Chapter 2

Circular Split Ring Resonator (C-SRR)

: Resonant Frequency and Magnetic Polarizability
2.1 INTRODUCTION

Realization of negative permeability was achieved using C-shaped conducting loops printed on a dielectric substrate known as Split Ring Resonator (SRR) by Pendry et al. in 1999 [1]. This was a significant breakthrough since realization of negative permittivity using thin nano-wires operating below plasma frequency, \( \omega_p \), was well established. Composite lattice structures with SRRs and nano-wires could now be used to realize double negative (DNG) media. Pendry and his group [1] described the structure as the first non magnetic resonator capable of exhibiting negative values of magnetic permeability around its resonant frequency. The structures also exhibited high magnetic polarizability around its resonant frequency as stated by Marquez et al. in [2]. The size of these structures is much smaller at around one tenth of free space wavelength at resonance [3]. The SRR and its complementary structure called complementary split ring resonator (CSRR) has found applications in designing narrow and wideband planar microwave filters [4],[5] novel phase shifter [6], microwave power divider [7] etc. Split ring resonator has also been considered as a potential candidate for designing compact high gain antennas [8] and leaky wave antennas [9], [10]. For all such applications, however, the accurate estimation of the resonant frequency of the SRR becomes imperative.

The circular SRR is the most common configuration studied and analyzed by researchers. The structure is formed using two metallic open rings in edge coupled (EC) or broadside coupled (BC) configurations as shown in Figs. 2.1(a) and 2.1 (b), respectively. The merits and demerits of the configurations have been described in Chapter 1. The splitted circular rings are formed with metallic strips of width, \( c \), and radii \( r_0 \) and \( r_{\text{ext}} \) forming the inner and outer rings, respectively with inter ring spacing, \( d \). The splits on the inner and outer rings have identical gap dimensions \( g \) which lies
on diametrically opposite sides of the same axis. When an external magnetic field is applied along the z-axis, an electromotive force appears around the SRR and couples the two metallic rings with the induced current passing from one ring to the other ring through a distributed capacitance formed due to the inter ring spacing. The gap within the rings formed by the splits help to obtain a resonant structure with a much smaller dimension compared to a quarter of a wavelength for a closed, unsplitted ring [3]. Thus the electrical size of the SRR can be considered small compared to the free space wavelength and a quasi-static model is plausible.

A comprehensive analysis of the circular SRR in both the configurations have been presented by Marquez and his group in [2] and in [11]. However, the complexity in the computation of ring inductance and the limitation in accurately predicting the resonant frequency is the motivation in deriving simple close form expression based on a modified equivalent circuit in this chapter. This chapter presents a versatile CAD formulation based on a quasi-static model to compute the resonant frequency and magnetic polarizability of the circular SRR. The model incorporates the effect of the

![Schematic Diagram of Circular SRR](image-url)
capacitance and inductance due to the change in the split gap dimensions yielding more accurate estimation of the resonant frequency and magnetic polarizability of the circular SRR. The presented model is applicable to edge coupled (EC) SRR.

Figure 2.2(a) shows the schematic diagram of a circular SRR and its equivalent circuit model is shown in Fig. 2.2(b). Application of an external magnetic field along the z-axis of the SRR induces an electromotive force around the SRR with induced currents passing from one ring to the other through the inter ring spacing, $d$ and the structure behaves as a $LC$ circuit. As shown in the equivalent circuit in Fig. 2.2, the metallic rings contribute a total inductance, $L_T$ and distributed capacitances $C_1$

![Schematic diagram of a circular SRR and its equivalent circuit model](image)

**Fig. 2.2** (a) Schematic view of a circular split ring resonator formed with metallic rings of width, $c$ having radii, $r_0$ and $r_{ext}$ with inter ring spacing $d$ and split gap dimensions, $g_1 = g_2$, printed on a dielectric substrate having height, $h$ and dielectric constant $\varepsilon_r$. (b) Equivalent Circuit model of a circular SRR shown in (a).

