CHAPTER 2

DEVELOPMENTS OF FACTS TECHNOLOGY AND COMPUTATIONAL TECHNIQUES IN POWER SYSTEM

2.1 INTRODUCTION

The electrical power system is an interconnection of a large number of generators and consists of (large and small) consumer loads connected with complex transmission network, transformers, protective equipments and ancillary equipments. Greater demand has been placed on the transmission network and these demands will continue to increase the number of utilities. The power system will be forced to operate the existing network even though demands will increase in the level, because of the lack or restriction of expansion planning. However, the power system needs to provide quality and secure power supply to consumers. The FACTS technology is essential to achieve proper operation and overcome the difficulties. FACTS controllers based different technical approaches, resulting in both power stability and good quality by controlling one or more AC transmission system parameters will enhance the controllability and power transfer capability of the network.

The high power electronic based system is equipped with FACTS technology with its real-time operating control and deal with power systems issues like power flow transfer capability, enhancement of continuous control of voltage profile, improving the system stability and minimizing losses. The first generation FACTS devices work like passive elements using impedance or tap changer transformers controlled by thyristors and these devices provide control and fast response. The devices are Static Var Compensator (SVC),
Thyristor Controlled Series Capacitor (TCSC) and TCPAR. The second-generation FACTS devices work like angle and module controlled voltage sources and without inertia based in converters. The Static Synchronous Compensators (STATCOM), Static Synchronous Series Compensators (SSSC), Unified Power Flow Controllers (UPFC) and Interline Power Flow Controllers (IPFC) belong to the second-generation controllers.

The emergence of computational intelligence techniques, inspired by biological and social behavior algorithms, are one of the most exciting and important fields in engineering, which addresses the complex problems of real world applications and especially in multi disciplinary and multi-objective problems. Various algorithms namely ACO, BFA, PSO and improved or modified version of BFA have evolved over years to solve numerous optimization problems of the power system. The purpose of reliability analysis is to determine the point of voltage instability identification of critical lines and the weakest bus in the system.

2.2 SIGNIFICANCE OF VOLTAGE CONTROL AND REACTIVE POWER

The power supply and voltage at the consumer point must be held substantially constant. Maintaining the voltage profile at a desired level is an important task in the power system and voltage variations are mostly dependent on the reactive power demand. Therefore, the voltage level is kept constant by providing the reactive power in the correct sign at the system level or by remote control level. Hence, reactive power control is required to maintain the voltage within the acceptable limits. The supply of reactive power varies with load conditions and voltage variation. In general, terms, decreasing reactive power margin causes voltage fall, while increasing reactive power margin causes voltage rise.
The devices of a more obvious kind for controlling voltage are Tap settings transformer. Compensation methods and introducing the FACTS devices. It is the most effective way for utilities to improve the voltage profile and voltage stability margin.

22.1 Effect of On-Load Tap Changer (OLTC) Transformer on Voltage Stability

The secondary voltage of OLTC transformer is typically maintained at nominal value by the operation of the tap changer, and the tap changer level will be tuned according to the voltage drops in the primary of transmission system. The voltage stability enhancement is based on the operation with respect to the reactive power status. However, if the load demand becomes excessively heavy, the secondary voltage may become unstable even with tap changing, the voltage instability occurs due to reactive power shortage and it affects the performance of OLTC. Therefore, the OLTC operation is not suitable for voltage control under heavy loading conditions, and when the system is undergoing a complex state of affairs. The simplified model of Transmission line with Tap changer is shown in the Figure 2.1.

![Figure 2.1A Simplified Model of Transmission line with Tap changer](image)

22.2 Effect of Compensation Methods to Enhance Voltage Stability

The problem of maintaining the voltage within the allowable limits is complex because the system comprises of many sources and loads. The voltage control is not sufficient to maintain the voltage at any one specific point, it must be maintained at many points of the power system. The voltage
at a bus can be controlled by injection of reactive power. The compensation consists of injecting reactive power to improve power system operation, more specifically to keep voltages close to the nominal values, to reduce the line currents and network losses, which contribute to stability enhancement. Most often, compensation is provided by capacitors and the line reactance compensation.

### 22.2.1 Line Series Compensation

Series compensation is an introduction of power capacitors in series with transmission line in order to decrease the reactance of transmission lines as shown by a simple equivalent circuit in the Figure 2.2. The voltage stability is also improved by injecting the capacitive reactive power directly into the line.

![Series Compensation](image)

**Figure 2.2 Series Compensation**

The line reactance is given by, in the Equation (2.1).

\[
X_{\text{net}} = X + X_C + L \frac{1}{C}
\]  

(2.1)

### 22.2.2 Shunt Compensation

The static shunt capacitors or synchronous compensators are used to supply the reactive power to both the transmission lines and distribution levels, it is probably the simplest, and the most widely used
compensation. The simplified equivalent circuit for shunt compensation is shown in the Figure 2.3.

![Shunt Compensation Circuit](image)

**Figure 2.3 Shunt Compensation**

For a shunt capacitor compensated line, the real and reactive power equations at the receiving end bus are given in the Equations (2.2) and (2.3).

\[
P = \frac{EV}{X} \sin \delta \\
Q = \frac{EV}{X} \cos \delta - \frac{V^2}{X} \frac{V^2}{C} \]

(2.2)

The Jacobian matrix \([J]\) in this case is given by

\[
J = \frac{1}{X} \begin{bmatrix}
EV \cos \delta & E \sin \delta \\
EV \sin \delta & 2V & E \cos \delta & 2VX & C
\end{bmatrix}
\]

(2.3)

### 2.2.3 Introduction of FACTS Devices

Flexible AC Transmission System (FACTS) is defined as an alternating current transmission system incorporating a semiconductor based power electronics and other static controllers to enhance power transfer capacity and increase the controllability of transmission lines. FACTS accommodate changes in the operating conditions, maintaining steady state and transient stability margins with advanced power conversion and switching capabilities.
The static equipments based FACTS devices can improve the power transfer capability and power controllability of the network. The FACTS devices are converter based controllers and it consist of gate turn-off devices, and the principle of voltage source converters (VSI) and current source converters (CSI). In this VSI, unidirectional DC voltage of a DC capacitor is placed in AC side as AC voltage through proper switching technology. It is then possible to vary the AC output voltage in magnitude and in phase compared to the AC system voltage. The voltage source converters are given preference over the current source converters based on the cost factor. The FACTS devices are integrated in the power systems to control power flow, increase the system stability in the network, reduce the losses by providing compensation and reduce the cost of the production.

2.3 FACTS TECHNOLOGY

Advanced control technologies are required to improve the utilization of the existing power system. Because in the mechanical controls are not high-speed control. The Flexible Alternating Current (AC) Transmission Systems provides proven technical solution to address these new operating challenges of the power system. FACTS technologies offer benefits to the transmission system. It is compared to the construction of new transmission lines with minimal infrastructure investment, environmental impact and implementation time.

FACTS technologies provide advanced solutions as cost-effective alternatives to new transmission line erection. The potential benefits of FACTS equipment are now widely recognized by power systems engineering. With respect to FACTS equipment, Voltage Source Converter (VSC) technology, which utilizes self-commutated thyristors/transistors, has been successfully applied in a number of installations worldwide. FACTS
technology is a collection of controllers, which can control series impedance, current voltage and Phase angle. FACTS technology is to lighten difficulties of the power system by enhancing the control of the transmission of power and to enhance the grid reliability with the same existing line.

2.3.1 FACTS Controllers

FACTS controllers may be used on thyristor devices with power devices with gate turn-off capability. Most of the controllers with gate turn-off capability are DC to AC converters, which can exchange active or reactive power with the AC bus at which the FACTS controller is connected. When the task of the FACTS controller is to control the reactive power only, they are provided with a minimum storage at the DC side. However, if the need is to generate AC voltage or current more that 90 phase difference with corresponding quantities of AC system, the storage device needs augmentation using capacitors, batteries and superconducting magnets.

The FACTS controllers offer a great opportunity to regulate the AC transmission, increasing or diminishing the power flow in specific lines and responding almost instantaneously to the stability problems. The potential of this technology is based on the possibility of controlling the route of the power flow, and the ability to connect networks that are not adequately interconnected, thereby giving the possibility of trading energy between distant agents.

2.3.2 Benefits of FACTS Devices to Power Systems

Once, viable solutions/options are identified, the benefits of the added power system control must be determined. The following are some of the benefits,
It is fast and enables regulation under both steady and dynamic conditions.

More effective use of transmission lines are against loading condition and Environmental benefits.

Increases the system security by adding effective Power flow control.

Reduces the system losses and improves the voltage profile.

Reduces the reactive power flows thus allows the lines to carry more active power.

Increases System Security by providing secure tie line connections to neighboring utilities and decreases the overall generation reserve requirements.

Better utilization of the existing transmission system assets and increase the utilization of lowest cost generation and the need for new transmissions lines are eliminated.

Dynamic control of power flow in selected transmission lines within the network to enable optimal power flow conditions.

