CHAPTER 3

SLOW-SCALE INSTABILITY IN POWER ELECTRONIC CONVERTERS

3.1 INTRODUCTION

Slow-scale bifurcation is a phenomenon characterised by a sudden expansion of a stable fixed point to a long-period limit cycle in phase plane, with much wider amplitude of voltages and currents. As this bifurcation which results in low frequency oscillation in time domain, apparently affect the transfer efficiency and the switching stress of the converters, a better analysis of their nonlinear dynamics to avoid the occurrence of such undesirable bifurcations is vital for any system. In this chapter, the slow-scale bifurcation of a Photovoltaic (PV) powered hysteresis current-controlled two stage cascaded-boost converter which is a popular choice, for solar energy systems is investigated. The stability of the system is studied with the aid of nonlinear state equations derived from an averaged continuous model. Analysis of the describing autonomous equations reveals that the system loses stability via supercritical Hopf bifurcation. Numerical analysis is done by examining the movement of the complex eigenvalues of Jacobian matrix at the equilibrium point by varying the converter parameters. Extensive computer simulations are performed to capture the system’s dynamic behaviour and demarcate the bifurcation boundaries. Furthermore, the possibility of subcritical Hopf bifurcation for variation in input voltage is also examined. Trajectory and Poincaré section before and after the bifurcation are
shown. Experimental results are also provided to confirm the observed bifurcation scenario.

3.2 SOLAR PV POWERED HYSTERESIS CURRENT-CONTROLLED TWO STAGE CASCADED-BOOST CONVERTER

The schematic diagram of a solar powered two stage cascaded-boost converter under free-running current-mode control with an input inductance $L_1$, output inductance $L_2$, transfer capacitance $C_1$, output capacitance $C_2$ and load resistance $R$ is shown in Figure 3.1. A solar PV module with several solar cells connected together to generate more power is used as an input source. A simplified Thevenin’s equivalent circuit of solar cells is used for analysis. For algebraic brevity, the equivalent Thevenin’s resistance is neglected and the PV module voltage is considered as the input voltage $E$.

Figure 3.1 Schematic diagram of a hysteresis current-controlled two stage cascaded-boost converter
Assuming that the converter is operating in CCM, the circuit can be regarded as a variable structure that toggles its topology between two states depending on whether the switch $S$ is open or closed. In free-running cascaded-boost converter, the switch is turned ON and OFF in a hysteretic fashion, when the sum of inductor currents falls below or rises above a preset hysteretic band. The average value of the hysteretic band is set by the output voltage. Figures 3.2(a) and 3.2(b) show the equivalent circuit of a cascaded-boost converter in state 1 (switch closed) and state 2 (switch open) operation respectively. The system parameters and specifications of the PV module considered for analysis are given in Tables 3.1 and 3.2.

![Equivalent circuit of a hysteresis current-controlled cascaded-boost converter](image)

**Figure 3.2** Equivalent circuit of a hysteresis current-controlled cascaded-boost converter when (a) switch $S$ is ON; (b) switch $S$ is OFF

**Table 3.1 System parameters**

<table>
<thead>
<tr>
<th>Parameter Descriptions</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input and output inductance $L_1$, $L_2$</td>
<td>10 mH each</td>
</tr>
<tr>
<td>Transfer and output capacitance $C_1$, $C_2$</td>
<td>1500 μF each</td>
</tr>
<tr>
<td>Load resistance $R$</td>
<td>$50 , \Omega$</td>
</tr>
<tr>
<td>Control parameter $\mu$</td>
<td>0.125</td>
</tr>
</tbody>
</table>
Table 3.2 Specifications of PV module

