Abstract

Angular differential cross section for formation of H, as a result of electron capture by proton from He\(^+\) (1s) at the center-of-mass energies 4 to 24 KeV, are calculated using distorted wave Coulomb - Born approximation. The total interaction potentials in the prior-interaction channel and in the post-interaction channel are considered. Coulomb boundary conditions are taken care of with proper choice of the perturbation potentials in both the prior and the post channels. A post-prior discrepancy is noted. Existing theoretical results are presented along with the present results.
1. Introduction:

Charge transfer (CT) reaction $H^+ + He^* \rightarrow H^0 + He^{2+}$ is the simplest ion-ion collision process leading to charge transfer and formation of neutral hydrogen. The velocity of the projectiles are taken greater than the Bohr orbital velocity. There are theoretical investigations on the total cross sections for charge transfer for the above process by using various approximations. After many years of investigations, good agreement between theory and experiment was finally achieved with respect to total cross section. In order to deepen the insight into the physics of the one electron charge transfer process one should study the differential cross sections (DCS). Only theoretical calculation on DCS of this process is in the eikonal approximations. However, experiment on the DCS for the charge transfer in $H^+He^+$ collision is meager. We shall use non-relativistic Coulomb gauge and distorted-wave-Coulomb-Born approximation to study the angular differential cross section of the charge transfer process where Coulomb boundary conditions will be taken care of with the proper choice of perturbation potential. Post-prior discrepancy in the cross section is calculated.

We have studied the angular distribution of the reaction products in prior and in post interaction form of potentials at CM energies ranging from 4-14 KeV. From the results we have noticed that, from 0$^\circ$ to 0.1$^\circ$ scattering angle, the contribution to charge exchange scattering is maximum, and the discrepancy between results with post and prior interaction potential is almost nil. We have approached the problem quantum mechanically and used Coulomb-Born approximation [1] and total interaction potential to find out the angular differential cross section for charge exchange scattering.

Previously, Fujiwara [2] in 1976 has dealt with collision between proton and highly ionized atom to calculate total cross section. In fusion plasmas, Co, N, Ar, C, O, Cu, Nb, Mo, Fe are present in various charged states which forms neutralized energetic fuel particles (p, d, t) by charge transfer. These particles formed can escape from magnetic confinement In order to know the reaction rate and the total cross section for the charge transfer, Fujiwara [2] has
applied Coulomb-Born approximation instead of Born approximation. Earlier, Brinkman and Krammers [3] in 1930, Jackson and Schiff [4] in 1953 and Bates and Dalgarino [5] in 1952 have studied charge transfer reaction using Born approximation. Fujiwara has used the mathematical formulation used by Geltman [6] in 1971 i.e. Coulomb projected Born approximation for charge transfer reaction between p⁺-H atom. Fujiwara successfully has shown the failure of Born approximation over the wide range of energy. In energy range less than 1 MeV, Coulomb repulsion sufficiently affects collision and charge transfer processes unlike those in high energy range. But as the Coulomb-Born (CB) approximation takes care of this repulsion effect, it yields better results for cross sections both in high and low energies of projectile. Mukherjee et al [7] in 1979 have considered light ionized atoms (He and Li) colliding with proton to calculate the total charge transfer cross section using same CB-approximation as used by Fujiwara [2]. Mukherjee et al [7] calculated the total cross sections using only the post interaction form of potential. Mitchell et al [8] in 1977 also gave some results for the combined process of the ionization in p⁺-He⁺ collisions. But in their work, mainly, dominance of ionization was shown in the particular energy range. Capture of electron in that process was not highlighted. Angel et al [9] in 1978 also determined cross section for charge transfer into all final bound states of hydrogen for the same process, in CM-energy range of 60-180 KeV. They have compared results given by Olson et al [10] who used classical trajectory Monte Carlo method in 1977 to obtain cross section. All the process mentioned above are used to calculate integral cross section for charge transfer.

It is to be mentioned that we have also used Coulomb-Born approximation [11] in the light of Fujiwara’s [2] work. In the present work we have determined angular distribution in charge transfer scattering using post as well as prior forms of the interaction potentials. Coulomb boundary conditions [1] are achieved, which are necessary for asymptotic vanishing of the potential. A post-prior discrepancy, however small, is noted in the present case. Only other theoretical work on the angular differential cross section till the publication of our paper [11] in
Winter has done a theoretical calculation to study of $p^+ - He^+$ collision reaction at CM energies 4-14 KeV. The $p^+ - He^+$ process was difficult to treat theoretically, which was specially found by Winter [12], Fritsch & G.D Lin [13] in 1982, Bransden [14] in 1992, Noble [15] in 1980. They found that, unlike for $H^+_2 - H$ collisions, a purely bound state basis of atomic and molecular states is generally inadequate in higher intermediate energies and continuum intermediate states are needed to obtain an accurate integrated cross section. When these states were included, the cross section do in fact agree among themselves and with the experimental results for $He^{2+} - H$ collision. The angular differential cross sections for collision processes $p^+ - H^+$ and $He^{2+} - H$ provide more exacting test of theory. Winter [12] approached the two collision reactions using (i) triple center basis of eikonal process, (ii) double center basis of Sturmian-Pseudo states. Winter studied angular distribution of cross sections where one can see a disagreement between two results coming out of these two processes. We have compared the angular differential cross sections calculated by us using Coulomb-Born approximation with those by Winter using eikonal approximation. In absence of any other experimental results we found present results for the reaction

$$p^+ + He^*(I_s) \rightarrow H^*(I_s) + He^{2+}$$

follow similar trend as those of the experimental results by the Geissen group [15] in Germany for the reaction

$$He^* + He^* \rightarrow He^e + He^{2+}.$$
2. Mathematical Formulation

The charge transfer reaction under consideration is

\[ A^Z_A + B^{(Z_B-1)}_B \rightarrow A^{(Z_A-1)}_A + B^Z_B \]  (2.1)

\( A^Z_A \) is bare projectile ion, \( B^{(Z_B-1)}_B \) is target ion. Co-ordinates in laboratory system are shown in fig.1.\( R_A, R_B \) and \( r \) are relative coordinates of \( A^Z_A, B^{(Z_B-1)}_B \) and electron \( e \) respectively. \( C_A, C_B \) are center of mass of \( (A^Z_A, e^-) \) system and \( (B^{(Z_B-1)}_B, e^-) \) system respectively.

Fig 1. Coordinate system: \( A^Z_A, B^{(Z_B-1)}_B \) and \( e^- \) denote the bare ion of nuclear charge \( Z_A \), hydrogen type ion of nuclear charge \( Z_B \) and the electron, respectively. \( C_A \) is the centre of mass of the \( (e^-; A^Z_A) \) system and \( C_B \) is that of the \( (e^-; B^{(Z_B-1)}_B) \) system.

