Chapter 5

Characterization of SRR Loaded Shorted Coplanar Waveguide for Left Handed (LH) Propagation:

Resonant Frequency and Dispersion Relation
5.1 INTRODUCTION

Chapters 2, 3 and 4 of this dissertation have addressed three different SRR geometries, viz. circular, square and hexagonal SRRs [1-8]. They have been synthesized either by placing inside a rectangular waveguide [9] as described in Chapter 2 or by loading on the back side of a coplanar waveguide [10-12] as described in Chapters 3 and 4. In the former method, the SRRs are strategically placed inside a waveguide chamber with proper excitation for achieving negative permeability. However, to realize actual DNG material, a combination of both negative $\mu$ and negative $\varepsilon$ in the same frequency range is pertinent. This is theoretically possible in a waveguide since the waveguide below its cut-off frequency acts like effective plasmonic medium required for realizing ENG (epsilon negative) material. In that case the cut-off frequency for the dominant $\text{TE}_{10}$ mode becomes the effective plasma frequency ($\omega_p$) below which propagation is prohibited. However, signal below $\omega_p$ would fail to excite the SRRs due to non-propagation of signal below the cut-off frequency. On the other hand, the second method has the extra advantage of being applicable in planar circuitry, easy to fabricate and testing. This

![Fig. 5.1 Schematic view of a Square SRR loaded left handed CPW](image-url)
configuration necessarily does not provide any possibility of achieving negative \( \varepsilon \) unless perturbed otherwise. To do so i.e. achieve a planar double negative medium the previously studied SRR loaded CPW mediums are perturbed. Thin nano wires are placed along lateral direction (along Y axis) periodically along the CPW line so that the shunt wires effectively short circuits the CPW after a regular interval. This configuration of coplanar waveguide periodically loaded with short – circuiting shunt wires is called left handed coplanar waveguide as this short-circuited waveguide will allow propagation of signal in backward direction only due to the reflection of the signal from the metallic short. However, due to the combined electric and magnetic effect of shorting wires and pair of SRR loaded on the back side of the configuration, the structure behaves as a double negative material exhibiting complimentary characteristics as compared to the configuration with only SRR loading. The pass band and stop band are interchanged. Now, around SRR resonance frequency the one dimensional lattice of SRR and wires provides both negative \( \mu \) and \( \varepsilon \) giving rise to a real propagation constant \( k \). Thereby previous stop band around SRRs resonance are changed to pass band and vice versa. Schematic diagrams of these configurations with a linear array of square and hexagonal SRR is shown in Fig. 5.1 and 5.2 respectively.

![Schematic view of a Hexagonal SRR loaded left handed CPW](image.png)

**Fig. 5.2** Schematic view of a Hexagonal SRR loaded left handed CPW
5.2 Electromagnetics of a Double Negative Medium

To establish the concept of wave propagation in left-handed media, let us consider Helmholtz’s equation,

\[ \left( \nabla^2 - \frac{n^2}{c^2} \frac{\partial^2}{\partial t^2} \right) \psi = 0 \]  

(5.1)

where \( n \) is the refractive index, 
\( c \) is the velocity of light in vacuum, 
and \( n^2/c^2 = \mu \varepsilon \)

From eqn.( 5.1) it is observed that since squared refractive index \( n^2 \) is not affected by a simultaneous change of sign in both \( \varepsilon \) and \( \mu \), low-loss left-handed media must be transparent. Also from the same idea it can be concluded that solutions to above equation will remain invariant under a simultaneous change of the signs of \( \varepsilon \) and \( \mu \).

Maxwell’s first-order differential equations for a lossless medium can be expressed as,

\[ \vec{\nabla} \times \vec{E} = -j\omega \mu \vec{H} \]  

(5.2)

\[ \vec{\nabla} \times \vec{H} = j\omega \varepsilon \vec{E} \]  

(5.3)

For plane-wave fields are

\[ \vec{E} = \vec{E}_0 e^{j(\omega t - \vec{k} \cdot \vec{r})} \]
\[ \vec{H} = \vec{H}_0 e^{j(\omega t - \vec{k} \cdot \vec{r})} \]

Equations (5.2) and (5.3) can be written as

\[ \vec{k} \times \vec{E} = \omega \mu \vec{H} \]  

(5.4)

\[ \vec{k} \times \vec{H} = -\omega \varepsilon \vec{E} \]  

(5.5)
For conventional material i.e. materials with $\varepsilon$ and $\mu > 0$, $\vec{E}$, $\vec{H}$ and $\vec{k}$ form a right-handed orthogonal system of vectors.