<table>
<thead>
<tr>
<th>Electrical Characteristics</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum power</td>
<td>100 W</td>
</tr>
<tr>
<td>Voltage at maximum power</td>
<td>15 V</td>
</tr>
<tr>
<td>Current at maximum power</td>
<td>6.67 A</td>
</tr>
<tr>
<td>Module efficiency</td>
<td>15.6%</td>
</tr>
<tr>
<td>Open circuit voltage</td>
<td>22 V</td>
</tr>
<tr>
<td>Temperature coefficient of power</td>
<td>-(0.5±0.05)% / °C</td>
</tr>
<tr>
<td>Temperature coefficient of open circuit voltage</td>
<td>-(160±20)mV/ °C</td>
</tr>
<tr>
<td>Temperature coefficient of short circuit current</td>
<td>(0.065±0.015)% / °C</td>
</tr>
</tbody>
</table>

The three main methods available for the dynamic analysis of a self-oscillating DC-DC converter under hysteresis control are descriptive function method, Tsypkin method and sliding-mode approach. In the first two methods, the analysis of the hysteresis controlled system has to be performed in the frequency domain and by a numerical procedure. However, the sliding-mode approach based on the equivalent control method enables analysis in the time domain using an analytical approach and is used here to provide more insight into the system response.

3.2.1 Variable Structure System Model and Equivalent Control

The two stage cascaded-boost converter can be considered as a variable structure system and is represented by the following state-space equations where \( u = 1 \) when the switch \( S \) is turned ON and \( u = 0 \) when the switch \( S \) is turned OFF.
\[
\begin{align*}
\frac{di_{L1}}{dt} &= \frac{(1-u)}{L}v_{C1} + \frac{E}{L}, \\
\frac{di_{L2}}{dt} &= \frac{v_{C1}}{L} - \frac{(1-u)}{L}v_{C2}, \\
\frac{dv_{C1}}{dt} &= \frac{(1-u)}{C}i_{L1} - i_{L2}, \\
\frac{dv_{C2}}{dt} &= \frac{(1-u)}{C}i_{L2} - \frac{v_{C2}}{RC}. \\
\end{align*}
\] (3.1)

where \(i_{L1}\) and \(i_{L2}\) are the inductor currents and \(v_{C1}\) and \(v_{C2}\) are the capacitor voltages. Equation (3.1) could also be expressed in the following general form:

\[
\dot{x} = (Ax + f) + (Bx + c)u, \\
\] (3.2)

where \(x\) is the state vector given by \(x = [i_{L1} \ i_{L2} \ v_{C1} \ v_{C2}]^T\) and the system matrices \(A, B, c\) and \(f\) are given by:

\[
A = \begin{bmatrix}
0 & 0 & -1/L & 0 \\
0 & 0 & 1/L & -1/L \\
1/C & -1/C & 0 & 0 \\
0 & 1/C & 0 & -1/RC \\
\end{bmatrix}, \quad 
B = \begin{bmatrix}
E \\
0 \\
0 \\
0 \\
\end{bmatrix}, \quad 
f = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}, \quad 
\]

\[
c = 0
\]

The output voltage \(v_{C2}\) is fed back to set the average value of the hysteretic band, forcing the control variable to be related by the following control equation:

\[
i_{L1} + i_{L2} = g(v_{C2}), \\
\] (3.3)
where \( g(v_{C2}) \) is the control function. For a simple proportional controller of gain \( \mu \), the control law takes the form:

\[
\Delta (i_{L1} - i_{L2}) = -\mu \Delta v_{C2}. \tag{3.4}
\]

Assuming regulated output, Equation (3.4) has the following form:

\[
i_{L1} + i_{L2} = K - \mu v_{C2}, \tag{3.5}
\]

where \( \mu \) and \( K \) are the control parameter 1 and 2 respectively. To perform the time-domain analysis using the sliding-mode approach, the sliding surface \( \Sigma \) is defined as \( \Sigma = \{ x/\sigma(x) = 0 \} \), where \( \sigma(x) \) is given by:

\[
\sigma(x) = K - \mu v_{C2} = 0. \tag{3.6}
\]

Since \( i_{L1} + i_{L2} \) is related to \( v_{C2} \) by a linear algebraic Equation (3.5), the order of the system reduces by one as follows making the stability analysis more simple.