and $C_2$ forming at the two halves of the SRR structure above and below the split gaps. This new equivalent circuit also incorporates the gap capacitances, $C_{g1}$ and $C_{g2}$ formed due to the split within the inner and outer rings, respectively. Measured results are compared with the proposed theory showing good agreement and improvement over previously published models. Results obtained using a commercially available electromagnetic simulator [12] are also compared with the proposed theory and are

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presented in this chapter. Simulated and experimental data [2] are compared with theoretical results computed using the proposed model showing good correspondence. The present formulation show improved correspondence with the measured resonant frequency and magnetic polarizability constant as a function of ring radius, \( r_{\text{ext}} \) and ring width, \( c \) with less than 3% average error whereas previously published results reveal more than 5% average error.

2.2 THEORETICAL FORMULATIONS

2.2.1 Resonant Frequency

The resonant frequency \( f_0 \) of the circular SRR is given by

\[
f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{L_C C_{\text{eq}}}}
\]

(2.1)

where, \( C_{\text{eq}} \) is the total equivalent capacitance of the structure.

Again, from the equivalent circuit of Fig. 2(b), the total equivalent capacitance, \( C_{\text{eq}} \) can be evaluated as

\[
C_{\text{eq}} = \frac{(C_1 + C_{g1})(C_2 + C_{g2})}{(C_1 + C_{g1}) + (C_1 + C_{g2})}
\]

(2.2)

Since the split gaps are of identical dimensions \( g_1 = g_2 = g \), hence the gap capacitances \( C_{g1} = C_{g2} = C_g \) and the series capacitances \( C_1 = C_2 = C_0 \) and therefore eqn. (2.2) is modified as,

\[
C_{\text{eq}} = \frac{(C_0 + C_g)}{2}
\]

(2.3)

Considering a metal thickness, \( t \) of the strip conductors, the gap capacitances \( C_{g1} \) and
\( C \) can be represented as
\[
C_{g_1} = C_{g_2} = C_g = \frac{\varepsilon_0 c t}{g}
\] (2.4)

where, \( c \) and \( t \) are the width and thickness of the metallic rings, respectively and \( \varepsilon_0 \) is the free space permittivity.

The distributed capacitances \( C_1 \) and \( C_2 \) are also a function of the split gap dimensions \( g_1 = g_2 = g \) and the average ring radius \( r_{\text{avg}} \) and is given as
\[
C_0 = C_1 = C_2 = (\pi r_{\text{avg}} - g)C_{\text{pul}}
\] (2.5)

where,
\[
r_{\text{avg}} = r_{\text{ext}} - c - \frac{d}{2}
\] (2.6)

and \( C_{\text{pul}} \) is the capacitance per unit length and is calculated as
\[
C_{\text{pul}} = \sqrt{\varepsilon_r / \varepsilon_0 Z_0}
\] (2.7)

where, \( \varepsilon_0 = 3 \times 10^8 \text{m/s} \) is the velocity of light in free space, \( \varepsilon_r \) is the effective permittivity of the medium and \( Z_0 \) is the characteristic impedance of the line. The effective permittivity \( \varepsilon_r \) can be calculated as [13]
\[
\varepsilon_r = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k')K(k_1)}{K(k)K(k_1)}
\] (2.8)

where,
\[
k = \frac{c/2}{c/2 + d}
\] (2.9)
\[
k_1 = \frac{\sinh(\pi a / 2h)}{\sinh(\pi b / 2h)}
\] (2.10)
\[
a = \frac{c}{2}
\] (2.11)
\[
b = \frac{c}{2} + d
\] (2.12)

and
\[
k' = \sqrt{1 - k^2}
\] (2.13)
$K(k)$ is a complete elliptic function of the first kind and $K(k')$ is its complimentary function. An approximate expression for $K(k)/K(k')$ is given as

$$\frac{K(k)}{K(k')} = \frac{1}{\pi} \ln \left( \frac{2 + \sqrt{k}}{1 - \sqrt{k}} \right) \quad \text{for} \quad 0 \leq k \leq 0.7 \quad (2.14)$$

$$\frac{K(k)}{K(k')} = \frac{1}{\pi} \ln \left( \frac{2 + \sqrt{k}}{1 - \sqrt{k}} \right) \quad \text{for} \quad 0.7 \leq k \leq 1 \quad (2.15)$$

