aim to fit the observed skew surface by calibrating a function $\sigma_{\text{loc}}(S, t)$ (where $S$ is the stock price and $t$ the time) such that it reproduces actual market
prices for each $K$ and $T$. This function $\sigma_{\text{loc}}(S, t)$ is then used in conjunction with
the Black-Scholes model to perform other important operations such as the
pricing of exotic options, hedging, and so on. The problem is however, that while
$\sigma_{\text{loc}}$ by construction allows one to reproduce the set of prices to which it was
calibrated, it does not contain any information about the true dynamics of the
underlying asset; therefore it can actually lead to worse hedging strategies and
erroneous exotic prices than would have been obtained with simply using Black-
Scholes without any attempts to account for the smile [P.S. Hagan, D. Kumar,

\textsuperscript{153} Ahn, D., R. Dittmar, B. Gao, and A.R. Gallant. "Purebred or hybrid? Reproducing the volatility in term
\textsuperscript{154} Dupire, B., 1994 \textit{op cit}
\textsuperscript{155} Derman, E. and Kani, I. 1994, \textit{op cit}
A.S. Lesniewski and D.E. Woodward, (2002). Nonetheless, local volatility models have been widely used by many banks and trading desks.


---

159 Cont, R., Tankov, and P. Financial modelling with jump processes. CRC Press, 2004
161 Heston, S.L. 1993 op cit
162 Fouque et al.(2000), op cit
164 Hagan et al.(2002), op cit
Cont, and J.-P. Bouchaud (1998)\textsuperscript{170}, J.-P. Bouchaud and M. Potters (2004)\textsuperscript{171} constitute approaches which have been successful in capturing some of the features of real option prices (Some of these works have already been discussed above). For example the recent 'stochastic alpha, beta, rho' (SABR) model [P.S. Hagan, D. Kumar, A.S. Lesniewski and D.E. Woodward (2002)\textsuperscript{172}] can be well fit to empirical skew surfaces. It also provides a better model of the dynamics of the smile over time.

Borland (2002)\textsuperscript{173},\textsuperscript{174} derives Option pricing formulas from a non-Gaussian model of stock returns. Fluctuations are assumed to evolve according to a nonlinear Fokker-Planck equation, which maximizes the Tsallis non-extensive entropy of index q. A generalized form of the Black-Scholes differential equation is found, and a martingale measure is derived which leads to closed form solutions for European call options. The model is intrinsically more parsimonious than stochastic volatility models in the sense that they have just one source of randomness, which also allows it to remain within the framework of complete markets. The option pricing methodology yields closed form formulae based on such a model, thereby predicting option prices rather than fitting parameters to match observed market prices. They found very good agreement between theoretical and traded prices, lending support to the possible validity and

\textsuperscript{171} Bouchaud, J.-P. and M. Potters (2004), op cit
\textsuperscript{172} P.S. Hagan, D. Kumar, A.S. Lesniewski and D.E. Woodward (2002), op cit
\textsuperscript{173} Borland, L., (2002), op cit
\textsuperscript{174} For a detail discussion on this please refer to chapter 3
potential applicability of the model. Considering all this we decided to use this model as an alternative to the B-S-M Model based on the Geometric Brownian Motion assumption.

2.3 Literature on the Tools used for Empirical Research

We now proceed to present our survey on existing literature on the following tools of empirical investigation:

- Empirical Mode Decomposition (The tool which enables decomposition of the time series into constituent functions)
- Recurrence Plot & Recurrence Quantification Analysis (Which can detect regime changes and critical phases of a time series)
- Power Spectrum Analysis

2.3.1 Empirical Mode Decomposition

Many “paradoxes” exist in standard decomposition of time signals. To avoid them, Huang et al. (1998)\textsuperscript{175} have developed a method, termed the Hilbert view, for studying nonstationary and nonlinear data in nonlinear dynamics. This method uses the empirical mode decomposition technique to generate finite number of Intrinsic Mode Functions (IMFs) that assume well behaved Huang–Hilbert Transform. The key part of the method is the ‘empirical mode decomposition’ method with which any complicated data set can be decomposed into a finite and often small number of `intrinsic mode functions' that admit well-