Masses of \( A^Z_A, B^{(Z_B-1)}_B \), and \( e^- \) are \( M_A, M_B, m_e \) respectively. Total P.E. of the system is

\[ U = \frac{Z_A Z_B}{|\vec{R}_A - \vec{R}_B|} - \frac{Z_A}{|\vec{R}_A - \vec{r_e}|} - \frac{Z_B}{|\vec{R}_B - \vec{r_e}|} \]  (2.2)

The operator expressions for kinetic energies of relative motion of the interacting system for the initial and final channel are given below. For initial channel, from fig.1
Consider a \((\text{A}^z_1, (\text{B}^{\text{Z}_1\text{u}^1} + e^-))\) system; the position vectors are \((\vec{r}_s, \vec{p}_i, \vec{R})\), where, 
\(\vec{r}_s\) is relative position vector of electron with respect to \(\text{B}^{\text{Z}_1\text{u}^1}\) ion, \(\vec{p}_i\) is the position vector of \(\text{A}^z_1\) with respect to the (CM) of the subsystem \((\text{B}^{\text{Z}_1\text{u}^1} + e^-)\). 
\(\vec{R}\) is the position vector of the whole system. \(\mu_a, \mu_b, \mu_i\) and \(\mu_f\) are the reduced masses associated with the relative co-ordinates \(\vec{r}_e, \vec{r}_s, \vec{p}_i, \vec{p}_f\) respectively.

For initial channel, expression for kinetic energy of relative motion of the interacting system is

\[
T_i = -\frac{1}{2\mu_a} \nabla^2 r_e - \frac{1}{2\mu_i} \nabla^2 p_i - \frac{1}{2M} \nabla^2 \vec{R} \tag{2.3}
\]

And for final channel the system considered is \((\text{A}^{\text{Z}_1\text{u}^1}, \text{B}^{\text{Z}_1\text{u}^1})\). Hence, K.E. expression will be

\[
T_f = -\frac{1}{2\mu_a} \nabla^2 r_a - \frac{1}{2\mu_f} \nabla^2 p_f - \frac{1}{2M} \nabla^2 \vec{R} \tag{2.4}
\]

Where, \(\vec{r}_a\) is the relative position vector of electron with respect to \(\text{A}^{\text{Z}_1\text{u}^1}\), \(\vec{p}_f\) is the relative position vector of \(\text{B}^{\text{Z}_1\text{u}^1}\) with respect to the CM of the subsystem \((\text{A}^{\text{Z}_1\text{u}^1} + e^-)\).

Reduced masses are, \(\mu_a, \mu_b, \mu_i, \mu_f\) [Appendix:2A] and associated relative coordinates are \(r_e, r_s, p_i\) and \(p_f\) respectively. \(\vec{R}\) is the CM of the whole system.

\[
\vec{p}_i = \vec{R}_A - \frac{M_B \vec{R}_B + m_e \vec{r}_o}{M_B + m_e}, \quad \vec{R} = \frac{M_A \vec{R}_A + M_B \vec{R}_B + m_e \vec{r}_o}{M_A + M_B + m_e}, \quad \vec{r}_0 = \vec{R}_B - \vec{r}_o,
\]

\[
M = M_A + M_B + m_e, \quad C_1 = \frac{m_e}{M_B + m_e}, \quad A_2 = \frac{M_B}{M_B + m_e} \tag{2.5}
\]
In the relative and CM co-ordinates of the initial system, the potential energy (P.E.) of the initial system is (changing Eqn.(2.2) by using Eqn.(2.5))

\[ U_i = \frac{Z_A Z_B}{|\tilde{p}_1 - C_1 \tilde{r}_b|} - \frac{Z_A}{|\tilde{p}_1 + A_2 \tilde{r}_b|} \frac{Z_B}{|\tilde{r}_b|} \]  

(2.6)

Similarly P.E. in the final channel becomes,

\[ U_f = \frac{Z_A Z_B}{|\tilde{p}_f - C_1 \tilde{r}_b|} - \frac{Z_A}{|\tilde{p}_f + B_1 \tilde{r}_a|} \frac{Z_B}{|\tilde{r}_a|} \]  

(2.7)

Where,

\[ \tilde{p}_f = \tilde{r}_x - \frac{M_A \tilde{r}_a + m_e \tilde{r}_e}{M_A + m_e}, \]

\[ \tilde{r}_e = \tilde{R}_A - \tilde{r}_e, \]  

(2.8)

\[ C_1 = \frac{m_e}{M_A + m_e}, B_1 = \frac{M_A}{M_A + m_e} \]

The total Hamiltonian of the system, corresponding to the above two expressions, can be written as follows:

\[ H = T_i + U_i = H_i^f + V_i \]

\[ H = \left\{ \frac{1}{2\mu_a} \frac{v^2}{|\tilde{p}_a|} + \frac{1}{2\mu_i} \frac{v^2}{|\tilde{p}_i|} + \frac{Z_A(Z_B-1)}{|\tilde{p}_i - C_1 \tilde{r}_b|} \right\} + \frac{1}{2M} \frac{v^2}{|\tilde{r}_b|} + \]

\[ \left\{ \frac{Z_A(Z_B-1)}{|\tilde{p}_i - C_1 \tilde{r}_b|} \right\} + Z_A \left\{ \frac{1}{|\tilde{p}_f - C_1 \tilde{r}_b|} \right\} \]  

(2.9)

Or, \[ H = T_f + U_f = H_f^f + V_f \]

Accordingly,
\[ H = \left[ -\frac{1}{\mu_s} \nabla_s^2 - \frac{Z_A}{|r_s|} \right] + \left[ -\frac{1}{\mu_f} \nabla_f^2 + \frac{Z_B(Z_A-1)}{|\rho_f|} \right] - \frac{1}{2M} \nabla_x^2 \right] + \left[ Z_B(Z_A-1) \left\{ \frac{1}{|\rho_f - C_1 \tilde{r}_a|} - \frac{1}{|\rho_f|} \right\} + Z_B \left\{ \frac{1}{|\rho_f - C_1 \tilde{r}_a|} - \frac{1}{|\rho_f + B_1 \tilde{r}_a|} \right\} \right] \] (2.10)

