CHAPTER 6
SPEED CONTROLLER DESIGN FOR AN INDIRECT VECTOR CONTROLLED INDUCTION MOTOR DRIVE

6.1 INTRODUCTION

Nowadays, as a consequence of the important progress in power electronics and micro computing, the control of AC electric machines has seen considerable development and the possibility for industrial application (Hazzab et al 2005). The induction motor, known for its robustness, relatively low cost, reliability and efficiency, is the object of several research works. However the control of induction motor drive presents difficulties because of its high non-linearity and its coupled structure (Mansouri et al 2004). The technique known as vector control, first introduced by Blaschke and Hasse, has resulted in large change in the field of electrical drives. This is because, with this type of approach, the robust induction motor can be controlled for giving better performance. This control strategy can provide the same performance as obtained from a separately excited DC motor (Bousserhane and Hazzab 2006, Meziane et al 2008).

Next, Induction motor is the most used drive in all the industrial speed control applications. In this regard design of speed controller forms a major part of the motor control system. But an induction motor is a higher order, multivariable, nonlinear, uncertain system which seems to be very difficult to control. In practice many nonlinear processes are approximated by
reduced order models only, possibly linear which are clearly related to the underlying process characteristics. In this regard a new model order reduction technique is used to obtain the equivalent reduced order model of the inverter fed indirect vector controlled induction motor drive. System model is necessary for tuning controller coefficients in an appropriate manner (e.g., percent overshoot, settling time).

PI controller is the most commonly used control algorithm in industrial drives. The main reason is its relatively simple structure which can be easily understood and implemented in practice and that many sophisticated control strategies such as model predictive control, are based on it. In spite of its widespread use there exists no generally accepted design method for the controller (Wang and Shao 2000). This linear regulator is depends only on two parameters, namely the proportional gain ($K_p$) and the integral gain ($K_i$). In this work, the initial values of controller coefficient are obtained from reduced order model of the system with the help of pole zero cancellation technique. The obtained controller coefficients are tuned till the design specifications are meet out. The tuned controller is connected with the original system and the closed loop response is observed for stabilization process.

Several design techniques are used to obtain a perfect controller characteristic using artificial intelligent schemes and particle swarm technique (Nagaraj et al 2008). The designing includes new control schemes or betterment of existing controller by tuning them. One such popular existing conventional method of tuning is Ziegler Nichols (Z-N). This method is applied even when the transfer function of the system is unknown, but it is only an approximated tuning method which does not give optimized gain values. To overcome the drawbacks of Z-N method artificial intelligence techniques like Fuzzy Logic (FL), Neural Network (NN), Genetic Algorithm (GA) were introduced either offline or online (Arunima Dey et al 2009). GA scheme gives improved responses under normal conditions for vector controlled induction motor drive (Krishnan and Bharadwaj 1991).
The GA methods have been employed successfully to solve complex optimization problems. The use of GA methods in the determination of the different controller parameters is practical due to their fast convergence and reasonable accuracy (Wassim et al 2001). The parameters of the PI controller are determined by the minimization of an objective function. The goal of this work is to show that by the optimization of the parameters of the PI controller, a new class of optimization can be achieved. This can be seen by comparing the results of the Model Order Reduction (MOR) technique based PI controller using genetic algorithm tuned gains and the conventional Symmetric Optimum (SO) approximation method based PI controller.

6.2 INDIRECT VECTOR CONTROLLED INDUCTION MOTOR DRIVE MATHEMATICAL MODEL

Vector (Field oriented) control is widely used in industry for high performance IM drives as the same performance as separately excited DC motor. Here knowledge of synchronous angular velocity is often necessary in phase transformation to achieve favourable decoupling control between motor torque and rotor flux, the same as one used for separately excited DC motor. This is done by one of the two types of vector control, i.e., direct or indirect vector control. Both the methods have been implemented in industrial drives demonstrating performances suitable for a wide range of technological applications. But IM controlled performance is still affected by uncertainty such as mechanical parameter variation, external disturbance and unstructured uncertainty due to non ideal field orientation in a transient state.

In this section the indirect vector control induction motor parameters are derived from the dynamic equations of the induction machine in the synchronously rotating reference frames. To simplify the derivation, a current source inverter is assumed. In that case, the stator phase currents serve as inputs. Hence the stator dynamics can be neglected. In turn this can lead to omitting the stator equations from further consideration.
If the rotor flux linkages used as variables then the rotor circuit equations of the induction machine become

\[
R_s i_{qr}^e + p \dot{\lambda}_{qr}^e + \omega_s \dot{\lambda}_{dr}^e = 0
\]  

(6.1)

\[
R_s i_{dr}^e + p \dot{\lambda}_{dr}^e - \omega_s \dot{\lambda}_{qr}^e = 0
\]  

(6.2)

where

\[
\omega_d = \omega_s - \omega_r
\]  

(6.3)

The rotor flux linkage expressions can be given as

\[
\dot{\lambda}_{qr}^e = L_m i_{qr}^e + L_r i_{dr}^e
\]  

(6.4)

\[
\dot{\lambda}_{dr}^e = L_m i_{dr}^e + L_r i_{dr}^e
\]  

