Chapter 6

Application of SRR Loaded CPW Medium as Multi Frequency Notch Filter
6.1 INTRODUCTION

Potential applications of SRR in passive microwave planar circuits [1-3] and antennas [4-5] are gaining popularity of late. Because of the existence of the narrow band licensed spectrum like Wi-Max, W-LAN within the UWB spectrum recommended by FCC [6], such narrow band notch filters are becoming very popular for rejecting those narrow band spectra for UWB systems. Since SRR loaded coplanar waveguide gives a sharp dip in and around its magnetic resonance frequency, such structures have the inherent potential of frequency rejection. The main advantage of such SRR loaded CPW based filters are very low pass band loss, sharp transition from pass band to stop band and compactness [7], [8]. Another advantage of such structures is that they can be very easily improvised for designing multi notch filters by exploiting the multiple resonances of SSRs loaded on the back side of the planar CPW lines.

Chapters 2, 3 and 4 of this dissertation clearly have shown that the geometrical parameters of SRR ($r_{ext}$, $a_{ext}$, $c$, $d$ and $g$) and substrate dielectric constant $\epsilon_r$ controls SRRs magnetic resonance frequency. However apart from these structural parameters of the SRRs, the angular orientations of the two split gaps of the two rings of the SRR can be another tuning parameter to achieve multiple resonances. Because of the

![Schematic view of a linear array of circular SRR with gradually rotated inner ring loaded in CPW medium.](image)

**Fig. 6.1** Schematic view of a linear array of circular SRR with gradually rotated inner ring loaded in CPW medium.

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rotational symmetry of the configuration, the circular geometry is an automatic choice for using this parameter. Figure 6.1 shows a schematic diagram with varying angular orientations of the inner ring for shifting the individual resonances. The detailed analysis addressing the effect of such angular rotation is studied in chapter 2 and it can be recalled here that due to the increased angular rotation \( \theta \) (the conventional SRR with two splits on the diametrically opposite sides corresponds to \( \theta = 0^\circ \)) the SRRs resonance frequency increases. The same idea is presented in the schematic diagram of Fig. 6.1 in which the left most SRR pair is the conventional one corresponding to minimum resonance frequency. The 2\(^{nd}\) and 3\(^{rd}\) SRR pairs will give resonances at higher frequencies because of increased value of angular rotation \( \theta \).

Fig. 6.2 Schematic view of a linear array of Square SRR of varying dimension loaded in CPW medium.

Fig. 6.2 shows a schematic diagram of multiple square SRR pairs loaded impedance matched CPW lines. A similar schematic with hexagonal SRR loading is presented in Fig. 6.3. The detailed theoretical analysis and subsequently verified by simulation and experimental measurements for such configuration is addressed in chapters 3 and 4, respectively. As predicted in those chapters, the SRRs, depending on their geometrical configurations and dimension will have a magnetic resonance frequency
at which there will be maximum interaction of the propagating EM signal and the SRRs. Due to this interaction at resonance, there is a sharp fall in transmission and a corresponding notch is observed in $S_{21}$ plot. Since the size of the staggered SRR pairs is gradually varying they are excited at different frequencies and the structure will effectively act as multiple notch system and therefore can be efficiently used as multi notch filter. One can criticize this proposition because of the extremely narrow rejection bands of such structure. However this limitation can be mitigated with precise variations of SRR dimensions. This will cause the shift in the corresponding resonances by a very small amount and therefore the combined structure can give rise to a wider rejection band. So another merits of the structures presented in this chapter is their use for dual operation: multi notch filtering and achieving wider rejection band. Because of the versatility in designing the individual resonators with the theoretical design equations developed in chapters 2, 3, and 4 with all the configurations (circular, square and hexagonal ), any practical designer will have lot of choices to shift the resonances for such applications. This chapter deals with the design of multi notch filters employing the SRR resonances of all the configurations.

Fig. 6.3 Schematic view of a linear array of hexagonal SRR of varying dimension loaded in CPW medium.
introduced here. Prototypes for all the configurations are fabricated and measured giving an excellent matching with theoretically computed and simulated [9] results.

6.2 Basic Geometry and Dimensions
As already discussed in section 6.1, the chapter deals with all the SRR geometries and their theoretical designs for achieving multi notch and wide band rejection purpose. Section 6.2.1 and 6.2.2 discuss the detailed structural parameters of the fabricated prototypes of the rotational circular SRR, square SRR and hexagonal SRR loaded CPW lines considered for achieving these functionality.