Assuming the projectile as proton, \( Z_A = 1 \), and writing the interaction potentials \( V's \) in prior and in post interaction forms as \( V_i \) and \( V_f \) respectively, we obtain from (2.9) and (2.10)

\[ V_i = (Z_B - 1) \left\{ \frac{1}{|\tilde{\rho}_i - C_1 \tilde{r}_b|} - \frac{1}{|\tilde{\rho}_i|} \right\} + \left\{ \frac{1}{|\tilde{\rho}_i - C_1 \tilde{r}_b|} - \frac{1}{|\tilde{\rho}_i + A_2 \tilde{r}_b|} \right\} \] (2.11)

and

\[ V_f = Z_B \left\{ \frac{1}{|\tilde{\rho}_f - C_1 \tilde{r}_a|} - \frac{1}{|\tilde{\rho}_f + B_1 \tilde{r}_a|} \right\} \] (2.12)

\( V_i \) and \( V_f \) vanish at the asymptotic limit; i.e. at the limit where \( \tilde{\rho}_i, \tilde{\rho}_f \to \infty \), interaction vanishes and eventually satisfy the Coulomb boundary conditions[1]. Using equation (2.5) and (2.8) we write \( \tilde{\rho}_f \& \tilde{r}_a \) in terms of \( \tilde{\rho}_i \& \tilde{r}_b \) and hence we obtain

\[ \tilde{\rho}_f = A_1 \tilde{r}_b - B_1 \tilde{r}_i \]

\[ \tilde{r}_a = \tilde{\rho}_i + A_2 \tilde{r}_b \]

Where,

\[ A_1 = \frac{m_e (M_A + M_B + m_e)}{(M_A + m_e)(M_B + m_e)} \]

Hence, from equation (2.12),

\[ V_f = Z_B \left( \frac{1}{|\tilde{\rho}_i - C_1 \tilde{r}_b|} - \frac{1}{|\tilde{r}_b|} \right) \] (2.14)
Eigenfunctions of the channel Hamiltonian $H_0 \dot{\phi}$ and $H_0 \dot{\phi}$ as given by Fujiwara [2] are respectively

$$\psi_i = \phi_i(\bar{r}_b) \chi_i(\bar{k}_i, \bar{p}_i), \quad (2.15)$$

$$\psi_f = \phi_f(\bar{r}_a) \chi_f(\bar{k}_f, \bar{p}_f) \quad (2.16)$$

Where, $\chi$, the eigen-function for relative motion of the colliding system in the initial channel is expressed by Coulomb wave $F_c(\bar{k}_i, \bar{p}_i)$, while in the final channel $\chi$ is a plane wave. Here, $\bar{k}_i$ and $\bar{k}_f$ are respectively the relative momenta in the initial and final channels, respectively. The transition matrix element $V_{ji}^{\text{prior (post)}}$ of the prior (post) interaction $V_i(V_f)$ between the initial state $\psi_i$ and the final state $\psi_f$ is obtained in the Coulomb-Born approximation as follows. With the help of equations (2.13), (2.15) and (2.16), we get

$$V_{ji}^{\text{prior (post)}} = \int \psi_f(\bar{r}_a) \rho_i d^3 \bar{r}_a d^3 \bar{r}_b$$

$$= \int \phi_f(\bar{r}_a) \chi_f(\bar{k}_f, \bar{p}_f) \rho_i d^3 \bar{r}_a d^3 \bar{r}_b$$

Taylor's expansion (2A) as shown in the appendix is used to rewrite Eqn. (2.11) and (2.12) as follows.

Taking, $h = C_i \bar{r}_b$ and $V_p = i \hat{p}$, $\hat{p}$ being the momentum vector, so, $V_i$ and $V_f$ become

$$V_i = (Z_B - 1) \left( e^{-iC_1 \bar{r}_b \cdot \hat{p}} - 1 \right) \frac{1}{|\bar{p}_f|} + Z_A \left( e^{-iC_1 \bar{r}_b \cdot \hat{p}} - e^{-iA_2 \bar{r}_b \cdot \hat{p}} \right) \frac{1}{|\bar{p}_f|}$$

$$V_f = Z_B \left( e^{-iC_1 \bar{r}_b \cdot \hat{p}} \frac{1}{|\bar{p}_f|} \frac{1}{|\bar{r}_b|} \right)$$

210608
We compute the DCS for charge transfer between proton and $He^+(1s)$ ion to produce hydrogen atom in the ground state. Taking $Z_a=2$, the transition matrix element (2.17) with prior and post interaction potentials become respectively

\[
V_{j\ell}^{(\text{prior})}=2V_{j\ell}(1)-V_{j\ell}(2)-V_{j\ell}(3) 
\]
(2.20)
\[
V_{j\ell}^{(\text{post})}=2V_{j\ell}(1)-2V_{j\ell}(4) 
\]
(2.21)

Where,

\[
V_{j\ell}(n) = \int \phi_f^* (\vec{r}_a) \chi_j^* (\vec{k}_f, \vec{p}_f) \frac{e^{-i\vec{s}_b \cdot \vec{p}_f}}{\vec{p}_f} \phi_i (\vec{r}_b) F_c (\vec{k}_i, \vec{p}_i) d^3 \vec{k}_i d^3 \vec{p}_i d^3 \vec{r}_b 
\]
(2.22)

Such that, for $n=1$, $s=C_1$;
$n=2$, $s=0$ and
$n=3$, $s=-A_2$ (2.23)
and,

\[
V_{j\ell}(4) = \int \phi_f^* (\vec{r}_a) \chi_j^* (\vec{k}_f, \vec{p}_f) \frac{I}{\vec{p}_f} \phi_i (\vec{r}_b) F_c (\vec{k}_i, \vec{p}_i) d^3 \vec{k}_i d^3 \vec{p}_i d^3 \vec{r}_b 
\]
(2.24)

We use transformations (2.13) for $\vec{r}_a$ and $\vec{p}_f$ in computing $V_{j\ell}$. From space translation we obtain for the momentum operator $\hat{\vec{p}}$ in (2.18) and (2.19) the following Eqn.

\[
e^{-i\vec{k} \cdot \vec{x}} \chi_j^* (\vec{k}_f, \vec{p}_f) = e^{-i\vec{k} \cdot \vec{x}} \chi_j^* (\vec{k}_f, \vec{p}_f) 
\]
(2.25)

Where,

\[\vec{\lambda} = s\vec{s}_b\]

After some lengthy calculation, we obtain $V_{j\ell}(n)$ & $V_{j\ell}(4)$ as shown in Appendix (3A) to (39A). Consequently we get the expression for cross section as below.