\[
\begin{align*}
\frac{di_{L2}}{dt} &= \frac{v_{C1}}{L} - \frac{(1-u)}{L} v_{C2}, \\
\frac{dv_{C1}}{dt} &= \frac{(1-u)(K - \mu v_{C2} - i_{L2})}{C} - \frac{i_{L2}}{C}, \\
\frac{dv_{C2}}{dt} &= \frac{(1-u)}{C} i_{L2} - \frac{v_{C2}}{RC}.
\end{align*} \tag{3.7}
\]

For the sliding-mode to exist, the transversality condition given by the scalar product \( \nabla \sigma(\mathbf{Bx} + c) \neq 0 \) is also verified for our system. In this case, an expression of the continuous equivalent control \( u_{eq} \) is found by differentiating Equation (3.5) and direct substitution of the involving derivatives from Equation (3.7).
\[ u_{eq} = \frac{(v_{c2} - E)}{\alpha} - \frac{\mu L i_{L2}}{C} \left( \frac{v_{c2}}{RC} \right), \]  

(3.8)

where

\[ \alpha = v_{c1} + v_{c2} - \frac{\mu L i_{L2}}{C}. \]

This equivalent control is bounded by the minimum and maximum values of the control signal given by \( 0 < u_{eq} < 1 \). By substituting Equation (3.8) in \( 0 < u_{eq} < 1 \), the region of the design parameter space and the state space where sliding dynamics exist is determined. In this region, the equations governing the dynamics of the system are obtained by substituting the continuous equivalent control signal \( u_{eq} \) given by Equation (3.8) in place of the switched control signal given by Equation (3.7). It is to be noted that the equivalent control found at the equilibrium point corresponds to the duty cycle of the converter.

### 3.2.2 Dimensionless Equations

For algebraic brevity, the state-space equations given above are written in dimensionless form, with the dimensionless state variables defined as:

\[ x_1 = \frac{R i_{L2}}{E}; \quad x_2 = \frac{v_{c1}}{E}; \quad x_3 = \frac{v_{c2}}{E}. \]  

(3.9)

and the dimensionless time and parameters defined as:

\[ \tau = \frac{R t}{2L}; \quad \varphi = \frac{L/R}{CR}; \quad k_0 = \frac{KR}{E}; \quad k_1 = \mu R. \]  

(3.10)
Substitution of these new dimensionless state variables, time and parameters in Equations (3.7) and (3.8) results in the following dimensionless ‘d’ and state equations.

\[
d = \frac{x_3 - 1 - k_i \xi (x_1 - x_4)}{x_2 + x_3 - k_i x_i \xi}, \tag{3.11}
\]

\[
\begin{align*}
\frac{dx_1}{d\tau} &= 2(x_2 - x_3) + 2x_3 \left( \frac{x_3 - 1 - k_i \xi (x_1 - x_4)}{x_2 + x_3 - k_i x_i \xi} \right), \\
\frac{dx_2}{d\tau} &= \frac{2\xi(k_0 - k_i x_3 - x_1)(x_2 + 1 - k_i \xi x_3)}{x_2 + x_3 - k_i x_i \xi} - 2\xi x_1, \\
\frac{dx_3}{d\tau} &= \frac{2\xi x_1(x_2 + 1 - k_i \xi x_3)}{x_2 + x_3 - k_i x_i \xi} - 2\xi x_1. \tag{3.12}
\end{align*}
\]

Since \( d \) is bounded between 0 and 1 in a practical system, PWM saturates for \( d > 1 \) or \( d < 0 \) and the condition for saturation is given by:

\[
d > 1 \Leftrightarrow x_2 + 1 - k_i \xi x_3 < 0, \tag{3.13}
\]

\[
d < 0 \Leftrightarrow 1 + k_i \xi x_1 - x_3(k_i \xi - 1) > 0.
\]