behaved Hilbert transforms. This decomposition method is adaptive, and, therefore, highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it is applicable to nonlinear and non-stationary processes. With the Hilbert transform, the `intrinsic mode functions' yield instantaneous frequencies as functions of time that give sharp identifications of imbedded structures. The final presentation of the results is an energy-frequency-time distribution, designated as the Hilbert spectrum. In this method, the main conceptual innovations are the introduction of `intrinsic mode functions' based on local properties of the signal, which makes the instantaneous frequency meaningful; and the introduction of the instantaneous frequencies for complicated data sets, which eliminate the need for spurious harmonics to represent nonlinear and non-stationary signals. Examples from the numerical results of the classical nonlinear equation systems and data representing natural phenomena are given in this classic paper to demonstrate the power of this new method. Classical nonlinear system data are especially interesting, for they serve to illustrate the roles played by the nonlinear and non-stationary effects in the energy-frequency-time distribution. The results obtained from this technique can be further analysed to reveal more details about the time series. Since then this technique has been used to analyse a host of time signals including financial time series.
A thorough survey was done to understand the use and application of this method. Huang et al.\textsuperscript{176} (2003) elaborates on the use of this technique for financial time series. They show that this decomposition method is adaptive, and, therefore, highly efficient. Since the decomposition is based on the local characteristic time scale of the data, it is applicable to non-linear and non-stationary processes. With the Hilbert transform, the IMF yield instantaneous frequencies as functions of time that give sharp identifications of imbedded structures. The final presentation of the results is an energy-frequency-time distribution, which we designate as the Hilbert Spectrum. Comparisons with Wavelet and Fourier analyses show the new method offers much better temporal and frequency resolutions. The EMD is also useful as a filter to extract variability of different scales. In their work, HHT has been used to examine the changeability of the market, as a measure of volatility of the market.

Peel et al.\textsuperscript{177}, discuss the process of applying EMD to a time series is demonstrated using 10 years of monthly precipitation data from Melbourne Regional Office. The Melbourne monthly precipitation time series is decomposed into three intrinsic mode functions (IMFs) and a residual. The conditions defining an IMF are presented. The sifting process used to obtain each IMF and the

\begin{footnotesize}
\textsuperscript{176} Huang, Norden E., Wendong Wu Man-Li, Qu, Steven R. Long, and Samuel S. P. Shen. "Applications of Hilbert-Huang transform to non-stationary financial time series analysis." \textit{Applied stochastic models in business and industry} 19, no. 3 (2003): 101

\end{footnotesize}
The residual is described. The general features of IMFs are described along with the ability to combine IMFs and the residual to form low frequency or high frequency filters. Two key decisions in the EMD application process, the rule for deciding when to stop sifting for an IMF and the choice of cubic spline end condition rule are reviewed and discussed in detail. A new cubic spline end condition rule, based on the assumption that the slope of the cubic spline at the end point is equal to zero, is proposed and compared to two other end condition rules from the literature. The comparison is based on three applications of the EMD algorithm, each application with a different end condition rule, to 8135 annual precipitation time series from around the world. The annual precipitation time series have periods of record ranging from 30 up to 299 years and represent a wide range of climatic zones. The proposed end condition rule is found to be the most efficient of the three rules tested, due to the EMD algorithm producing less IMFs when using the proposed rule. The end condition rule proposed in this paper is recommended for future EMD applications.

The utility of EMD for financial time series is thoroughly exhibited by Wei Wang et al. (2009). Due to the fluctuation and complexity of the financial time series, it is difficult to use any single artificial technique to capture its non-stationary property and accurately describe its moving tendency. So a novel hybrid intelligent forecasting model based on empirical mode decomposition (EMD) and

---

support vector regression (SVR) is proposed by them. EMD can adaptively decompose the complicated raw data into a finite set of intrinsic mode functions (IMFs) and a residue, which have simpler frequency components and higher correlation. Tendencies of these IMFs and the residue are forecasted by SVR respectively, in which the kernel functions are appropriately chosen according to their different fluctuations. The final forecasting value can be obtained by the sum of these prediction results. Successful forecasting application of Shanghai-securities index demonstrates the feasibility and validity of the presented model.

Wu et al., 2007\textsuperscript{179} shows the utility of EMD for non-stationery data set. Determining trend and implementing detrending operations are important steps in data analysis. Yet there is neither precise definition of “trend” nor any logical algorithm for extracting it. As a result, various ad hoc extrinsic methods have been used to determine trend and to facilitate a detrending operation. In this article, a simple and logical definition of trend is given for any nonlinear and nonstationary time series as an intrinsically determined monotonic function within a certain temporal span (most often that of the data span), or a function in which there can be at most one extremum within that temporal span. Being intrinsic, the method to derive the trend has to be adaptive. This definition of trend also presumes the existence of a natural time scale. All these requirements suggest the Empirical Mode Decomposition (EMD) method as the logical choice of

algorithm for extracting various trends from a data set. Once the trend is
determined, the corresponding detrending operation can be implemented. With
this definition of trend, the variability of the data on various time scales also can
be derived naturally. Climate data are used to illustrate the determination of the
intrinsic trend and natural variability.