(a) For $n=1,2,3$; $V_{j\ell}(n)$ is [App.]

\[
V_{j\ell}(n) = G x \int \frac{1}{2\lambda^2(\lambda^2 + q^2)} \left\{ \frac{3}{\lambda^2} \frac{2}{\lambda^2 + q^2} + \frac{1}{\lambda(\lambda^2 + q^2)} \left\{ \frac{1}{\lambda^2} \frac{1}{(\lambda^2 + q^2)} \right\} \right\} 
\]

(b) For $n=4$, $V_{\mu}(4)$ is as follows [App.]

$$V_{\mu}(4) = G x^4 \int \frac{1}{2} x^2 \left( \frac{3}{x^2} + \frac{2}{x^2 + q^2} \right) + \frac{1}{x^2} \left( \frac{1}{x^2 + q^2} \right)$$

(2.26)

Now, we can calculate $V_{\mu}^{(\text{prior})}$ and $V_{\mu}^{(\text{post})}$ and then with the help of it, the charge transfer cross sections for both prior and post interaction potentials as the form stated below:

$$\sigma(E_i) = \int \frac{d^3 k_f}{|k_i|} \frac{d^3 k_f}{|k_i|} \frac{1}{2\pi^2} \delta(E_i - E_f) = \int \frac{1}{2\pi^3} \left|\frac{k_f}{k_i}\right| 4\pi^2 |V_{\mu}|^2 \sin \theta d\theta$$

(2.28)

3. Results and Discussion:

The charge transfer cross-section in the center of mass coordinate becomes

$$\sigma(E_i) = \int \frac{d^3 k_f}{|k_i|} \frac{1}{2\pi^3} \delta(E_i - E_f) = \int \frac{1}{2\pi^3} \left|\frac{k_f}{k_i}\right| 4\pi^2 |V_{\mu}|^2 \sin \theta d\theta$$

We compute the differential cross-section with respect to the CM scattering angle $\theta$ and obtain

$$\frac{d\sigma(\theta)}{d\theta} = \int \frac{d^3 k_f}{|k_i|} \frac{1}{2\pi^3} \delta(E_i - E_f)$$

(2.28)
In the charge transfer reaction (1), the energy and momentum transfers are very small. Eventually, the maximum contribution to the reaction comes from the small range of angles in the forward direction. We computed with both the post and the prior form of the interaction potential. In the prior channel the interacting systems are charged, while in the post channel one of the interacting systems is neutral (H (1s)) while the other one is the fully stripped charged nucleus. With no target recoil in the projectile energy range under consideration, we find from (10A)

\[|\delta_1/\delta_2| << 1\]

And approximate

\[\Psi(\lambda) = 4\pi/\delta_1\]

There is discrepancy, however small, between the post and the prior form of the cross sections. This addresses to the question of non-orthogonality when the initial and the final wave functions (eqs. (15), (16) ) as given in refs. [7] and [2], are used for the present problem. However, the discrepancies being small, the use of the mentioned wave functions in the present calculations is justified. The range of the angles giving maximum contribution is from 0° to 0.1° in both the prior and the post interaction potentials. As the angle increases to 0.5°, the DCS value falls by a factor of 10^{-3} from the value at 0.1°. At 4 keV and 5 keV CM-energies, Winter's results are comparable with those of the present results in the range 0° to 0.1°. Beyond those angles, the present values for the DCS decrease exponentially, while results given by Winter maintain almost steady value with very little variation (fig.2-6). With the increase in the collision energies the position of the peaks shifts towards the forward direction from 0.05° at 4 keV to 0.026° at 24 keV, irrespective of the post and the prior interaction forms. Decrease in the peak values of the \(d\sigma/d\theta\) are also noted with the increase in the CM energy. To see the variation of the cross sections with solid angle we computed the result at 10 keV (Fig. 7). It is surprising to note that the behavior like
rapid drop of the cross sections with solid angle, corroborates with the experimental results given by Giessen group (inset in fig. 7), for a similar type of charge capture reaction [14]

\[ \text{He}^+ + \text{He}^+ \rightarrow \text{He}^+ + \text{He}^2+ \]

4. Conclusion:

In this paper we have used distorted wave Coulomb-Born approximation with total interaction potential in the prior and post channels. The DCS has its maxima close to the forward direction and decreases exponentially with angle. The small discrepancy in the DCS for the post and the prior interaction forms indicates the amount of non-orthogonality present in the wave functions as given in ref. [7] & [1]. Eventually, to eliminate post prior discrepancy, the exact solution of Schrodinger eqn. with the unperturbed Hamiltonians \( H_0^p \) and \( H_0^p \) (eqn. (9), (10)) is required. However, the discrepancy at small angle scattering being small, one may use the wave functions by Fujiwara, without much loss of generality, for calculating the charge transfer reactions. The necessary boundary conditions, for asymptotic vanishing of the long range interaction potentials, are achieved. The differential cross section \( d\sigma/d\Omega \) plotted as a function of the CM scattering angle at 10 keV CM energy, shows similar characteristic variation as obtained in the experiment [14] for \( \text{He}^+ \) capturing electron from \( \text{He}^+ \). This depicts the validity of our theoretical approach in computing angular differential cross section for electron capture in ion-ion collision. However, experimental results on the present problem are necessary before coming to a final conclusion regarding our theoretical approach.
Reduced masses are,
\[
\mu_a = \frac{M_a M_b}{M_a + M_b}, \quad \mu_b = \frac{M_a M_b}{M_a + M_b}, \quad \mu_c = \frac{M_c}{M_a + M_b + M_c}
\]

\[
\mu_d = \frac{M_d}{M_a + M_b + M_c + M_d}
\]

Taking  \( f(\hat{p}_i) = \frac{1}{|\hat{p}_i|} \); \( Taylor's expansion is used to rewrite Eqn. (2.11) and (2.12) as follows.

Taking, \( h = C, \hat{r}_b \) and \( \nabla_\rho = i\hat{p}, \hat{p} \) being the momentum vector,

\[
\begin{align*}
&= f(\hat{p}_i - C'_1 \hat{r}_b) = \frac{1}{|\hat{p}_i - C'_1 \hat{r}_b|} = f(\hat{p}_i) - \frac{h}{\partial \hat{p}_i} f(\hat{p}_i) + \frac{h^2}{2!} \frac{\partial^2}{\partial \hat{p}_i^2} f(\hat{p}_i) - \ldots \\
&= \frac{1}{|\hat{p}_i|} \left( 1 - \frac{h}{\hat{p}_i} + \frac{h^2}{2!} (\hat{p}_i)^2 \right) f(\hat{p}_i) \quad (2A)
\end{align*}
\]

Similarly, other functions can also be transformed.