The characteristic impedance $Z_0$ is given as

$$Z_0 = \frac{120\pi}{\sqrt{\varepsilon_r}} \frac{K(k)}{K(k')} \quad (2.16)$$

Substituting the values of $C_0$ and $C_g$ in eqn. (2.3) we get,

$$C_{eq} = \frac{(\pi \sigma_0 - g)C_{pal}}{2} + \frac{\varepsilon_0 c r}{2g} \quad (2.17)$$

Hence, the resonant frequency is computed as

$$f_0 = \frac{1}{2\pi \sqrt{L_T C_{eq}}} = \frac{1}{2\pi \sqrt{L_T \left[ \frac{(\pi \sigma_0 - g)C_{pal}}{2} + \frac{\varepsilon_0 c r}{2g} \right]}} \quad (2.18)$$

A simplified formulation for the evaluation for the total equivalent inductance $L_T$ for a wire of rectangular cross section having finite length $l$ and thickness $c$ is proposed as [14], and [15]

$$L_T = 0.0002l(2.303 \log_{10} \frac{4l}{c} - \gamma) \mu H \quad (2.19)$$

where, the constant $\gamma = 2.451$ for a wire loop of circular geometry. The length $l$ and thickness $c$ are in mm. This inductance is a function of the winding geometry due to the varying current vectors as described in [15]. The evaluation of the wire length $l$ is straightforward as
For close proximity wires at high frequencies, the current is confined to the wire surfaces and effectively reduces the spacing between them [16]. The finite length $l$ is calculated considering a single loop with $r_{ext}$ as the radius.

### 2.2.2 Magnetic Polarizability

Polarizability is an intrinsic property of a medium which describes the response of the medium towards an incident electric or magnetic field [2], [17]. Magnetic polarizability of the SRR is a measurement of magnetic response of the SRR in which a magnetic resonance occurs when a magnetic field vertical to SRR plane is incident. The response of the SRR element to incident EM wave is crucial for metamaterial application. Around the resonant frequency, the magnetic polarizability gives rise to a strong diamagnetic behaviour near and above the SRR resonance [1], [2] and [11].

Neglecting the effect of cross-polarization, which results in a bianisotropic behaviour of using this structure, the experimental determination of the magnetic polarizability, $\alpha_{mm}$ in [2] have been simulated using an EM simulator to determine the resonant frequency, $\omega_0$ and the magnetic polarizability constant $\alpha_0$ where,

$$\alpha_{mm} = \alpha_0 \left( \frac{\omega_0^2}{\omega^2} - 1 \right)^{-1},$$  
(2.21)

and

$$\alpha_0 = \frac{\pi^2 r_{avg}^4}{L_T} \times 10^{-12}.$$  
(2.22)

Here $L_T$ is the total equivalent inductance computed using (2.19) and $r_{avg}$ is the average radius in mm computed using (2.6). The computed resonant frequencies and
magnetic polarizability constants are compared with the simulated and measured results in the following section.

2.3 EXPERIMENTAL AND SIMULATION RESULTS

The proposed theoretical formulations for the evaluation of the resonant frequency and magnetic polarizability were validated using measured results published in [2] and simulated results obtained using an electromagnetic simulator [12]. As proposed in [2], a simulation model was created by placing a single element SRR inside a small circular aperture having radius $R_s$ drilled in a metallic screen and located in the middle of a hollow rectangular waveguide as shown in Fig. 2.3. The waveguide was then excited by using waveports with a $\gamma$-directed E-field propagating along $x$. This configuration yielded a transmission coefficient $|S_{21}|$ as shown in Fig. 2.4 for different variation of $r_{ext}$ placed in a circular aperture with $R_s = 4$mm.

