CHAPTER 6

RECURRANCE ANALYSIS OF CRITICAL REGIMES OF STOCK MARKET

6.1. Introduction

In this chapter we establish the ability of Recurrence Plot analysis to capture the nonlinear dynamics and time evolution of a financial time series. Earlier versions of this study are published in Guhathakurta et al (2009\textsuperscript{362}, 2010a\textsuperscript{363}, 2010b\textsuperscript{364}).

Stock market investment lies at the core of any modern market economy. The graphs charting movement of stock market indices are synonymous with the ECG of the economic heart of even a country like India. The fear of every investor is a sudden and steep drop of asset prices: the occurrence of a stock market crash. Crashes are rare events but can happen even in mature markets. The study of critical phenomena like financial crashes has been the focus of much recent research work.

One area of work studies the Stock Market Crashes considering stock price movement following a complex power law series only in case of endogenous crashes. Motivated by this, we then use a technique evolved from nonlinear time

\textsuperscript{362} Guhathakurta, Kousik, Bhattacharya, Basabi and Roychowdhury, A. (2009), op cit
\textsuperscript{363} Guhathakurta, Kousik, Bhattacharya, Basabi and Roychowdhury, A. (2010a), op cit
\textsuperscript{364} Guhathakurta, Kousik, Bhattacharya, Basabi and Roychowdhury, A. (2010b), op cit
series analysis in the study of deterministic chaos to find out whether we can
distinguish between endogenous and exogenous crashes.

There has been considerable research work towards modelling financial crashes,
most suggesting that, close to a crash, the market behaves like a thermodynamic
system which undergoes phase transition. Some propose a picture of stock
market crashes as critical points in a system with discrete scale invariance. The
critical exponent is then complex, leading to log-periodic fluctuations in stock
market indices. This picture is in the spirit of the known earthquake-stock market
analogy and of recent work on log-periodic fluctuations associated with
earthquakes [Feigenbaum(1996)\textsuperscript{365}, Feigenbaum(1998)\textsuperscript{366}]. Some has shown
that stock market crashes are caused by the slow build up of long-range
correlations between traders, leading to a collapse of the stock market in one
critical instant. A crash is interpreted as a critical point [Sornette(1999\textsuperscript{367},
1996\textsuperscript{368})].

Mostly, these recent works have shown an analogy between crashes and phase
transition [Sornette(1999\textsuperscript{369}, 1996\textsuperscript{370}), Johansen(1999)\textsuperscript{371}, Vandewalle (1998\textsuperscript{372},


\textsuperscript{368} Sornette, D., A. Johansen, and J. P. Bouchaud. "Stock Market Crashes, Precursors and Replicas ."  

\textsuperscript{369} \textit{Ibid}
1998a\textsuperscript{373}); as in earthquakes, log periodic oscillations have been found before some crashes [Sornette (1996\textsuperscript{374}), Johansen(2000\textsuperscript{375})], and then it was proposed that an economic index \( y(t) \) increases as a complex power law, whose representation is

\[
y(t) = A + B \ln(t_c - t) \{1 + C \cos[\omega \ln(t_c - t) + \varphi]\}
\]

[6.1.1]

where A, B, C, \( \omega \), \( \varphi \) are constants and \( t_c \) is the critical time (rupture time).

An endogenous crash is preceded by an unstable phase where any information is amplified; this critical period is called the speculative bubble.

As discussed in chapter 4, Recurrence Plots are graphical tools elaborated by Eckmann, Kamphorst and Ruelle in 1987 and are based on Phase Space Reconstruction [Eckmann(1987)\textsuperscript{376}]. In 1992, Zbilut and Webber [Zbilut(1992)\textsuperscript{377}] proposed a statistical quantification of RPs and gave it the name of Recurrence Quantification Analysis (RQA). RP and RQA are good at working with non-stationarity and noisy data, in detecting changes in data behaviour, in particular in detecting breaks, like a phase transition [Lambertz(2000)\textsuperscript{378}], and in informing