Suling Jia et al (2009)\(^\text{180}\)(Suling Jia, Yanqin Guo, Qiang Wang, Jian Zhang, Trend
Extraction and Similarity Matching of Financial Time Series Based on EMD
Method, 2009 WRI World Congress on Computer Science and Information
Engineering) proposes a trend extraction method of financial time series which
are non-stationary and non-linear. And this article also gives the definition of
trend of time series based on different time intervals and the criterion to measure
the precision of the trend extracted. Experiments of trend extraction based on
EMD method are conducted and the result verifies the effectiveness of EMD
method in trend extraction of financial time series.

Wu et al.\(^\text{181}\) use EMD process to correlate two financial time series. The scaling,
phase distribution, and phase correlation of financial time series are investigated
based on the Dow Jones Industry Average and NASDAQ 10-min intraday data
for a period from 1 Aug. 1997 to 31 Dec. 2003. The returns of the two indices are

---

180 Jia, Suling, Yanqin Guo, Qiang Wang, and Jian Zhang. "Trend Extraction and Similarity Matching of
Financial Time Series Based on EMD Method." WRI World Congress on Computer Science and
Information Engineering. 2009. 526-530
181 Wu, Ming-Chya, Ming-Chang Huang, Hai-Chin Yu, and Thomas C. Chiang. "Phase distribution and phase
shown to have nice scaling behaviours and belong to stable distributions according to the criterion of Lévy’s $\alpha$ stable distribution condition. An approach catching characteristic features of financial time series based on the concept of instantaneous phase is further proposed to study the phase distribution and correlation. Analysis of the phase distribution concludes that return time series fall into a class which is different from other nonstationary time series. The correlation between returns of the two indices probed by the distribution of phase difference indicates that there was a remarkable change of trading activities after the event of the 9/11 attack, and this change persisted in later trading activities.

Guhathkurta et al.\textsuperscript{182} apply the EMD technique successfully on financial time series keeping in mind all the aspects brought out in the works discussed above. Analysis of financial time series with a view to understanding its underlying characteristic features has been the recent focus of scientists and practitioners studying the financial market. One of the key attributes of a time series is its periodicity. Because of their quasi-periodic nature, the financial time series do not reveal their periodicity clearly. Hilbert–Huang empirical mode decomposition (EMD) method elegantly brings out the underlying periodicity of any time-series. In the present study, the authors have used the EMD technique to analyse two different financial time series, viz., the daily movement of NIFTY index value of National Stock Exchange, India, and that of Hong Kong AOI, Hong Kong Stock

Exchange from July 1990 to January 2006. The returns of the two indices are shown to have strikingly similar probability distribution. The IMF phase and amplitude probability distribution of the two indices also reveal striking similarity. This indicates a remarkable similarity of trading behaviour in the two markets. Considering the geographical and political separation of the two, this indeed is an important discovery.

2.3.2 Recurrence Analysis

Eckmann et al. (1987)\(^{183}\) have introduced a tool which can visualize the recurrence of states \(x_i\) in a phase space. Usually, a phase space does not have a dimension (two or three) which allows it to be pictured. Higher dimensional phase spaces can only be visualized by projection into the two or three dimensional sub-spaces. However, Eckmann’s tool enables us to investigate the \(m\)-dimensional phase space trajectory through a two-dimensional representation of its recurrences. Such recurrence of a state at time \(i\) at a different time \(j\) is marked within a two-dimensional squared matrix with ones and zeros dots (black and white dots in the plot), where both axes are time axes. This representation is called recurrence plot (RP).

Zilbut and Webber (1992)\(^{184}\) further examine their tool in 1992 wherein they proposed a statistical quantification of RPs and gave it the name of Recurrence Quantification Analysis (RQA). While Recurrence plots have been advocated as

\(^{183}\)Eckmann J.P., S.O. Kamphorst, D. Ruelle (1987), *op cit*

\(^{184}\)Zbilut J. P, Webber C. L (1992.), *op cit*
a useful diagnostic tool for the assessment of dynamical time series, they extend
the usefulness of this tool by quantifying certain features of these plots which
may be helpful in determining embeddings and delays.

The theoretical and implementation aspects of RP and RQA have been enriched
by many important works. The most natural question relates to the way of
choosing an appropriate value for the time delay $d$ and the embedding dimension
$m$ while implementing the Recurrence Plot. Several methods have been
developed to best guess $m$ and $d$. The most often used methods are the Average
Mutual Information Function (AMI) for the time delay, as introduced by Fraser
and Swinney (1986)$^{185}$ and the False Nearest Neighbours (FNN) method for the
embedding dimension developed by Kennel et al. (1992)$^{186}$. The mutual
information $I$ is examined for a model dynamical system and for chaotic data from
an experiment on the Belousov-Zhabotinskii reaction. An algorithm for calculating
$I$ is presented. As proposed by Shaw, a minimum in $I$ is found to be a good
criterion for the choice of time delay in phase-portrait reconstruction from time-
series data. This criterion is shown to be far superior to choosing a zero of the
autocorrelation function. We examine the issue of determining an acceptable
minimum embedding dimension by looking at the behaviour of near neighbours
under changes in the embedding dimension from $d\rightarrow d+1$. When the number of
nearest neighbours arising through projection is zero in dimension $d_E$, the