The advances in this list are important to achieve overall planning and secure operation of the power systems. However, for justifying the performance of the conventional controller, solutions are compared with the FACTS controllers and more specific metrics of the benefits added to the power system. This allows for a direct quantification of the benefits of adding power system control and provides a means to compare such benefits by the various critical conditions by optimal placing of FACTS devices.
2.4 TYPES OF FACTS CONTROLLERS

FACTS controllers can be categorized as Series controllers, Derivation controllers, Serial to Serial controllers, and Serial-Derivation controllers (Abido (2008), Kundur (1994)).

2.4.1 Series Controller

Series controllers are used to inject voltage in series with the line and consist of variable impedance controllers, such as capacitors and reactors. Series voltage injection in the line is adjusted by varying the impedance and current flow, on the condition that the supply or consumption of the reactive power variation depends on the voltage that is in phase quadrature with the line current. The Series controllers are TCSC, SSSC, Thyristor Controlled Series Reactor (TCSR), Thyristor Switched Series Capacitor (TSSC) and Thyristor Switched Series Reactor (TSSR). The Figure 2.4 describes the schematic connection of series FACTS controller.

![Figure 2.4 Series FACTS Controller](image)

2.4.2 Derivation Controllers

Derivation controllers consist of variables impedance, variable source, or a combination of both. Derivation controllers inject current into the system at the point of connection and it is like a current source, which draws from, or injects currents in the bus. The injected current may lead or lag
depending upon the load. This type of controller only supplies or consumes variable reactive power. This device is installed at the desired bus at which controllability needs to be enhanced. Shunt type FACTS devices are very effective in voltage control and damping of voltage oscillations. The SVC, STATCOM, Thyristor Controlled Braking Resistor (TCBR) belongs to shunt controllers or Derivation controllers. The schematic connection diagram of derivation FACTS controller is shown in the Figure 2.5.

![Diagram](image)

**Figure 2.5 Derivation FACTS Controller**

### 2.4.3 Combined Series – Series Controller

Any standard series FACTS controller can be connected with another type of series FACTS controller to form a series-series controller. The schematic connection diagram of the Combined Series – Series FACTS Controller is shown in the Figure 2.6. This type of controllers can be a combination of coordinated serial controllers in a multiline transmission system or it could be a unified controller. The serial controller provides independent series reactive compensation for each line, but also transfers active power among the lines via the power link. The active power transmission capacity of the unified serial controller makes possible the active and reactive power flow balance. The unified arrangement gives the DC terminals of the converters of all the controllers, which are connected to
achieve a transfer of active power between each other. A typical controller is the Interline Power Flow Compensator (IPFC).

![Diagram of Series-Series FACTS Controller]

**Figure 26 Series-Series FACTS Controller**

2.4.4 Combined Serial-Derivation Controllers

This device can be a combination of serial and derivation controllers. The principle of operation of the serial-derivation controllers is to inject current to the system through the component in derivation of the controller. The serial and derivation controllers are unified such that they can exchange active power between them. A typical Serial-derivation controller is called as Unified Power Flow Controller (UPFC).

2.4.5 Modeling of FACTS Devices and Power Injection Model

Modeling of FACTS devices (Abido (2008), Kundur (1994)) is essential due to the power transfer capability of long transmission lines being limited by stability considerations. The performance of transmission lines can be enhanced by this, thereby reducing the effective reactance of lines by using series compensation methods, which is an economical solution but a slow process. The Figure 2.7 shows a schematic representation of the VSC diagram.
The introduction of fast acting FACTS devices allow fast and effective control of series compensation using Thyristor Controlled Series Capacitor (TCSC) and Static Synchronous Series Compensator (SSSC). TCSC is a variable impedance device based on thyristors, while SSSC is based on Voltage Source Converter (VSC). In addition, the VSCs inject variable reactive voltage with self-commutating devices like GTOs. The schematic diagram of VSC based FACTS device is shown in the Figure 2.8.

The static models of these FACTS controllers have been considered for solving the MO-OPD problem. For static applications, FACTS controllers are designed as an injection model and are to inject a certain amount of active and reactive power to a node. Thus, the FACTS device is represented as PQ elements. The advantage of power injection model is that it does not destroy
the symmetrical characteristic of the admittance matrix and allows efficient and convenient integration of FACTS devices into the existing power system.

The incorporation of FACTS devices depend on the line flows and parameters of the particular transmission line. The simple transmission line represented by its lumped parameters and the equivalent parameters connected between bus-i and bus-j is shown in the Figure 2.9. The bus voltages and angles of the buses i and j are given by $V_i$, $\delta_i$ and $V_j$, $\delta_j$ respectively. Under the normal operating conditions, the reactance $X_{ij}$ value is small, so the line voltage $V_i$ and $V_j$ are equal and the control of power (active and reactive power) flow in the line depends on the control of $X_{ij}$ as the resistance value is small compared to the reactance. The reactance of the line can be controlled by FACTS devices such as TCSC and SVC and used to control the reactive power and power flows in the power system network. In general, the real and reactive power flow between the buses i to j are given in the Equations (2.4) and (2.5).

![Figure 2.9 Modeling of Transmission line](image)

\[
\begin{align*}
P_{ij} & = V_i^2 G_{ij} V_i V_j \left[ G_{ij} \cos \delta_{ij} \quad B_{ij} \sin \delta_{ij} \right] \quad (2.4) \\
Q_{ij} & = V_i^2 B_{ij} B_{ij} V_i V_j \left[ G_{ij} \sin \delta_{ij} \quad B_{ij} \cos \delta_{ij} \right] \quad (2.5)
\end{align*}
\]
Where, \( \delta_{ij} \), \( \delta_i \), \( \delta_j \), similarly the real and the reactive power flow between the bus \( j \) to bus \( i \) is given by the Equations (2.6) and (2.7).

\[
P_{ji} = V_i^2 G_{ij} + V_i V_j \left[ G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij} \right] \tag{2.6}
\]

\[
Q_{ij} = V_i^2 B_{ij} + B_{sh} V_i V_j \left[ G_{ij} \sin \delta_{ij} + B_{ij} \cos \delta_{ij} \right] \tag{2.7}
\]

The approximate equation of active power in the line is given by the Equation (2.8).

\[
P = \frac{V_i V_j}{X_{ij}} \sin \delta_{ij} \tag{2.8}
\]

If the TCSC is integrated in the transmission line for OPD problem, the line data is given by the Equation (2.9). The \( X_s \) source reactance and new line reactance \( X_{new} \) is given as follows; line losses, susceptance and conductance values are detailed below in the Equation (2.10).

\[
X_{new} = X_{ij} - X_s \tag{2.9}
\]

Where, \( G_{ij} \), \( R_{ij} / R_{ij}^2 \), \( X_{new} \), \( B_{ij} \), \( X_{new} / R_{ij}^2 \), \( X_{new} \) \( \delta_{ij} \) is the voltage angle difference between bus \( i \) and \( j \).

The active and reactive power loss on each line is formulated as and is given by Equations (2.11)-(2.14).

\[
P_i = P_{ij} + P_{ji} \tag{2.11}
\]

\[
P_i = V_i^2 G_{ij} + V_j^2 G_{ij} + 2V_i V_j G_{ij} \cos \delta_{ij} \tag{2.12}
\]
\[ Q_i \quad Q_{ij} \quad Q_j \]  
\[ Q_i \quad V_i^2 B_{ij} \quad V_j^2 B_{ji} \quad 2V_i V_j B_{ij} \cos \delta_{ij} \]  
(2.14)

The incorporation of FACTS devices into the load flow solution is based on the power-injected model. The power injection model gives the solution of the load flow without any modification of the existing impedance matrix \( Z \). In fact, the injected power model is well situated and as much as necessary for the power system with FACTS devices. The mathematical models of the FACTS devices are developed mainly to perform the steady state research. The power flow equations with FACTS devices are given as below in the Equations (2.15)-(2.18).

\[ P_{Gi} \quad P_{Di} \quad \sum_{j=1}^{n} |V_i| |V_j| \quad G_{ij}^{FACTS} \cos \delta_{ij} \quad B_{ij}^{FACTS} \sin \delta_{ij} \quad 0 \]  
(2.15)

\[ Q_{Gi} \quad Q_{Di} \quad \sum_{j=1}^{n} |V_i| |V_j| \quad G_{ij}^{FACTS} \sin \delta_{ij} \quad B_{ij}^{FACTS} \cos \delta_{ij} \quad 0 \]  
(2.16)

\[ |V_i|_{\text{min}} \quad |V_i| \quad |V_i|_{\text{max}} \]  
(2.17)

\[ \delta_{ij} \quad \delta_{ij}^{\text{max}} \]  
(2.18)

Where,  
\( P_{Gi}, Q_{Gi} \) - Generated real and reactive power at bus \( i \)  
\( P_{Di}, Q_{Di} \) - Real and reactive power of load at bus \( i \)  
\( n \) - Number of buses  
\( G_{ij}^{FACTS} \) - Real part of \((i, j)\)th element of network admittance matrix included FACTS devices  
\( B_{ij}^{FACTS} \) - Imaginary part of \((i, j)\)th element of network admittance matrix included FACTS devices
\[ \delta_{ij} \] - Difference of phase angle between buses i and j

\[ |V_{i_{\text{min}}}|, |V_{i_{\text{max}}}| \] - Maximum and minimum voltage magnitude at bus i