![Fig 2.3 Schematic of the simulation setup for computing $|S_{21}|$ of a circular SRR. The SRR have been placed inside a metallic screen with a circular aperture of radius $R_s = 4$mm.](image)
As shown in the figure, the curve peaks corresponds to the resonant frequency, $\omega_0$ and the dips are denoted by $\omega_d$. This dip in transmission is caused due to the destructive interference between the magnetic dipole associated with the SRR and the magnetic dipole associated with the circular aperture [2]. The magnetic dipole associated with the SRR is rapidly varying with frequency and is positive below the resonant frequency [2]. The magnetic dipole associated with the circular aperture is smoothly varying with frequency and is negative and can be evaluated using Bethe hole theory of diffraction for small holes [18]. At a particular frequency, $\omega_d$, below the resonant frequency, the magnetic dipoles of the hole and the SRR cancel each other and no radiation is transmitted.

With the values of $\omega_0$ and $\omega_d$, we can now compute the measured $\alpha_{\text{meas}}$ as [2]
The computed resonant frequencies for different $r_{\text{ext}}$ values using the present formulation and those presented in [2] are compared with the measured results and are shown in Fig. 2.5. The gap dimension $g$ though not mentioned in [2] is assumed as $g = 0.1\text{mm}$. The present model show excellent agreement with the experimental data for the entire range of $r_{\text{ext}}$ values as revealed in the figure. The computed values are also presented in Table 2.1 along with the present computation of the resonant frequency for different gap dimension, $g$. The average error of the present model is 2.14\% when compared with the simulated values computed for $g = 0.1\text{mm}$.

\[
\alpha_{0,\text{meas}} = \frac{8R_j}{3\mu_0} \left( \frac{\omega_0^2}{\omega_j^2} - 1 \right)
\] (2.23)
Table 2.1 depicts the measured and computed values reported in [2]. Similar comparison of the resonant frequencies of the SRR using both the theoretical model for different strip width, $c$ is depicted in Fig. 2.6 showing excellent correspondence of the present formulation when compared with the measured values.

Table 2.1 Comparison of measured, simulated and computed resonant frequencies for different $r_{ext}$ of the circular SRR. $\varepsilon_r = 2.43$, $c = 0.5\text{mm}$, $d = 0.2\text{mm}$, $h = 0.49\text{mm}$, $t = 35\mu\text{m}$

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Fig. 2.6 Computed, simulated and measured resonant frequency of a circular SRR as a function of $c$. $r_{ext} = 2.6\text{mm}$, $\varepsilon_r = 2.43\text{mm}$, $d = 0.2\text{mm}$, $g = 0.1\text{mm}$, $h = 0.49\text{mm}$, $t = 35\mu\text{m}$. 

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The evaluated values of the total equivalent inductance $L_T$ and the capacitance per unit length $C_{pu}$ using the present formulation for different $r_{ext}$ and gap dimension $g$ of the SRR are shown in Table 2.2. Fig. 2.7 shows the computed and simulated

Table 2.2 Computed values of the total equivalent inductance $L_T$ and the capacitance per unit length $C_{pu}$ for various $r_{ext}$ and different gap dimensions $g$ of the circular SRR. $\varepsilon_r = 2.43$, $c = 0.5\text{mm}$, $d = 0.2\text{mm}$, $h = 0.49\text{mm}$, $t = 35\mu\text{m}$

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<th>$r_{ext}$ (mm)</th>
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<th>$C_{pu}$ (pF/m)</th>
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<td>9.60</td>
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Fig. 2.7 Computed and simulated resonant frequency of a circular SRR as a function of $\varepsilon_r$. $r_{ext} = 2.2\text{mm}$, $r_{ext} = 2.6\text{mm}$, $\varepsilon_r = 2.43$, $c = 0.5\text{mm}$, $d = 0.2\text{mm}$, $g = 0.1\text{mm}$, $h = 0.49\text{mm}$, $t = 35\mu\text{m}$. 