Wave function \( \Phi_I (\hat{r}_b) = N_2 e^{-\alpha_2 \hat{r}_b} \) is for \( He^+(Is) \) in the initial channel and in the final channel wave function of \( H(Is) \) is,

\[
\Phi_f (\hat{r}_a) = N_1 e^{-\alpha_1 [\hat{p}_i + A_2 \hat{r}_b]}
\]

Where, \( \hat{p}_i + A_2 \hat{r}_b = \hat{r}_a \)
$N_1, N_2$ are normalizing factors for respective wave function. The Coulomb distorted wave function in attractive field of the bound electron is

$$F_c(\vec{k}_i, \vec{\rho}_i) = e^{-\eta / 2} I(1 + i\eta) e^{i\eta} \frac{1}{\sqrt{2\mu E(i)}}$$

Relative motion of the colliding system in the initial channel is expressed by Coulomb wave function. $\eta$ is repulsive Coulomb parameter $(Z_B/k)$ and $F_c$ is confluent hypergeometric function. $\vec{k}_i$ is relative initial momentum of $He^+(1s)$ with respect to $H^+$. $\vec{k}_f$ is final relative momentum of hydrogen atom with respect to $He^+$. Plane wave function for relative motion in final channel is

$$\varphi_f(\vec{k}_f, \vec{\rho}_f) = (2\pi)^{-3/2} e^{-i\vec{k}_f \cdot \vec{\rho}_f}$$

$\vec{\rho}_f = A\vec{r}_b - B\vec{r}_i$

Scattering amplitude of the reaction in Coulomb-Born approximation is calculated using Eqns.(2.25), (3A) and (4A).

(A) The expression for $V_n(n)$ for $n=1,2,3$

$$V_n(n) = N_1 N_2 \int \frac{1}{|\vec{r}|} \exp[-\alpha_1 \vec{r} + A_2 \vec{r}] + i\vec{k}_f \cdot (A_i \vec{r}_s - B_i \vec{r}_i) - isB_i \vec{r}_s \cdot \vec{k}_f$$

Taking

$$I(n) = \int \frac{1}{|\vec{r}|} \exp[-\alpha_1 \vec{r} + A_2 \vec{r}] + i\vec{k}_f \cdot (A_i \vec{r}_s - B_i \vec{r}_i) - isB_i \vec{r}_s \cdot \vec{k}_f$$

$$F_c(\vec{k}_i, \vec{\rho}_i) d^3 \vec{\rho} d^3 \vec{r}_s$$

(6A)
Applying Fourier integral formulae,
\[
e^{-\alpha_1 A_1} = \frac{\alpha_1}{\pi^2} \int \frac{e^{-\frac{\epsilon(A+A_A)}{2}}}{(\alpha_i^2 + k^2)^2} \, d^4k
\]  

The integral to be computed is,
\[
I(n) = \int \int \int \exp(-\alpha_1 |\vec{r}|) \frac{e^{-\alpha_1 |A+A_A|}}{\pi^2} \left( \frac{A_i^2}{\alpha_i^2 + k^2} \right) \exp\left[ i \left( (A_i - B_i)s \right) \cdot \vec{k} - B_i \vec{p} \cdot \vec{k} \right] \, d^4\vec{k}
\]

First let us consider the integration over \( r_b \); since from (2.8) \( B_i \approx I \), the integral over \( r_b \) is
\[
I_{rb} = \int e^{-\alpha_1 |\vec{r}|} e^{i \frac{(A_i - B_i)s}{k^2}} d^4\vec{r}
\]
\[
= \int \exp\left[ -\alpha_1 |\vec{r}| + i \left( A_i \vec{k} + (A_i - B_i) \vec{k}_f \right) \cdot \vec{r} \right] d^4\vec{r}
\]
\[
= 8\pi\alpha_2 \frac{1}{\left[ \alpha_2^2 + |A_2 \vec{k} + (A_1 - B_1) \vec{k}_f|^2 \right]^2}
\]

So, the integration over \( k \) starts here, integral is
\[
I_k = \int e^{i \vec{k} \cdot \vec{r}} \frac{d^4\vec{k}}{\left[ \alpha_i^2 + |A_i \vec{k} + (A_i - B_i) \vec{k}_f|^2 \right]^2}
\]
\[
= \frac{1}{2 \alpha_2} \int \frac{d\alpha}{d\alpha_2} \int e^{i \vec{k} \cdot \vec{r}} \frac{d^4\vec{k}}{\left[ \alpha_i^2 + |A_i \vec{k} + (A_i - B_i) \vec{k}_f|^2 \right]^2}
\]

Hence,
\[
I(n) = \int \frac{8\pi\alpha_2 I_k}{\pi^2} \frac{F_c(\vec{k}, \vec{p}, \vec{p})}{|\vec{p}|} \, d^4\vec{p}
\]

Using Feynman’s identity
\[
\frac{1}{a^2b} = \frac{1}{2} \int \frac{x}{(ax + b(1-x))^3} \, dx
\]
The integral $I_k$ (10A) turns to the form (12A) on considering
\[ a = \alpha^2 + k^2 \]
\[ b = \alpha^2 + |A_2 k + (A_1 - s) k_f|^2 \]
as we get,
\[
I_k = - \frac{1}{2\alpha_1 \alpha_2} \int e^{-\frac{\alpha^2}{2} d^2 k} \int_{0}^{1} \frac{2x}{2} dx \left\{ \frac{\alpha^2}{2} + \left| A_2 k + (A_1 - s) k_f \right|^2 \right\} (1-x) \]
(12A)

\[
= - \frac{1}{2\alpha_1 \alpha_2} \int e^{-\frac{\alpha^2}{2} d^2 k} \int_{0}^{1} \frac{2x}{2} dx \left\{ \frac{\alpha^2}{2} + \left| A_2 k + (A_1 - s) k_f \right|^2 \right\} (1-x)
\]
\[
g = (\alpha^2 + k^2)x + (\alpha^2 + |A_2 k + (A_1 - s) k_f|^2)(1-x)
\]
Which is factorised into
\[
= (\alpha^2 + k^2)x - (\alpha^2 + |A_2 k + (A_1 - s) k_f|^2)x + (\alpha^2 + |A_2 k + (A_1 - s) k_f|^2)
\]
\[
= ((\alpha^2 + k^2) - (\alpha^2 + |A_2 k + (A_1 - s) k_f|^2)x + (\alpha^2 + |A_2 k + (A_1 - s) k_f|^2)
\]
\[
= \tilde{k}^2 \left\{ 1 - A_2^2 \right\} x + A_2^2 \left\{ A_1 (A_1 - s) k_f - 2A_2 (A_1 - s) k_f x \right\}
\]
\[
+ \left\{ \alpha^2 - \alpha^2 \cdot (A_1 - s)^2 k_f^2 \right\} x + (A_1 - s)^2 k_f^2
\]
\[
= \tilde{k}^2 + 2\tilde{t}, \tilde{k} + h_1 \left\{ A_2^2 (1-x) + x \right\}
\]
where,
\[
t_1 = \frac{A_2 (A_1 - s) k_f (1-x)}{A_2^2 (1-x) + x} \]
(13A)
\[
h_1 = \frac{\left\{ \alpha^2 - \alpha^2 \cdot (A_1 - s)^2 k_f^2 \right\} x + \alpha^2 + (A_1 - s)^2 k_f^2}{x + A_2^2 (1-x)}
\]