The dimensionless state equations for saturation obtained with the direct substitution of \( d = 1 \) or \( d = 0 \) in Equation (3.6) are

\[
\begin{align*}
\frac{dx_1}{d\tau} &= 2(x_2 - x_3), \\
\frac{dx_2}{d\tau} &= \frac{2\xi(k_0 - k_i x_3 - x_1)}{x_2 + x_3 - k_i x_i \xi} \quad \text{for } d > 0; \\
\frac{dx_3}{d\tau} &= \frac{2\xi x_1}{x_2 + x_3 - k_i x_i \xi}. \tag{3.14}
\end{align*}
\]
\[
\begin{align*}
\frac{dx_1}{d\tau} &= 2x_2, \\
\frac{dx_2}{d\tau} &= -2\xi x_1, \quad \text{for } d > 1. \\
\frac{dx_3}{d\tau} &= -2\xi x_3,
\end{align*}
\] (3.15)

### 3.2.3 Derivation of Equilibrium Point

The equilibrium point \((X_1, X_2, X_3)\) is found by setting all the time-derivatives to zero \((dx_1/d\tau = dx_2/d\tau = dx_3/d\tau = 0)\) in Equation (3.11), which yields the following set of simultaneous equations:

\[
\begin{align*}
 x_2 (x_2 + x_3 - k_1 x_3^\xi) - x_3 (x_2 + 1 - k_1 x_3^\xi) &= 0, \\
 x_1 (x_2 + x_3 - k_1 x_3^\xi) - (x_2 + 1 - k_1 x_3^\xi)(k_0 - k_1 x_3 - x_1) &= 0, \\
 x_3 (x_2 + x_3) - x_1 (x_2 + 1) &= 0.
\end{align*}
\] (3.16)

Solving Equation (3.16) simultaneously using an algorithm written in MATLAB based on Newton’s method and considering only the restricted sign of state variables, the equilibrium point \((X_1, X_2, X_3)\) is found.

### 3.3 Stability of Equilibrium Point and Bifurcation Analysis

The Jacobian matrix which provides a means to evaluate the dynamics of the system is derived in this section to examine the bifurcation phenomena.
3.3.1 Derivation of Jacobian

The Jacobian matrix $J(X)$ for the dimensionless system evaluated at the equilibrium point is given by:

$$J(X) = \begin{bmatrix}
J_{11} & J_{12} & J_{13} \\
J_{21} & J_{22} & J_{23} \\
J_{31} & J_{32} & J_{33}
\end{bmatrix},$$

where

$$J_{11} = -\frac{2X_3 k_1 \xi (X_2 + 1 - k_1 \xi X_3)}{(X_2 + X_3 - k_1 \xi X_1)^2},$$

$$J_{12} = 2 - \frac{2X_3 (X_3 - 1 - k_1 \xi (X_1 - X_3))}{(X_2 + X_3 - k_1 \xi X_1)^2},$$

$$J_{13} = \frac{\left( (X_2 + X_3 - k_1 \xi X_1) (2X_3 + 2X_3 k_1 \xi - 1 - k_1 X_1 \xi) \right)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2},$$

$$J_{21} = -2 + \frac{\left( (X_2 + X_3 - k_1 \xi X_1) (-X_2 - 1 + k_1 \xi X_3) \right)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2},$$

$$J_{22} = \frac{\left( X_2 + X_3 - k_1 \xi X_1 \right) (k_0 - k_1 X_3 - X_1)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2},$$

$$J_{23} = \frac{\left( X_2 + X_3 - k_1 \xi X_1 \right) (k_0 - k_1 X_3 - X_1)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2},$$

$$J_{31} = \frac{\left( X_2 + X_3 - k_1 \xi X_1 \right) (k_0 - k_1 X_3 - X_1)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2},$$

$$J_{32} = \frac{\left( X_2 + X_3 - k_1 \xi X_1 \right) (k_0 - k_1 X_3 - X_1)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2},$$

$$J_{33} = \frac{\left( X_2 + X_3 - k_1 \xi X_1 \right) (k_0 - k_1 X_3 - X_1)}{\left( X_2 + X_3 - k_1 \xi X_1 \right)^2}.$$
3.3.2 Identification of Hopf Bifurcation

The stability of the system and the bifurcation behaviour exhibited by the period-1 limit cycle are then analysed, by studying the movement of the eigenvalues of the Jacobian matrix under the variation of selected parameters. The usual procedure to find the eigenvalues of the system at the equilibrium point is to solve the following equation for $\lambda$.