(6.5)

where

- \( R_s \) = rotor resistance per phase
- \( L_m \) = magnetizing inductance per phase
- \( L_r \) = rotor inductance per phase referred to stator
- \( i_{dr}^e \) = direct axis rotor current
- \( i_{qr}^e \) = quadrature axis rotor current
- \( p \) = differential operator d/dt.
- \( \omega_d \) = slip speed in rad/sec,
- \( \omega_s \) = electrical stator speed in rad/sec.
- \( \omega_r \) = electrical rotor speed in rad/sec
- \( \dot{\lambda}_{dr}^e \) = direct axis rotor flux linkages and
- \( \dot{\lambda}_{qr}^e \) = quadrature axis rotor flux linkages
The resultant rotor flux linkage, $\lambda_r$, also known as the rotor flux-linkage phasor, is assumed to be on the direct axis to reduce the number of variables in the equations by one. Moreover, it corresponds with the reality that the rotor flux linkages are a single variable. Hence aligning the $d$ axis with rotor flux phasor yields

$$\dot{\lambda}_r = \dot{\lambda}_{d_r}$$  \hspace{1cm} (6.6)
$$\dot{\lambda}_{qr} = 0$$ \hspace{1cm} (6.7)
$$p \dot{\lambda}_{qr} = 0$$ \hspace{1cm} (6.8)

Substituting equations (6.6) to (6.8) in (6.1) and (6.2) causes the new rotor equations

$$R_r i_{qr}^e + \omega_{sd} \dot{\lambda}_r = 0$$ \hspace{1cm} (6.9)
$$R_r i_{dr}^e + p \dot{\lambda}_r = 0$$ \hspace{1cm} (6.10)

The rotor currents in terms of the stator currents are derived from Equations (6.4) and (6.5) as

$$i_{qr}^e = -\frac{L_m}{L_r} i_{qr}$$ \hspace{1cm} (6.11)
$$i_{dr}^e = \frac{\dot{\lambda}_r}{L_r} - \frac{L_m}{L_r} i_{ds}$$ \hspace{1cm} (6.12)

Substituting for $d$ and $q$ axes rotor currents from Equations (6.11) and (6.12) into equations (6.9) and (6.10), the following are obtained.

$$i_f = \frac{1}{L_m} \left[ 1 + T_{r,p} \right] \dot{\lambda}_r$$ \hspace{1cm} (6.13)
$$\omega_{sd} = K_s \left[ \frac{L_r}{T_r} \frac{T_r}{\dot{\lambda}_r^2} \right] = K_s R_f \left[ \frac{T_r}{\dot{\lambda}_r^2} \right] = \frac{L_m}{T_r} \frac{i_f}{\dot{\lambda}_r}$$ \hspace{1cm} (6.14)
where

\[ i_f = i_{ds}^e \]  \hspace{1cm} (6.15)  \\
\[ i_T = i_{qs}^e \]  \hspace{1cm} (6.16)  \\
\[ T_r = \frac{L_r}{R_r} \]  \hspace{1cm} (6.17)  \\
\[ K_w = \frac{2}{3} \frac{2}{P} \]  \hspace{1cm} (6.18)

The q and d axes currents are relabeled as torque current \( i_T \) and flux current \( i_q \) producing components of the stator current phasor respectively. \( T_r \) denotes the rotor time constant. The Equation (6.13) resembles the field equation in a separately excited dc machine whose time constant is usually on the order of seconds. Likewise the induction motor rotor time constant is also on the order of a second is to be noted.

Similarly by the same substitution of the rotor currents from Equations (6.11) and (6.12) into the torque expression, the electromagnetic torque is derived as

\[ T_e = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} (\dot{\lambda}^e_{dr} i_{qs}^e - \dot{\lambda}^e_{qs} i_{ds}^e) = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} (\dot{\lambda}^e_{dr} i_{qs}^e) = K_w \omega \dot{\lambda}^e_{qs} \] \[ = K_w \lambda^e_{qs} i_T \]  \hspace{1cm} (6.19)

where the torque constant \( K_w \) is defined as

\[ K_w = \frac{3}{2} \frac{P}{2} \frac{L_m}{L_r} \]  \hspace{1cm} (6.20)

The electromagnetic torque of the induction motor is proportional to the product of the rotor flux linkages and the stator q axis current. This resembles the air gap torque expression of dc motor which is proportional to the product of the field flux linkage and the armature current. If the rotor flux linkage is maintained constant then the torque is simply proportional to the torque producing component of the stator current as in the case of the
separately excited dc machine with armature current control where the torque is proportional to the armature current when the field current is constant. Similar to the dc machine armature time constant the rotor time constant is of the order of few milliseconds. The time constant of the torque current is proved to be also on the same order in a later section and is equal to the stator transient time constant. The rotor flux linkages and air gap torque equations given in Equations (6.14) and (6.19) respectively, complete the transformation of the induction machine parameters into an equivalent separately excited dc machine parameters from a control point of view. The stator current phasor which is the phasor sum of the \(d\) and \(q\) axes stator currents in any frames; it is given by

\[
i_s = \sqrt{(i_{qs}^*)^2 - (i_{ds}^*)^2}
\]  

(6.21)

and the \(dq\) axis to \(abc\) phase current relationship is obtained from

\[
\begin{bmatrix}
i_q^* \\
i_d^*
\end{bmatrix} = \frac{2}{3} \begin{bmatrix}
\cos \theta_f & \cos \left( \theta_f - \frac{2\pi}{3} \right) & \cos \left( \theta_f + \frac{2\pi}{3} \right) \\
\sin \theta_f & \sin \left( \theta_f - \frac{2\pi}{3} \right) & \sin \left( \theta_f + \frac{2\pi}{3} \right)
\end{bmatrix} \begin{bmatrix}
i_{as} \\
i_{bs} \\
i_{cs}
\end{bmatrix}
\]

(6.22)

which is compactly expressed as

\[
i_{qd} = [T] [i_{abc}]
\]  

(6.23)

and

\[
i_{qd} = \begin{bmatrix} i_q \\ i_d \end{bmatrix}
\]

(6.24)

\[
i_{abc} = \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}
\]