$^{185}$ Fraser, Andrew, M. Swinney, and Harry L. "Independent coordinates for strange attractors from

$^{186}$ Kennel, Matthew B., Reggie Brown, and Henry D. I. Abarbanel. "Determining embedding dimension for
attractor has been unfolded in this dimension. The precise determination of \( d_E \) is clouded by “noise,” and we examine the manner in which noise changes the determination of \( d_E \). Our criterion also indicates the error one makes by choosing an embedding dimension smaller than \( d_E \).

Gao and Cai \(^{187}\) (2000) discusses the interpretation of Recurrence Plots. Recurrence plots (RPs) often have fascinating structures, especially when the embedding dimension is 1. They identify four basic patterns of a RP, namely, patterns along the main (45\(^\circ\)) diagonal, patterns along the 135\(^\circ\) diagonal, block-like structures, and square-like textures. They also study how the structures of and quantification statistics for RPs vary with the embedding parameters. By considering the distribution of the main diagonal line segments for chaotic systems, they relate some of the known statistics for the quantification of a RP to the Lyapunov exponent. This consideration enables us to introduce new ways of quantifying the diagonal line segments. Furthermore, they categorize recurrence points into two classes. A number of new quantities are identified which may be useful for the detection of nonstationarity in a time series, especially for the detection of a bifurcation sequence. A noisy transient Lorenz system is studied, to demonstrate how to identify a true bifurcation sequence, to interpret false bifurcation points, and to choose the embedding dimension.

Casdagli(1997) discusses on various aspects of Recurrence plot analysis. They show that recurrence plots (RPs) give detailed characterizations of time series generated by dynamical systems driven by slowly varying external forces. For deterministic systems we show that RPs of the time series can be used to reconstruct the RP of the driving force if it varies sufficiently slowly. If the driving force is one-dimensional, its functional form can then be inferred up to an invertible coordinate transformation. The same results hold for stochastic systems if the RP of the time series is suitably averaged and transformed. These results are used to investigate the nonlinear prediction of time series generated by dynamical systems driven by slowly varying external forces. They also consider the problem of detecting a small change in the driving force, and propose a surrogate data technique for assessing statistical significance. Numerically simulated time series and a time series of respiration rates recorded from a subject with sleep apnea are used as illustrative examples.

Mawan(2003) in his PhD thesis does the most comprehensive analysis of the theoretical and implementation aspects of recurrence plot and quantification analysis. In this work, different aspects and applications of the recurrence plot analysis are presented. First, a comprehensive overview of recurrence plots and their quantification possibilities is given. New measures of complexity are defined by using geometrical structures of recurrence plots. These measures are capable

to find chaos-chaos transitions in processes. Furthermore, a bivariate extension to cross recurrence plots is studied. Cross recurrence plots exhibit characteristic structures which can be used for the study of differences between two processes or for the alignment and search for matching sequences of two data series. The selected applications of techniques introduced to the various kinds of data demonstrate their ability. Analysis of recurrence plots can be adopted for the specific problem and thus opens a wide field of potential applications.

The complexity measures the RQA provides have been useful in describing and analysing a broad range of data. But one key question in empirical research concerns the confidence bounds of measured data. However, Schinkel et al. (2009)\textsuperscript{190} have recently presented one for RQA-measures. The complexity measures the RQA provides have been useful in describing and analysing a broad range of data. It is known to be rather robust to noise and non-stationarities. Yet, one key question in empirical research concerns the confidence bounds of measured data. In this work we propose a straightforward extension to the existing RQA framework which allows us to not only compute these complexity measures, but also to estimate their confidence bounds. They do this by using a well-known re-sampling paradigm – the bootstrap. They show that the confidence bounds of RQA measures come with the regular analysis at virtually no extra costs and that the method can be useful for comparing

univariate time series in a statistically sound fashion. They study the applicability of the suggested method with model and real-life data.

Marwan and Kurths (2005)\textsuperscript{191} elaborate interpretation and use of Recurrence Plot. Recurrence plots exhibit line structures which represent typical behaviour of the investigated system. The local slope of these line structures is connected with a specific transformation of the time scales of different segments of the phase-space trajectory. This provides us a better understanding of the structures occurring in recurrence plots. The relationship between the time-scales and line structures are of practical importance in cross recurrence plots. Using this relationship within cross recurrence plots, the time-scales of differently sampled or time-transformed measurements can be adjusted. An application to geophysical measurements illustrates the capability of this method for the adjustment of time-scales in different measurements.