\textsuperscript{370} Ibid
\textsuperscript{373} Vandewalle, N., M. Ausloos, Ph. Boveroux, and A. Minguet. "How the financial crash of October 1997 could have been predicted." \textit{European Physics Journal B} 4 (19998): 139-141.
\textsuperscript{374} Sornette, D., A. Johansen, J. P. Bouchaud(1996), \textit{op cit}
\textsuperscript{376} Eckmann J.P., S.O. Kamphorst, D. Ruelle (1987), \textit{op cit}
\textsuperscript{377} Zbilut J. P, Webber C. L (1992.), \textit{op cit}
\textsuperscript{378} Lambertz, M., R. Vandenhouten, R. Grebe, P. Langhorst(2000),\textit{op cit}
about other dynamic properties of a time series [Eckmann(1987)]\(^{379}\). Most of the applications of RP and RQA are at this time in the field of physiology and biology, but some authors have already applied these techniques to financial data [Holyst(2000)\(^{380}\), 2001\(^{381}\), Antoniou(2000)\(^{382}\)]. We have used RP and RQA techniques for examining both endogenous and exogenous crash data to find out the distinction between the two phenomena. Then we have used these techniques in detecting critical regimes preceding an endogenous crash seen as a phase transition and hence give an estimation of the initial bubble time. We have worked with data from the US, Indian and Hong Kong Stock markets. To the best of our knowledge this combination of three different categories of market in terms of efficiency has not been analysed together before. However, an important work on Indian Stock exchanges was carried out by Pan and Sinha [Pan(2008)]\(^{383}\).

6.2. Recurrence Analysis: A recapitulation

In this section we recapitulate the basic concepts of recurrence analysis before proceeding with the implantation. A recurrence plot (RP) is a graph that shows all those times at which a state of the dynamical system recurs. In other words, the RP reveals all the times when the phase space trajectory visits roughly the same area in the phase space. Natural processes can have a distinct recurrent

---

\(^{379}\) Ibid

\(^{380}\) Holyst, J. A., M. Zebrowska(2000), op cit

\(^{381}\) J. A. Holyst, M. Zebrowska, K. Urbanowicz(2001), op cit

\(^{382}\) Antoniou, A., C. E. Vorlow(2000), op cit

behaviour (e.g. periodicities (as seasonal or Milankovich cycles)) but also irregular cyclicities (as El Nino Southern Oscillation). Moreover, the recurrence of states, in the sense that states are arbitrarily close after some time, is a fundamental property of deterministic dynamical systems and is typical for nonlinear or chaotic systems. The recurrence of states in nature has been known for a long time and has also been discussed in early publications\(^{384}\).

Usually, a phase space does not have a dimension (two or three) which allows it to be pictured. Higher dimensional phase spaces can only be visualised by projection into the two or three dimensional sub-spaces. However, Eckmann's tool (1987\(^{385}\)) enables us to investigate the m-dimensional phase space trajectory through a two-dimensional representation of its recurrences. Such recurrence of a state at time \(i\) at a different time \(j\) is marked within a two-dimensional squared matrix with one and zero dots (black and white dots in the plot), where both axes are time axes. This representation is called the recurrence plot (RP). Such an RP can be mathematically expressed as

\[
R_{i,j} = \Theta(\varepsilon_i - \|x_i - x_j\|), x_i \in \mathbb{R}^m, i, j = 1, \ldots, N
\]  

\[\text{[6.2.1]}\]

Where \(R_{i,j}\) is the recurrence plot, \(N\) is the number of considered states \(x_i\), is a threshold distance, \(\|\cdot\|\) a norm and \(\Theta(\cdot)\) the Heaviside function. The Heaviside step function is given by:

\(^{384}\) Refer chapter 4.

\(^{385}\) Eckmann et al., (1987), op cit
\[ \Theta(x) = 0 \text{ if } x < 0 \]
\[ \Theta(x) = 1 \text{ if } x \geq 0 \]  

[6.2.2]

The threshold distance is called the delay factor and the number of considered states is called the embedding dimension.

