CHAPTER 2

MULTI-OBJECTIVE REACTIVE POWER OPTIMIZATION

2.1 INTRODUCTION

In this chapter, a fundamental knowledge of the Multi-Objective Optimization (MOO) problem and the methods to solve are presented. The various objectives and constraints of the Reactive Power Optimization (RPO) problem are discussed. The Multi-Objective Reactive Power Optimization (MORPO) problem is formulated as a constrained, non differential complex problem with competing objectives namely; minimization of the real power loss, minimization of voltage deviation, minimization of the investment cost of the compensating devices and minimization of the voltage stability index (L-index). The MORPO problem is subjected to the equality and inequality constraints. The equality constraints is the load flow equations and the inequality constraints like voltage constraints, generator reactive power constraint, transformer tap setting constraint and line flow limits are considered.

2.2 MULTI-OBJECTIVE OPTIMIZATION

2.2.1 Single Objective Optimization

A Single Objective Optimization (SOO) seeks the best solution of a well defined objective. The SOO problem has the formulation as:

\[ \text{Min/Max } f(x) \]
Subjected to \( g_j(x) \leq 0 \), where \( j = 1, 2, 3, \ldots, L \)

\[ h_k(x) = 0, \quad \text{where} \ k = 1, 2, 3, \ldots, M \]

where \( x \) is the decision vector comprising the decision variables, \( g_j(x) \) is the inequality constraint with index \( j \) and \( h_k(x) \) is the equality constraint with index \( k \). If the problem is convex for a minimization problem or convex for maximization problem there will be a single optimal solution for the problem. If the problem is non-convex or non-concave the problem will have more than one global optimal solution. The SO problem provides a powerful tool to explore trade off space of a given objective. But most of the practical problems are multi-objective.

2.2.2 Multi-Objective Optimization Problem

MOO problem has \( N \) number of conflicting and competing objectives functions which has to be solved simultaneously. The MOO problem with \( N \) number of objective function is formulated as:

\[
\text{Minimize } \hat{y} = \tilde{F}(\bar{x}) = [f_1(\bar{x}), f_2(\bar{x}), f_3(\bar{x}), \ldots, f_N(\bar{x})]^T
\]

Subject to constraints, \( g_j(\bar{x}) \leq 0, j = 1, 2, 3, \ldots, M \)

where, \( \bar{x} = [x_1, x_2, x_3, \ldots, x_p]^T \in Q \), \( \hat{y} \) is the objective vector and \( \bar{x} \) is a \( P \)-dimensional vector which represents the decision variables within the search space \( \Omega \). The space spanned by the objective vector is called as the objective space and the subspace of the objective space which satisfy the constraints is called as the feasible space.

The optimal solutions of all the objectives are called as the utopian solution. The utopian solution normally does not exist as the individual objective functions are conflicting. So in a MO optimization problem there is
a possibility of uncountable set of solutions called as the non-dominated solutions. In a non-dominated solution a solution cannot be improved by degrading at least a single objective. The non-dominated solution represents different compromise or trade-offs between the objectives.

### 2.2.3 Concepts of Pareto Optimality

The concept of Pareto optimality or Pareto dominance is used to compare the solution of a candidate. A solution belongs to a Pareto set if there exists no other solution that can improve at least one objective without degrading any other objective. In a minimization problem a decision vector \( \vec{i} \) is said to pre-dominate the decision vector \( \vec{j} \) if and only if:

\[
\forall i \in \{1,2,3 \ldots N\}, f_i(\vec{i}) \leq f_i(\vec{j}) \quad \text{and} \quad \exists j \in \{1,2,3 \ldots N\}: f_j(\vec{i}) < f_j(\vec{j}).
\]

A solution \( \vec{a} \) is said to be pareto-optimal if there exists another solution that dominates it. The corresponding objective vector \( \vec{F}(\vec{a}) \) of \( \vec{a} \) is called as the

![Figure 2.1 Pareto front for a bi-objective optimization](image)

Pareto dominant vector or non-dominated vector. The set of all the Pareto optimal solutions are called as the Pareto optimal set and the
corresponding objective vector is the Pareto front. The Pareto front of a bi-objective is illustrated in Figure 2.1.

2.2.4 Methods to Solve MOO Problem

The MOO problems are solved by classical methods by initially converting the MOO problem into a Single Objective Optimization (SOO) problem and then a traditional technique is used. The method is ideal for the cases where the preferential information about the objectives is known in advance. Another approach to solve the MOO problem is by determining the Pareto frontier by optimizing all the objectives separately.

2.2.4.1 Traditional methods

In weighted aggregation method the MOO problem is converted into an SOO problem by using a function operator to the objective vector. The conversion is by the linear combination of all the objectives. The major disadvantage in this method is choosing the weights which cannot be done without prior information. The method is improved by dynamic weighted aggregation in which the weights are incrementally changed. For each combination of weights, the problem is solved and compromise solutions are generated. The disadvantage of this method is the inability to yield the non-dominating front and it misses the concave portion of the frontier.