Taking,
\[
\lambda^2 = -t_1 + h_1
\]
(14A)

We get
\[
g = \left[ \tilde{k} + \tilde{t}_1 \right]^2 + \lambda^2 \left[ \tilde{k} + A_2^2 (1-x) \right]
\]
(15A)

The integral part of eqn. (12A) can be written as
\[
I_k = -\frac{1}{2\alpha_2} \int \frac{x dx}{(x+A_2(1-x))^2} \int \frac{e^{i\vec{p} \cdot \vec{r}} d^3 \vec{k}}{\left(\vec{k} + \vec{i}\right)^2 + \lambda^2 / \lambda^3}
\]

Using the formula

\[
\int \frac{e^{i\vec{p} \cdot \vec{r}} d^3 \vec{r}}{\left(\eta^2 + t^2\right)^2} = -\frac{\pi^2}{4\eta} \frac{d}{d\eta} \left(\frac{e^{-\lambda \eta}}{\eta}\right)
\]  (16A)

The \(k\) dependent integral part becomes

\[
\int \frac{e^{i\vec{p} \cdot \vec{r}} d^3 \vec{k}}{\left(\vec{k} + \vec{i}\right)^2 + \lambda^2 / \lambda^3} = -\frac{\pi^2}{4\lambda} \frac{d}{d\lambda} \left[\frac{e^{-\lambda \eta}}{\lambda}\right]
\]

So (12A) becomes

\[
I_k = -\frac{1}{2\alpha_2} \int \frac{x dx}{(x+A_2(1-x))^2} e^{-i\vec{p} \cdot \vec{r}} \left(-\frac{\pi^2}{4\lambda} \frac{d}{d\lambda} \left[\frac{e^{-\lambda \eta}}{\lambda}\right]\right)
\]  (17A)

Using (10A2) and (17A) in (6A2), the transition matrix element \(V_g(n)\) becomes

\[
V_g(n) = \frac{8\pi N_1 N_2 \alpha_1 \alpha_2}{\pi^2} \left(-\frac{1}{2\alpha_2}\right) \int \frac{x dx}{(x+A_2(1-x))^2} e^{-i\vec{p} \cdot \vec{r}}
\]

\[
\left(-\frac{\pi^2}{4\lambda} \frac{d}{d\lambda} \left[\frac{e^{-\lambda \eta}}{\lambda}\right]\right) \left[\frac{e^{-\lambda \eta_1}}{\lambda_1} \right] e^{-i\vec{p}_1 \cdot \vec{r}_1} e^{-i\lambda_1 \vec{r}_1}
\]

\[
\int F_1(-i\eta, l, -i\vec{k}, \vec{p}_1 + ik_1 \rho_1) e^{-\eta^2 / 2 \Gamma(1+i\eta)} d^3 \rho_1
\]

where, \(\int F_1(-i\eta, l, -i\vec{k}, \vec{p}_1 + ik_1 \rho_1)\) is taken to be equal to unity.

Let, \(G = \frac{8\pi N_1 N_2 \alpha_1 \alpha_2}{\pi^2} e^{-\eta^2 / 2 \Gamma(1+i\eta)}\)
\[
\phi(\lambda) = \frac{1}{\pi} \int_{\bar{\rho}_i} e^{-\frac{i\lambda}{\rho_i}} e^{-i(\pi_i, \xi, \phi, \xi_1, \phi_1)} F_i (-i\eta, i\xi, -i\bar{k}, \bar{\rho} + \mathbf{i}k, \rho_i) d^3 \rho_i
\]

Let,
\[
\begin{align*}
q_e &= t_1 + B_1 k_f \\
q &= t_2 - B_1 k_f - k
\end{align*}
\]
\[
\delta_i = \vec{k}^2 + \bar{q}^2
\]
\[
\delta_2 = 2(\bar{q} \cdot \vec{k} - \bar{k}^2 - i2k_1)
\]

(18A)

Using Nordseick integral [15]
\[
\phi(\lambda) = \frac{4\pi}{\delta_i} \left(1 + \frac{\delta_2}{\delta_1}\right)^n
\]

Since, \(\frac{\delta_2}{\delta_1} \ll 1\), is very small, higher terms of the above expansion can be dropped

\[
\phi(\lambda) = \frac{4\pi}{\delta_i}
\]

(19A)

\[
V_p(n) = G \left(- \frac{1}{2\alpha_2} \frac{d}{d\alpha_2} \right) \left[ x dx \left( x + A_x^2 (1-x) \right) \left( \frac{\pi^2}{4\lambda} \right) d\lambda \left( \frac{\phi(\lambda)}{\lambda} \right) \right]
\]

Since,
\[
\frac{d}{d\lambda} \left[ \frac{\phi(\lambda)}{\lambda} \right] = 4\pi \left[ \frac{d}{d\lambda} \left( \frac{1}{\lambda(\lambda^2 + q^2)} \right) \right]
\]
\[
= -\frac{4\pi}{\lambda^2 (\lambda^2 + q^2)} \left[ 1 + \frac{2\lambda^2}{(\lambda^2 + q^2)} \right]
\]

(20A)

And from (2.5) \(A_2 \approx 1\), and hence (19A) becomes

\[
V_p(n) = \left(- \frac{1}{2\alpha_2} \frac{d}{d\alpha_2} \right) \left[ x dx \left( \frac{\pi^2}{4\lambda} \right) \left( -\frac{4\pi}{\lambda^2} \right) \left( \frac{1}{\lambda(\lambda^2 + q^2)} \right) \left[ 1 + \frac{2\lambda^2}{(\lambda^2 + q^2)} \right] \right]
\]

(21A)

\[
= \left(- \frac{1}{2\alpha_2} \frac{d}{d\alpha_2} \right) \left[ x dx \left( \frac{\pi^2}{4\lambda} \right) \left( \frac{1}{\lambda^2} \right) \left[ \frac{2\lambda^2}{(\lambda^2 + q^2)} \right] \right]
\]

(22A)
Let, \( \lambda^2 = -L \lambda^2 + Mx + N \)
\[ \lambda^2 + q^2 = Bx + C \] (23A)