6.2.1. Embedding parameters\textsuperscript{386}

The most natural question relates to the way of choosing an appropriate value for the time delay \( d \) and the embedding dimension \( m \). Several methods have been developed to best guess \( m \) and \( d \). The most often used methods are the Average Mutual Information Function (AMI) for the time delay, as introduced by Fraser and Swinney [Fraser(1986)\textsuperscript{387}] and the False Nearest Neighbours (FNN) method for the embedding dimension developed by Kennel et al. [Kennel (1992)\textsuperscript{388}].

First of all, the time delay has to be estimated. In the first one, the value for which the autocorrelation function \( C(d) \) first passes through zero is searched, which gives \( d \). In the second, one chooses the first minimum location of the average mutual information function, where the mutual information function \( S(d) \) is defined in the usual manner.

\textsuperscript{386} For details please refer chapter 4. Here we just recapitulate certain portions.
\textsuperscript{387} Fraser, Andrew M. Swinney, and Harry L. (1986), \textit{op cit}
\textsuperscript{388} Kennel , Matthew B., Brown, Reggie and Abarbanel , Henry D. I (1992), \textit{op cit}
The value of $d$ that firstly minimizes the quantity $S(d)$ is the method choice for finding a reasonable time delay.

The difference between these two methods resides in the fact that while the first looks for linear independence, the second measures a general dependence of two variables. For this reason, the second method seems to be preferred in nonlinear time series analysis.

On the other hand, the method used to find the embedding dimension is based on the concept of a false neighbour. A false neighbour is a point in the data set that looks like a neighbour to another because the orbit is seen in an overly small embedding space. For example, two points on a circle can appear close to each other, even though they are not, if, for example, the circle is seen sideways (as a projection), thus is appearing as a line segment. Therefore, increasing by one the dimension $m$ of the reconstructed space often permits one to differentiate between the points of the orbit; i.e. those which are true neighbours and those which are not.

Let $y(i)$ be a point of the reconstructed space. Let us note as $y_r(i)$ the $r$-th nearest neighbour and compute the Euclidean distance $L^2$ between them

$$R_m^2(y(i), y(i)^r) = \sum_{k=1}^{m-1} [y(i + kd) - y_r(i + kd)]^2$$  \[6.2.1.1\]
Next increase $m$ to $m + 1$ and compute the new distance, i.e. $R^2$

$$R_{m+1}^2(y(i), y'(i)) = R_m^2(y(i), y'(i)) + \left[ y(i + kd) - y'(i + kd) \right]^2$$  \[6.2.1.2\]

The point $y_t(i)$ is said a false nearest neighbour if

$$\left[ \frac{R_{m+1}^2(y(i), y'(i)) - R_m^2(y(i), y'(i))}{R_m^2(y(i), y'(i))} \right] > R_{tol}$$  \[6.2.1.3\]

where $R_{tol}$ is a predetermined threshold. Note that the number of false nearest neighbours depends on $R_{tol}$. The sensitivity of the criterion to $R_{tol}$ is not discussed here. Kennel et al. found that for $R_{tol} = 10$ the false nearest neighbour is clearly identified and we stick by this value below, but for a more profound discussion we suggest to read Ref. [Kennel(1992)].

### 6.2.2. A short note on structures in recurrence plots

The initial purpose of RPs is the visual inspection of higher dimensional phase space trajectories. The view on RPs gives hints about the time evolution of these trajectories. The advantage of RPs is that they can also be applied to rather short non-stationary data.

---

[^389]: We recapitulate certain aspects of RP structure already discussed in chapter 4 because of their relevance to this work.
The RPs exhibit characteristic large scale and small scale patterns. The first patterns were denoted by Eckmann et al. (1987) as typology and the latter as texture. The typology offers a global impression which can be characterized as homogeneous, periodic, drift and disrupted.

The closer inspection of the RPs reveals small scale structures (the texture) which are single dots, diagonal lines as well as vertical and horizontal lines (the combination of vertical and horizontal lines obviously forms rectangular clusters of recurrence points).

Single, isolated recurrence points can occur if states are rare, if they do not persist for any time or if they fluctuate heavily. However, they are not a unique sign of chance or noise (for example in maps).