6.2.1 Multi Notch Filter

Circular SRR Array

Figure 6.4 and 6.5 show the fabricated prototype of two SRR pairs with circular geometry. For both the structures the first (left most) SRR pair is conventional with $\theta = 0^\circ$ while the second one differs in angular orientation of $\theta = 60^\circ$ and $90^\circ$ respectively. Both the prototypes are fabricated on 3 M substrate with $\varepsilon_r = 2.33$ and $h$

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Fig. 6.4 Two element C-SRR linear array with rotated inner ring for multi notch filter application. For first SRR pair $\theta = 0^\circ$, for the second pair $\theta = 60^\circ$, $r_{cu} = 2.2\text{mm}$, $c = 0.5\text{ mm}$, $d = 0.2\text{mm}$, $g = 0.2\text{ mm}$, $\varepsilon_r = 2.33$.
Fig. 6.5 Two element C-SRR linear array with rotated inner ring for multi notch filter application. For first SRR pair $\theta = 0^\circ$, for the second pair $\theta = 90^\circ$, $r_{ex} = 2.2\text{mm}$, $c = 0.5\text{mm}$, $d = 0.2\text{mm}$, $g = 0.2\text{mm}$, $\varepsilon_r = 2.33$.

Fig. 6.6 Three element C-SRR linear array with rotated inner ring for multi notch filter application. For first SRR pair $\theta = 0^\circ$, for the second pair $\theta = 60^\circ$, for the third pair $\theta = 90^\circ$, $r_{ex} = 2.2\text{mm}$, $c = 0.5\text{mm}$, $d = 0.2\text{mm}$, $g = 0.2\text{mm}$, $\varepsilon_r = 2.33$.

$= 1.575\text{mm}$ and having dimensions $a_{ex} = 2.2\text{mm}$, $c = 0.5\text{mm}$, $d = 0.2\text{mm}$ and $g = 0.2\text{mm}$. The coplanar waveguide is designed to provide nearly $50\Omega$ matched impedance. The width of the central strip is $5\text{mm}$, while the slot widths between the central strips and two ground planes are $0.5\text{mm}$ each. Because of two elements
resonating at different frequencies the structure is supposed to provide double notch characteristics as will be revealed in the result section. This is due to increase in resonance frequency in rotational SRR as discussed in chapter 2. Figure 6.6 shows a three element C-SRR linear array with different angular orientations $\theta = 0^\circ$, $\theta = 60^\circ$ and $\theta = 90^\circ$ respectively. For all of them other geometrical parameters are same and are as specified in Fig. 6.6. Because of varying angular rotation in three pairs, the configuration will provide three resonances. This is discussed in the result section of this chapter. For designing such array another consideration is to make them isolated by sufficient distance so as to nullify the effect of mutual inductance. Otherwise two resonances can be mutually reinforced and give an unwanted resonance profile.

**Square SRR Array:**

Figure 6.7 shows the fabricated prototypes of three pair of square SRR loaded coplanar waveguide. The substrate material and the CPW dimension is as mentioned earlier. For each pair of SRRs, geometrical parameters are varied judiciously to obtain three distinct notch frequencies. This can be achieved by varying any parameters like $a_{ext}$, $c$, $d$ and $g$. In the present design the $a_{ext}$ and $g$ are varied to achieve three distinct resonances. The detailed design parameters are summarized in the TABLE 6.1

<table>
<thead>
<tr>
<th>TABLE 6.1</th>
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<tbody>
<tr>
<td>Dimensions of the S-SSR elements for multi notch application</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Element</th>
<th>$a_{ext}$ (mm)</th>
<th>$c$ (mm)</th>
<th>$d$ (mm)</th>
<th>$g$ (mm)</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>2.6</td>
<td>0.35</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>2.2</td>
<td>0.35</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>0.35</td>
<td>0.6</td>
<td>0.4</td>
</tr>
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</table>

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The SRR pairs are separated (distance between two adjacent outer rings along the length of the array) by 0.5mm along the length of the array.