However, if $\varepsilon < 0$ and $\mu < 0$, then eqns. (5.4) and (5.5) are modified as,

$$\vec{k} \times \vec{E} = -\omega |\mu| \vec{H} \quad (5.6)$$
$$\vec{k} \times \vec{H} = \omega |\varepsilon| \vec{E} \quad (5.7)$$

Equations (5.6) and (5.7) show that now $\vec{E}$, $\vec{H}$ and $\vec{k}$ form a left-handed triplet as shown in Fig. 5.3.

![Fig. 5.3 Orientations of $\vec{E}$, $\vec{H}$ and $\vec{k}$ and $\vec{S}$ for a plane TEM wave in (a) conventional and (b) left-handed medium](image)

Though a left handed triplet is formed between $\vec{E}$, $\vec{H}$ and $\vec{k}$ in a medium with simultaneous negative $\varepsilon$, and $\mu$, the direction of time averaged flux of energy determined by the real part of Poynting vector,

$$\vec{S} = \frac{1}{2} \vec{E} \times \vec{H}^* \quad (5.8)$$

is unaffected by simultaneous negative $\varepsilon$ and $\mu$. Therefore, in a Left handed media,
direction of energy flow and wave front, such propagation is also called backward wave propagation. Another interesting aspect of such medium is anti-parallel phase and group velocity as proved below.

The expression of time-averaged density of energy in a non-dispersive media is given by

\[ U_{nd} = \frac{1}{4} \left\{ \varepsilon |E|^2 + \mu |H|^2 \right\} \]  \hspace{1cm} (5.9)

For \( \varepsilon < 0 \) and \( \mu < 0 \) the energy density becomes negative which is physically invalid. Also we must remember that practically any media other than vacuum must be dispersive. Keeping these two points in mind we modify the expression for the energy density of eqn. (5.9) as [12],

\[ U = \frac{1}{4} \left\{ \frac{\partial (\omega \varepsilon)}{\partial \omega} |E|^2 + \frac{\partial (\omega \mu)}{\partial \omega} |H|^2 \right\} \] \hspace{1cm} (5.10)

where the derivatives are evaluated at the central frequency. Thus the physical requirement of positive energy density requires,

\[ \frac{\partial (\omega \varepsilon)}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial (\omega \mu)}{\partial \omega} > 0 \] \hspace{1cm} (5.11)

This is possible with \( \varepsilon < 0 \) and \( \mu < 0 \) provided

\[ \frac{\partial \varepsilon}{\partial \omega} > \left| \frac{\varepsilon}{\omega} \right| \quad \text{and} \quad \frac{\partial \mu}{\partial \omega} > \left| \frac{\mu}{\omega} \right| \] \hspace{1cm} (5.12)

This suggests that physical left handed medium is highly dispersive.
Differentiating the squared wave number \( k^2 \) with respect to \( \omega \) we get,

\[
\frac{\partial k^2}{\partial \omega} = 2k \frac{\partial k}{\partial \omega} = 2 \frac{\omega}{v_p v_g}
\]  \hspace{1cm} (5.13)

where \( v_p = \frac{\omega}{k} \) and \( v_g = \frac{\partial \omega}{\partial k} \), are phase and group velocities respectively.

Again from \( k^2 = \omega^2 \varepsilon \mu \) we get

\[
\frac{\partial k^2}{\partial \omega} = \omega \varepsilon \frac{\partial (\omega \mu)}{\partial \omega} + \omega \mu \frac{\partial (\omega \varepsilon)}{\partial \omega}
\]  \hspace{1cm} (5.14)

RHS of last equation is negative for \( \varepsilon < 0 \) and \( \mu < 0 \) as \( \frac{\partial (\omega \mu)}{\partial \omega} > 0 \) and \( \frac{\partial (\omega \varepsilon)}{\partial \omega} > 0 \)

From equation (5.13) and (5.14) we get

\[ v_p v_g < 0 \]  \hspace{1cm} (5.15)

This property implies that wave packets and wavefronts travel in opposite direction and can work as an additional proof of backward-wave propagation in physical left handed media.

