CHAPTER 8
CONCLUDING OBSERVATIONS

In this thesis we have presented an empirical work on financial time series using non-traditional methods drawn from the field of nonlinear dynamics. The purpose of using such methods was to capture the nonlinear dynamics of the time series. While the same can be modelled quite efficiently using traditional methods, these alternative methods help to capture the dynamics in a manner that enables us to compare time series effectively. Also, there is a body of works existing which have used these traditional methods to analyse financial time series in both India and abroad. However, there is very little work which uses the alternative methods to capture the nonlinear dynamics of financial time series of Indian market. Our study thus attempts to bridge this gap as well.

We use various tools to analyse different aspects of financial time series like their underlying periodicity, regime changes and time evolution. Finally we use these methods to compare and comment on the ability of two continuous time models to capture the nonlinear dynamics of financial time series in Indian market.

Most important results in modern finance are based on the assumption that the stock prices follow Geometric Brownian motion. The most famous amongst the theories is the Option Pricing Theory by Fischer Black, Myron Scholes (1973).\textsuperscript{420}

\textsuperscript{420} Black, Fischer and Scholes, Myron (1973), op cit
and Robert Merton (1973)\textsuperscript{421}. Our objective was to examine the empirical validity of this assumption with respect to stock price movement in India. Our endeavour was to simulate a Geometric Brownian Motion based on data from stock indices of India and examine its validity with respect to actual price. To evaluate the efficacy of the model we decided to choose an alternative model and compare the relative performance of the two models, using specific tools. The choice of alternative model was critical. Our selection was based on one major consideration. Since the Black-Scholes-Merton option pricing model gives a closed end solution which makes it so easy to comprehend and use, the alternative model must also result in a closed end solution. To our knowledge, none of the alternate models to Black-Scholes-Merton result in manageable closed form solutions, which make the Black and Scholes approach so useful and popular. However, Borland (2002)\textsuperscript{422} succeeds in obtaining closed form solutions for European options. Their approach is based on a new class of stochastic processes (Tsallis 1988)\textsuperscript{423}, which models the underlying stock returns allowing for statistical feedback. This was our motivation to use the Borland model as an alternative to Black-Scholes-Merton model. However, the point to be noted is that we have not investigated the option pricing model but the asset pricing model of the underlying.

Our analyses were based on S & P CNX NIFTY (NIFTY) stock index from the NSE (National Stock Exchange, India) website database and SENSEX stock

\textsuperscript{421} Merton, Robert (1973), \textit{op cit}
\textsuperscript{422} Borland, Lisa (2002), \textit{op cit}
\textsuperscript{423} Tsallis C. (1988), \textit{op cit}
index from the Bombay stock exchange, India. The choice of two indices was based on the fact that these are the two main indices reflecting the stock price movement in India. Based on the data from the indices we simulated a Geometric Brownian Motion data set and another data set based on the Borland model which uses the Tsallis distribution. We first tested all the three series for presence of nonlinearity using various established methods for testing nonlinearity in data. We then used the Cross-Correlation study and Power Spectrum Analysis to find out the linear relationship between the original and the simulated series respectively. Since the nonlinearity in the data sets are established we then examined the nonlinear dynamic properties of each of the simulated time series using Empirical Mode Decomposition and Recurrence Analysis.

Empirical Mode Decomposition is a method that helps decompose a time series into several constituent series, which, if added, will lead to the original time series. The number and nature of such constituent series called the intrinsic mode function (IMF) depend on the nonlinear dynamic properties of the original data set itself. Thus by looking at the number and properties of such IMFs one can compare two or more time series effectively. If the time series have similar nonlinear dynamic properties the number and nature of the IMFs will also be approximately same. We use this property of EMD to good effect by studying two different financial time series from two markets, viz., India and Hong Kong.
Recurrence Plot (RP) is essentially done to reveal the dynamic characteristics of the time series under investigation. For an economic time series, the evolving patterns tell us whether the series is disrupted, non-stationary or nonlinear in nature. By comparing the RPs of two economic time series we can make out whether the dynamical systems governing the time series are similar, or not. Recurrence Quantification Analysis (RQA) is a statistical quantification of RPs. RP and RQA are good at working with non-stationarity and noisy data, in detecting changes in data behavior, in particular in detecting breaks, like a phase transition and in informing about other dynamic properties of a time series. Endogenous Stock Market Crashes have been modelled as phase changes in recent times. Motivated by this, we have used RP and RQA techniques for detecting critical regimes preceding an endogenous crash seen as a phase transition and hence give an estimation of the initial bubble time. We have used a new method for computing RQA measures with confidence intervals.

Finally we have used these methods to compare two models of stock price movement, namely the GBM and the Borland model. Taking help of the properties of the methods as mentioned above, we can clearly comment on the similarity/ dissimilarity of the time series under study – the original data set from Indian stock market, and time series generated from simulations using the two models and the original data. This helps us in detecting which model is closer to the original data set in terms of nonlinear dynamics.
Before we worked on the empirical investigation of the data series under consideration we proceeded to establish the utility of the tools from nonlinear dynamics, viz., Empirical Mode Decomposition (EMD) and Recurrence Plot and Quantification analysis. In the first study in chapter 5, we have used the EMD technique to analyse two different financial time series, viz., the daily movement of NIFTY index value of National Stock Exchange, India, and that of Hong Kong AOI, Hong Kong Stock Exchange from July 1990 to January 2006. We found that this new technique could decompose the original time series into several intrinsic mode functions. With the help of these functions we could clearly compare and establish the similarity and dissimilarity between two different time series. This technique helped us in detecting the underlying periodicity and proved to be a more efficient tool that other spectral analysis tool in doing so.