(6.25)

\[
[T] = \frac{2}{3} \begin{bmatrix}
\cos \theta_f & \cos \left( \theta_f - \frac{2\pi}{3} \right) & \cos \left( \theta_f + \frac{2\pi}{3} \right) \\
\sin \theta_f & \sin \left( \theta_f - \frac{2\pi}{3} \right) & \sin \left( \theta_f + \frac{2\pi}{3} \right)
\end{bmatrix}
\]

(6.26)

where \(i_{as}, \ i_{bs}\) and \(i_{cs}\) are the three phase stator currents.
It is known that the elements in the T matrix are cosinusoidal functions of electrical angle, $\theta_k$. The electrical field angle in this case is that of the rotor flux-linkages phasor and is obtained as the sum of the rotor and slip angles.

$$\theta_f = \theta_r + \theta_{sl}$$  \hspace{1cm} (6.27)

and the slip angle is obtained by integrating the slip speed and is given as

$$\theta_{sl} = \int \omega_{sl} \, dt$$  \hspace{1cm} (6.28)

Further this mathematical model is used to design the speed controller for an indirect vector controlled induction motor drive.

### 6.3 SPEED CONTROLLER DESIGN USING CONVENTIONAL SYMMETRIC OPTIMUM METHOD

The direct vector control method for induction motor is now a days possible by decoupling nonlinear controller as its principles and implementation are very much familiar. This is also applicable to indirect vector control which has made possible the independent control of field flux and torque of the induction machine. Torque control and speed control of induction machine are frequently implemented by industries in a large number of applications. For speed regulation of induction motor which finds numerous applications included in the speed control circuit then the design of the speed controller is of importance. For speed control application the speed signal is considered in the outer loop of the proposed control system. An analytical method using the transfer function is considered in the design of the speed controller.
The vector controller modifies the induction motor drive into a linear control system irrespective of the magnitude of the input signal when the flux linkages are maintained constant and hence this becomes similar to separately excited dc motor drive in all respects and make the analysis simpler besides the development of the block diagram as well as synthesis of speed controller. This section indicates step by step the systematic development of the transfer function for the speed controlled indirect vector controlled induction motor drive. Similar derivations are possible for the direct vector controller by deriving block diagram of the system. Based on the transfer function the speed controller is designed by using symmetric optimum method. Symmetric optimum is often used to maintain the uniformity of the speed controller design for all ac and dc drive systems in this work.

6.3.1 Block Diagram Development

The transfer functions of the various subsystems such as the induction machine, inverter, speed controller and that of feedback transfer functions are developed step by step by showing the block diagram of indirect vector controlled machine. By block diagram reduction technique the overall block diagram of the induction motor derive is obtained. There is an overlap between torque current feedback loop and induced emf feedback loop. This overlap is decoupled again by block diagram reduction technique, making the inner current loop totally independent of the motor mechanical transfer function. This approach lends itself to a simpler synthesis of the current controller.

The speed controller for the indirect vector controlled induction motor drive is carried out by the key assumption of constant rotor flux linkages. The assumption leads to
\[ \lambda_r = \text{a constant} \quad (6.29) \]

\[ p \dot{\lambda}_r = 0 \quad (6.30) \]

The stator voltage equations of the motor are

\[ V_{qs} = (R_s + L_s p) i_{qs}^e + \omega_s L_s i_{ds}^e + L_m p i_{qr}^e + \omega_s L_m i_{dr}^e \quad (6.31) \]

\[ V_{ds} = -\omega_s L_s i_{qs}^e + (R_s + L_s p) i_{ds}^e - \omega_s L_m i_{qr}^e + L_m p i_{dr}^e \quad (6.32) \]

From the vector controller the following relationships of the rotor q and d axes flux linkages are made use of to recast the stator voltage equations as

\[ i_{qr}^e = \frac{L_m}{L_r} i_{qs}^e \quad (6.33) \]

\[ i_{dr}^e = \frac{\dot{\lambda}_r}{L_r} - \frac{L_m}{L_r} i_{ds}^e \quad (6.34) \]

Substitution of the rotor currents into the stator voltage equations results in

\[ V_{qs} = (R_s + \sigma L_s p) i_{qs}^e + \sigma L_s \omega_s i_{ds}^e + \sigma \frac{L_m}{L_r} \lambda_r \quad (6.35) \]

\[ V_{ds} = (R_s + \sigma L_s p) i_{ds}^e - \sigma L_s \omega_s i_{ds}^e + \frac{L_m}{L_r} p \dot{\lambda}_r \quad (6.36) \]

where \( \sigma \) is the leakage coefficient. It is known that the flux component of the stator current is constant in steady state and that is the d axis stator current in the synchronous frames. Its derivative is also zero giving the following.

\[ i_f = i_{ds}^e \quad (6.37) \]

\[ p i_{ds}^e = 0 \quad (6.38) \]
The torque component of the stator current is the q axis current in the synchronous frames which is given by

\[ i_T = i_{qs} \] (6.39)

Substituting these equations (6.37), (6.38) and (6.39) into the q axis voltage Equation (6.35) and (6.36) gives

\[ V_{qs}^e = (R_s + L_a p)i_T + \omega_s L_s i_f + \omega_s \frac{L_m}{L_r} \lambda_r \] (6.40)

where \( L_a \) is given by

\[ L_a = \sigma L_r = \left( L_r - \frac{L_s^2}{L_r} \right) \] (6.41)

Substituting for \( \lambda_r = L_m i_f \) gives the q axis stator voltage in synchronous reference frames as

\[ V_{qs}^e = (R_s + L_a p)i_T + \omega_s L_a i_f + \omega_s \frac{L_m^2}{L_r} i_f = R_s + L_a p i_T + \omega_s L_s i_f \] (6.42)