### 2.4.5.1 Power Injection Model of TCSC

The corresponding power injection model of TCSC incorporated within the transmission line is shown in the Figure 2.10. The line admittance is given in the Equation (2.19). The change in flow without series capacitance with additional power injections at the receiving \( (S_{i_r}) \) and sending \( (S_{r_s}) \) ends are shown in the Figure 2.10.

\[ \begin{align*}
\text{Bus}_i & \quad Z_{ij} = r_{ij} + jx_{ij} \quad \text{Bus}_j \\
S_{ic} & \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad S_{jc}
\end{align*} \]

**Figure 2.10 Power Injection model of TCSC**

\[ y_{ij} \quad y_{ij} \quad y_{ij} \quad g_{ij} \quad jb_{ij} \quad g_{ij} \quad jb_{ij} \]  \quad (2.19)

Where, \[ g_{ij} = \frac{r_{ij}}{\sqrt{r_{ij}^2 + x_{ij}^2}}, \quad b_{ij} = \frac{x_{ij}}{\sqrt{r_{ij}^2 + x_{ij}^2}} \]  \quad (2.20)

\[ g_{ij} = \frac{r_{ij}}{\sqrt{r_{ij}^2 + x_{ij}^2 x_{TSCC}^2}}, \quad b_{ij} = \frac{x_{ij}}{\sqrt{r_{ij}^2 + x_{ij}^2}} \]  \quad (2.21)

When TCSC is installed in the line between buses i and j, the new admittance matrix is obtained from Equation (2.22).
\[ Y_{Bus}^* Y_{Bus} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & y_{ij} & 0 & \ldots & 0 & y_{ij} & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 0 & y_{ij} & 0 & \ldots & 0 & y_{ij} & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 & 0 \end{bmatrix} \]  

\hspace{1cm} (2.22)

The capacitive or inductive mode of the TCSC gives a decrease or increase effect of branch impedance (reactance) of line \((X_i)\)

### 2.4.5.2 Power Injection Model of SVC

The main purpose of SVC is to control the voltage at weak points in a network and SVC is installed in the centre of the transmission line. The reactive power of a SVC can be defined as in the Equation (2.23).

\[ Q_{SVC} = \frac{V_i V_i V_r}{X_{S1}} \]  

\hspace{1cm} (2.23)

Where, \(X_{S1}\) is the equivalent slope reactance in p.u and \(V_r\) are reference voltage magnitude.

The total loss of the FACTS device one at a time is written in the Equation (2.24) and (2.25).

\[ P_{i,k}^c P_{i,k} \begin{bmatrix} P_i \text{ com} & P_j \text{ com} \end{bmatrix} \]  

\hspace{1cm} (2.24)

More than one device used at the time can be expressed as,

\[ P_{i,k}^c P_{i,k} \sum_{d=1}^{N_{d}} \begin{bmatrix} P_i \text{ com} & P_j \text{ com} \end{bmatrix} \]  

\hspace{1cm} (2.25)

Where, \(N_{d}\) is the number of devices to be located at various lines.
When the SVC is installed at node \(j\), the admittance matrix can be expressed as in Equation (2.26).

\[
\begin{bmatrix}
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & Y_{SVC} & \ldots & 0 & 0 & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0 \\
0 & 0 & 0 & \ldots & 0 & 0 & 0
\end{bmatrix}
\]  

(2.26)

---

**Figure 2.11** Power Injection model of SVC

### 2.4.5.3 Power Injection Model of TCPAR

The mathematical equations are used to model the power injection model of the TCPAR, the injected real and reactive powers for TCPAR at bus \(i\) and bus \(j\) are given in the Equations (2.27)-(2.30).

\[
P_i = V_i^2 t G_{ij} - V_i V_j t G_{ij} \sin(d_{ij}) - V_i V_j B_{ij} \cos(d_{ij})  
\]

(2.27)

\[
Q_i = V_i^2 t B_{ij} - V_i V_j t G_{ij} \cos(d_{ij}) - V_i V_j B_{ij} \sin(d_{ij})  
\]

(2.28)

\[
P_j = V_i V_j G_{ij} \sin(d_{ij}) - V_i V_j B_{ij} \cos(d_{ij})  
\]

(2.29)

\[
Q_j = V_i V_j G_{ij} \cos(d_{ij}) - V_i V_j B_{ij} \sin(d_{ij})  
\]

(2.30)

Where \(t = \tan(\alpha_{ij})\)
The real and reactive power loss in the line having the TCPPAR can be expressed as in Equations (2.31) and (2.32).

\[
P_l \quad P_{ji} \quad P_{ij}
\]  
\[Q_l \quad Q_{ij} \quad Q_{ji}
\]  
\hspace{2cm} (2.31) \hspace{2cm} (2.32)

Where, \( U = \cos(\alpha_p) \) and \( \delta = \delta_l = \alpha_p \).

These equations will be used to model the TCPPAR in the power flow calculation.

### 2.4.5.4 Power Injection Model of SSSC

The complex power exchanged by the series converter with the line is expressed in (2.33).

\[
S_c = V_x I
\]  
\hspace{2cm} (2.33)

![Mathematical model of SSSC](image)

**Figure 2.12** Mathematical model of SSSC

Where, \( V_x \) is the complex voltage injected by the device and \( I \) is the current through the line.

The active and reactive powers exchanged with the lines are modeled as resistance and reactance associated as in Equation (2.34).

\[
R = \frac{V_x^2}{P_s} \quad \text{and} \quad X = \frac{V_x^2}{Q_s}
\]  
\hspace{2cm} (2.34)
The resistance $R$ and the reactance $X$ are equivalent series combination.

### 2.4.5.5 Power Injection Model of UPFC

A series inserted voltage, phase angle of inserted voltage can model the effect of UPFC on network, and it is connected to the system through two coupling transformers. The real and reactive power injected at buses $i$ and $j$ can expressed as follows in the equations (2.35)-(2.38).

\[
P_i \cos \theta \quad V_i^2 \left( G_{ij} \cos \phi_{upfc} + \delta_{ij} + B_{ij} \sin \phi_{upfc} \right) \quad (2.35)
\]

\[
Q_i \cos \theta \quad V_i V_j \left[ G_{ij} \sin \phi_{upfc} \right] \quad (2.36)
\]

\[
P_j \cos \theta \quad V_j \left[ G_{ij} \cos \phi_{upfc} \right] \quad (2.37)
\]

\[
Q_j \cos \theta \quad V_j V_i \left[ G_{ij} \sin \phi_{upfc} \right] \quad (2.38)
\]

### 2.4.5.6 Power Injection Model of IPFC

In the IPFC scheme, the active power is exchanged via series converters through a common DC link and it is noted that the sum of the active power outputted from VSCs to the transmission lines should be zero when the losses of the converter circuits are ignored. The injection magnitude and the phase angle is controlled by a combination of the series connected VSCs. It also maintains the fundamental frequency for controlling the DC link voltage at a desired level. The common DC link is represented by a bidirectional link for active power exchange between the voltage sources. The placement of IPFC in a transmission line as a power injection model and the power injection model of an IPFC is shown in the Figure 2.13. The power injections at buses are summarized and expressed in the Equations (2.39) - (2.42).
\[ P_{\text{inj},i} = \sum_{n,j,k} V_{i} V_{\text{se,}n} b_{\text{in}} \cos n \sin \theta_{\text{in}} \]  
(2.39)

\[ Q_{\text{inj},i} = \sum_{n,j,k} V_{i} V_{\text{se,}n} b_{\text{in}} \cos n \sin \theta_{\text{in}} \]  
(2.40)

\[ P_{\text{inj},n} = V_n V_{\text{se,}n} b_{\text{in}} \sin n \sin \theta_{\text{in}} \]  
(2.41)

\[ Q_{\text{inj},n} = V_n V_{\text{se,}n} b_{\text{in}} \cos n \sin \theta_{\text{in}} \]  
(2.42)

Where \( n = j, k \)

Based on the equivalent circuit, the voltage source impedances, the active and reactive power equations are expressed and the node equations are given by in the Equations (2.43) - (2.52).