### 5.3 SRR Loaded in a CPW with Shorting Wires

The proposed left handed CPW loaded with split ring resonators on the back side provides DNG nature around the resonance frequency of SRR. It must be recalled from chapter 1, that for single negative medium no propagation is allowed in the band of negative \( \varepsilon \) or \( \mu \). But with double negative parameters the medium becomes transparent and propagation is allowed in the region of simultaneous negative
parameters. This simple but very much physically insightful idea is proved here by loading a pair of SRR on the back side of a shorted CPW. The shorting wires placed laterally on the CPW acts as the nano wires and provide the negative \( \varepsilon \) below the plasma frequency. The schematic diagram with square and hexagonal SRR is shown in Figs. 5.1 and 5.2, respectively. Figure 5.4 shows the fabricated square SRR and hexagonal SRR loaded on the back side of CPW which is shorted by thin shunt wires of rectangular cross-section. On the other hand the SRRs loaded on the opposite side provide negative \( \mu \) around its resonance frequency. Thus, in the vicinity of SRRs

![Photographs of the fabricated square and hexagonal SRR loaded left handed CPW. (a) Top and back surface with S-SRR loaded LH CPW (b) top and back surface with H-SRR loaded LH CPW](image)

**Fig. 5.4** Photographs of the fabricated square and hexagonal SRR loaded left handed CPW. (a) Top and back surface with S-SRR loaded LH CPW (b) top and back surface with H-SRR loaded LH CPW
resonance the structure exhibit DNG property and should be transparent to the incident EM signal. Thus, previous stop band around SRRs resonance will now be a pass band and previous pass band at off resonant frequencies will now be converted into a stop band. In this chapter this physical idea is verified for two different SRR geometries, square and hexagonal shaped SRRs, using simulations and experiments.

5.4 Theoretical Formulations

5.4.1 Resonant Frequency

The magnetic resonance frequency of square and hexagonal SRRs is derived in chapter 3 and chapter 4 and are given by

\[
f_{0s-\text{SRR}} = \frac{1}{2\pi \sqrt{L_T C_{eq}}} = \frac{1}{2\pi \sqrt{L_T \left[ \left( \frac{2a_{eq}}{2} - \frac{g}{2} \right) C_{pol} + \frac{\varepsilon_s c t}{2g} \right]}} \tag{5.16}
\]

\[
f_{0h-\text{SRR}} = \frac{1}{2\pi \sqrt{L_T C_{eq}}} = \frac{1}{2\pi \sqrt{L_T \left[ \frac{3a_{eq} - g}{2} C_{pol} + \frac{\varepsilon_s c t}{2g} \right]}} \tag{5.17}
\]

Due to the SRRs loaded on the left handed CPW at the magnetic resonance frequency a sharp peak is observed in $S_{21}$. $S_{11}$ plot will exhibit a sharp dip at this frequency corresponding to low reflection. Thus theoretically computed magnetic resonance frequencies using eqns. (5.16) and (5.17) are verified by observing simulated [13] and measured reflection and transmission parameters. These are discussed in details in section 5.5.1 and 5.5.2 of this chapter.
5.4.2 Dispersion Analysis

Chapters 3 and 4 have dealt with dispersion diagram for SRR loaded CPW lines. The presence of a narrow stop band around SRRs resonance frequency was revealed theoretically for different SRR dimensions. The resonance frequency was counter verified by electromagnetic simulation and experimental measurement.

In the present configuration of SRRs loaded on the back side of the left handed CPW, the nature of the dispersion diagram is changed. Here due to double negativity of the medium around SRRs resonance, a small pass band is observed making all other frequencies forbidden. To develop the concept of pass band around the magnetic resonance frequency of the SRRs placed in a left handed CPW medium, the dispersion diagram is obtained using the equivalent circuit model of the SRR and host LH CPW line as done in [10] and [12]. The equivalent circuit model for the present configuration would be identical to that of SRR loaded in CPW lines discussed in Fig. 3.6 of chapter 3, but with addition of shunt inductance to take care of the effect of shunt wires between the signal and ground lines of the CPW. The equivalent circuit model with the inclusion of shunt inductance $L_p$ is shown in Fig. 5.5. $L_p$ is calculated as in [10], [12] and can be used to find the plasma frequency of the structure.