Marwan et al.(2007),\textsuperscript{192} provides a classic summary of Recurrence analysis which is a must read for any one working with these tools. This report is a comprehensive overview covering recurrence based methods and their applications with an emphasis on recent developments. After a brief outline of the theory of recurrences, the basic idea of the recurrence plot with its variations is

presented. This includes the quantification of recurrence plots, like the recurrence quantification analysis, which is highly effective to detect, e. g., transitions in the dynamics of systems from time series. A main point is how to link recurrences to dynamical invariants and unstable periodic orbits. This and further evidence suggest that recurrences contain all relevant information about a system's behaviour. As the respective phase spaces of two systems change due to coupling, recurrence plots allow studying and quantifying their interaction. This fact also provides with a sensitive tool for the study of synchronization of complex systems. In the last part of the report several applications of recurrence plots in economy, physiology, neuroscience, earth sciences, astrophysics and engineering are shown. The authors succeed in their aim of this work to provide the readers with the know-how for the application of recurrence plot based methods in their own field of research. They detail the analysis of data and indicate possible difficulties and pitfalls. Marwan et al. (2008), also give readers a brilliant overview on this subject matter. Another work that was thoroughly surveyed was Marwan’s (2008) historical perspective of Recurrence Plot analysis.


Atay and Altıntaş (1999) explored the possibilities of this method as an analytical tool and established its utility. In their work, a graphical method based on recurrence plots is used in the reconstruction of the phase space from a time series of measurements. It is demonstrated that if the embedding delay and dimension are correctly chosen, the recurrence plot of a smooth dynamical system has a particularly simple form. It is shown how to use recurrence plots to determine the correct embedding parameters so that reliable quantitative information can be drawn about the system generating the time series. The average line length in the plot is shown to be directly related to the prediction horizon. Furthermore, it is a numerical characteristic of the embedded series independent of the threshold used in the plot.

RP and RQA are good in working with non-stationary and noisy data, in detecting changes in data behaviour, in particular in detecting breaks, like a phase transition, and in informing about other dynamic properties of a time series. This has been explored in a different context by Lambertz et al., (2000). Neuronal activities of the reticular formation (RF) of the lower brainstem and the nucleus tractus solitarii (NTS, first relay station of baroreceptor afferents) were recorded together in the anesthetized dog with related parameters of EEG, respiration and cardiovascular system. The RF neurons are part of the common

---

brainstem system (CBS) which participates in regulation and coordination of cardiovascular, respiratory, somatomotor systems, and vigilance. Multiple time series of these physiological subsystems yield useful information about internal dynamic coordination of the organism. Essential problems are nonlinearity and instationarity of the signals, due to the dynamic complexity of the systems. Several time-resolving methods are presented to describe nonlinear dynamic couplings in the time course, particularly during phase transitions. The methods are applied to the recorded signals representing the complex couplings of the physiological subsystems. Phase transitions in these systems are detected by recurrence plots of the instationary signals. The pointwise transinformation and the pointwise conditional coupling divergence are measures of the mutual interaction of the subsystems in the state space. If the signals show marked rhythms, instantaneous frequencies and their shiftings are demonstrated by time frequency distributions, and instantaneous phase differences show couplings of oscillating subsystems. Transient signal components are reconstructed by wavelet packet time selective transient reconstruction. These methods are useful means for analysing coupling characteristics of the complex physiological system, and detailed analyses of internal dynamic coordination of subsystems become possible. During phase transitions of the functional organization (a) the rhythms of the central neuronal activities and the peripheral systems are altered, (b) changes in the coupling between CBS neurons and cardiovascular signals, respiration and the EEG, and (c) between NTS neurons (influenced by baroreceptor afferents) and CBS neurons occur, and (d) the processing of
baroreceptor input at the NTS neurons changes. The results of this complex analysis, which could not be done formerly in this manner, confirm and complete former investigations on the dynamic organization of the CBS with its changing relations to peripheral and other central nervous subsystems.

Antoniou and Vorlow(2000) effectively use the recurrence analysis in financial time series. They explore their applicability in the graphic investigation of financial time series and their volatility. They examine the recurrence plots of various stock market indices in different frequencies and search for indications of deterministic nonlinearities. They provide a comparison of recurrence plots of observed financial time series with theoretically relevant sequences such as the Brownian motion, white noise and ARCH processes. Their conclusion is that recurrence plots can at least be used as a very accurate weak form efficiency visual test.