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resonant frequencies of circular SRRs having $r_{ext} = 2.2$ mm and 2.6 mm as a function of dielectric constant $\varepsilon_r$ of the laminate on which they are printed. The plot reveals excellent agreement of the present formulation for SRR printed on laminates with dielectric constant ranging from 2 to 13.

Fig. 2.8 Computed and measured magnetic polarizability constant $\alpha_0$ of a circular SRR as a function of $r_{ext}$. $\varepsilon_r = 2.43$, $c = 0.5$ mm, $d = 0.2$ mm, $g = 0.1$ mm, $h = 0.49$ mm, $t = 35 \mu$m.

The evaluated values for the magnetic polarizability constant $\alpha_0$ using (2.22) are compared with the measured and computed values reported in [2] and are plotted in Figs. 2.8 and 2.9 as a function of SRR radius $r_{ext}$ and strip width $c$ respectively. The theoretical evaluation of the present model show good agreement with the measured results and significant improvement over previously reported formulation.
2.4 Rotational Circular Split Ring Resonators

The split ring resonator with a rotated inner or outer ring was studied by Wang et. al [19] in 2008. When a magnetic field is applied along the z-axis, an electromotive force appears around the SRR and induces current which passes from one ring to the other through the inter ring spacing, \( d \) and the structure behaves as an \( LC \) circuit. A rotational edge coupled circular SRR and its equivalent circuit model is shown in Figs. 2.10 (a) and 2.10 (b) respectively. Present model works with the same geometry and uses simple formulations to accurately estimate the change in resonance frequency of the rotated SRR.
2.4.1 THEORETICAL FORMULATIONS

As shown in the equivalent circuit, the metallic rings contribute a total inductance, $L_T$ and distributed capacitances $C_1$ and $C_2$ forming at the two halves of the SRR structure above and below the split gaps. This new equivalent circuit also incorporates the gap capacitances, $C_{g1}$ and $C_{g2}$ formed due to the split within the inner and outer rings, respectively.

![Fig. 2.10](image)

**Fig. 2.10** (a) Schematic view of a circular split ring resonator with rotated inner ring formed with metallic rings of width, $c$ having radii, $r_0$ and $r_{ex}$, with inter ring spacing $d$ and split gap dimensions, $g_1 = g_2 = g$, printed on a dielectric substrate having height, $h$ and dielectric constant $\varepsilon_r$. (b) Equivalent circuit of SRR with rotated inner ring.

The resonance frequency $f_0$ of the circular SRR, thus is given as

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{1}{L_T C_{eq}}}$$  \hspace{1cm} (2.24)

where, $C_{eq}$ is the total equivalent capacitance of the structure.

For a conventional SRR, the split gaps $g_1$ and $g_2$ lies on the same axis. Hence, $C_1$ and $C_2$, the capacitances of the upper and lower half of the rings are equal. For rotational SRR where the inner ring is rotated by an angle $\theta$, the values of $C_1$ and $C_2$ become unequal. For an angle of rotation $\theta$ of the inner ring, the capacitance of the portion within OA and OB of the SRR as shown in Fig. 2.10(a) is given as
\[ C_1 = (\pi - \theta)r_{\text{avg}}C_{\text{pd}} \quad (2.25) \]

where, \( C_{\text{pd}} \) is the per unit length capacitance between the inner and outer rings of the SRR and the uniform average dimension of the inner and outer rings of the SRR, \( r_{\text{avg}} \) is calculated as

\[ r_{\text{avg}} = r_{\text{ext}} - c - \frac{d}{2} \quad (2.26) \]

The capacitance of the remaining portion of the SRR is given by,

\[ C_2 = (\pi + \theta)r_{\text{avg}}C_{\text{pd}} \quad (2.27) \]

The gap capacitance \( C_g \) and per unit length capacitance \( C_{\text{pd}} \) between the inner and outer rings of the SRR is calculated as equation 2.4 and 2.7 for the conventional circular SRR.