6.2.2 Enhancing the Rejection Bandwidth

To enhance the rejection band of SRR loaded CPW line, the geometrical parameters of the loaded elements are varied very smoothly so that their resonances are very close to each other. Due to the proximity of the multiple resonances, the configuration will contribute to a wider rejection band. Figs. 6.8 and 6.9 show the fabricated prototypes of two such structures having square and hexagonal geometry, designed with this concept. In Fig. 6.8 three pairs of square SRR are loaded symmetrically with slightly varying $a_{ext}$ and $g$, keeping $c$ and $d$ constant to 0.35mm 0.6mm respectively. For the first pair $a_{ext} = 2.7$mm and $g = 0.1$mm, for the second one $a_{ext} = 2.6$mm and $g = 0.3$mm and for the third pair $a_{ext} = 2.55$mm and $g = 0.6$mm. Due to the gradual
Fig. 6.8 Three element S-SRR linear array of varying dimension for frequency rejection band. For the first SRR pair: $\alpha_{ext} = 2.7\text{mm}$, $g = 0.1\text{mm}$; for the second pair: $\alpha_{ext} = 2.6\text{mm}$, $g = 0.3\text{mm}$; for the third pair: $\alpha_{ext} = 2.55\text{mm}$, $g = 0.6\text{mm}$, $c = 0.35\text{mm}$, $d = 0.6\text{mm}$.

Fig. 6.9 Two element H-SRR linear array of varying dimension for notch filter application. For the first SRR pair: $\alpha_{ext} = 2.6\text{mm}$; for the second pair: $\alpha_{ext} = 2.65\text{mm}$, $c = 0.6\text{mm}$, $d = 0.2\text{mm}$, $g = 0.2\text{mm}$.
decrease of $a_{ext}$ and simultaneous increase of the split gap $g$ for the successive SRR pairs their resonances will be slowly increasing as desired. The corresponding measured and simulated results are elaborated in the next section. In Fig. 6.9 two HSRR pairs of slightly varying $a_{ext}$ are printed on the back side of the CPW line. Their dimensions are:

H-SRR pair 1: $a_{ext} = 2.6$mm, $c = 0.6$mm, $d = 0.2$mm, $g = 0.2$mm

H- SRR pair 2: $a_{ext} = 2.65$mm, $c = 0.6$mm, $d = 0.2$mm, $g = 0.2$mm

This structure because will have dual resonances because different $a_{ext}$ choices but their separation will be narrow as the variation is fine. This gives rise to a wider rejection band and the corresponding results are discussed in the next section.

6.3 Measured and Simulated Results

The fabricated prototypes of different shapes, sizes and configuration described in previous section are measured using VNA (Agilent E8363B). The measured S parameter result exhibits an excellent matching when compared with the theoretical results. All the prototypes are simulated in [9] in a commercial EM solver tools yielding a great correspondence between the two. However, as the number of SRR pairs increases the simulation mesh number increases and the run time increases; and to handle such complexity more efficient computer memory is required which was beyond our scope. Due to some constrain in this regard, in few cases the closeness of measured and simulated result is not that good, though the overall correspondence is quite impressive. Figure 6.10 shows the measured and simulated reflection and transmission parameters for the circular SRR array of Fig. 6.4. Due to the dual resonances contributed by two pairs, corresponding to $\theta = 0^0$ and $\theta = 60^0$, two distinct
notches are observed both in the measured and simulated $S_{21}$ curve of Fig. 6.10. The

![Graph showing $S_{21}$ and $S_{11}$ curves.](image)

Fig. 6.10 Measured and simulated $S_{21}$ and $S_{11}$ of two element C-SRR linear array of varying angular orientations of $\theta = 0^\circ$ and $\theta = 60^\circ$. The dimensions are as given in Fig. 6.4

first resonance measured at 7.62 GHz corresponds to the un-rotated SRR while the
second one measured at 8.28 GHz is due to the rotational SRR with $\theta = 60^\circ$. The corresponding simulated resonance frequencies are 7.71 GHz and 8.25 GHz respectively exhibiting an excellent matching between measured and simulated data.