Where, using eqns. (13A) in (14A) and equating with the coefficients of eqn. (23A), we get, considering \( B_j \ll 1 \), for small momentum transfer

\[ L = (A_j - s)k_j^2 \]
\[ M = (A_j - s)k_j^2 + \alpha_j - \alpha_j^2 \]
\[ N = \alpha_j^2 \] (24A)

Again, using eqns. (13A) & (14A) in (18A) and then with the help of (24A) and equating the coefficients with those of (23A), we get

\[ B = \alpha_j^2 - \alpha_j^2 - L - 2(A_j - s)k_j, (\bar{k}_j - \bar{k}_j) \]
\[ C = \alpha_j^2 + L + 2i_1, (\bar{k}_j - \bar{k}_j) + (\bar{k}_j - \bar{k}_j) \] (25A)

Writing,

\[ \frac{d}{d\alpha_2} \left[ \frac{1}{\lambda^2 (\lambda^2 + q^2)} + \frac{2}{\lambda (\lambda^2 + q^2)} \right] \]
\[ = -\frac{3}{\lambda^2 (\lambda^2 + q^2)} \frac{d\lambda}{d\alpha_2} + \frac{3}{\lambda^2 (\lambda^2 + q^2)} \frac{d}{d\alpha_2} \left( \frac{1}{\lambda^2 + q^2} \right) \frac{d\lambda}{d\alpha_2} - \frac{2}{\lambda^2 (\lambda^2 + q^2)} \frac{d\lambda}{d\alpha_2} \]
\[ = \frac{4}{\lambda (\lambda^2 + q^2)} \frac{d}{d\alpha_2} \left( \lambda^2 + q^2 \right) \]

From eqn. No. (24A) & (23A), we get

\[ \lambda^2 = -L' x^2 + (L' + \alpha_j^2 - \alpha_j^2) x + \alpha_j^2 \]
\[ M = L' + \alpha_j^2 - \alpha_j^2 \]
\[ N = \alpha_j^2 \]

So, we get

\[ \frac{d\lambda}{d\alpha_2} = \frac{\alpha_j}{\lambda} (1 - x) \]
\[ \frac{d}{d\alpha_2} \left( \lambda^2 + q^2 \right) = 2\alpha_j (1 - x); \]

where,
The transition matrix element (22A) becomes

\[
V_{\mu}(n) = G^{1/4} \int_0^1 2x(1-x) \left[ \frac{1}{2x'(x^2 + q^2)} \left( \frac{3}{x^2 + x^2 + q^2} + \frac{1}{Ax(x^2 + q^2)} \right) \right]
\]

(Suitable program is made to solve the above integration and to get the value of \( V_{\mu}(n) \), where \( n=1,2,3 \).

(B) For \( n=4 \) calculation of \( V_{\mu}(4) \) is as follows: where \( s=0 \):

\[
V_{\mu}(4) = \left( (2\pi)^{3/2} N_1 N_2 \right) \int e^{i\phi} \left( e^{-i|\phi + h_\lambda|} e^{i\phi} \right) \frac{1}{|s|} F_{e}(\bar{k}, \bar{r}) d^3 r d^3 \rho
\]

Using integral formula (7A)

\[
e^{-\alpha|\phi + h_\lambda|} = \frac{\alpha_j}{\pi^2} \int \frac{e^{i\phi(h + h_\lambda)}}{\alpha_j^2 + k^2} d^3 k
\]

The integral part of eqn. (27A) is

\[
I_4 = \frac{\alpha_j}{\pi^2} \int \frac{e^{i\phi(h + h_\lambda)}}{\alpha_j^2 + k^2} d^3 k
\]

The integration over \( r_b \) is given below:
\[ I_\omega = \int \frac{e^{-\omega_0 \lambda} e^{\mu \hat{\mathbf{k}} \cdot \hat{\mathbf{r}}}}{\hat{\mathbf{r}}_0} \, d^3 \hat{\mathbf{r}}_0 \]

\[ = \int e^{-\omega_0 \lambda} \frac{1}{|\hat{\mathbf{r}}_0|} e^{i(\hat{\mathbf{a}} \cdot \hat{\mathbf{r}}_0 + \hat{\mathbf{b}} \cdot \hat{\mathbf{r}}_0) \lambda} \, d^3 \hat{\mathbf{r}}_0 \]

Using the integral formula

\[ \int e^{-e^{\hat{\mathbf{r}}^2}} \, d^3 \hat{\mathbf{r}} = \frac{4\pi}{(\alpha^2 + k^2)} \]

\[ I_\omega = \frac{4\pi}{\alpha^2 + |A_2 \hat{k} + A_1 \hat{k}_f|^2} \]

So,

\[ I_\omega = \frac{4\pi}{\alpha^2 + |A_2 \hat{k} + A_1 \hat{k}_f|^2} \int e^{-i(\hat{\mathbf{a}} \cdot \hat{\mathbf{r}}_0 + \hat{\mathbf{b}} \cdot \hat{\mathbf{r}}_0) \lambda} \frac{e^{-i(\hat{\mathbf{a}} \cdot \hat{\mathbf{r}}_0 + \hat{\mathbf{b}} \cdot \hat{\mathbf{r}}_0) \lambda}}{(\alpha_1^2 + k^2)(\alpha_2^2 + |A_2 \hat{k} + A_1 \hat{k}_f|^2)} \]

\[ e^{-\nu \gamma^2} \Gamma(1 + i\eta), F_1(-i\eta, 1, i\hat{k} \cdot \hat{\mathbf{r}}_0 + k, \rho\cdot \rho) \, d^3 \hat{\mathbf{r}}_0 \rho\cdot \rho \] (30A)

Using Feynman's identity

\[ \frac{1}{a^2b} = \int \frac{2xdx}{(ax + b(1-x))^2} \]

and taking,

\[ a = \alpha_1^2 + k^2 \]

\[ b = |A_2 \hat{k} + A_1 \hat{k}_f|^2 \]

\[ I_\omega = \frac{4\pi}{\alpha^2 \int \frac{2xdx}{(\alpha_1^2 + k^2)(\alpha_2^2 + |A_2 \hat{k} + A_1 \hat{k}_f|^2)} } \]

\[ e^{-\nu \gamma^2} \Gamma(1 + i\eta), F_1(-i\eta, 1, i\hat{k} \cdot \hat{\mathbf{r}}_0 + k, \rho\cdot \rho) \, d^3 \hat{\mathbf{r}}_0 \rho\cdot \rho \] (31A)