A diagonal line \( R_{i+k,j+k} = 1 \) (for \( k = 1 \ldots l \), where \( l \) is the length of the diagonal line) occurs when a segment of the trajectory runs parallel to another segment, i.e. the trajectory visits the same region of the phase space at different times. The length of this diagonal line is determined by the duration of such similar local evolution of the trajectory segments. The direction of these diagonal structures can differ. Diagonal lines parallel to the Line of identity (LOI) (angle \( \pi \)) represent the parallel running of trajectories for the same time evolution. The diagonal structures

\[ ^{390} \text{Eckmann et al. (1987), op cit} \]
perpendicular to the LOI represent the parallel running with contrary times (mirrored segments; this is often a hint for an inappropriate embedding). Since the definition of the Lyapunov exponent uses the time of the parallel running of trajectories, the relationship between the diagonal lines and the Lyapunov exponent is obvious.

A vertical (horizontal) line $R_{i+k,j+k} = 1$ (for $k = 1 \ldots v$, where $v$ is the length of the vertical line) marks a time length in which a state does not change or changes very slowly. It seems that the state is trapped for some time. This is a typical behaviour of laminar states (intermittency).

These small scale structures are the base of a quantitative analysis of the RPs. The visual interpretation of RPs requires some experience. The study of RPs from paradigmatic systems gives a good introduction into characteristic typology and texture. However, their quantification offers a more objective way for the investigation of the considered system. With this quantification, the RPs have become more and more popular within a growing group of scientists who use RPs and their quantification techniques for data analysis. A detailed discussion on the application and interpretation of RP and the various structures revealed by RP is found in Marwan(2007).

---

391 In mathematics the Lyapunov exponent of a dynamical system is a quantity that characterizes the rate of separation of infinitesimally close trajectories
392 Marwan, N. Romano, M. C., Thiel, M., Kurths, J. (2007) op cit
6.3. Quantification of recurrence plots (recurrence quantification analysis) with confidence intervals

Definition. The recurrence quantification analysis (RQA) is a method of nonlinear data analysis which quantifies the number and duration of recurrences of a dynamical system presented by its phase space trajectory.

A quantification of recurrence plots was developed by Zbilut and Webber Jr. [Zbilut(1992)] and extended with new measures of complexity by Marwan(2003). Measures which are based on diagonal structures are able to find chaos-order transitions [Trulla(1996)], and measures based on vertical (horizontal) structures are able to find chaos-chaos transitions (laminar phases) [Marwan(2002)].

These measures can be computed in windows along the main diagonal. This allows us to study their time dependence and can be used for the detection of transitions [Trulla(1996)]. Another possibility is to define these measures for each diagonal parallel to the main diagonal separately [Marwan(2002)]. This approach enables the study of time delays, unstable periodic orbits (UPOs; [Lathrop (1989), Gilmore (1998)]), and by applying to cross recurrence plots,

---

393 Zbilut (1992), op cit
394 Marwan, N. 2003, op cit
395 Marwan, Norbert and Kurths , Jurgen (2002), op cit
396 L. L. Trulla, A. Giuliani, J. P. Zbilut, C. L. Webber Jr. (1996), op cit
397 Ibid
the assessment of similarities between processes [Marwan(2002\textsuperscript{400})]. The complexity measures the RQA provides have been useful in describing and analysing a broad range of data. But one key question in empirical research concerns the confidence bounds of measured data. However, Schinkel et al. [Schinkel(2009\textsuperscript{401})] have recently presented one for RQA-measures. We have used the following RQA measures: (a) Determinism (DET) (b) average lengths of the diagonal lines (L) (c) Laminarity (LAM) (d) Trapping Time (TT). For a detailed discussion of these measures we recommend Marwan (2007\textsuperscript{402}), Schinkel(2009\textsuperscript{403}) and Chapter 4.

We have used 95% confidence level for computation of these measures.

6.4. Data, methodology and software

The stock market index data used are the daily closing price of Indian stock market index NIFTY based on stocks traded in National Stock Exchange (NSE), India, (NSE being the trading volume wise largest stock exchange of India), the Hong Kong AOI index based on stocks traded at Hong Kong Stock Exchange and Dow Jones Industrial Average (DJIA) index based on stocks traded at the New York Stock Exchange. The period spanned is from Jan., 1998 to Sep., 2001 for NIFTY (as depicted by the time series graph in Fig. 3(Top)) and August; 1998 to Sep., 2001 for Hong Kong AOI (as depicted by the time series graph in Fig.