![Fig. 5.5 Lumped equivalent circuit model of the SRR loaded shorted CPW](image-url)
Figure 5.5 shows the lumped element equivalent circuit model of the SRR loaded coplanar waveguide for unit section. This lumped element circuit approach does not violate the concept of distributed parameter in microwave circuits as the electrical size of the SRR at resonance is very small. The present equivalent circuit model does not consider the losses (ohmic and dielectric) and the inter SRRs coupling. From the above equivalent circuit model the dispersion relation derived by [10] and [12] is given by,

\[
\cos \beta l = 1 - \frac{L_p \omega - \frac{1}{C_\omega}}{4L_p} \left\{ \frac{L_s' \omega - \frac{1}{C_s'}}{2L_s' \omega - \frac{1}{C_s'}} \right\}
\]

(5.18)

where

\[
L_s' = \omega_0^2 M^2 C_s
\]

(5.19)

\[
C_s' = L_s' / \omega_0^2 M^2
\]

(5.20)

\[
\omega_0 = \frac{1}{\sqrt{L_s' C_s'}} = \frac{1}{\sqrt{L_s C_s'}}
\]

(5.21)

The shunt wires loaded in the CPW give rise to the plasma frequency which can be evaluated by [12],

\[
f_p = \frac{1}{2\pi} \frac{1}{\sqrt{L_s C_s'}}
\]

(5.22)

The mutual inductance \( M \) and the shunt wire inductance \( L_p \) is determined as that of [12]. Per section CPW inductance and capacitance \( L \) and \( C \), SRRs inductance and capacitance \( L_s \) and \( C_s \) are calculated exactly by the same way as discussed in chapters 3 and 4.
5.5 Experimental and Simulation Results

5.5.1 Scattering Parameters

A conventional coplanar waveguide is a low dispersion planar transmission line suitable for MMIC applications. It is effectively an all pass system with very low attenuation. The reflection ($S_{11}$) and transmission ($S_{21}$) co-efficient for an electromagnetic signal passing through impedance matched CPW medium is shown in Fig 5.6. Here $S_{21}$ indicated by violet color is ‘0’ dB for the entire frequency band of interest indicating a 100% transmittance from one port to another port. Fig. 5.7 indicates the scattering parameters for a LH CPW which is shorted by a shunt wire placed midway in the CPW line. As discussed in section 5.2 of this chapter due to the
shunt wire the all pass normal CPW now rejects the entire band of frequency giving rise to a very low $S_{21}$ (nearly -20 dB). This is clearly demonstrated in Fig. 5.7. Figure 5.8 shows a representative plot of the reflection and transmission parameters of SRR loaded coplanar waveguide. This is discussed in details in chapter 3 and 4 with square and hexagonal SRR respectively. The plot clearly shows a dip in $S_{21}$ at resonance frequency of the SRR. The reflection and transmission parameters shown in figure 5.9 for SRR loaded left handed CPW is just the complement of the same shown in figure 5.8. This result is of particular interest from metamaterial (DNG) or
left handed material perspective. The presence of pass band around the SRR resonance is due to the DNG property achieved by the combination of wire (shunt strip) and split ring resonator. Figure 5.10 show the fabricated prototype of the proposed S-SRR and H-SRR loaded left handed coplanar waveguide. The S-SRR loaded prototype shown in fig. 5.10 (a) was fabricated on Taconic substrate (TLY-3, \( \tan \delta = 0.0009 \)) with \( \varepsilon_r = 2.33 \) and \( h = 1.55 \) mm. The H-SRR loaded prototype was fabricated on 3M laminate with same dielectric constant and substrate height and is shown in Fig. 5.10(b).
Figure 5.11 shows a screen shot of measured $S_{11}$ and $S_{21}$ taken from Vector Network Analyzer for S-SRR loaded LH CPW shown in Fig. 5.10 (a). Figure 5.12 shows the measured (a) $S_{11}$ and (b) $S_{21}$ for a S-SRR loaded LH CPW medium as a function of frequency using VNA. $a_{eff} = 2.5\text{mm}, c = 0.35\text{mm}, d = 0.6\text{mm}, g = 0.7\text{mm}, \varepsilon_r = 2.33$. 