The second stator equation does not require the solution of either which will yield \( i_T \) that is the variable under control in the system. The stator frequency is represented as

\[ \omega_s = \omega_r + \omega_d = \omega_r + \frac{i_T}{i_f} \left( \frac{R_r}{L_r} \right) \] (6.43)

The electrical equation of the motor is obtained by substituting for \( \omega_s \) from (6.43),

\[ V_{qs}^e = (R_s + L_a p)i_T + \omega_r (L_s i_f) + \omega_d L_s i_f \]

\[ = (R_s + L_a p)i_T + \omega_r (L_s i_f) + i_T \frac{R_s L_m}{L_r} \]
\[\dot{T} = (R_s + \frac{R_s L_s}{L_p} + L_a p) i_T + \omega_s L_s i_f \quad (6.44)\]

from which the torque component of the stator current is derived as

\[i_T = \frac{V_{qs} - \omega_s L_s i_f}{R_s + \frac{R_s L_s}{L_p} + L_a p} = \frac{K_u}{(1 + sT_a)} \left\{ \frac{V_{qs} - \omega_s L_s i_f}{L_s} \right\} \quad (6.45)\]

where

\[R_u = R_s + \frac{L_u}{L_p} \quad (6.46)\]

\[K_u = \frac{1}{R_u} \quad (6.47)\]

\[T_a = \frac{L_u}{R_u} \quad (6.48)\]

From this block which converts the voltage and speed feedback into the torque current, the electromagnetic torque is written as

\[T_e = K_i i_T \quad (6.49)\]

where the torque constant is defined as

\[K_i = \frac{3 P L_m^2}{2 2 L_s^2} \quad (6.50)\]

The load dynamics can be represented, given the electromagnetic torque and a load torque that is considered to be frictional for this particular case as

\[J \frac{d\omega_m}{dt} + B\omega_m = T_e - T_i = K_i i_T - B_i \omega_m \quad (6.51)\]
which in terms of the electrical rotor speed is derived by multiplying both sides by the pair of poles.

\[
J \frac{d\omega}{dt} + B(\omega, r) = \frac{P}{2} K_r i_r - B_r(\omega, r) \tag{6.52}
\]

and hence the transfer function between the torque and the speed producing current is derived as

\[
\frac{i_r(s)}{\omega_r(s)} = \frac{K_m}{1 + sT_m} \tag{6.53}
\]

where

\[
K_m = \frac{P K_r}{2 B_r}
\]

\[
B_r = B + B_i
\]

\[
T_m = \frac{J}{B_i} \tag{6.54}
\]

The inverter delivers the stator \(q\) axis voltage with a command input that is the error between the torque current reference and the torque current feedback. This current error is amplified through a current controller. The gain of the current controller is considered unity here but any other gain can be incorporated in the subsequent development. The inverter is modeled as a gain, \(K_{in}\) with a time lag of \(T_{in}\). The gain is obtained from the dc link voltage to the inverter, \(V_{dc}\) and maximum control voltage, \(V_{cm}\) as

\[
K_{in} = 0.65 \frac{V_{dc}}{V_{cm}} \tag{6.55}
\]

The constant 0.65 here is introduced to account for the maximum peak fundamental voltage obtainable from the inverter with a given dc link voltage. The torque current error is restricted within the maximum control
voltage, $V_{cm}$. The time lag in the inverter is equal to the average carrier switching cycle time, i.e., half the period and is expressed in terms of the PWM switching frequency as

$$T_{in} = \frac{1}{2f_c} \quad (6.56)$$

As usual a Proportional plus Integral (PI) controller is used to process the speed error between the speed reference and filtered speed feedback signals. The transfer function of the speed controller is given as

$$G_s(s) = \frac{K_s (1 + sT_s)}{sT_s} \quad (6.57)$$

where $K_s$ and $T_s$ are the gain and time constants of the speed controller respectively.

The feedback signals are current and speed which are processed through the first order filters. They are given in the following.

Very little filtering is common in the current feedback signal. The signal gain is denoted by

$$G_c(s) = H_c \quad (6.58)$$

The speed feedback signal is processed through a first order filter given by

$$G_{\omega}(s) = \frac{\omega_m(s)}{\omega_r(s)} = \frac{H_{\omega}}{1 + sT_{\omega}} \quad (6.59)$$

where $H_\omega$ is the gain and $T_\omega$ is the time constant of the speed filter.
The speed filter accepts the speed signal as input and produces a modified speed signal for comparison to the speed reference signal, $\omega_b$. This completes the inclusion of all the subsystems of the vector controlled induction motor drive with constant rotor flux linkages. By incorporating Equations (6.44), (6.45), (6.47) and from (6.55) to (6.59) with the mechanical impedance of the load, speed filter, speed controller and $i_T$ loop, the block diagram shown in Figure 6.1 is derived.