From the equivalent circuit of Fig. 2.9 (b) the equivalent capacitance is given as

\[ C_{\text{eq}} = \frac{(C_1 + C_g)(C_2 + C_g)}{(C_1 + C_g) + (C_1 + C_g)} \quad (2.28) \]

Substituting, \( C_1 \) and \( C_2 \) from equation (2.25) and (2.27) in equation (2.28) we get

\[ C_{\text{eq}} = \frac{[(\pi - \theta)r_{\text{avg}}C_{\text{pd}} + C_g][(\pi + \theta)r_{\text{avg}}C_{\text{pd}} + C_g]}{[(\pi - \theta)r_{\text{avg}}C_{\text{pd}} + C_g] + [(\pi + \theta)r_{\text{avg}}C_{\text{pd}} + C_g]} \quad (2.29) \]

\[ C_{\text{eq}} = \frac{(\pi + q)^2 - \theta^2}{2(\pi + q)} - r_{\text{avg}}C_{\text{pd}} \quad (2.30) \]

where,

\[ q = \frac{C_g}{r_{\text{avg}}C_{\text{pd}}} \quad (2.31) \]

As the angle of rotation increases and the angular separation between the split gap decreases, the area of the portion of rings between OA and OB decreases and the area of the remaining portion increases. This leads to decrease in \( C_1 \) and increase in \( C_2 \).
which results in decrease of the equivalent capacitance $C_{eq}$ of the structure, hence resulting in increase of the resonance frequency of the SRR structure.

So, the resonance frequency of the rotated SRR can be obtained from eqn. (2.24) as

$$f'_0 = \frac{1}{2\pi \sqrt{L_T \left[ \frac{(\pi + q)^2 - \theta^2}{2(\pi + q)} \right] r_{avg} C_{pul}}} \quad (2.32)$$

For conventional SRR, resonance frequency is given by,

$$f_0 = \frac{1}{2\pi \sqrt{L_T \left[ \frac{\pi \epsilon_{avg} C_{pul}}{2} + \frac{\epsilon_0 \epsilon_{ch}}{2g} \right]}} \quad (2.33)$$

$$f_0 = \frac{1}{2\pi \sqrt{L_T \left( \frac{\pi + q}{2} \right) r_{avg} C_{pul}}} \quad (2.34)$$

Simplifying (2.34) we get,

$$f_0 = \frac{1}{2\pi \sqrt{L_T \left( \frac{\pi + q}{2} \right) r_{avg} C_{pul}}} \quad (2.35)$$

A simplified formulation for the evaluation for the total equivalent inductance $L_T$ is already derived in section 2.2.1 of this chapter.

From equation (2.32) and (2.23) we get,

$$\frac{f'_0}{f_0} = \frac{\pi + q}{\sqrt{(\pi + q)^2 - \theta^2}} \quad (2.36)$$

If gap dimension ($g < 0.2\text{mm}$) is very small we may neglect $q$. So we get

$$\frac{f'_0}{f_0} \approx \frac{\pi}{\sqrt{\pi^2 - \theta^2}} \quad (2.37)$$

where, $\theta$ and $\pi$ are in radian. Equation (2.37) is applicable for the rotation of both
2.4.2 EXPERIMENTAL MEASUREMENTS AND SIMULATION RESULTS

The proposed theoretical formulations for the evaluation of the resonance frequency was validated using simulated results obtained using an electromagnetic simulator HFSS v11 and measurements reported in [2] and are presented in this section for different parametric variation. Different methods to excite the magnetic resonance of a single element SRR are proposed in [2], [5] and [20]. When excited by an electromagnetic wave propagating along x-direction with a z-oriented magnetic field result in a transmission coefficient $|S_{21}|$ as shown in Fig. 2.10 for different angles of rotation of inner ring of the SRR. The dip in transmission coefficient $S_{21}$ corresponds to the resonance frequency for different angular separations of $0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ$ and $105^\circ$. For further increase in $\theta$ and thus higher angular rotation beyond $105^\circ$, the two split gaps appear in close proximity yielding very weak resonance diminishing the resonance characteristics of the SRR with less current flowing through the rings.