Holyst and Zebrowska(2000) apply these techniques on Foreign exchange data and get very good results. Selected data from Polish and USA stock and bond markets as well as foreign-exchange data have been analysed by the use of recurrence plots and the Hurst method. It has been found that there exist significant correlations in some of analysed data chains. Values of recurrence

ratios and ratios of determinism calculated from recurrence diagrams decrease significantly if one shuffles the data. The corresponding values of Hurst exponents are in the range 0.56–0.74 and they also decrease after shuffling. The lowest values of the Hurst exponent have been found for single shares at Polish stock market while the highest values are related to foreign-exchange data. The mean length of the cycle calculated from the behaviour of the Hurst exponent for Dow Jones index and S&P500 index is about 5 years while the Warsaw Stock Index WIG possesses the corresponding cycle of order of 11 months. The performed analysis shows that in the economical dynamics the main role is played by stochastic behaviour but traces of deterministic origin can be also seen.

Antoniou and Vorlow\textsuperscript{(2004)} further explore the use of RP in determining the underlying nonlinear dynamics of financial time series. In this paper they investigate for the presence of non-stochastic, possibly nonlinear deterministic dynamical cycles in financial time series. Evidence of nonlinear dynamics is revealed in denoised daily stock market index returns for six countries by combining Recurrence Quantification Analysis and wavelet filtering. Quantitative and qualitative results indicate that through wavelet pre-filtering they obtain a clearer view of the underlying dynamical structure of returns generating processes. Their results also suggest the existence of high dimensional deterministic dynamics, unstable periodic orbits and chaos.

Holyst and Zebrowska (2001) have tried to investigate chaos in financial time series using Recurrence Plots. Several economical time series such as exchange rates US$/British £, USA Treasury Bonds rates and Warsaw Stock Index WIG have been investigated using the method of recurrence plots. The percentage of recurrence REC and the percentage of determinism DET have been calculated for the original and for shuffled data. They have found that in some cases the values of REC and DET parameters are about 20% lower for the surrogate data which indicates the presence of unstable periodical orbits in the considered data. A similar result has been obtained for the chaotic Lorenz model contaminated by noise. Their investigations suggest that real economical dynamics is a mixture of deterministic and stochastic chaos. They show how a simple chaotic economic model can be controlled by appropriate influence of time-delayed feedback.

Bandt et al. (2008) show how recurrence analysis can be used to detect behaviour of coupled systems. In the analysis of coupled systems, various techniques have been developed to model and detect dependencies from observed bivariate time series. Most well-founded methods, like Granger-causality and partial coherence, are based on the theory of linear systems: on

---


correlation functions, spectra and vector autoregressive processes. In this paper they discuss a nonlinear approach using recurrence analysis.

Belaire-Franch et al.(2002) use recurrence plot analysis with exchange rate data. Purchasing power parity (PPP) is an important theory at the basis of a large number of economic models. However, the implication derived from the theory that real exchange rates must follow stationary processes is not conclusively supported by empirical studies. In this work, they follow a two-stage testing procedure to test for nonlinearities and chaos in real exchange rates, using recurrence quantification analysis (RQA). Belaire-Franch(2004) further explores recurrence analysis of financial markets. In this paper, earlier work on testing for non-linear dynamics on realizations from an artificial financial market is extended in two ways. On the one hand, Hinich’s bispectral test and White’s neural network test are computed. On the other hand, a recently developed methodology to test for hidden structures in data (Recurrence analysis), inherited from Physics, is successfully applied on the realizations of the artificial market. Results among alternative tests are compared.

In this connection, one must cite some related works which established the motivation for using Recurrence plot and quantification analysis more with time

---


series data. Gilmore(1993)\textsuperscript{204} examines this new approach in 1993. Whether certain financial and economic time series are better described by linear stochastic models or are appropriately characterized by deterministic chaos is an issue of great current interest. Empirical research to detect the presence of chaos in such time series has been hampered by lack of an adequate test for chaotic behaviour in small, noisy data sets. His paper presents a new, topological test for chaos, demonstrates its advantages over current testing methodology, and compares the results with earlier analyses based on metric tests of economic and financial series. A short-term forecasting approach for chaotic processes is also described. Gilmore(1996)\textsuperscript{205} further explores the methods for financial time series analysis. Interest in the relevance of nonlinear dynamics to fields such as finance and economics has spurred the development of new methods of analysis for time series data. Early tests for chaos led to problems when applied to financial and economic data. This motivated development of the BDS family of statistics to test for nonlinearity generally. He discusses another method of analysis which has been introduced into the scientific literature which is topological analysis related to Recurrence Plot. It uses a test for chaos which is relatively simple and appropriate for financial data. A quantitative version of this test is developed here and is used to analyse stock return data. Gilmore(2001)\textsuperscript{206} further examines the method on analysis of exchange rates.