The goal attainment or goal programming is a variant of the weighted aggregation method. The goal attainment method seeks to minimize deviation from the prespecified goals. The drawback of this method is that it requires prior information of the priorities and targets.

The $\varepsilon$-constraint method determines the Pareto optimal solutions by optimizing only one objective while the other objectives are treated as
constraints bounded by some allowable range $\varepsilon_i$. The problem is repeatedly solved for different values of $\varepsilon_i$ to generate the Pareto set. The drawback of this method is that the solutions obtained are not globally non-dominated.

In lexicographic method the objectives are arranged in order of importance and the objectives are solved one by one.

The other traditional methods like the weighted min-max method, exponential weightage criterion method, weighted product method, Bounded objective function method, the MOO problem is converted into a SOO problem.

### 2.2.4.2 Intelligent techniques

Intelligent Techniques have the potential to generate multiple solutions in a single run. In an intelligent technique, a set of solution candidates is maintained, which undergoes a selection process and is manipulated using a genetic operator. When an intelligent technique is applied to an MOO problem fitness assignment and selection should be accomplished properly and the diverse population should be maintained. Fitness assignment and selection is accomplished properly inorder to guide the search towards the Pareto-optimal set. Fitness assignment and selection is performed by switching objectives or aggregation selection with parameter variation or Pareto based selection. The diverse population is maintained inorder to prevent premature convergence and to achieve a well distributed non-dominated set. The diverse population is maintained by any one means, namely, fitness sharing, restricted mating, isolation by distance, over-specification, re-initialization and crowding.

The procedure for solving the MOO problem is illustrated in figure 2.2. The procedure for solving MOO using the intelligent techniques can be
classified into Pareto based and non-Pareto based methods. Vector Evaluated Genetic Algorithm (VEGA) is one of the non Pareto based algorithm which varies only in the selection step. In the selection step of VEGA, the population is divided into as many as equal size subgroups as there are objectives and the fittest individual for each objective are selected.

![Figure 2.2 Generation of Pareto front using population based techniques](image)

The Pareto based intelligent techniques uses Pareto ranking to determine the probability of replication of an individual. A set of non-dominated individual are identified and are assigned with ranks and
eliminated from further contention. The process is repeated with the remaining individuals until the entire population is ranked and assigned a fitness value. The various Pareto based techniques are the MOGA (Multiobjective Genetic algorithm), NSGA (Non-dominated Sorting Genetic Algorithm), NPGA (Niched Pareto Genetic Algorithm), SPEA (Strength Pareto Genetic Algorithm).

2.3 REACTIVE POWER OPTIMIZATION PROBLEM

RPO is a single objective problem whose objective is generally minimization of real power loss. The RPO deals with the minimization of the real power loss and thereby improves the system voltage profile, system power transfer capability and overall system operation. The RPO is subjected to equality and inequality constraints. The equality constraint is the power balance equations and the inequality constraints are the operational constraints. The operational constraints are the voltage magnitude, real and reactive power generations, transformer tap settings and maximum VAR injection.

The major objective of the RPO problem is the minimization of the real power loss in the system given by,

$$ P_L = \sum_{i=1}^{NL} Loss_i $$  \hspace{1cm} (2.1)

The RPO problem is subjected to the power flow equations,

$$ P_{Gi} - P_{Di} = V_i \sum_{j=1}^{N_B} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \hspace{1cm} i = 1,2,...N_B $$ \hspace{1cm} (2.2)

$$ Q_{Gi} - Q_{Di} = V_i \sum_{j=1}^{N_B} V_j (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \hspace{1cm} i = 1,2,3,...N_B $$ \hspace{1cm} (2.3)
The RPO problem is subjected to the inequality constraints,

(i) Voltage Constraints

\[ V_i^{\min} \leq V_i \leq V_i^{\max} \quad i \in N_B \]  \hspace{1cm} (2.4)

(ii) Generator reactive power generation limit

\[ Q_i^{\min} \leq Q_i \leq Q_i^{\max} \quad i \in N_G \]  \hspace{1cm} (2.5)

(iii) Transformer tap setting limit

\[ T_i^{\min} \leq T_i \leq T_i^{\max} \quad m \in N_T \]  \hspace{1cm} (2.6)

(iv) Line flow limits

\[ S_l \leq S_l^{\max} \quad l \in NL \]  \hspace{1cm} (2.7)

### 2.4 MULTI-OBJECTIVE REACTIVE POWER OPTIMIZATION

The RPO problem is a synergic problem with multiple objectives, variables and constraints. RPO in traditionally a single objective problem with objective as minimization of real power loss subjected to equality and inequality constraints. The objective of an RPO problem is also formulated based on cost (Minimization of Investment Reactive Power) and deviation in voltage. The equality constraint is the power flow equation and the inequality constraints are the variable generator node voltage, transformer tap ratios and switching number of compensation capacitors. In this section the RPO is formulated as a MORPO with competing objectives namely the minimization of real power loss, voltage deviation, investment cost and the stability index.
2.4.1 Objectives

The objectives considered are the real power loss which influences the operational efficiency, investment on reactive power sources which deals with the minimum investment, voltage deviation and voltage stability index which deals with the system quality and system services.