\textsuperscript{399} Gilmore, R. (1998), \emph{op cit}
\textsuperscript{400} Ibid
\textsuperscript{401} Schinkel, S., Marwan, N., Dimigen, O., Kurths, J., 2009, \emph{op cit}
\textsuperscript{402} Marwan(2007), \emph{op cit}
\textsuperscript{403} Ibid
4(Top)); June, 2005 and August, 2008 for DJIA (as depicted by the time series graph in Fig. 1(Top)) resulting in a total of 921 daily observations for NIFTY, 766 daily observations for Hong Kong AOI and 815 observations for DJIA. For the exogenous crash we have chosen a period covering the famous 9/11 event in US market. The stock market data used are the daily closing prices of DJIA from the period April 1999 to March 2002 (as depicted by the time series graph in Fig. 2(Top)) consisting of 752 observations. All the time series have been first transformed into the classical momentum divided by the maximum $X_{\text{max}}$ of the time series over the time window which is considered in the specific analysis:

$$X_i = \frac{X_i - X_1}{X_{\text{max}}}$$

RP$s have been plotted for each time series, RQA variables have been computed on different epochs.

For Recurrence Plot a Cross Recurrence Plot Toolbox, Version 5.15 (R28.4) 21-Jul-2009 is used (Copyright (c) 1998-2008 Norbert Marwan, Potsdam University, Germany; http://www.agnld.uni-potsdam.de.) For computation of RQA measures with confidence intervals we have used the software available for download at http://tocsy.agnld.uni-potsdam.de. It is freely available on the internet.
6.5. Analysis of empirical results

6.5.1. Endogenous & exogenous crashes compared

6.5.1.1. Dow Jones industrial average: Endogenous

Fig. 6.1(Bottom) is the RP of Dow Jones Industrial Average between June, 2005 and August, 2008. A look at the index movement in the first observation confirms that it is not homogeneous. It is concurrent with the fact that a financial time series covering a period of three years can hardly display such stationarity. Let us now examine the RP more thoroughly and deliberate on the interpretation of each band. Between co-ordinates 0 and 267 RP appears quite white. However, there is a small dark band between co-ordinates 153 and 250. This may be interpreted as follows: during the first one and half years nothing significant happens except for a small period of boom and bust represented by the dark band. The most interesting spectrum evolves between the ordinates 297 and 690. A dark blue band emerges which encircles a green band. At the centre we have a dark band between the ordinates 419 and 631. This represents the fact that there has been a bullish trend which emerges from March 2007 until the bubble bursts on July 2007 and the bearish trend takes over which continues up to March 2008. In correspondence of the period in which the bubble grows, RQA variables take the highest absolute values and then just before the bubble bursts they drop down (Table 6.1).
Fig. 6.1. Top: Time series graph of DJIA for the period June, 2005 to August, 2008. Bottom: Recurrence plot of Dow Jones industrial average between June, 2005 and August, 2008.

Table 6.1

<table>
<thead>
<tr>
<th>Epoch</th>
<th>Start</th>
<th>DET</th>
<th>L</th>
<th>LM</th>
<th>TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.82503</td>
<td>4.1604</td>
<td>0.86002</td>
<td>4.3957</td>
</tr>
<tr>
<td>2</td>
<td>101</td>
<td>0.88335</td>
<td>4.929</td>
<td>0.91888</td>
<td>5.2042</td>
</tr>
<tr>
<td>3</td>
<td>201</td>
<td>0.86108</td>
<td>4.8623</td>
<td>0.91251</td>
<td>4.9312</td>
</tr>
<tr>
<td>4</td>
<td>301</td>
<td>0.95546</td>
<td>10.855</td>
<td>0.97667</td>
<td>8.9417</td>
</tr>
<tr>
<td>5</td>
<td>401</td>
<td>0.94474</td>
<td>5.9664</td>
<td>0.97131</td>
<td>6.3034</td>
</tr>
<tr>
<td>6</td>
<td>501</td>
<td>0.85775</td>
<td>4.4153</td>
<td>0.89437</td>
<td>5.0149</td>
</tr>
<tr>
<td>7</td>
<td>601</td>
<td>0.92234</td>
<td>6.5682</td>
<td>0.94628</td>
<td>6.331</td>
</tr>
<tr>
<td>8</td>
<td>701</td>
<td>0.91932</td>
<td>9.5165</td>
<td>0.95488</td>
<td>8.1036</td>
</tr>
</tbody>
</table>

Table 6.1 RQA of DJIA (95% CI) for the period June 2005 to August 2008 on the sub series studied of 100 days with a data shift of 100 days.