He discusses the fact that interest in the relevance of nonlinear dynamics to finance and economics has spurred the evolution of new ways to analyse time series data. Tests for chaos, based on a metric approach which measures spatial correlations, led to the development of the correlation dimension test for chaos and the BDS test for nonlinearity. More recently, a topological method has been introduced into the scientific literature which employs a simple qualitative test for chaos that is adaptable to the characteristics of financial data. A quantitative version is also presented there. Conflicting evidence exists about the presence of chaotic behaviour in exchange-rate data. The qualitative topological test does not support evidence of a chaotic generating mechanism in these series. The quantitative form finds nonlinear dependence and is a useful diagnostic to determine the adequacy of ARCH-type models for this nonlinear structure.

Gilmore (1998) examines chaotic dynamical systems. Topological methods have been developed for the analysis of dissipative dynamical systems that operate in the chaotic regime. They were originally developed for three-dimensional dissipative dynamical systems, but they are applicable to all “low-dimensional” dynamical systems. These are systems for which the flow rapidly relaxes to a three-dimensional subspace of phase space. Equivalently, the associated attractor has Lyapunov dimension $d_L < 3$. Topological methods supplement methods previously developed to determine the values of metric and dynamical invariants. However, topological methods possess three additional

---

features: they describe how to model the dynamics; they allow validation of the
topological invariants are robust under changes in
control-parameter values. The topological-analysis procedure depends on
identifying the stretching and squeezing mechanisms that act to create a strange
attractor and organize all the unstable periodic orbits in this attractor in a unique
way. The stretching and squeezing mechanisms are represented by a caricature,
a branched manifold, which is also called a template or a knot holder. This turns
out to be a version of the dynamical system in the limit of infinite dissipation. This
topological structure is identified by a set of integer invariants. One of the truly
remarkable results of the topological-analysis procedure is that these integer
invariants can be extracted from a chaotic time series. Furthermore, self-
consistency checks can be used to confirm the integer values. These integers
can be used to determine whether or not two dynamical systems are equivalent;
in particular, they can determine whether a model developed from time-series
data is an accurate representation of a physical system. Conversely, these
integers can be used to provide a model for the dynamical mechanisms that
generate chaotic data. In fact, the author has constructed a doubly discrete
classification of strange attractors. The underlying branched manifold provides
one discrete classification. Each branched manifold has an “unfolding” or
perturbation in which some subset of orbits is removed. The remaining orbits are
determined by a basis set of orbits that forces the presence of all remaining
orbits. Branched manifolds and basis sets of orbits provide this doubly discrete
classification of strange attractors. In this review the author describes the steps
that have been developed to implement the topological-analysis procedure. In addition, the author illustrates how to apply this procedure by carrying out the analysis of several experimental data sets. The results obtained for several other experimental time series that exhibit chaotic behaviour are also described.

Some recent applications of recurrence plot and quantification analysis establishes the utility of these tools firmly in time series analysis. Bigdeli and Afshar (2009)\textsuperscript{208}. Price forecasting in the current deregulated power markets is an important requirement for deriving proper bidding strategy and profit maximization of producers. On the other hand, the energy price in the power market experiences lots of fluctuations which may affect the accuracy of the price forecasting seriously. Seeking for predictability, in this paper, the characteristics of these fluctuations are investigated through time series analysis methods. The results of analyses are representative of the existence of a deterministic chaos in the system with a mimic predictability. Besides, it is observed that because of existing the seasonality and non-stationarity in the system dynamics, a fixed model cannot perform properly even in case of normalized input data, but the developed models should be updated regularly.

Bigdeli et al. (2009) extends their study in a later study. Market data analysis and short-term price forecasting in Iran electricity market as a market with pay-as-bid payment mechanism has been considered in this paper. The data analysis procedure includes both correlation and predictability analysis of the most important load and price indices. The employed data are the experimental time series from Iran electricity market in its real size and is long enough to make it possible to take properties such as non-stationarity of market into account. For predictability analysis, the bifurcation diagrams and recurrence plots of the data have been investigated. The results of these analyses indicate existence of deterministic chaos in addition to non-stationarity property of the system which implies short-term predictability. In the next step, two artificial neural networks have been developed for forecasting the two price indices in Iran's electricity market. The models' input sets are selected regarding four aspects: the correlation properties of the available data, the critiques of Iran's electricity market, a proper convergence rate in case of sudden variations in the market price behaviour, and the omission of cumulative forecasting errors. The simulation results based on experimental data from Iran electricity market are representative of good performance of the developed neural networks in coping with and forecasting of the market behaviour, even in the case of severe volatility in the market price indices.