The computed resonance frequencies for a wide range of angular rotation is plotted against the simulated one and is shown in Fig. 2.11. The plot in Fig. 2.12 explicitly shows the increase in the resonance frequency with the increase in the rotational angle $\theta$. This can be attributed to the decreasing equivalent capacitance ($C_{eq}$) within the rings of the SRR with increasing angle of rotation. The computed plot shows excellent agreement with the simulation data for rotational angle of the inner ring up to $\pm 95^\circ$.
Fig. 2.11 Simulated $S_{21}$ of a circular SRR as a function of frequency for different angle of rotation $\theta$ of the inner ring. $r_{\text{ext}} = 2.2$ mm, $r_o = 1.5$ mm, $\varepsilon_r = 2.43$, $c = 0.5$ mm, $d = 0.2$ mm, $h = 0.49$ mm.

Fig. 2.12 Computed, simulated and measured resonance frequency of a circular SRR as a function of angle of rotation of the inner ring for different $r_{\text{ext}}$. $\varepsilon_r = 2.43$, $c = 0.5$ mm, $d = 0.2$ mm, $g = 0.1$ mm, $h = 0.49$ mm.
Computed results are also compared with the measured datum [11] reported for $\theta = 0^\circ$ (conventional SRR). The simulated current distribution of the inner and outer rings of the circular SRR is studied in Fig. 2.13, for six different angle of rotation. As depicted form the plots, the magnitude of the current distribution on the outer ring is maximum at $\theta = 0^\circ$ and diminishes with increasing $\theta$. For $\theta > 100^\circ$ the excitation of the magnetic resonance of the SRR weakens due to diminishing flow of current in the inner and outer rings. A comparison of the computed resonance frequencies with simulated values for different $r_{\text{ext}}$ is shown in Fig. 2.14. Results for two different angle of rotation, $\theta = 0^\circ$ and $60^\circ$ is reported. The computed values are compared with the measured datum [2] for $\theta = 0^\circ$. The present formulations show better coherence with the measured and simulated data. With increase in $r_{\text{ext}}$ the total inductance ($L_T$) and equivalent capacitance ($C_{eq}$) of the SRR increases thereby decreasing resonance frequency. Similar comparison of the measured, simulated and computed data for two

![Simulated current distribution on the inner and outer rings of a circular SRR for different angle of rotation of the inner ring](image)

Fig. 2.13 Simulated current distribution on the inner and outer rings of a circular SRR for different angle of rotation of the inner ring

different angular rotation, for varying width of the conducting strip, $c$, is shown in Fig. 2.15. The present computation shows excellent match with the simulated and measured values. The variation of the inter ring spacing, $d$, and its effect on the resonance frequency is presented in Figure 2.16.

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Fig. 2.14 Computed and simulated resonance frequency of a circular SRR as a function of \( r_{\text{ext}} \) for different angle of rotation of the inner ring. \( \varepsilon_r = 2.43 \), \( c = 0.5 \text{ mm} \), \( d = 0.2 \text{ mm} \), \( g = 0.1 \text{ mm} \), \( h = 0.49 \text{ mm} \).

Fig. 2.15 Computed and simulated resonance frequency of a circular SRR as a function of \( c \) for different angle of rotation of the inner ring. \( \varepsilon_r = 2.43 \), \( r_{\text{ext}} = 2.6 \text{ mm} \), \( d = 0.2 \text{ mm} \), \( g = 0.1 \text{ mm} \), \( h = 0.49 \text{ mm} \).
The present computation shows excellent agreement with the measured and simulated data. The present formulation has also been verified for prediction of the resonance frequency due to change in split gap dimension, \( g \) of the circular SRR. Figure 2.17 shows the comparison of the computed resonance frequency compared with the simulated values as a function of the split gap dimension, \( g \) for a circular SRR with two different angular rotation of the inner ring.