It should be noted here that the theoretical computation for rotational SRRs in chapter 2, suggests the resonance frequencies for the pairs 7.72 and 8.18 GHz respectively. Considering this as the reference the error for the measured data is 1.1% and 1.3% for conventional and rotational SRRs respectively. The corresponding simulations errors are 0.12% and 0.9% respectively. The $S_{11}$ curve shows peaks at resonance corresponding to high reflection (around -2 dB). Figure 6.11 shows the measured and simulated reflection and transmission parameters of another fabricated prototype with
a different set of angular orientations of $\theta = 0^\circ$ and $\theta = 90^\circ$. As revealed from the plot the first resonance corresponding to un-rotated SRR is unchanged (compared to Fig. 6.10) but the second one is shifted to higher side for higher angular rotation of $90^\circ$. Here too an excellent matching is obtained between measured and simulated results.

Figure 6.12 shows a similar plot of simulated and measured transmission and reflection parameters with three C-SRRs pairs with angular orientations of $\theta = 0^\circ$, $\theta = 60^\circ$ and $\theta = 90^\circ$ respectively. Here theoretically three notches in $S_{21}$ are expected. Measured and Simulated results deviate a bit from the theoretical prediction. The first resonance is clear from both measured and simulated plot. The next two resonances have actually overlapped with each other due the mutual reinforcement. This mismatch can be caused by some fabricational mismatch and simulation inaccuracy.
Fig. 6.12 Measured and simulated $S_{21}$ and $S_{11}$ of three element C-SRR linear array of varying angular orientation of $\theta = 0^\circ$, $\theta = 60^\circ$ and $\theta = 90^\circ$. The dimensions are as given in Fig. 6.6.

Figure 6.13 shows a screen shot from the VNA used to measure the transmission parameter ($S_{21}$ in dB) for the three pairs of S-SRR array symmetrically loaded CPW lines. The measured $S_{21}$ trace clearly show three distinct notches contributed by each pair of different dimensions. The first notch at 5.8 GHz is contributed by left most SRR pair (The configuration is shown in inset of Fig. 6.13) of highest size. The next notch at a frequency of 6.94 GHz is due to the middle pair of intermediate size and last notch at 9 GHz is due to resonance of the smallest SRR pair of the configuration. Because of the judicial choice of the SRR dimension, the individual resonance is sufficiently isolated and unperturbed by any mutual coupling. Figure 6.14 show the measured and simulated transmission and reflections parameters in the same graph.
Fig. 6.13 Measured $S_{21}$ of three element S-SRR linear array of varying dimension for triple notch application. The dimensions are as given in Fig. 6.7

Fig. 6.14 Measured and simulated reflection and transmission of three element S-SRR linear array of varying dimension for triple notch application. The dimensions are as given in Fig. 6.7
Fig. 6.15 Current distributions on the SRRs array loaded CPW medium at different frequencies and the scale, (a) at $f = 5.96$ GHz (b) at $f = 7.27$ GHz (c) at $f = 9.39$ GHz

The plot shows a reasonably good correspondence between measured and simulated data. Another method to confirm the proposed concept is to identify the resonances contributed by each pairs, by observing the induced surface currents at different frequencies. This is shown in Fig. 6.15 at three different frequencies, corresponding to
the three resonances, contributed by three SRR pairs. In Fig. 6.15 (a) the induced surface current is plotted at $f = 5.96$ GHz, the resonance frequency corresponding to the largest size SRR pair. This is confirmed from Fig. 6.15 (a), which clearly indicates the excitation of the left most SRR pair (largest size) due to which the sharp dip is contributed. At this frequency other SRR pairs are not excited as their resonance is far away from this frequency. Similarly Fig. 6.15 (b) indicates the intermediate SRR is strongly excited by the incoming signal, other two remaining passive. This is a clear indication of the fact that the second resonance is contributed by the intermediate SRR pair (medium sized SRR pair). Similarly from Fig. 6.15 (c) we can infer that the third resonance is contributed by third SRR pair (right most, smallest size SRRs). The

![Graph showing measured and simulated reflection and transmission of three element S-SRR linear array of varying dimensional for wide rejection band. The dimensions are as given in fig. 6.8.](image)

**Fig. 6.16** Measured and simulated reflection and transmission of three element S-SRR linear array of varying dimensional for wide rejection band. The dimensions are as given in fig. 6.8
physical concept of widening the rejection band using multiple SRR pairs loaded CPW lines is discussed in section 6.1. The corresponding design idea and detailed geometrical parameters are discussed in section 6.2. The measured and simulated result for the prototype shown in Fig. 6.9 is shown in Fig. 6.16. Due to the close proximity of the dimensions \((a_{ex} \text{ and } g)\), the three resonances corresponding to three SRR pairs are very close which results in a wider rejection band. The simulated result also matches pretty closely with the measured results. The dotted lines in the figure correspond to -10 dB line for \(S_{21}\) and it can be concluded that configuration provides a 500 MHz rejection band from 5.6 GHz to 6.1 GHz.