Recurrence Plots (RP) and Recurrence Quantification Analysis (RQA) are good at working with non-stationarity and noisy data, in detecting changes in data behavior, in particular in detecting breaks, like a phase transition and in informing about other dynamic properties of a time series. Endogenous Stock Market Crashes have been modelled as phase changes in recent times. Motivated by this, in our next work, in Chapter 6 we have used RP and RQA techniques for detecting critical regimes preceding an endogenous crash seen as a phase transition and hence give an estimation of the initial bubble time. We have used a new method for computing RQA measures with confidence intervals. We have also used the techniques on a known exogenous crash to see if the RP reveals a
different story or not. The analysis is made on Nifty, Hong Kong AOI and Dow Jones Industrial Average, taken over a time span of about 3 years for the endogenous crashes. Then the RPs of all time series have been observed, compared and discussed. All the time series have been first transformed into the classical momentum divided by the maximum $X_{\text{max}}$ of the time series over the time window which is considered in the specific analysis. RPs have been plotted for each time series, and RQA variables have been computed on different epochs. Our studies reveal that, in the case of an endogenous crash, we have been able to identify the bubble, while in the case of exogenous crashes the plots do not show any such pattern, thus helping us in identifying such crashes. In our study we also established that this tool may also be used to test the predictive power of models by comparing the RP of model simulations and actual data.

With the powers of these new tools established we proceeded with our examination of the Geometric Brownian Motion and the alternate model proposed by Borland in the manner described earlier. Our findings helped us to compare and comment on the relative advantage of Borland (2002)\textsuperscript{424} proposed model over the Black-Scholes model. Firstly, the cross-correlation study gave us an initial indication in favour of the Borland model as we found that for all values of the delay factor the correlation for Borland and NIFTY remained higher than that of GBM and NIFTY. The same was also true for the SENSEX data. The Power Spectrum Analysis also showed the Borland model to be closer to actual

\textsuperscript{424} Borland, Lisa (2002), op cit
data set in terms of distribution of peaks and troughs than the Geometric Brownian Motion Model. This was true for both NIFTY and SENSEX data set. Our Recurrence Plot Analysis showed that out of the two models, Borland model captures the essential dynamics of the original data set much better. An inspection of the Recurrence Quantification Analysis revealed a similar picture. Our findings indicated that at micro epoch level the pricing model using Tsallis distribution captures the time series dynamics of the actual data set much better than the simulation using Geometric Brownian Motion assumption. Our findings were valid for both NIFTY and SENSEX. The EMD analysis clearly shows that the underlying periodicity of the simulated time series based on the Borland model were much closer to the periodicity of actual return series of NIFTY and SENSEX than that of the simulations based on BSM model. The closer inspection of the phase and amplitude distribution of the 1st two IMFs also revealed the same picture.

This acts as an empirical validation of the underlying theoretical framework of the option-pricing model. The Borland model being a major closed end solution to Option Pricing can become an alternative to the BS model as a benchmark tool for the investors. The result of our findings establishes that the Borland model captures the non-linear dynamics of the time series representing the underlying better than the BS model. Thus through our work we established that for the Indian market, Geometric Brownian Motion model performs worse than the model based on Tsallis distribution insofar as capturing the nonlinear dynamics.
of the original data set is concerned. Taking all findings in to account we can propose that the Geometric Brownian Motion assumption needs to be replaced by the model based on Tsallis distribution. The implication for option pricing model is that the same assumption from the Black- Scholes- Merton pricing theory is to be replaced by the new one based on Tsallis distribution. This further means that the solution derived by Borland(2002)\(^{425}\) may be considered as a viable alternative, at least for Indian conditions.

We have thus used alternative methods to bring out the nonlinear dynamics of Indian market and also compared it with the same of some foreign markets. Using the Empirical Mode Decomposition we found that the underlying periodicity of two different markets , viz., Indian stock market and Hong Kong stock market exhibit similar characteristics , pointing towards a possibility of using a single model to capture the stock price movement in both the markets. Recurrence Plot and Recurrence Quantification Analysis helped us to detect critical periods in stock markets, thus acting as a signal to bubbles and bust. Finally using different methods we found that the nonlinear dynamics of Indian stock market as revealed through the time series representing the data from the two main indices of Indian stock market can be represented by the Borland model using Tsallis distribution better than the model using Geometric Brownian Motion. Our results showed that the data generated from the Borland model had the same number of IMFs with approximately same properties. On the other hand the data generated

\(^{425}\) Borland, Lisa (2002), *op cit*
from the GBM model had slightly different results. The recurrence analysis also showed that the time evolution of the data set generated from the Borland model followed the time evolution of the original data set much closer than the data generated from the GBM model.

Our work thus established the alternate methods as useful tools for understanding the nonlinear dynamics of financial time series. Secondly, we were able to detect critical regimes of financial time series, viz., bubbles and crashes before time. Next, our work bridged the gap that exists in India in using these methods bringing out the nonlinear dynamics of Indian market. Finally we were able to compare and comment on the efficacy of two asset pricing models using these methods. The study therefore establishes its relevance for application of finance theories in terms of investment decisions in the backdrop of portfolio theory as well as for strategic decisions relating to multidimensional financial linkage across national boundaries.