At node i:

\[ P_i = V_i V_j B_{ij} \sin i j + V_i V_{\text{ser,}i} B_{i\text{sr}} \sin i \text{ sr} + V_i V_{\text{ser,2}i} B_{i\text{sr2}} \sin i \text{ sr2} \]  
(2.43)

\[ Q_i = V_i^2 b_{ij} - V_i V_j B_{ij} \sin i j + V_i V_{\text{ser,}i} B_{i\text{sr}} \sin i \text{ sr} + V_i V_{\text{ser,2}i} B_{i\text{sr2}} \sin i \text{ sr2} \]  
(2.44)

At node j:

\[ P_j = V_j V_i B_{ij} \sin j i + V_j V_{\text{ser,}j} B_{j\text{sr}} \sin j \text{ sr} \]  
(2.45)

\[ Q_j = V_j^2 b_{ij} - V_j V_i B_{ij} \sin j i + V_j V_{\text{ser,}j} B_{j\text{sr}} \sin j \text{ sr} \]  
(2.46)

At node k:

\[ P_k = V_k V_i B_{ik} \sin k i + V_k V_{\text{ser,}k} B_{i\text{sr}} \sin k \text{ sr} \]  
(2.47)

\[ Q_k = V_k^2 b_{ik} - V_k V_i B_{ik} \sin k i + V_k V_{\text{ser,}k} B_{i\text{sr}} \sin k \text{ sr} \]  
(2.48)
Series converter 1:

\[ P_{\text{ser1}} = V_{\text{ser1}} B_{i \text{j}} \sin \theta_{\text{ser1}} + i \quad Q_{\text{ser1}} = V_{\text{ser1}} B_{j \text{i}} \sin \theta_{\text{ser1}} \quad (2.49) \]

Series converter 2:

\[ P_{\text{ser2}} = V_{\text{ser2}} B_{k \text{i}} \sin \theta_{\text{ser2}} + i \quad Q_{\text{ser2}} = V_{\text{ser2}} B_{i \text{k}} \sin \theta_{\text{ser2}} \quad (2.51) \]

Where, \( B_{ij}, B_{ik}, 2X_{ser1}, B_{ij}, B_{ik}, X_{ser1}, B_{ik}, B_{ik}, X_{ser1} \)

The set of non-linear equations considered to obtain the IPFC control variables after power flow equation, are equations (2.43-2.52). In the case of power injection model, the IPFC neither absorbs nor injects active power with respect to the AC system, the active power exchange between the converters through the DC link is zero.

![Power Injection model of IPFC](image)

**Figure 2.13 Power Injection model of IPFC**

The real and reactive power injection at each bus is computed by representing IPFC as current source. For the purpose of simplifying the calculation, the resistance of the transmission lines and the series coupling transformers are neglected in the mathematical model of IPFC. As IPFC
either absorbs or injects active power with respect to the system, the active power exchange between the converters via the DC link is zero, the equation shows injection model in (2.53).

\[
\text{Re} \ V_s e_{ji} I_{ji}^* - V_s e_{ki} I_{ki}^* = 0 \tag{2.53}
\]

Where, the superscript * denotes the conjugate of a complex number. If the resistances of series transformers are neglected, (2.53), (2.54) can be written as,

\[
\sum_{m} P_{\text{inj},m} = 0 \tag{2.54}
\]

Normally, in the static operation, the IPFC is used to control the real and reactive power flows in the transmission lines in which it is placed. Constraints of the real and reactive power flow control are given in the Equations (2.55) and (2.56).

\[
P_{\text{inj}} \leq P_{\text{spec}} \tag{2.55}
\]

\[
Q_{\text{inj}} \leq Q_{\text{spec}} \tag{2.56}
\]

Where, \( n = j, k \); \( P_{\text{spec}} \), \( Q_{\text{spec}} \) are the specified active and reactive power flow control references respectively, and

\[
P_{\text{inj}} \leq \text{Re} \ V_{n} I_{n}^* \tag{2.57}
\]

\[
Q_{\text{inj}} \leq \text{Im} \ V_{n} I_{n}^* \tag{2.58}
\]

Thus, the power balance equations are as follows in (2.59) and (2.60)

\[
P_{\text{gen}} + P_{\text{inj},m} + P_{\text{line},m} = 0 \tag{2.59}
\]
\[ Q_{gm} \quad Q_{mj.m} \quad Q_{lm} \quad Q_{line.m} \quad 0 \] (2.60)

Where, \( P_{gm} \) and \( Q_{gm} \) are generation active and reactive powers, \( P_{lm} \) and \( Q_{lm} \) are load active and reactive powers, \( P_{line.m} \) and \( Q_{line.m} \) are conventional transmitted active and reactive powers at the bus \( m = j, k \) and \( i \).

**Figure 2.14 Connection and Power Injection diagram of IPFC**

The IPFC is connection diagram in the transmission lines at the lines \( i \) to \( f \) and \( i \) to \( h \) which \( j \) and \( k \) are the buses to connect the IPFC. The Figure 2.14 shows the connection and power injection diagram of IPFC. The active power, the reactive power loads in the bus \( j \) and the real power load in the bus \( k \) are set to the values being controlled by the IPFC (\( P_{j}, Q_{j}, P_{k} \)).

### 2.5 METHOD FOR OPTIMAL PLACEMENT OF FACTS DEVICES

The solution of this MO-OPD problem along with the optimal placement of various FACTS devices is an important problem in power systems operation for secure operation. In the past, most researchers had utilized dynamic considerations for the placement of the FACTS devices, as these devices have been utilized mainly to improve the stability of the power system networks. In the present research, the FACTS devices have been considered from a static point of view to reduce the total system transmission loss and enhance the stability of the system.
Hence, a new method based on reliability analysis approach, as described below, has been suggested for placement of the FACTS devices. When the FACTS devices are included in the system, it will modify the power flow between two transmission lines. Therefore, these devices should be placed on the most sensitive lines. A more flexible formulation of the problem can be accomplished by stating the problem in a manner of multiobjective optimization. More formally, a multiobjective nonlinear programming model for the MO-OPD with optimal placement of FACTS devices constraints is given in Equation (2.61).

\[
\text{Minimize } \{F, S_p\} \\
X_{L \text{ line}}, X_{\text{TUSC}}, X_{\text{TSC}} \\
Q, Q_{\text{VC}}
\]

(2.61)

\(F\) is the number of objectives (to be optimized), \(S_p\) the system sensitive index by reliability analysis and \(X_{L \text{ line}}\) - reactance of transmission line. The above formulation is meant for simultaneously optimizing the objective functions and if there is no conflict between the objective functions, a solution can then be found where simultaneous optimization of several objective functions is possible. The reliability analysis is done to address the optimal location of FACTS devices. In addition to this, the RBFA algorithm requires particle representation and is given in the Equation (2.62)

\[
Z = \begin{bmatrix} P_g, \ldots, P_{gN}, Q_g, \ldots, Q_{gN}, V_{g}, \ldots, V_{gN}, V_{L}, \ldots, V_{LN}, Q_{gL}, \ldots, Q_{gLN} \end{bmatrix}
\]

(2.62)

Where,

- \(V_{gi}\) - the voltage magnitude of generator bus
- \(P_{gi}\) - the active power generations at bus \(i\).
- \(Q_{gi}, Q_{ci}\) - the reactive powers and reactive power compensation.
26 RELIABILITY ANALYSIS AND STABILITY INDICES

The optimal location of FACTS devices are based on critical identification, and the identification process by reliability analysis along with various stability indices.

26.1 Voltage Stability Analysis by Fast Line Flow Index

The Fast Line Flow Index (FLFI) method is used to ensure the power flow control and stability index between the receiving and sending end power in the interconnected power system network. In this method, the set of power flow equations is to co-ordinate the real and reactive power flow control over a transmission line in both directions of flow. The set of equations was used for analysis and identification of critical lines and weak bases.

The maximum voltage deviations are pointed out in the particular systems in the view of voltage stability analysis. The analysis of line flow approach is given for a two bus system in the Equation (2.63).

\[ L_{\text{fl}} = \frac{4XQ_{rl}}{\sqrt{(V_{r} \sin \delta)^2}} \]  \hspace{1cm} (2.63)

Where, \( L_{\text{fl}} \) - Fast Line Flow Index, \( \theta \) - impedance angle from impedance triangle, \( \delta \) - Influence of the vector diagram, the angle between sending end and receiving end voltage, X - line reactance, \( Q_{rl} \) - reactive power flow at the receiving end and \( V_{r} \) - sending end voltage. The \( Q_{rl} \) - reactive power injection is estimated by Q-V analysis and voltage stability by VSA approach (Nandha & Renuga (2010)).
26.2 Voltage Stability Approach

The Voltage Stability Approach (VSA) comprises of a Voltage Stability Index (VSI) against voltage collapse and line stability based on the concept of maximum power transferred through a transmission line flow. The optimal location and control variables of FACTS devices are based on the voltage stability index of each transmission line. The loading of real or reactive power leads to identification of the critical transmission paths via weak buses. A voltage stability index deals with the maximum voltage deviation via power flow in transmission, which is a lead to maintain the voltage profile against loading condition. Therefore, VSA gives the corrected voltage drop of a line segment and is defined as the projection of the receiving end bus voltage of that segment on the voltage phasor of the generator, which is the starting point of that transmission path. This index (Nandha & Renuga (2010)) is given in the Equation (2.64).

\[ \text{VSI} = hV_{\text{act}} V \]  \hspace{1cm} (2.64)

Where, \( V_{\text{act}} \) is the actual generator voltage; \( h \) is the parameter to correct the desired constant value 0.5. The real power and reactive power flow in transmission line is defined as a sequence of connected buses with declining voltage magnitudes again starting from a generator bus.

The FLFI and VSA are analyses are used to carry out the real and reactive power loading and with address of critical lines and weak buses. The voltage deviation and the voltage stability enhancement happen for placing the FACTS devices. Further Q-V analysis deals with voltage stability and reactive power compensation in FACTS devices.
26.3 Q-V Analysis: Reactive Power Control and Voltage Stability Index

Q-V analysis includes voltage stability analysis, reactive power control variables. VAR compensation design is given in the matrix form, the equation is given below (2.65).

\[
\begin{bmatrix}
J_{p8} & J_{pV} \\
J_{q8} & J_{qV}
\end{bmatrix}
\begin{bmatrix}
\delta \\
v
\end{bmatrix}
= 
\begin{bmatrix}
P \\
Q
\end{bmatrix}
\]  \hspace{1cm} (2.65)

Where, \( P \) and \( Q \) are incremental real, reactive power.
\( \delta \) and \( v \) are incremental bus voltages and bus angles.