![Graph showing resonance frequency comparison](image)

**Fig. 2.16** Computed and simulated resonance frequency of a circular SRR as a function of \( d \) for different angle of rotation of the inner ring. \( r_{in} = 2.2 \text{mm}, \varepsilon_r = 2.43, c = 0.5 \text{ mm}, g = 0.1 \text{ mm}, h = 0.49 \text{ mm} \).

The effect of varying dielectric constant on the resonance frequency for different angular rotation \( \theta \) is depicted in Figure 2.18. Staggering multiple SRRs in linear array format on the back side of the CPW medium with gradually shifted angular orientations helps in achieving multiple resonances and a simulated result is plotted in Fig. 2.19.
Fig. 2.17 Computed and simulated resonance frequency of a circular SRR as a function of $g$ for different angle of rotation of the inner ring. $r_{ext} = 2.2\text{mm}$, $\varepsilon_r = 2.43$, $c = 0.5 \text{ mm}$, $d = 0.2\text{mm}$, $h = 0.49 \text{ mm}$.

Fig. 2.18 Computed and simulated resonance frequency of a circular SRR as a function of dielectric constant of substrate $\varepsilon_r$ for different angle of rotation of the inner ring. $r_{ext} = 2.2\text{mm}$, $c = 0.5 \text{ mm}$, $d = 0.2\text{mm}$, $g = 0.1 \text{ mm}$, $h = 0.49 \text{ mm}$.
Fig. 2.19 Simulated $S_{21}$ of a circular SRR one dimensional array as a function of frequency $r_{ext} = 2.4 \text{ mm}$, $r_e = 1.7 \text{ mm}$, $\varepsilon = 2.43$, $c = 0.5 \text{ mm}$, $d = 0.2 \text{ mm}$, $h = 0.49 \text{ mm}$. $g = 0.1 \text{ mm}$. $\theta = 0^\circ$, $60^\circ$ and $90^\circ$.

Fig. 2.20 Extracted magnetic permeability (real) of split ring resonator for different angle of rotation between the rings as a function of frequency. $r_{ext} = 2.4 \text{ mm}$, $c = 0.5 \text{ mm}$, $d = 0.2 \text{ mm}$, $g = 0.1 \text{ mm}$, $\theta = 0^\circ$, $60^\circ$ and $90^\circ$. 

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The extraction of negative permeability is computed using a method proposed by Smith et.al. [21] and the extracted values as a function of frequency is presented in Fig.2.20. An one-dimensional array of SRR is designed and simulated using HFSS v.11 yielding $S_{21}$ (magnitude and phase) and $S_{11}$ (magnitude and phase) values. The effective permeability of the medium is extracted using the values of $S_{21}$ and $S_{11}$ in the closed form expressions of [21]. This is done for different angular orientations of the inner ring $\theta = 0^\circ, 60^\circ$ and $90^\circ$. The resultant permeability curve follows the Lorentzian profile exhibiting negative values around the resonance frequency of the SRR.

2.5 CONCLUSION

Circular SRR is the basic SRR geometry used for metamaterial design and also in designing novel passive planar circuits. The theories available for resonance frequency of circular SRR are based on rigorous mathematical analyses and huge computational steps. A simplified formulation for estimation of the resonant frequency and magnetic polarizability of a circular SRR is proposed in this chapter. The present formulation involving minimum mathematical steps and computational time predicts comparatively more accurate theoretical values than previously published models. The closed form expressions developed in this chapter thus should be very useful to a design engineer working with circular SRR with or without rotated inner ring. Change in the resonance frequency due to change in split gap dimension is also verified using the proposed formulation. The theory is widely applicable for all parametric changes of the SRR geometry. The versatility of the present formulation of this chapter is also checked against the various dielectric constant of the host substrate. Some of the measured data reported earlier have been compared with the
present theory revealing excellent agreement with the present theory. Multiple resonances is achieved using an array of SRR with different angular separation between the gaps and can be used for frequency tunability. The extracted constitutive parameter show negative values of permeability around the resonance of the SRR. The present formulation can be easily implemented for designing tunable SRR structures useful for NIM applications.
REFERENCES