**Fig. 5.11** Measured (a) $S_{11}$ and (b) $S_{21}$ for a S-SRR loaded LH CPW medium as a function of frequency using VNA. $a_{eff} = 2.5\text{mm}, c = 0.35\text{mm}, d = 0.6\text{mm}, g = 0.7\text{mm}, \varepsilon_r = 2.33$. 

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Fig. 5.12 Simulated Reflection and Transmission coefficients of the S-SRR loaded left handed CPW with $\varepsilon_r = 2.33$, $\alpha_{cut} = 2.5 \text{mm}, c = 0.35 \text{ mm}, d = 0.6 \text{ mm}, g = 0.7 \text{ mm}, h = 1.575 \text{ mm}, t = 35 \mu\text{m}$.

Fig. 5.13 Measured and simulated Reflection and Transmission coefficients of the S-SRR loaded left handed CPW with $\varepsilon_r = 2.33$, $\alpha_{cut} = 2.5 \text{ mm}, c = 0.5 \text{ mm}, d = 0.2 \text{ mm}, g = 0.2 \text{ mm}, h = 1.575 \text{ mm}, t = 35 \mu\text{m}$.
0.6mm, \( g = 0.7\)mm and theoretically corresponds to a resonance frequency of 6.28 GHz. The violet line corresponds to the \( S_{21} \) plot exhibiting a sharp peak in transmission at resonance frequency while the red one represents the reflection or \( S_{11} \).

Figure 5.13 shows the measured and simulated reflection and transmissions for the same structure. The simulated and measured resonance frequency observed for the fabricated S-SRR loaded prototypes are 6.34 and 6.39 GHz respectively. Thus an excellent matching is observed between the simulated and measured S parameters.

![Graph showing measured and simulated reflection and transmission parameters](image)

**Fig. 5.14** Measured and simulated Reflection and Transmission coefficients of the H-SRR loaded left handed CPW with \( e = 2.33, \ a_{ext} = 2.5\)mm, \( c = 0.5\) mm, \( d = 0.2\)mm, \( g = 0.2\)mm, \( h = 1.575\)mm, \( t = 35\)um

Figure 5.14 shows a similar plot of the measured and simulated reflection and transmission parameter with H-SRR loading. The fabricated prototype with \( a_{ext} = 2.5\)mm, \( c = 0.5\)mm, \( d = 0.2\)mm and \( g = 0.2\)mm correspond to a computed resonance
frequency of 7 GHz. The measured and simulated S parameter plots shown in Fig. 5.14 yield resonance at 6.79 GHz and 6.82 GHz indicating a good matching.

**5.5.2 Dispersion Diagram**

The theoretical concept of lumped equivalent circuit based dispersion relation developed in section 5.5.3 for SRR loaded LH CPW line is verified here for both square and hexagonal configurations. Figure 5.15 show the dispersion diagram for square SRR loading with $a_{ext} = 2.3$ mm, $c = 0.5$ mm, $d = 0.6$ mm and $g = 0.2$ mm printed on the back side of a LH CPW with substrate height $h = 1.575$ mm and dielectric constant $\varepsilon_r = 2.33$. Here the longitudinal length of the SRR = 4.6 mm. Considering some inter SRR distance for linear array so that their mutual coupling is negligible, we consider the period = 6 mm. This configuration provides:

![Dispersion Diagram](image)

**Fig. 5.15** Dispersion diagram of the S-SRR loaded left handed CPW with $\varepsilon_r = 2.33$, $a_{ext} = 2.3$ mm, $c = 0.5$ mm, $d = 0.6$ mm, $g = 0.2$ mm, $h = 1.575$ mm, $t = 35 \mu m$
\[ f_0 = 7.21 \text{ GHz} \quad L = 1.564 \text{ nH} \quad C = 0.3802 \text{ pF} \]
\[ L_s = 7.74 \text{ nH} \quad C_s = 62.4 \text{ fF} \quad M = 1.38 \text{ nH}, L_p = 0.12 \text{ nH} \]