\( J_{p8} \), \( J_{pV} \), \( J_{qV} \) and \( J_{q8} \) are sub matrices of Jacobian in power flow equation.

The Q-V analysis is a method to identify FACTS devices for compensation at a particular point after identification of weak buses and critical lines, to improve the voltage stability and finally provides information to enhance voltage stability by taking necessary actions. This analysis gives a detailed view of stability enhancement by modification and rescheduling of control variables like real and reactive power controls.

Power flow equations after the increments in bus voltage magnitude and angle, real and reactive power can be written as in matrix form and mentioned in the Equation (2.66).

\[
\begin{bmatrix}
J_{p8} & J_{pV} \\
J_{q8} & J_{qV}
\end{bmatrix}
\begin{bmatrix}
\delta \\
v
\end{bmatrix}
= 
\begin{bmatrix}
P \\
Q
\end{bmatrix}
\] \hspace{1cm} \text{or} \hspace{1cm}
\[
\begin{bmatrix}
J_{11} & J_{12} \\
J_{31} & J_{32}
\end{bmatrix}
\begin{bmatrix}
\delta \\
v
\end{bmatrix}
= 
\begin{bmatrix}
P \\
Q
\end{bmatrix}
\]  \hspace{1cm} (2.66)
In fact, system voltage stability is affected by P and Q variation. However, at each operating point, keep P as a constant and evaluate the voltage stability by considering the incremental relationship between Q and V. Based on the above considerations, in Equation (2.66), let ∆P=0, equation then becomes and is given in Equation (2.67).

\[
\begin{bmatrix}
0 \\
\Delta V
\end{bmatrix} = \begin{bmatrix}
P \\
\Delta Q
\end{bmatrix} \begin{bmatrix}
P \\
Q
\end{bmatrix}^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix} \begin{bmatrix}
0 \\
\Delta V
\end{bmatrix}
\]

(2.67)

From a stability point of view, according to point of operation real power is kept constant. The incremental relationship of Q-V analysis is given in the Equation (2.68).

\[V \quad J_R^{-1} \quad Q\]

(2.68)

Where, \(J_R\) is known as reduced Jacobian and is given in the Equation (2.69).

\[
\begin{bmatrix}
J_R & J_{QV} & J_{ΩΦ} & J_{PV} & J_{ΦV}
\end{bmatrix}
\]

(2.69)

The voltage stability analysis is further done with the help of sub-matrix Jacobian, is given in the following Equation (2.70).

\[
L_k \quad \frac{Q_k}{V_k} \quad \frac{Q_{ik}}{V_k} \quad B_{kk} \quad V_k
\]

(2.70)

Where, \(q_{ik}\) - Reactive power demand at \(i^{th}\) bus, \(L_k\) - Voltage stability index at \(i^{th}\) bus and \(B_{kk}\) - Imaginary part of Admittance matrix.

Using the reduced Jacobian matrix, the sensitivity of Voltage stability index with respect to VAR injection at \(k^{th}\) bus written as, in Equations (2.71) and (2.72).
\[
L_k^f \frac{Q_k}{V_k} V
\]  \hspace{1cm} (2.71)

Where, \[ V \rightarrow J_R \begin{bmatrix} Q_{ij} \end{bmatrix} \]  \hspace{1cm} (2.72)

Voltage stability index depends upon the following parameters, namely, the voltage profile improvement, reactive power demand, voltage at \(K^\text{th}\) bus, i.e., \(B_{kk}\). Generally, the product \(B_{kk}V_k\) is important and dominant. If \(B_{kk}\) is larger, then relatively lesser voltage magnitude may be sufficient to give the required voltage stability margin in the following Equation (2.73).

\[
L_k \rightarrow L_k^f \sum_{k=1}^{NC} A_{kj} C_k
\]  \hspace{1cm} (2.73)

Where, \(C_k\) is \(k^\text{th}\) bus change in reactive power control variables.

\(N_C\) is the total number of reactive power control variables, which includes PV buses, tap changers and switchable shunt reactors. \(A_{kj}\) is the sensitivity coefficient of VSIs with respect to the change in reactive power control variables (0.25 to 0.55). In order to improve the voltage stability and maintain the voltage profile end results of Q-V analysis is required to inject a reactive power at the critical and weak buses.

The results of reliability analysis, Power flow by TCSC and TCPPAR should be placed according to the critical condition and maintain constant bus voltage by the placement of SVC or SSSC. The control parameters for the FACTS devices include line net series reactance (the reactance \(X_k\) placed in transmission line k) for TCSC, phase angle shift (angle placed in line k) for TCPPAR.
27  OPTIMIZATION TECHNIQUES

Electric power has become an essential part of the modern world. Due to the increasing power demand with various constraints, such as economic considerations and environmental regulations, the power systems are forced to operate closer to their operating limits. The power system countenances the stability and quality problems. To undertake the complex problems in the real world, optimization is the only solution. Here, the optimization techniques are focused to the solution of generated power and it is distributed economically in a secured manner. In the present research, the optimization techniques addressed are the Multi-Objective based Optimal Power Dispatch problem, optimal parameters and optimal locations of FACTS devices.

To reduce the level of congestion, either the complex link or the neighborhood lines, are the potential locations for installing the FACTS controllers. This set point can be used for incorporating any one of FACTS in the MO-OPD problem.

28  BACTERIAL FORAGING ALGORITHM

The BFA is a new algorithm from the family of evolutionary computation, known as Bacteria Foraging Algorithm (BFA). The idea of BFA is due to the fact that natural selection tends to eliminate animals with poor foraging strategies and favor those having successful foraging strategies. After many generations, poor foraging strategies are either eliminated or reshaped into good ones. The E.coli bacteria that are present in the human intestines have a foraging strategy governed by four processes, namely, chemotaxis, swarming, reproduction, and elimination - dispersal.
2.8.1 Chemotaxis

This is a process achieved through swimming and tumbling by flagella. The rotation of flagella in each bacterium decides its pattern, whether it should move in a predefined direction as swimming or altogether in different directions as tumbling in the entire lifetime. During the entire lifetime, bacteria are set to two modes of operation. These modes enable the bacteria to move in random directions. An E. coli bacterium can move in two alternate ways: tumble and run. A unit walk with random direction represents a tumble, a unit walk with the same direction as the previous step indicates a run. A chemotactic process is started by one step of tumble and followed by uncertain steps of run, depending on the variation of the environment.

In a tumble, the position of the i-th bacterium is updated in the Equation (2.74).

\[ ^i (j \quad 1, k, l) \quad i \quad j, l \quad C \quad i < j \quad (2.74) \]

Where, \( ^i (j, k, l) \) is the position of the i-th bacterium at the j-th chemotactic step of the k-th reproduction loop in the l-th elimination dispersal event. \( C \) (i) is the size of the step taken in the random direction specified by the tumble. The \( \Phi(j) \) is the angle of the direction, which is randomly generated in the range of \((0, 2\pi)\). The fitness value of the i-th bacterium at \( ^i (j, k, l) \) is represented by \( j' (j, k, l) \). If \( \theta^i (j+1, k, l) \) is the better cost, it means lower than at \( \theta^i (j, k, l) \). Now, the next step of step size (C (i)) in the same direction will be taken and once again, if the step resulted in a position with better cost than the previous step, another will be taken. This swim is continues as long as it continues to reduce the cost and it depends upon maximum number of steps (Ns). The \( N_c \) is the number of chemotaxis steps.
282 Swarming

During the process, the E-coli bacterium produces attraction convergence characteristics and has to decide when anyone bacteria reaches the best position or location, it should attract other bacteria. Therefore, they converge in that location and this will happen by generation of attraction signal and in the meantime, each bacterium releases repellent to warn other bacteria to keep a safe distance. The E-coli bacterium has its own specific sensing, cell to cell signaling, actuation and decision-making mechanism. These properties provide global search. BFA simulates this social behavior by representing the cell to cell signaling by the Equation (2.75).

\[ j \alpha ^{i, k, l}, j, k, l \quad \sum_{i=1}^{S} j_{\alpha} \frac{d_{\text{attract}}}{m_{i, m}} \exp \left( w_{\text{attract}} \frac{P}{m_{i, m}^2} \right) \]

\[ \sum_{i=1}^{S} j_{\alpha} \frac{d_{\text{repellant}}}{m_{i, m}} \exp \left( w_{\text{repellant}} \frac{P}{m_{i, m}^2} \right) \]

(2.75)

Where,
- \( d_{\text{attract}} \) depth of the attractant effect
- \( \omega_{\text{attract}} \) measure of the width of the attractant
- \( h_{\text{repellant}} = d_{\text{attract}} \) height of the repellent effect
- \( \omega_{\text{repellant}} \) measure of the width of the repellent
- \( P \) number of parameters to be optimized
- \( S \) number of bacteria

Where, \( [1, 2, \ldots] \) is a point optimization domain and it is the location of the \( i \) th bacterium on the \( P \)-dimensional optimization domain, \( i_{m} \) is the \( m \)th component of \( i \) th bacterium position, \( \{1, 2, \ldots, S\} \) represents the position of each member in the population of the \( S \) bacteria, \( i_{m} \) is the \( m \)th
component of \( \mathbf{i} \). \( i_m \) is the \( m \)th component of position \( \mathbf{i} \) for the \( t \)th bacterium. The \( d_{\text{attract}} \) is a quantification of how much attractant is released. \( d_{\text{repellant}} \) is a measure of the diffusion rate of the chemical signal, \( h_{\text{repellant}} \) and \( h_{\text{attract}} \) are the magnitude and width of the repelling effect respectively. \( J^i_{ce}(\cdot,\theta(j,k,l)) \) indicates the signals released by the \( i \)th bacterium and \( J_{ce}(\cdot) \) is time-varying function. \( J_{ce}(\cdot) \) represents the combined attraction and repelling effects received by the \( i \)th bacterium.