**Figure 6.1** Block diagram of the vector controlled induction motor with constant rotor flux linkages

### 6.3.2 Block Diagram Reduction

The speed signal pickoff point for the electrical system can be moved to the $i_T$ point resulting in the diagram shown in Figure 6.2 (i) which can be further simplified as in Figure 6.2 (ii) where the current closed loop transfer function is

$$G_i(s) = \frac{K_a K_m (1 + s T_m)}{\left\{ (1 + s T_{in}) \left[ \left( 1 + s T_{e} \right) \left( 1 + s T_m \right) + K_a K_b \right] + H_m K_a K_{in} (1 + s T_m) \right\}} \quad (6.60)$$

where the emf constant is given by

$$K_b = K_a L_i \frac{1}{f} \quad (6.61)$$
Figure 6.2 Block diagram Reduction of Figure 6.1
6.3.3 Current Loop Transfer Function

The third order current transfer function \( \frac{\tau_{in}}{H_c i_T} \), can be approximated to a first order transfer function as follows. \( T_{in} \) is usually negligible compared to \( T_1 \), \( T_2 \) and \( T_m \) and in the vicinity of the crossover frequency the following approximations are valid.

\[
1 + sT_{in} \approx 1
\]

(6.62)

\[
(1 + sT_u)(1 + sT_{in}) \approx 1 + s(T_u + T_{in}) \approx 1 + sT_{ar}
\]

(6.63)

where

\[
T_{ar} = T_u + T_{in}
\]

Substitution of these into \( G_i(s) \) results in

\[
G_i(s) = \frac{K_a K_m (1 + sT_m)}{(1 + sT_{ar})(1 + sT_m) + K_a K_b + H_c K_a K_m (1 + sT_m)}
\]

(6.64)

which is written compactly as

\[
G_i(s) = \frac{T_1 T_2 K_a K_m}{T_u T_m} \left( \frac{1 + sT_m}{(1 + sT_1)(1 + sT_2)} \right)
\]

(6.65)

where

\[
- \frac{1}{T_1} - \frac{1}{T_2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

(6.66)

\[
a = T_{ar} T_m
\]

\[
b = T_{ar} T_m + H_c K_a K_m T_m
\]

\[
c = 1 + K_a K_b + H_c K_a K_m
\]
The transfer function $G_i(s)$ is reduced by using the fact that $T_1 < T_2 < T_m$ and by considering the crossover frequency the following approximations are valid.

$$1 + sT_m \simeq sT_m \quad (6.67)$$

$$1 + sT_2 \simeq sT_2 \quad (6.68)$$

Substituting Equations (6.67) and (6.68) to Equation (6.65) gives

$$G_i(s) = \frac{K_i K_m T_1}{T_{ar}} \frac{1}{(1+sT_1)} = \frac{K_i}{1+sT_i} \quad (6.69)$$

where $K_i$ and $T_i$ are the gain and time constants of the simplified current loop transfer function given by

$$K_i = \frac{K_m K_m T_i}{T_{ar}} \quad (6.70)$$

$$T_i = T_1 \quad (6.71)$$

The model reduction of the current loop is necessary to synthesize the speed controller. The loop transfer function of the speed is given by the substitution of this simplified transfer function of the current loop as shown in Figure 6.2 (iii) and by combining all the blocks to obtain the final block diagram as shown in Figure 6.52 (iv).

### 6.3.4 Speed Controller Design

The loop transfer function of the speed loop is given by

$$GH(s) \simeq \frac{K_x K_y}{T_i} \frac{1+sT_i}{s^2(1+sT_m)} \quad (6.72)$$
where approximation \( 1 + sT_m \approx sT_m \) is made and the current loop time constant and speed filter time constant are combined into a single time constant.

\[
T_{cT} = T_{cT} + T_i \quad (6.73)
\]

\[
K_g = K_v K_m \frac{H_{\alpha}}{T_m} \quad (6.74)
\]

The transfer function of the speed to its command is derived as

\[
\frac{\omega_s(s)}{\omega_r(s)} = \frac{1}{H_\omega} \frac{1 + sT_s}{1 + sT_s + \frac{T_s}{K_g K_s} s^2 + \frac{T_s T_{cT}}{K_g K_s} s^3} \quad (6.75)
\]

and by equating the coefficient of the denominator polynomial to the coefficient of the symmetric optimum function \( K_s \) and \( T_s \) can be evaluated. The symmetric optimum function for damping ratio of 0.707 is given by

\[
\frac{\omega_s(s)}{\omega_r(s)} = \frac{1}{H_\omega} \frac{1 + sT_s}{1 + (T_s)s + \left( \frac{3}{8} T_s^2 \right) s^2 + \left( \frac{1}{16} T_s^3 \right) s^3} \quad (6.76)
\]

from which the speed controller constants are derived as

\[
T_s = 6T_{cT} \quad (6.77)
\]

\[
K_s = \frac{4}{9} \frac{1}{K_g T_{cT}} \quad (6.78)
\]
The proportional and integral gains of the speed controller are respectively obtained as

\[ K_p = K_s = \frac{4}{9} \frac{1}{K_g T_{s\alpha}^2} \]  
\[ K_i = \frac{K_s}{T_s} = \frac{2}{27} \frac{1}{K_g T_{s\alpha}^2} \]

The overshoot of the speed on the drive can be suppressed by canceling the zero with the addition of a pole \((1+sT_s)\) in the path of the speed command. The following example is considered to test the validity of the various assumptions made in the derivation of the speed controller design.

### 6.3.5 Example

The induction motor with the inverter and load parameters are given below.

\( I_f = 6 \text{A}, f_c = 2000 \text{ Hz}, B_1 = 0.05, H_{\alpha} = 0.05, T_{\alpha} = 0.002, V_{cm} = 10 \text{V}, \)
\( J = 0.0165 \text{ kg}\cdot\text{m}^2, V_{dc} = 285 \text{V}, H_c = 0.333 \text{ V/A}. \) Symmetric optimum based speed controller is to be designed and the validity of the assumptions in design is verified as follows.