Figure 6.17 shows the measured and computed transmission and reflection for the HSRR pair loaded CPW line discussed in Fig. 6.9. This plot also exhibits a -10 dB measured rejection band of 250 MHz and excellent matching between the measured
and simulated S parameters. Though Fig 6.16 and 6.17 shows a wider rejection band the transmission loss is not that low over the band of interest. To overcome this problem the interaction between the CPW and SRRs element needs to be stronger. This can be achieved by using a higher dielectric constant \( (\varepsilon_r) \) and or reducing the substrate height \( (h) \). For first case the SRR-CPW interaction becomes stronger due to more confinement of the electric and magnetic field within the substrate where as in the second case the interaction is high because of the proximity of the SRR with the CPW line. This idea can be successfully used to mitigate the low isolation problem in the stop band. Fig. 6.18 show the simulated \( S_{21} \) and \( S_{11} \) plot for the same geometrical parameters of three pair S-SRR array of Fig. 6.16 but with a substrate having a high dielectric constant of \( \varepsilon_r = 13 \) and \( h = 0.49 \text{mm} \). Figure 6.18 shows a deep in \( S_{21} \) profile.

![Simulated reflection and transmission of two element H-SRR linear array of varying dimensional for frequency notch application. The dimensions are as given in Fig.6.9. \( \varepsilon_r = 13, h = 0.49 \text{mm} \)](image)

**Fig. 6.18** Simulated reflection and transmission of two element H-SRR linear array of varying dimensional for frequency notch application. The dimensions are as given in Fig.6.9. \( \varepsilon_r = 13, h = 0.49 \text{mm} \).
going up to -40 dB and achieves a -30 dB 400 MHz rejection band from 3.82 to 4.22 GHz. The shift in the band with respect to the case of Fig. 6.16 is due to change in the substrate height $h$ and dielectric constant $\varepsilon_r$.

Figure 6.19 shows the surface current profile for this configuration at 4 GHz, the centre of the rejection band. The plot shows all the SRRs are strongly excited due to the close proximity of SRR with the CPW and high dielectric constant of the substrate.

### 6.4 Conclusion

Application of SRR loaded planar transmission line for designing frequency rejection system is thoroughly developed in this chapter. Because of the inherent advantage of higher electromagnetic coupling with SRRs, coplanar waveguide is the optimum
choice as the transmission line. All shapes and configurations of SRR developed in previous chapters are used for designing such novel filters. The design is based on the theoretical models for estimation of magnetic resonance frequency for rotational circular SRRs, square SRRs and hexagonal SRRs developed in chapter 2, 3, and 4 respectively. Two common requirements required for worker in UWB band viz. notching out few small frequency band and widening the rejection band (for rejecting a practical licensed spectrum like Wi-MAX or W-LAN within the UWB spectrum) is addressed here. The advantage of the present work for designing multi frequency notch filter is its strong theoretical background and wide choice of parametric variations for shifting the resonances of SRRs pairs. This is clearly demonstrated with all three SRR configurations by repeated experimental and simulation results for different dimensions and configurations. A series of C-SRR, S-SRR and H-SRR linear array loaded CPW prototypes are fabricated with different parameters of the respective SRR for designing such novel filters. Scattering parameter measurements of those fabricated prototypes in VNA reveals an excellent agreement with the theoretically predicted results. The results obtained from the simulator are also equally impressive and show nice correspondence with the measured results. The proposed designs can be extremely useful and handy for researchers and EM engineers for designing multi notch filter because of its compactness and strong theoretical correlation and flexibility in choosing the SRR configuration and the set of its geometrical parameters to be varied for achieving multiple resonances. The simulated current distributions on different SRR pairs help in visualizing and understanding the concept of multiple resonances at different frequencies for the staggered geometry considered here. Method of achieving a better rejection in the stop band is also discussed and the corresponding results exhibited here strongly
recommends for the practical use of such filters. The theoretical analysis and the
design methodology can be used to customize the rejection band or design the
multiple notches as per the requirement of the designer.
REFERENCES