### 2.8.3 Reproduction

After chemotactic steps \( N_c \), a reproduction step is taken. Assume that, \( N_r \) is the number of reproduction steps taken. The final population of bacteria undergoes the reproduction process. Here, the least healthy bacteria die and other healthiest bacteria split into two at the same location. All the bacteria are sorted according to their fitness \( S_r \) (\( S_r=S/2 \), for convenience \( S \) is assumed to be a positive even integer) and the step fitness during the life and fitness values for the \( i \)th bacterium in the chemotactic loop are accumulated and calculated by the Equation (2.76).

\[
\sum_{j=1}^{N_r} j_{i,j} \sum_{j=1}^{N_r} j_{i,j,k,l}
\]

\[
j_{i,j} = \frac{1}{\sum_{j=1}^{N_r} j_{i,j,k,l}}
\]

(2.76)

Where, \( j_{i,j} \) represents the health of the \( i \)th bacterium.

For simplification, the number of the bacteria is kept constant in each chemotaxis process. The characters including location and step length of the mother bacterium reproduced to the children bacteria. Through this selection process, the remaining unhealthier bacteria \( S_r \) are eliminated and discarded.
28.4 Elimination-Dispersal

The process of chemotaxis and reproduction are not enough for finding the global solutions and to improve the process of global search ability, the elimination-dispersal event is introduced after \( N_c \) steps of reproduction. This elimination-dispersal event helps the bacterium avoid being trapped into local optima dispersal events may place bacteria near global solutions, and the behavior of bacteria seeks out in favorable environments. The bacteria are eliminated and dispersed by random positions in the optimization domain according to the probability, \( p_{cd} \). The number of the event is denoted as \( N_{cd} \).

28.5 Parameter Settings of BFA

The BFA requires specification of a variety of parameters. The bacteria size, \( S \) is important. The correct selection of \( S \) will lead the solution effectively, otherwise the solution becomes complex, the step length calculation is another one important process. If the step length \( C \) is too large, the bacteria may miss the global position and if step length \( C \) is too small, it takes more time to reach the global position. The size of the values of the parameters that define the cell-to-cell attractant functions \( J_{ac} \) will define the characteristics of swarming. The chemotactic step \( N_c \) will be chosen according to the complexity of problem. The larger value of \( N_c \) increases the computational complexity.

28.6 Algorithm

**Step 1** Initialize parameters \( p, S, N_c, N_s, N_{re}, N_{ed}, P_{cd}, C \) (i) \( (i=1, 2, \ldots S) \), \( \Theta^i \)

**Step 2** Elimination-dispersal loop: \( l=l+1 \)

**Step 3** Reproduction loop: \( k=k+1 \)
Step 4  Chemotaxis loop: $j = j + 1$

[a] For $i = 1, 2, \ldots, S$ take a chemotactic step for bacterium $i$ as follows.

[b] Compute fitness function, $J(i, j, k, l)$. Let $J(i, j, k, l) + J_{cc} + \theta(j, k, l, P(j, k, l))$

[c] Let $J_{\text{last}} = J(i, j, k, l)$ to save this value since one may find a better cost via a run.

[d] Tumble: generate a random vector $D(i) \in \mathbb{R}^p$ with each element $D_m(i), m=1, 2, \ldots, p$, a random number on $[-1, 1]$.

[e] Move: Let

$$i \quad j \quad k \quad l \quad i \quad j \quad k \quad l \quad C \quad i \quad \frac{i}{\sqrt{i}}$$

This result in a step of size $C(i)$ in the direction of the tumble for bacterium $i$ and is shown in the Equation (2.77).

[f] Compute $J(i, j + 1, k, l)$ and let

$$J i, j, k, l \quad J i, j, k, l \quad J_{cc} \quad i \quad j \quad k \quad l \quad P \quad j \quad 1 \quad k \quad l$$

[g] Swim

1) Let $m = 0$ (counter for swim length).

2) While $m < s N$ (if have not climbed down too long).

   i) Let $m = m + 1$.

   ii) If $J(i, j + 1, k, l) < J_{\text{last}}$ (if doing better let

       $$J_{\text{last}} = J(i, j + 1, k, l)$$ and let
\[
\begin{align*}
\theta^i(j + 1, j, k) & \quad \text{and use this } \theta^i(j + 1, j, k) \text{ to compute the new } J(i, j + 1, k, l) \text{ as in} \\
& \quad \text{[h] Else, let m = } N_S. \text{ This is the end of the while statement.} \\
& \quad \text{[i] Go to next bacterium (i+1) if } i \neq S \text{ (i.e., go to [b] to process the next bacterium).}
\end{align*}
\]

**Step 5** If \( j < N_C \) go to step 4. In this case, continue chemotaxis since the life of the bacterium is not over.

**Step 6** Reproduction

[a] For the given \( k \) and \( l \), and for each \( i = 1, 2, ..., S \), let

\[
J_{\text{health}} = \sum_{j=1}^{N_C} J(i, j, k, l)
\]  

(2.78)

be the health of the bacterium \( i \) (a measure of how many nutrients it got over its lifetime and how successful it was at avoiding noxious substances). Sort bacteria and chemotactic parameters \( C(i) \) in order of ascending cost \( J_{\text{health}} \) (higher cost means lower health) is described in the Equation (2.78).

[b] The \( S_f \) bacteria with the highest \( J_{\text{health}} \) values die and the remaining \( S_r \) bacteria with the best values split (this process is performed by the copies that are made placed at the same location as their parent).

**Step 7** If \( k < N_{re} \), go to step 3. In this case, if it has not reached the number of specified reproduction steps means, start the next generation of the chemotactic loop.
Step 8  Elimination-dispersal: For \( i = 1, 2, \ldots, S \) with probability \( P_{el} \), eliminate and disperse each bacterium (this keeps the number of bacteria in the population constant). To do this, if a bacterium is eliminated, simply disperse another one to a random location on the optimization domain.

Step 9  If \( l < N_{el} \) then go to step 2; otherwise end.

29  MODIFIED BACTERIAL FORAGING ALGORITHM

From the basic formulation of BFA, the modifications, enhancements and implementations have been proposed and investigated. In the concern, the Modified BFA (MBFA) algorithm is introduced. The unit step length of the basic BFA is a constant parameter, which may guarantee good searching results for small optimization problems. However, when applied to complex, large-scale problems with high dimensionality, it shows poor performance. The run-length parameter is the key factor for controlling the search ability of the BFA. From this perspective, several dynamic functions are implemented to control the local and global search ability of the algorithm, and for balancing the exploration and development of the search that could be achieved by adjusting the run-length.

At the initial point of the MBFA process, bacteria calculate the food focus and then tumble to take a random direction and swim for a distance for food identification. The chemotactic step contains tumble and swim processes. If the focus is greater at the next location, they then take another step in that direction. When the focus at the next location is lesser than that of the previous location, they tumble to find another direction and swim in this new direction. This process is carried out up to a certain number of steps, which is limited by the lifetime of the bacteria. At the end of its lifetime, the bacteria that have gathered good health and are in a better focus
region, divided into two groups. In the next reproductive step, the next generations of bacteria start from a healthy position. The better half reproduces to generate the next generation and the other half dies. The specifications such as the number of reproductive steps and the number of chemotactic steps consist of run and tumble. When the step length calculations are given for a particular problem, the variable can then be optimized using Modified Bacteria Foraging Optimization technique.

The MBFA propose variable step length instead of the constant step. For swarming, the distances of all the bacteria in a new chemotactic stage is evaluated from the global optimum bacterium until that point and not the distances of each bacterium from the rest of the others.

**Step length calculation:** The function is expressed in the Equation (2.79).

\[ C_{i,j} = 1 \left( \frac{C_{i,j} C_{N_c}}{N_c} \right)^{N_c-j} \]  

(2.79)

Where \( j \) is the chemotactic step and \( N_c \) is the maximum number of chemotactic steps while \( C(N_c) \) is a predefined parameter.