2.4.1.1 Real power loss

The real power loss of the system influences the operational efficiency of the power system. The objective is to minimize the total real power losses arising from line branches. The total real power loss ($F_i$) is given by,

$$P_L = \sum_{i=1}^{NL} Loss_i$$

(2.8)

2.4.1.2 Voltage deviation

It is related to maintaining the voltage profile of the power system. The bus voltage is one of the most important security and service quality index. Normally the voltage profile will be considered as a constraint in practice. But when the voltage profile is treated as a constraint, after the complete optimization procedure the voltage profile will move towards the maximum limit. So it can be avoided by considering the voltage profile as an objective. The minimum voltage deviation is given by.

$$F_2 = \sum_{e \in E} |V_e - V_e' | / N_{PQ}$$

(2.9)

2.4.1.3 Cost of the compensating devices
The cost of the compensating devices \( F_3 \) is considered as an objective which deals with the installation cost of the compensating equipments either capacitor bank or reactor bank such that the cost is optimal.

\[
\min F_3 = \sum_{i=1}^{NC} k|Q_i| \tag{2.9}
\]

Subjected to \( 0 \leq F_3 \leq F_{3m} \) and \( 0 \leq Q_i \leq Q_m \)

### 2.4.1.4 Voltage stability index

\( L \)-index is one among the different voltage stability index which can also takes into account generator buses reaching reactive power limits.

In a multi-node system, \( I_{bus} = Y_{bus} \times V_{bus} \) \tag{2.10}

When the load buses are segregated from the generator buses, the equation becomes

\[
\begin{bmatrix}
I_L \\
I_G
\end{bmatrix} =
\begin{bmatrix}
Y_i & Y_2 \\
Y_3 & Y_4
\end{bmatrix}
\begin{bmatrix}
V_L \\
V_G
\end{bmatrix} \tag{2.11}
\]

Let \( H_1, H_2, H_3 \) and \( H_4 \) be the sub-matrices generated from the partial inversion of \( Y_{bus} \)

\[
\begin{bmatrix}
V_L \\
I_G
\end{bmatrix} =
\begin{bmatrix}
H_1 & H_2 \\
H_3 & H_4
\end{bmatrix}
\begin{bmatrix}
I_L \\
V_G
\end{bmatrix} \tag{2.12}
\]

\[
H_2 = -Y_j \times Y_2 \tag{2.13}
\]

Let \( \overline{V}_{ak} = \sum_{i=1}^{N_c} H_{2i}.V_i \)
The L-index for bus k, \( L_k = \left| 1 + \frac{V_{ak}}{V_k} \right| \) \hfill (2.14)

The value of \( L_k \) should be less than 1 on a continuous basis for a system to be stable. Hence a global system indicator \( L \) describing the stability of the complete system is \( L = L_{\text{max}} \{ L_k \} \), where \( \{ L_k \} \) all the indexes are listed. The \( L_k \) must be lower than a threshold value. The threshold value will be predetermined based on the system policy and the utility policy. The fourth objective is the minimization of L-index (\( F_4 \)).

\[
\text{Min } F_4 = L \hfill (2.15)
\]

### Constraints

Depending on the network structure and the loading condition, the VAR planning does not affect the real power flow. Also mentioning the real power flow is difficult in VAR planning problem. So in case of RPO problem it is worthwhile to include the reactive power limits, voltage limits and transformer tap setting limits as the constraints. The equality constraints are the power flow equations given in (2.2) and (2.3). The inequality constraints are the voltage constraints, generator reactive power limit, transformer tap setting limit and limit flow limits given in equations (2.4) to (2.7) respectively.

### SUMMARY

A brief introduction to the SOO problem and MOO problem were presented. The various methods to solve the MOO problem is discussed along with their disadvantages. Then the MORPO problem is formulated with four competing objectives namely, minimization of real power loss, minimization of voltage deviation, minimization of the cost of the compensating devices...
and minimization of the L-index. The MORPO is formulated such that it is subjected to equality and inequality constraints. The equality constraints are the power flow equations and the inequality constraints are the voltage magnitude at the buses, generator reactive power limit, transformer tap setting limit and the line flow limit.