Plasma frequency for the shunt wires, \( f_p = 30.96 \text{ GHz} \). The plot in figure 5.15 shows a very narrow pass band (7.3476 to 7.3806 GHz) just above the resonance frequency of 7.21 GHz and well below the plasma frequency \( f_p \). This is the small frequency range at which the composite medium of SRR and shunt wire together provides a DNG property resulting in a narrow backward wave pass band. Beyond this band the propagation constant is imaginary corresponding to no propagation or stop band. This is due to single negative constitutive parameter (negative \( \varepsilon_r \) only). Another similar dispersion diagram is shown in figure 5.16 for \( a_{ex} = 2.5 \text{ mm} \). Due to the increased \( a_{ex} \) the space periodicity here is different and chosen as 7 mm.

**Fig. 5.16** Dispersion diagram of the S-SRR loaded left handed CPW with \( \varepsilon_r = 2.33, a_{ex} = 2.5 \text{ mm}, c = 0.5 \text{ mm}, d = 0.2 \text{ mm}, g = 0.2 \text{ mm}, h = 1.575 \text{ mm}, t = 35 \mu \text{m} \)
Corresponding circuit parameters are given by:

\[ f_0 = 5.13 \text{ GHz} \quad L = 1.82 \text{ nH} \quad C = 0.4436 \text{ pF} \]

\[ L_s = 8.76 \text{ nH} \quad C_s = 100.9 \text{ fF} \quad M = 1.38 \text{ nH}, \quad L_p = 0.12 \text{ nH} \]

As theoretically predicted here also the plot shows a pass band from 5.2679-5.2989 GHz just above the resonance frequency of 5.13 GHz.

**Fig. 5.17** Dispersion diagram of the H-SRR loaded left handed CPW with \( \varepsilon_r = 2.33, \quad a_{ext} = 2.1 \text{mm}, c = 0.5 \text{ mm}, d = 0.2 \text{mm}, g = 0.2 \text{mm}, h = 1.575 \text{mm}, t = 35 \mu\text{m} \)

Figure 5.17 and 5.18 show the similar kind of dispersion plots with H-SRR loading on the opposite side of LH CPW for two different \( a_{ext} \) dimensions, 2.1cm and 2.5mm respectively. Both the plots exhibits a narrow pass band around their respective resonance frequencies of 9.02 and 7.00 GHz respectively as revealed from the plots.
Fig. 5.18 Dispersion diagram of the H-SRR loaded left handed CPW with $c_r = 2.33$, $a_{cu} = 2.5\text{mm}$, $c = 0.5\text{ mm}$, $d = 0.2\text{mm}$, $g = 0.2\text{mm}$, $h = 1.575\text{mm}$, $t = 35\text{μm}$

5.6 Conclusion

Characterization of SRR loaded shorted coplanar waveguide for left handed propagation is performed extensively in this chapter. Two SRR configurations, S-SRR and H-SRR, discussed in chapters 3 and 4 have been dealt with. The theoretical dispersion curve obtained using the existing lumped equivalent circuit model and our calculations verify the existence of a narrow backward wave pass band due to double negative constitutive parameters. This is also verified with simple experimental measurements and simulation with both square and hexagonal geometries. Few prototypes of square and hexagonal SRR loaded left handed CPW lines were fabricated and measured using network analyzer. The measured scattering parameters are as per our theoretical predication and also match with reasonably good accuracy.
with electromagnetic simulations. The existence of pass band in the region of simultaneously negative $\varepsilon$ and $\mu$ essentially proves the metamaterial characteristics or left handed propagation. Away from the magnetic resonance frequency of SRR the material is opaque due to the single negative parameter and does not allow propagation. This idea is verified with repeated measurements and simulations. The existence of pass band around the magnetic resonance frequency of the SRR loaded in LH CPW line can also be used for designing narrow band pass filter with very good and sharp isolation between stop and pass band.
REFERENCES