**2.9.1 Parameters Settings of MBFA**

The parameters selected for the Modified BFA are similar to Bacteria Foraging Algorithm and the parameters are also listed in the Table 2.2. The parameters of step size can be adjusted according to the number of objectives in the problem. The various experiments are designed to evaluate the adaptability of MBFA for various multi modal functions and adjust the step size for various environments to optimize the performance. Hence, the step size of each bacterium is the main determining factor for both the speed
of convergence and deviation in the final output. MBFA method decides to control the step size depending upon the reach to the global position.

2.9.2 Pseudo code for MBFA

Begin

Randomly Initialize each bacteria and other parameters

FOR (Elimination/ Dispersal loop, l=l+1)

FOR (Reproduction loop, k=k+1)

FOR (Chemotaxis loop, j=j+1)

Update the run length unit using step length calculation

For the given k,l, evaluate the health of each bacterium

Evaluate the cost function J(i,j,k,l).

Generate a random vector and compute

\[ i^{\mu} \text{ and } \Delta_{\mu}(i), \text{ m } = 1, 2, ..., p \text{ is a random number in the range } [-1, 1] \]

new \[ J(i,j+1,k,l) \]

END FOR (Chemotaxis loop)

Update the step length

END FOR (Reproduction)

Randomly eliminate and disperse each bacterium i, keeping the size of the population constant. If one eliminates a bacterium, simply disperse it to a random location on the optimization domain.

END FOR (Elimination/Dispersal)

END
2.10 ANT COLONY OPTIMIZATION ALGORITHM

Ant Colony Optimization (ACO) is a natural computation method that mimics the behaviors of ant colony. It is a very good combination of optimization method. ACO algorithm is a recently proposed algorithm. It has strong robustness as well as good distributed calculative mechanism and it is easy to combine with other methods, and good performance has shown on resolving the complex optimization problem. ACO is a paradigm for designing metaheuristic algorithms for combinatorial optimization problems and is inspired by the behavior of real ants. ACO algorithm adapts genetic operations to enhance ant movement towards solution state and the algorithm converges to the optimal final solution, by accumulating the most effective sub-solutions.

A colony of ants is able to succeed in a task to find the shortest path between the nest and the food source. It was found that ants deposit a chemical substance trail, called pheromone on the ground when they move. This pheromone can be observed by other ants, and motivates them to follow the path with a high probability. The real ants lay down in some quantity an aromatic substance, known as pheromone, on their way to the food source. The pheromone quantity depends on the length of the path and the quality of the discovered food source. An ant chooses an exact path in connection with the intensity of the pheromone. The pheromone trail evaporates over time if no more pheromone is laid down, other ants are attracted to follow the pheromone trail. Therefore, the path will be marked again and it will attract more ants to use the same path.

The pheromone trail on paths leading to the rich food sources close to the nest will be more frequented, and will, therefore, grow faster. In this way, the best solution has more intensive pheromone and higher probability to be chosen. ACO algorithm has been used to solve combinatorial optimization
problem involving initialization, state transition rule, fitness evaluation, local updating rule and global updating rule.

**Pheromone Update:** Global updating rule is a process used to update the amount of pheromones generated by the ant, which has constructed the shortest route from the start of the tour. Only one ant that is allowed to update the amount of pheromone, which determines the best fitness.

**Reduce Pareto set by Clustering:** The concept of the Pareto dominance set is used to compare the whole set of solutions with each other. The Pareto optimal set states that the set of feasible solutions that are not dominated are known as the Pareto optimal set. In some of the problems, the Pareto optimal set can be extremely large. In this case, it is desirable to reduce the set of non-dominated solutions without destroying the characteristics of the tradeoff, clustering is applied to reduce the set of the Pareto optimal set with a manageable size. It works iteratively by joining the adjacent clusters until the required number of groups is obtained. If the given set exceeds the maximum allowable size, it is required to form a subset with the size that is needed.

### 2.10.1 Parameters Settings of ACO

In any social or bio-inspired algorithms, the important process is to find out the parameters for the optimization process and is based on the balance of examination and exploitation. The ACO determines the relative influence of pheromone trail and heuristic information. In order to evaluate the performance of the algorithm, the proposed algorithm contains four parameters namely, the number of ants (A), the number of iterations (I), Pheromone weight (α), Heuristic information weight (β), and Pheromone evaporation weight (ρ). Experiments based on the objectives, boundary limits of the problem and evaluation approach are conducted to find the suitable
values for parameters. Based on the results of the performance of the algorithm and these experiments, appropriate values for these parameters are found as \( A = 110 \), \( \alpha = 1.5 \), \( \beta = 5 \) and \( \rho = 0.55 \) considering the interaction factor.

### 2.10.2 Algorithm

The difference of ACO with the other optimization algorithm is clearly presented.

**Step 1**

**Initialization:** Generate an initial population with a heuristic initialization procedure. Create two empty sets \( P_{\text{known}} \) and \( SP_{\text{known}} \) external Pareto Optimal Set. [Note: Where, \( SP_{\text{known}} \) is a new empty, which is not been taken in any of the multi objective and reactive power compensation problem]

Heuristic Initialization: It is used to generate the initial population to obtain electrically well compensated individuals. The proposed heuristic is based on encouraging compensation at bus bars with large number of branches and voltage profile far from the desired values. This is done by using a method summarized as follows.

Choose a total amount of compensation \( B_{\text{total}} \). Calculate the compensation for each bus bar and is shown in the Equation (2.80).

\[
B_i = H_j B_{\text{total}} \tag{2.80}
\]

Amount of reactive compensation to be allocated \([i].B_i\) indicates the size of the reactive bank in the power system.

\[
B_i, B_1, B_2, \ldots, B_n, B_i, B_{\text{max}} \tag{2.81}
\]
The $B_i$ and $H_i$ are shown in the Equations (2.81) and (2.82). $B_i$ is the compensation at bus bar $[i]$ (Mvar) $B_{max}$ is the maximum amount of compensation at a single bus bar of the system (Mvar)

$$H_i^1 = H_i / \sum_{i=1}^{n} H_i$$

$$H_i = \begin{cases} V_{act} / V_{des} & \text{if } V_{act} > V_{des} \\ 0 & \text{if } V_{act} < V_{des} \end{cases}$$  \hspace{1cm} (2.82)$$

Where, $l_i$ = number of branches connected to node $(i)$

Step 2

**External Set Updating**: The solution in the population is formulated using the procedure in heuristic method.

The external Pareto-optimal set is updated as follows.

A: Search the population for the non-dominated individuals $P_{known}$ and copy them to the external Pareto set $SP_{known}$.

B: Search the external Pareto set $SP_{known}$ for non-dominated individuals and removes all the dominated solutions from the set.

Step 3

The external Pareto set $SP_{known}$ will have some particular solutions, which are non-dominated solutions from the set. Compare the external Pareto set $SP_{known}$ with the $P_{known}$ where the $P_{known}$ is values taken from the Step 2. While comparing the $SP_{known}$ with the $P_{known}$ remove the solutions with in $P_{known}$, which are dominated by the external Pareto set.
Step 4

If the number of the individuals externally stored in the Pareto set exceeds the pre-specified maximum size, reduce the set by clustering to reduce the size of the external population.

Step 5

Calculate the fitness value of individuals in both the external Pareto set and the population as follows:

Assign a real value called strength for each individual that is proportional to the number of individuals covered by it. The strength of a Pareto solution is at the same time its fitness. The fitness of each individual in the population is the sum of the strength of all the external Pareto solutions by which it is covered. To guarantee that Pareto solutions are most likely to be produced, and a small positive number is added to the resulting value.

Step 6

Select individuals from population $S_{P_{\text{known}}}$ external Pareto set $S_{P_{\text{known}}}$ select the two individuals at random, compare their fitness, select the better one, and copy it to the mating pool. Select individuals until the mating pool is filled.

Step 7

Perform the cross over and mutation operations according to their probabilities to generate the new population. Apply the probabilities to determine whether an individual is locally optimized or selected for cross over and mutation.
Step 8

Check for stopping criteria. If anyone is satisfied, then stop or else copy the new population solution set to the old population solution and go to step 2. The search will be stopped if the generation counter exceeds its maximum number.

2.11 PARTICLE SWARM OPTIMIZATION ALGORITHM

It is a foraging based optimization algorithm developed by Kennedy & Eberhart (1995). The algorithm was developed on the simulation of the social behavior of bird flocks. PSO is a population-based search process where individuals initialized with a population of random solutions, referred to as particles, are grouped into a swarm. Each particle in the swarm represents a candidate solution to the optimization problem, and if the solution is made up of a set of variables, the particle can correspondingly be a vector of variables. PSO has been successfully applied in many applications and research areas. It is not affected by the size and non-linearity of the problem, and can converge to optimal solution where most analytical methods fail to converge. In PSO system, each particle is "flown" through the multidimensional search space, adjusting its position in the search space according to its own experience and that of the neighboring particles. The particle, therefore, makes use of the best position encountered by itself and that of its neighbors to position itself toward an optimal solution. The performance of each particle is evaluated using a predefined fitness function, which encapsulates the characteristics of the optimization problem. The merits of PSO over the other optimization techniques are, easier to apply, lesser parameters to adjust, effective memory, more efficient in diversity of swarm. Additional operations such as crossover and mutation are not needed.
2.11.1 Parameters Settings of PSO

The control parameters of PSO are population size, maximum velocity, acceleration constants and inertia weight. Population size is between 20 and 100. Maximum velocity, \( V_{\text{max}} \), determines the maximum change that one particle can undergo in its positional coordinates during iteration. If \( V_{\text{max}} \) is too small, particles may not explore sufficiently beyond the local solutions. \( V_{\text{max}} \) is determined (Abido 2002) using Equation (2.83).

\[
V_{\text{max}} = \frac{x_{\text{max}} - x_{\text{min}}}{N_i}
\]  

(2.83)

Where \( N_i \) - number of intervals

Acceleration constant \( c_1 \) and \( c_2 \) controls the movement of each particle towards its best \( P_{\text{best}} \) and \( G_{\text{best}} \) positions. In general, the range is 0 to 4. Here, \( c_1 \) and \( c_2 \) are set to 2, according to experience.