#### Solution

Armature resistance, \( R_a = R_s + R_r \frac{L_s}{L_r} = 0.457 \Omega \)

Armature constant, \( K_a = \frac{1}{R_a} = 2.1885 \)

Armature inductance, \( L_a = L_s - \frac{L_m^2}{L_r} = 0.0047H \)
Armature time constant, \( T_a = \frac{L_a}{R_a} = 0.0104 \text{sec} \)

\( T_m = \frac{J}{B_i} = 0.33 \text{sec} \)

Mechanical time constant,

Mechanical constant, \( K_m = \frac{P K_i}{2 B_i} = 36.224 \)

Torque constant, \( K_t = 3 \frac{P}{2} \frac{L_m^2}{L_r} I_f = 0.9056 \)

Induced emf constant, \( K_b = \frac{P}{2 B_i} I_f L_r I_f = 11.9672 \text{v/rad/sec} \)

\( T_{in} = \frac{T_a}{2} = \frac{1}{2 f_c} = 0.00025 \text{sec} \)

\( K_m = 0.65 \frac{V_{dc}}{V_{cm}} = 18.525 \text{ V/V} \)

\( T_{ar} = T_a + T_{in} = 0.01065 \text{sec} \)

\( T_1 = 0.00074 \text{sec} \)

\( T_2 = 0.1173 \text{sec} \)

**Approximated current loop**

\( K_i = \frac{K_m}{R_a} = 2.8708 \)

\( T_i = T_1 = 0.00074 \text{sec} \)

**Speed controller**

\( K_g = \frac{K_i K_m H_{\alpha i}}{T_m} = 104.10 \)
\[ T_{o2} = T_{o1} + T_i = 0.00274 \text{sec} \]

\[ T_s = 6T_{o2} = 0.0164 \text{sec} \]

\[ K_s = \frac{4}{9K_g T_{o2} = 1.5605} \]

Proportional gain, \( K_{ps} = K_s = 1.5605 \)

Integral gain, \( K_i = \frac{K_s}{T_s} = 95.0655 \)

With these calculated values, the following transfer functions are obtained.

**Exact current loop transfer function,**

\[ G_t(s) = \frac{G_{in}(s)G_{12}(s)}{1 + H_cG_{in}(s)G_{12}(s)} \]

where

\[ G_{in}(s) = \frac{K_{in}}{1 + sT_{in}} \]

\[ G_{12}(s) = \frac{G_1(s)}{1 + G_1(s)G_2(s)} \]

\[ G_1(s) = \frac{K_a}{1 + sT_a} \]

\[ G_2(s) = \frac{K_b}{1 + sT_{in}} \]
The Simplified current loop transfer function is

\[ G_{ia}(s) = \frac{K_i}{1 + sT_i} \]

and exact speed loop transfer function is

\[ G_{\omega e}(s) = \frac{\omega_r(s)}{\dot{\omega}_r(s)} = \frac{G_{\omega e}(s)}{1 + G_f(s)G_{\omega i}(s)} \]

where

\[ G_{\omega e}(s) = \frac{K_m}{(1 + sT_m)} \cdot \frac{K_i(1 + sT_i)}{sT}G_f(s) \]

\[ G_{\omega i}(s) = \frac{H_{\omega i}}{1 + sT_{\omega i}} \]

**Simplified speed loop transfer function**

It is obtained from simplified current loop transfer function and is given by

\[ G_{\omega s}(s) = \frac{G_f(s)}{1 + G_f(s)G_{\omega i}(s)} \]

where

\[ G_f(s) = \frac{K_m}{(1 + sT_m)} \cdot \frac{K_i(1 + sT_i)}{sT_s} \cdot \frac{K_i}{(1 + sT_i)} \]

All these transfer functions are computed and their gain, phase plots and step response are given in Figures 6.3 to 6.8. Even though there seems to be a significant discrepancy between the gains of the simplified and
the exact current loop transfer functions note that their phases are identical in the frequency range of interest. The discrepancy due to the approximation of the current loop from third order to first order has hardly affected the accuracy of the speed loop transfer functions, as is clearly seen from the gain and phase plots. This justifies the assumptions made in various approximations. From step response plot of the exact and simplified current loop and speed loop transfer functions the time domain specifications is listed in Table 6.1 and Table 6.2, respectively.

![Frequency response of exact and simplified current loops](image)

Figure 6.3 Frequency response of exact and simplified current loops
Phase plots for exact and simplified current loops

Figure 6.4 Phase plots of exact and simplified current loops

Step Response of exact and simplified current loops

Figure 6.5 Step response of exact and simplified current loops
### Table 6.1 Comparison of time domain specifications of current loop

<table>
<thead>
<tr>
<th>Strategy of Control</th>
<th>Rise time ( (t_r) ) in sec</th>
<th>Settling time ( (t_s) ) in sec</th>
<th>% Overshoot</th>
<th>Peak amplitude</th>
<th>Peak time in sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact Current loop</td>
<td>0.000398</td>
<td>0.461</td>
<td>180</td>
<td>2.79</td>
<td>0.00259</td>
</tr>
<tr>
<td>Simplified current loop</td>
<td>0.00163</td>
<td>0.0029</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Figure 6.6** Comparison of speed loop frequency responses with exact and simplified current loops
Comparison of speed loop phase plots with exact and simplified current loops

Figure 6.7

Comparison of speed loop step responses with exact and simplified current loops

Figure 6.8
Table 6.2 Comparison of time domain specifications of speed loop

<table>
<thead>
<tr>
<th>Strategy of Control</th>
<th>Rise time (t_r) in sec</th>
<th>Settling time (t_s) in sec</th>
<th>% Overshoot</th>
<th>Peak amplitude</th>
<th>Peak time in sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exact speed loop</td>
<td>0.028</td>
<td>0.26</td>
<td>34.5</td>
<td>26.9</td>
<td>0.0703</td>
</tr>
<tr>
<td>Simplified speed loop</td>
<td>0.0251</td>
<td>0.321</td>
<td>45</td>
<td>29</td>
<td>0.066</td>
</tr>
</tbody>
</table>