6.5.1.2. Dow Jones industrial average: Exogenous

Fig. 6. 2(Bottom) is the RP of Dow Jones Industrial Average between April 1999 and March 2002. A look at the index movement in the first observation again confirms that it is not homogeneous. It is concurrent with the fact that a financial
time series covering a period of three years can hardly display such stationarity. Let us now examine the RP more thoroughly and deliberate on the interpretation of each band. We see that for most of the period the RP is reasonably random in nature without displaying any kind of pattern or trend or any shift in trend unlike the plot for the endogenous crash. Only a thin dark band appears between the x coordinate 618 and 625 indicating that some extreme event took place during September 10 2001 and the end of September 2001. This corresponds to the actual phenomena of 9/11.

We can also clearly observe the difference in RP of endogenous and exogenous crashes. There is no indication of any dark band appearing before the crash as was evident in the previous case indicating an absence of any bubble formation. We can therefore reasonably conclude that RP helps us in identifying whether a bubble is forming in the market beforehand. The RQA variables computed with 95% confidence level also confirm our findings. There is no well defined change in values that point towards a bubble being built up (Table 6.2).
Fig. 6.2. Top: Time Series Graph of DJIA for the period April 1999 and March 2002 Bottom: Recurrence Plot of Dow Jones Industrial Average between April 1999 and March 2002.

Table 6.2 RQA of DJIA (95% CI) for the period April 1999 to March 2002 on the sub series studied of 100 days with a data shift of 100 days.
6.5.2. Investigation of endogenous crashes

Inspired by our findings in the DJIA endogenous crash, we now examine two other cases in the Indian and Hong Kong stock market to find out whether we can similarly detect the presence of a bubble in those two cases as well.

6.5.2.1. NIFTY

Fig. 6. 3(Bottom) is the RP of NIFTY between January 1998 and September 2001. The first observation is that it is not homogeneous. It is concurrent with the fact that a financial time series covering a period of three years can hardly display such stationarity. Let us now examine the RP more thoroughly and deliberate on the interpretation of each band. Between co-ordinates 0 and 310 RP appears red and yellow. However, there is a small dark band between co-ordinates 75 and 205. This may be interpreted as follows: during the first one and half years nothing significant happens except for a small period of boom and bust represented by the dark band. The most interesting spectrum evolves between the ordinates 300 and 770. A dark blue band emerges which encircles a light green band. At the centre we have a dark band between the ordinates 460 and 540. This represents the fact that there has been a bullish trend which emerges from June 1999 until the bubble bursts on February 2000 and the bearish trend takes over which continues up to March 2001. In correspondence of the period in which the bubble grows, RQA variables take the highest absolute values and drop down just before the bubble bursts.
Considering that each coordinate in RP is linked with the time series, the border line of a blue or green band reveals the time when the data behaviour starts to change. Noting that the dates here above fall in the same time interval as the bubble and the subsequent crash, it can be supposed that the initial bubble time occurs at \( x = 341 \) (Sept., 1998). We can thus deduce that on such a day the evolution of the system changes; i.e. the evolution passes from a normal regime to a critical regime. This is a posteriori estimation of the initial bubble time, but through the analysis of the sub series one can argue that one is able to recognize the beginning of the bubble with some delay before the bubble grows.

In fact, while the RPs of the first (I) and the second (II) sub series do not present any remarkable pattern (however, one may notice a sharp dark band in (I) which represents the local maxima and minima within a short period of time) the fourth and fifth sub series present an interesting pattern: the RP in Fig 6.3(Bottom) shows the characteristic shape typical of the strong trend of a speculative bubble.