Inertia weight, \( \omega \) controls the momentum of particle. It is an important parameter for exploration and exploitation. Suitable selection of inertia weight provides a balance between the global, local exploration and exploitation, resulting in lesser iterations to find a successful optimal solution. As originally developed, \( \omega \) decreases linearly from about 0.9 to 0.4 during a run.

2.11.2 Algorithm

The basic PSO algorithm consists of two steps namely, initialization of population and updation of particles.

Step 1: Initialization

Each particle in PSO files in a D-Dimensional real parameter space. The particle position \( X_i = (x_{i1}, x_{i2}, ..., x_{iD}) \) and velocity \( V_i = (v_{i1}, v_{i2}, ..., v_{iD}) \) are
randomly generated according to population size, \( N_p \), within the search space, constrained by specified minimum and maximum bounds.

**Step 2: Particle updation**

Each particle's position and velocity are updated using the Equations (2.84) and (2.85). The best position that is associated with the best fitness encountered so far is called \( P_{i\text{es}} \). \( G_{i\text{es}} \) is the best position among all the individual best position achieved so far.

\[
V_{id}^{(t+1)} = V_{id}^{(t)} + c_1 \cdot r_1() \cdot (P_{i\text{es}}^{(t)} - X_{id}^{(t)}) + c_2 \cdot r_2() \cdot (P_{gd}^{(t)} - X_{id}^{(t)}) \tag{2.84}
\]

\[
X_{id}^{(t+1)} = X_{id}^{(t)} + V_{id}^{(t+1)} \tag{2.85}
\]

Where, \( c_1, c_2 \) - two acceleration constants known as self confidence and swarm confidence. The \( r_1(), r_2() \) - two random numbers between 0 and 1. \( P_{i\text{es}} \) - best position of the \( i \) th individual (\( P_{i\text{es}} \)). \( P_{gd} \) - best position among the individual (\( G_{i\text{es}} \)). \( V_{id} \) - velocity of \( i \) th individual of \( d \) dimension. \( X_{id} \) - position of \( i \) th individual of \( d \) dimension and \( t \) - iteration number.

The inertia weight, \( w \) is updated using the equation to enable quick convergence as pointed in the Equation (2.86).

\[
\max \left( \frac{\text{max} \cdot w \cdot \text{min}}{\text{iter max}} \right) \cdot \text{iter} \tag{2.86}
\]

Where, \( \text{iter max} \) - maximum number of iterations, \( \text{iter} \) - current iteration number

\( w_{\text{max}} \) - initial value of inertia weight

\( w_{\text{min}} \) - final value of inertia weight
2.12 TOOLS AND TEST SYSTEMS

Programming codes for BFA, MBFA, ACO, PSO and RBFA are developed using MATLAB 7.10.0 simulink environment. The initial parameter values adopted for various algorithms are presented in the Table 2.1, 2.2 and 2.3 respectively.

Table 2.1 Common Simulation Parameters for PSO

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Size</td>
<td>100</td>
</tr>
<tr>
<td>Number of Iterations</td>
<td>200</td>
</tr>
<tr>
<td>Number of Variables</td>
<td>3</td>
</tr>
<tr>
<td>$c_1$, $c_2$</td>
<td>2,2</td>
</tr>
<tr>
<td>wmin, wmax</td>
<td>0.9, 0.4</td>
</tr>
<tr>
<td>Convergence criteria</td>
<td>$10^6$</td>
</tr>
</tbody>
</table>

Table 2.2 Common Simulation Parameters of BFA and MBFA

<table>
<thead>
<tr>
<th>Control Parameters</th>
<th>BFA</th>
<th>MBFA</th>
<th>Control Parameters</th>
<th>BFA</th>
<th>MBFA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bacteria (S)</td>
<td>100</td>
<td>25</td>
<td>Size of step</td>
<td>0.1</td>
<td>Initial-0.1</td>
</tr>
<tr>
<td>Swimming length, $N_s$</td>
<td>12</td>
<td>12</td>
<td>$d_{\text{attractive}}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Number of chemotactic steps, $N_c$</td>
<td>100</td>
<td>100</td>
<td>$W_{\text{attractive}}$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Number of reproduction steps, $N_{re}$</td>
<td>16</td>
<td>16</td>
<td>$W_{\text{repellent}}$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Number of elimination-disperse steps, $N_{ed}$</td>
<td>4</td>
<td>4</td>
<td>$h_{\text{repellent}}$</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Probability, $P_e$</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3 Common Simulation Parameters of RBFA

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of bacteria, $s$</td>
<td>100</td>
</tr>
<tr>
<td>Number of chemotactic steps $N_c$</td>
<td>100</td>
</tr>
<tr>
<td>Limits the length of a swim $N_s$</td>
<td>12</td>
</tr>
<tr>
<td>The number of reproduction steps $N_{re}$</td>
<td>16</td>
</tr>
<tr>
<td>The number of elimination dispersal events $N_{ed}$</td>
<td>4</td>
</tr>
</tbody>
</table>
The performance of the proposed RBFA and PSO algorithms are tested with the standard benchmark problems, and the BFA, MBFA, ACO, and RBFA algorithms are tested on IEEE 6 bus system, IEEE 9 bus system, IEEE 30 bus system, New England 39 bus system and IEEE 57 bus system. The data for all the test systems are acquired from standard IEEE systems and is given in the Appendix, with 100MVA base and the up to 150 percentage loading condition is used.

From the Appendix 1, IEEE 30-bus system consists of 6 generator buses, 24 load buses and 41 transmission lines of which four transformer with off nominal tap ratio in lines 6 9, 6-10, 4-12, 27-2 and 9 buses of compensations 10, 12, 15, 17, 20, 21, 23, 24 and 29. The line data, bus data and other data are given in the Appendix. The lower voltage magnitude limits at all the buses are 0.95 p.u. and the upper limits 1.1 p.u. for the generator buses 2, 5, 8, 11 and 13, and 1.05 p.u. for the remaining buses including the swing bus 1. The lower limits and the upper limits of the transformer tap settings are 0.9 and 1.1 p.u. respectively. The capacitor bank ratings are set as 0-15 MVAR.

From the Appendix 2, the IEEE 57-bus test system consist of 80 transmission lines, seven generators at the buses 1, 2, 3, 6, 8, 9 and 12, and fifteen branches under load tap setting transformer branches. The shunt reactive power sources are considered at buses 18, 25 and 53. The total load demand of system is 1250.8MW and 336.4 MVAR. The maximum and minimum values for voltages of all the generator buses and tap setting transformer control variables are considered 0.9-1.1 in p.u. The maximum and minimum values of shunt reactive power sources are 0.0 and 0.3 in p.u. The maximum and minimum values for voltages of all the load buses are 1.06 and 0.95 in p.u and upper limits the PV buses are 1.1 p.u.
As given in the Appendix 3, the New England IEEE 39 bus system consists of the 10-machines, 39-bus system and 46 transmission branches. From the Appendix 4, IEEE 6 Bus system contains two generator and four load buses, seven transmission lines. Bus 1 is the swing bus, bus 2 is a generator bus, and the reactive power compensation buses are 3 and 6. The two branches with tap-setting transformer are branch 1-4 and 6-5.

From the Appendix 5, IEEE 9 Bus system contains three generator buses including the swing bus and six load buses, nine transmission lines. The three branches with tap-setting transformer are connected in the system. The tap settings branches are 1-4, 2-7 and 3-9. The mathematical models of various FACTS are detailed in the Appendix 6.

2.13 SUMMARY

Nowadays, demand for electricity is rapidly increasing. Environmental concerns, restrictions of new planning and economic problem necessitates enhancement of loadability of existing power transmission network. FACTS devices are used to enhance the stability and secure operation. Among the FACTS devices, TCSC, SVC, SSSC, TCPAR, UPFC and IPFC are commonly employed for voltage stability and loadability enhancement. Various computational algorithms like BFA, MBFA, ACO, PSO and RBFA are evolved to solve multi-objective optimization problem, but BFA’s improved version, RBFA was not applied for multi-objective optimization problem earlier. Hence, the present work, the RBFA is focused for solving power system problems. The performance of RBFA is compared with the other algorithms, the RBFA and its upgrading is focused to solve the multi-objective OPD problems. The next chapter presents a detailed view of the novel Refined Bacterial Foraging Algorithm.