6.4 SPEED CONTROLLER DESIGN BY MODEL ORDER REDUCTION METHOD WITH GENETIC ALGORITHM

The design of speed controller for inverter fed Vector controlled Induction Motor (VCIM) drive is quite difficult because it has practically complexity in mathematical models and of higher order. The design of controllers for higher order system involves computationally difficult and cumbersome tasks. Hence there is a need for the design of a higher order system through reduced order models. Here, a model order reduction technique is used for reducing higher order model into reduced order model. The controller designed on the basis of reduced order model should effectively control the original higher order system. A controller is designed for the reduced second order model to meet the desired performance specifications. This controller is attached with the reduced order model and closed loop response is observed. The parameters of the controller are tuned using genetic algorithm optimization technique to obtain a response with desired performance specifications. The tuned controller is attached with the original higher order system and the closed loop response is observed for stabilization process.
Here, a PI type controller is used to correct the motor speed. The proportional term does the job of fast acting correction which will produce a change in the output as quickly as the error arises. The integral action takes a finite time to act but has the capability to make the steady state speed error zero. A further refinement uses the rate of change of error speed to apply an additional correction to the output drive. This is known as Derivative approach. It can be used to give a very fast response to sudden changes in motor speed. In simple PID controllers it becomes difficult to generate a derivative term in the output that has any significant effect on motor speed. It can be deployed to reduce the rapid speed oscillation caused by high proportional gain. However, in many controllers, it is not used. The derivative action causes the noise (random error) in the main signal to be amplified and reflected in the controller output. Hence the most suitable controller for speed control is PI type controller.

6.4.1 Current Loop

Using the parameters in section 6.3.5, the current loop transfer function of the exact system found from Figure 6.2 (i) is

\[
G_i(s) = \frac{13.38s + 40.542}{8.58 \times 10^{-7}s^3 + 3.5171 \times 10^{-3}s^2 + 4.802s + 40.69} \tag{6.81}
\]

This is a third-order system. To reduce the order of the system for analytical design of speed controller, model order reduction technique serves. Using the model order reduction technique (Ramesh et al 2011), the reduced (second) order system \( G_n(s) \) is obtained which is suitable for use in the design of a speed loop.

Hence,

\[
G_n(s) = \frac{3801s + 11520}{s^2 + 1364s + 11560} \tag{6.82}
\]
The step response, gain and phase plots of the exact and reduced current loop transfer functions is shown in Figure 6.9, 6.10 and 6.11.

![Step Response](image1.png)

**Figure 6.9** Step response of exact and reduced current loop transfer functions

![Frequency Response](image2.png)

**Figure 6.10** Frequency response of exact and reduced current loop transfer functions
Figure 6.11 Phase plots for exact and reduced current loop transfer functions

The step response can be analyzed with the help of time domain specifications such as rise time, settling time, overshoot and peak value which are given in Table 6.3. This reduced order current loop transfer function is substituted in the design of the speed controller as follows.

**Table 6.3 Comparison of step response of current loop transfer Functions**

<table>
<thead>
<tr>
<th>Strategy of Control</th>
<th>Rise time ($t_r$) in sec</th>
<th>Settling time ($t_s$) in sec</th>
<th>% Overshoot</th>
<th>Peak amplitude</th>
<th>Peak time in sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original higher order system</td>
<td>0.000398</td>
<td>0.461</td>
<td>180</td>
<td>2.79</td>
<td>0.00259</td>
</tr>
<tr>
<td>Reduced order system</td>
<td>0.000441</td>
<td>0.464</td>
<td>175</td>
<td>2.74</td>
<td>0.00391</td>
</tr>
</tbody>
</table>
6.4.2 Speed Controller

The speed loop with the simplified current loop is shown in Figure 6.12. The open loop speed transfer function with the reduced current loop is given by

\[
G_{m}(s) = G_{n}(s)G_{m}(s) = \frac{137700s + 417300}{0.33s^3 + 451.1s^2 + 5179s + 11560} \tag{6.83}
\]

By using the Pole-Zero cancellation technique the initial values of \(K_p\) and \(K_i\) are obtained from the reduced second order current loop transfer function as:

\[
K_p = 1364, \quad K_i = 11560.
\]

The initial values of \(K_p\) and \(K_i\) obtained through the reduced order model are fine tuned using GA based on the minimal settling time criteria. The resultant values of \(K_p\) and \(K_i\) are obtained as,

\[
K_p = 8.7928, \quad K_i = 18.1848.
\]

These controller gains are used for the design of speed controller for reduced system and exact system.

Figure 6.12 The speed loop with the reduced order current loop
From Figure 6.12, the closed loop speed transfer function with the reduced order current loop is obtained as

$$G_{v(o)}(s) = \frac{\omega_r(s)}{\omega_r'(s)} = \frac{6677s^3 + 3360000s^2 + 11010000s + 2696000}{0.00066s^3 + 1.232s^2 + 461.5s^3 + 172100s^2 + 561900s + 134800}(6.84)$$

![Figure 6.13: The speed loop with the original order current loop](image)

From Figure 6.13, the closed loop speed transfer function with the original order current loop is obtained as

$$G_{v(o)}(s) = \frac{\omega_r(s)}{\omega_r'(s)} = \frac{23.5s^3 + 11830s^2 + 38760s + 9492}{5.662 \times 10^{-6}s^6 + 2.697 \times 10^{-6}s^5 + 0.004338s^4 + 1.624s^3 + 605.8s^2 + 1978s + 474.60}(6.85)$$