The trend starts to be significant in the middle of Oct. 1998. This indicates that the RP has indeed changed when the bubble has started. Even the RQA variables, in Table 1, show the highest values in this period. From the fifth sub series we find the values of RQA variables have gone down, indicating the normal regime of the stock market.
The RQA variables that are computed on different epochs are shown in Table 6.3 for the NIFTY. It can be seen that the lowest absolute values are found in epochs 3 & 4. This substantiates our findings in the RP.

Fig. 6.3. Top: Time Series Graph of NIFTY for the period January, 1998 and September, 2001 Bottom: Recurrence Plot of NIFTY between January, 1998 and September, 2001.
Table 6.3 RQA of NIFTY (95% CI) for the period January 1998 to September 2001 on the sub series studied of 100 days with a data shift of 100 days.

6.5.2.2. Hong Kong AOI

Fig. 6.4 (Bottom) is the RP of Hong Kong AOI between August 1998 and September 2001. A look at the index movement in Fig. 6.4 reveals a lot of local minima and maxima before reaching a peak. It is interesting to note that we have a situation of double peak (double rounded top).

It will be interesting to note what emanates from the corresponding RP. Once again, no homogeneity is displayed confirming the non-stationarity of financial time series. Here we find that between the ordinates 0 and 125 the band is white representing random behaviour. This means no particular trend has emerged during this period except some local maxima and minima. A dark band emanates from ordinate x D 140. The dark band encircles two blue bands (351 to 451) and (456 to 516). This represents the bullish trends during October 99 and March 2000 and the bearish trend from July 2000. Carrying the logic explained in the subsection above, it can be supposed that the initial bubble time occurs at x = 203 (October, 1999). We can thus again deduce that on such a day the evolution
of the system changes; i.e. the evolution passes from a normal regime to a critical regime. As has been explained, this is an a posteriori estimation of the initial bubble time, but through the analysis of the sub series one can argue that one is able to recognize the beginning of the bubble with some delay before the bubble grows. A look at the RQA table also reveals similar patterns as the variables take maximum values during this period only (Table 6.4).

The ROA variables that are computed on different epochs are shown in Table 6.4 for the AOI. We can see that the lowest values are found in the epochs 4 to 5. This is in line with our findings in the RP. Using the software available at http://tocsy.agnld.uni-potsdam.de, we have given the graphical representation of the DET values for Hong Kong AOI and NIFTY data series in Fig. 5. We can clearly observe that the values take a maximum in the epoch representing the time period about 3-4 months prior to bubble formation and drop down just about the time the bubble burst.
Fig. 6.4. Top: Time Series Graph of Hong Kong AOI for the period August 1998 and September 2001. Bottom: Recurrence Plot of Hong Kong AOI between August 1998 and September 2001.

Table 6.4 RQA of AOI (95% CI) for the period August 1998 to September 2001 on the sub series studied of 200 days with a data shift of 100 days.
6.6. RP & RQA as an effective signalling tool

The study establishes that using RP and RQA techniques it is possible to distinguish between endogenous and exogenous crashes. It has also been shown that, with some delay with respect to the beginning but enough time before the crash (3 to 4 months in this particular case), such that a warning could be given, RP and RQA detect a difference in state and recognize the critical regime. This opens up a possibility of using this alternate approach in analysing financial time signals- especially critical regimes like crashes and bubbles. This tool may also be used to test the predictive power of models by comparing the

Fig. 6.5. DET values of AOI and Nifty.
RP of model simulations and actual data. Future studies may be conducted on various models and actual data for various periods, and RP and RQA comparison may be done to select the most effective model. We can see that recurrence analysis captures the entire time evolution of the time series. We can detect whenever there is a change of regime of the time series by the change in colour in the graph and values of the RQA variables. If the underlying nonlinear dynamics of two time series are similar then the Recurrence Plot and the value of RQA variables must move in a similar manner. The results have policy implications for adoption of appropriate measures for the upcoming crashes identified and also for formulating international financial strategies. With the capability of our methods fully established we now proceed with testing the GBM and Tsallis model in the next chapter.