Let,

\[ g = (\alpha_1^2 + k^2)x + (\alpha_2^2 + |A_2 \hat{k} + A_1 \hat{k}_f|^2)(1-x) \]

which is factorised as
\[ g = k^2 + 2\tilde{k} \cdot \tilde{i} + h_j = \frac{k^2}{2} + \left( \frac{k+i}{2} \right)^2 - \tilde{i}_j^2 + h_j = \left( k + i \right)^2 + \tilde{\lambda}^2 \]

where:
\[ \tilde{\lambda}^2 = -\tilde{i}_j^2 + h_j \]
\[ \tilde{i}_j = \frac{2kA_1A_2k_f(1-x)}{x + A_2^2(1-x)} \]
\[ h_j = \frac{\left( \alpha_1^2 - \alpha_2^2 - A_1^2k_f^2 \right)x + \alpha_1^2 + A_1^2k_f^2 }{x + A_2^2(1-x)} \; \text{for} \; s = 0 \]

(32A)

\[ G^s = 2^2 \pi N \frac{\alpha_1}{\pi^2} e^{-\eta^2/2} F(1+i\eta) \], taking \( F \) as unity.

\[ I_s = G^s \int \frac{xdx}{x + A_2^2(1-x)^\frac{1}{2}} \left[ e^{\frac{i(k+i)}{2}} - e^{-\frac{i(k+i)}{2}} \right] d^3 k d^3 \rho_i \]

Using the integration
\[ \int \frac{e^{s^2 d^4 t}}{\left( \eta^2 + t^2 \right)^\frac{3}{2}} = \frac{\pi^2}{4\eta} d\eta \left( \frac{e^{-\eta}}{\eta} \right) \]

and considering \( A_2 = 1 \) eqn. (31A) becomes

\[ I_s = \int xdx \left( -\frac{x^2}{4x} \right) \left[ d^3 \rho_i \frac{d}{d\lambda} \left( \frac{e^{-2\lambda}}{\lambda} \right) e^{4(k+i-\delta_k)} \right] d^3 k \]
\[ \phi(x') = \int d^3 \rho_i \frac{d}{d\lambda} \left( \frac{e^{-2\lambda}}{\lambda} \right) e^{4(k+i-\delta_k)} \; d^3 k \]
\[ \phi(x') = \frac{4\pi}{\delta_1} (1 + \frac{\delta_2}{\delta_1})^n \]
where,
\[
\delta_2 \ll 1; \quad \phi(\lambda') = \frac{4\pi}{\lambda'^2 + q'^2}
\]

Using, eqns. (34A), (33A) in (27A) we can get
\[
\therefore V_p(4) = G'(-\pi^2/4\lambda') \frac{d}{d\lambda'} \left( \frac{\phi(\lambda')}{\lambda'} \right)^2 \int x dx
\]

(34A)

Like prior interaction potential calculation, \( \frac{d}{d\lambda'} \left( \frac{\phi(\lambda')}{\lambda'} \right) \) is same. Where
\[
\lambda^2 = L'x'^2 + M'x + N'
\]
(35A)
\[
\lambda^2 = q'^2
\]
(36A)

With the help of eqns. (32A) & (35A) we get
\[
L' = A_1^1 k_f^2
\]
\[
M' = A_1^1 k_f^2 + \alpha_1 \alpha_2^2
\]
\[
N' = \alpha_2^2
\]
(37A)

using eqns. (18A), (32A) & (37A) in (36A), we can have
\[
B' = \alpha_1^2 - \alpha_2^2 - L'' - 2 A_1 \tilde{k}_f (\tilde{k}_f - \tilde{k}_i)
\]
\[
C' = \alpha_2^2 + L'' + 2 \tilde{k}_i (\tilde{k}_f - \tilde{k}_i) + (\tilde{k}_f - \tilde{k}_i)^2
\]
(38A)

Again, with the help of (37A)
\[
\lambda'^2 = -L'' x'^2 + (L'' + \alpha_1^2 - \alpha_2^2) x + \alpha_2^2
\]
\[
M' = L'' + \alpha_1^2 - \alpha_2^2;
\]
\[
N' = \alpha_2^2
\]

So, we get
\[
\frac{d\lambda'}{d\alpha_2} = \frac{\alpha_2}{\lambda'} (1 - x)
\]
\[
\frac{d}{d\alpha_2} (\lambda'^2 + q'^2) = 2 \alpha_2 (1 - x);
\]

Where,
\[
B' x + C' = \lambda'^2 + q'^2 = (\alpha_1^2 - \alpha_2^2 - L'') x + (\alpha_2^2 + L'')
\]

36
Now, (34A) becomes

\[ V_{\mu}(4) = G' x^2 \int_0^1 2x(1-x) \left[ \frac{3}{2\lambda^2 (\lambda^2 + q'^2)} + \frac{2}{\lambda (\lambda^2 + q'^2)} \right] \left[ \frac{1}{\lambda^2 + q^2} \right] \left[ \frac{1}{\lambda^2 + q^2} \right] \]

(39A)

Now, we can calculate \( V_{\mu(\text{prior})} \) and \( V_{\mu(\text{post})} \) and then with the help of it, the charge transfer cross sections for both prior and post interaction potentials as the form stated below:

The cross section of charge transfer phenomena is

\[ \sigma(E_i) = \int \frac{d^4 k_f}{2\pi^2} 2\pi \delta(E_i - E_f) = \int \frac{\nu_p^2 \mu_p^2}{(2\pi)^3} d^3 k \left| \frac{k_f}{k_i} \right| V_{\mu}^2 \sin \theta d\theta \]

(40A)
6. References:

Fig. 2. Angular Differential cross-section for charge transfer at 4 keV CM-energy for the reaction: \( \text{H}^+ + \text{He}^+(1s) \rightarrow \text{H}(1s) + \text{He}^2+ \). Present paper in: (i) prior interaction potential \( \ldots \ldots \), (ii) post interaction potential \( \ldots \ldots \). Theoretical results by Winter [1] in Eikonal approximations using triple-centre, atomic state approach \( \ldots \ldots \).
Fig. 3. Angular Differential cross-section for charge transfer at 5 keV CM-energy for the same reaction. Present paper: same as in Fig. 2. Theoretical results by Winter [1] in Eikonal approximations using: (i) triple-centre, atomic state approach ——, (ii) double-centre, Sturmian pseudostate approach ——.
Fig. 4. The same as in Fig. 3 for CM collision energy 12.5 keV.
Fig. 5  The same as in Fig. 2 for CM collision energy 14 keV.
Fig. 6. The same as in Fig. 2 for CM collision energy 24 keV.
Fig. 7. The present results for $d\sigma/d\Omega$ as a function of CM-scattering angle at CM-energy 10 keV. Inset in the figure shows the experimental result of Ref. [15] for the reaction: $\text{He}^+ + \text{He}^+ \rightarrow \text{He}^{2+} + \text{He}^0$ at 10 KeV. CM-energy.