The step response of closed loop speed transfer function with the reduced and original order current loop is shown in Figure 6.14. The steady state response of the closed loop speed transfer function with reduced order current loop is exactly matching with that of the original current loop speed transfer function. This can be analyzed with the help of time domain specifications such as rise time, settling time, steady state value and peak value which are given in Table 6.4. The magnitude plot and phase plot of speed transfer function with original and reduced current loop are shown in Figure 6.15 and Figure 6.16 respectively.
Figure 6.14  Step response of speed loop transfer functions with original and reduced current loop transfer function

Table 6.4  Comparison of step response of speed loop with original and reduced current loop

<table>
<thead>
<tr>
<th>Strategy of Control</th>
<th>Rise time ($t_r$) in sec</th>
<th>Settling time ($t_s$) in sec</th>
<th>% Overshoot</th>
<th>Peak amplitude</th>
<th>Peak time in sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Speed loop transfer function with original current loop</td>
<td>0.00268</td>
<td>0.035</td>
<td>31.7</td>
<td>26.3</td>
<td>0.00711</td>
</tr>
<tr>
<td>Speed loop transfer function with reduced current loop</td>
<td>0.00291</td>
<td>0.035</td>
<td>30.3</td>
<td>26.1</td>
<td>0.00711</td>
</tr>
</tbody>
</table>
Figure 6.15  Gain plots of speed loop transfer functions with original and reduced current loop transfer function

Figure 6.16  Phase plots of speed loop transfer functions with original and reduced current loop transfer function
6.5 COMPARISON OF CONVENTIONAL METHOD AND PROPOSED MODEL ORDER REDUCTION TECHNIQUE

Figure 6.17 shows the comparison of step response of speed loop transfer function using original current loop with symmetric optimum principle and model order reduction technique with genetic algorithm tuned controller gains. This can be analyzed with the help of time domain specifications such as rise time, settling time, steady state value and peak value which are given in Table 6.5. The step response of the speed loop transfer function with original current loop using proposed model order reduction technique with genetic algorithm tuned controller gains method gives better time domain specifications than the conventional symmetric optimum principle method. The peak time results state that Genetic Algorithm based PI controller is 9 times lesser than SO PI speed controller. With consideration over the rise time the Genetic Algorithm PI controller is efficient giving 10.45 times lesser time.

![Comparison of speed loop transfer function using original current loop with symmetric optimum and MOR-GA methods](image)

**Figure 6.17** Comparison of speed loop transfer function using original current loop with symmetric optimum and MOR-GA methods
Table 6.5  Comparison of step response of speed loop using original current loop transfer function

<table>
<thead>
<tr>
<th>Strategy of Control</th>
<th>Rise time ($t_r$) in sec</th>
<th>Settling time ($t_s$) in sec</th>
<th>% Overshoot</th>
<th>Peak amplitude</th>
<th>Peak time in sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric optimum principle</td>
<td>0.028</td>
<td>0.26</td>
<td>34.5</td>
<td>26.9</td>
<td>0.0703</td>
</tr>
<tr>
<td>MOR technique with GA tuned gains</td>
<td>0.00268</td>
<td>0.307</td>
<td>31.7</td>
<td>26.3</td>
<td>0.00711</td>
</tr>
</tbody>
</table>

Figure 6.18 shows the comparison of step response of speed loop transfer function using reduced order current loop with symmetric optimum method and model order reduction technique with genetic algorithm tuned controller gains. This can be analyzed with the help of time domain specifications such as rise time, settling time, steady state value and peak value which are given in Table 6.6. It is shows that peak amplitude and percentage of overshoot is considerably reduced with the comparison of conventional symmetric optimum method. It is observed that the conventional method has peak overshoot 45% while that of the proposed method is 30.3%. The rise time for the conventional method is around 0.00251 sec whereas the proposed method has the rise time of 0.00291.
Figure 6.18 Comparison of speed loop transfer function using reduced current loop with symmetric optimum and MOR-GA methods

Table 6.6 Comparison of step response of speed loop using reduced current loop transfer function

<table>
<thead>
<tr>
<th>Strategy of Control</th>
<th>Rise time ($t_r$) in sec</th>
<th>Settling time ($t_s$) in sec</th>
<th>% Overshoot</th>
<th>Peak amplitude</th>
<th>Peak time in sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetric optimum method</td>
<td>0.00251</td>
<td>0.321</td>
<td>45</td>
<td>29</td>
<td>0.066</td>
</tr>
<tr>
<td>MOR technique with GA</td>
<td>0.00291</td>
<td>0.33</td>
<td>30.3</td>
<td>26.1</td>
<td>0.00711</td>
</tr>
</tbody>
</table>
6.6 SUMMARY

In this chapter cross multiplication of polynomials model order reduction method is used to reduce the inverter fed indirect vector controlled induction motor drive higher order system into an equivalent reduced second order system and controllers designed to the reduced second order model. Genetic algorithm optimization tuning technique is used for obtaining optimal coefficients of the reduced order model. The tuned controller is attached with the original higher order system and the closed loop response is observed for stabilization process.

The steady state performance of proposed PI controller with the help of GA has been compared with the conventional (SO) PI controller. It is observed that the conventional symmetric optimum method has peak overshoot 34.5% while that of the proposed method is 31.7%. The rise time for the conventional method is around 0.028sec, whereas the proposed method has the settling time around 0.00268sec. Peak amplitude is also considerably reduced with that of conventional method.

The step response of the speed loop transfer function with reduced order current loop using proposed model order reduction technique with genetic algorithm tuned controller gains method gives better time domain specifications than the conventional symmetric optimum method.