---

Fabretti and Ausloos (2005) apply this technique to detect critical regime in financial time series. Recurrence Plot (RP) and Recurrence Quantification Analysis (RQA) are signal numerical analysis methodologies able to work with nonlinear dynamical systems and nonstationarity. Moreover, they well evidence changes in the states of a dynamical system. The authors recall their features and give practical recipes. It is shown that RP and RQA detect the critical regime in financial indices (in analogy with phase transition) before a bubble bursts, whence allowing to estimate the bubble initial time. The analysis is made on DAX and NASDAQ daily closing price between January 1998 and November 2003. DAX is studied in order to set-up overall considerations, and as a support for deducing technical rules. The NASDAQ bubble initial time has been estimated to be on 19 October 1999.

Following this, Guhathakurta et al. (2009, 2010a, 2010b), have done some important works to establish this method as a technique for determining critical phases in financial time series and also brought out the ability of recurrence analysis to capture the underlying nonlinear dynamics of a time series. RP and

---


RQA are good at working with non-stationarity and noisy data, in detecting changes in data behaviour, in particular in detecting breaks, like a phase transition and in informing about other dynamic properties of a time series. Endogenous Stock Market Crashes have been modelled as phase changes in recent times. Motivated by this, the authors have used RP and RQA techniques for detecting critical regimes preceding an endogenous crash seen as a phase transition and hence give an estimation of the initial bubble time. They have used a new method for computing RQA measures with confidence intervals. They have also used the techniques on a known exogenous crash to see if the RP reveals a different story or not. The analysis is made on Nifty, Hong Kong AOI and Dow Jones Industrial Average, taken over a time span of about 3 years for the endogenous crashes. Then the RPs of all time series have been observed, compared and discussed. All the time series have been first transformed into the classical momentum divided by the maximum $X_{\text{max}}$ of the time series over the time window which is considered in the specific analysis. RPs have been plotted for each time series, and RQA variables have been computed on different epochs. Our studies reveal that, in the case of an endogenous crash, we have been able to identify the bubble, while in the case of exogenous crashes the plots do not show any such pattern, thus helping us in identifying such crashes.

2.3.3 Power Spectrum Analysis

One of the simplest and useful tools to investigate nonlinear dynamics is the power spectrum analysis. For a given data series, the power spectrum gives a plot of the portion of a signal’s power (energy per unit time) falling within given
frequency bins. The most common and effective way of generating a power spectrum is by using a discrete Fourier transform. The classic text for Fourier analysis by Bracewell (2000)\textsuperscript{214} was our starting point in understanding the basics of the tool. Of particular interest to us was the Chapter on Discrete Fourier Transform and the FFT (Chapter 11). This chapter describes in detail the various theoretical and practical aspects of Fast Fourier Transform and discusses at length on the critical aspects of power spectra analysis.

Important aspects of FFT has been discussed in detail by Bergland (1969)\textsuperscript{215}. This article was written as an introduction to the Fast Fourier Transform. The basic concepts were introduced using specific examples. The utilities and implementation aspects of the tool is elaborated in this classic article.

Another important treatise on Fourier analysis is by Bloomfield (2000)\textsuperscript{216}. Bloomfield provides in-depth discussions of harmonic regression, harmonic analysis, complex demodulation, and spectrum analysis. All methods are clearly illustrated using examples of specific data sets, while ample exercises acquaint readers with Fourier analysis and its applications. The chapter on Fast Fourier Transform clearly brings out the theoretical and practical aspects of FFT in analysing time series.

\textsuperscript{214} Bracewell, Ronald. \textit{The Fourier Transform and its Applications}. Mcgraw-Hill Higher Education.
Giampaoli et al.,\textsuperscript{217} (2009) discuss the application of advanced methods from Fourier analysis to the study of ultra-high-frequency financial data. The use of Lomb–Scargle Fourier transform provides a robust framework to take into account the irregular spacing in time, minimising the computational effort. Likewise, it avoids complex model specifications (e.g. ACD or intensity models) or resorting to traditional methods, such as (linear or cubic) interpolation and regular resampling, which not only cause artifacts in the data and loss of information, but also lead to the generation and use of spurious information. This paper was studied to explore the possibilities of using some of these techniques.

\subsection*{2.4 Alternative techniques for empirical research reviewed}

From the survey of existing literature on the methodology we found that the methods have been very useful in bringing out the dynamics of time series and helped in detailed analysis of the same. It is also clear that most of the empirical work in this direction has been done with data from developed market and very little work has been done in Indian market. This provides motivation for the present work to bridge the gap in this direction.

Our purpose of studying the existing literature on option pricing model was to arrive at a model which gives a closed end solution like the Black Scholes model and at the same time uses an alternative to the Geometric Brownian Motion

Model while modelling the underlying. Our survey shows that the model proposed by Borland (2002) using Tsallis Distribution can be chosen as a viable alternative, it is to be noted that our purpose is to empirically check the validity of the model of the underlying and not the option pricing model itself. We now proceed with elaboration of the theoretical framework of the two chosen models viz., Geometric Brownian Motion Model and the one based on Tsallis distribution in the next chapter.