$$\det[\lambda I - J(X)] = 0.$$  \hspace{1cm} (3.18)

Then the existence of Hopf bifurcation is established, if at the critical parameter value, the following conditions are satisfied by the complex eigenvalue pair:

$$\text{Re}(\lambda)_{\text{crit}} = 0,$$  \hspace{1cm} (3.19)

$$\text{Im}(\lambda)_{\text{crit}} \neq 0.$$  \hspace{1cm} (3.20)
The cascaded-boost converter designed with the values given in Tables 3.1 and 3.2 is examined for its stability numerically. The numerical calculations of eigenvalues performed for the practical range of parameters $6.25 < k_0 < 125$, $6.25 < k_1 < 125$, $0.00053 < \zeta < 0.01$ reveal the presence of one negative real eigenvalue and a complex conjugate eigenvalue pair. The complex eigenvalue pair has either positive or negative real part depending upon the values of $k_0$, $k_1$ and $\zeta$. Table 3.3 shows the variation of the eigenvalues for various values of $k_0$. The following observations are made:

- For small values of $k_0$, the complex eigenvalue pair has a negative real part indicating the fixed point is a stable focus.

- As $k_0$ increases, the real part of the complex eigenvalue pair gets less negative and at a critical value, the real part crosses zero and becomes positive satisfying Equations (3.19) and (3.20), which implies an unstable focus.

- The locus of the complex eigenvalue pair plotted for various values of $k_0$ in Figure 3.3 clearly depicts that their movement does not slide along the imaginary axis ensuring the third condition for Hopf bifurcation given by Equation (3.21).

- The movement of the locus from the left plane to the right plane shows that the system loses its stability when the load is decreased.
Table 3.3 Eigenvalues at $\zeta = 0.00053$

<table>
<thead>
<tr>
<th>$k_0$</th>
<th>$k_1 = 6.25$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>-0.0036, -0.0007±0.0698i</td>
<td>Stable fixed point</td>
</tr>
<tr>
<td>20</td>
<td>-0.0026, -0.0007±0.0518i</td>
<td>Stable fixed point</td>
</tr>
<tr>
<td>30</td>
<td>-0.0024, -0.0006±0.0470i</td>
<td>Stable fixed point</td>
</tr>
<tr>
<td>40</td>
<td>-0.0023, -0.0005±0.0442i</td>
<td>Stable fixed point</td>
</tr>
<tr>
<td>50</td>
<td>-0.0022, -0.0003±0.0422i</td>
<td>Stable fixed point</td>
</tr>
<tr>
<td>60</td>
<td>-0.0021, -0.0002±0.0408i</td>
<td>Stable fixed point</td>
</tr>
<tr>
<td>70</td>
<td>-0.0021, ±0.0396i</td>
<td>Slow-scale bifurcation</td>
</tr>
<tr>
<td>75</td>
<td>-0.0021, 0.0001±0.0391i</td>
<td>Unstable fixed point</td>
</tr>
</tbody>
</table>

Figure 3.3 Loci of the complex eigenvalue pair for variations in $k_0$ of a hysteresis current-controlled cascaded-boost converter

3.3.3 Margin of Stability Curve

The critical value of reference current at which bifurcation occurs, depends on various circuit parameters. But more practically, as the values of
$k_0$ depend on the values of $k_1$ and $\zeta$ the operation boundaries are investigated considering these parameters. Figure 3.4 shows the operation boundaries in the parameter space of $(k_0, k_1)$ for different values of $\zeta$, where, the sign of the real part of the complex eigenvalues changes and the system loses its stability via Hopf bifurcation. The following observations are made:

- For $\zeta = 0.00053$, the critical point of $k_0$ at which Hopf bifurcation occurs is 100.
- For $\zeta = 0.001$, the critical point of $k_0$ at which Hopf bifurcation occurs is 80.
- For $\zeta = 0.01$, the critical point of $k_0$ at which Hopf bifurcation occurs is 74.7.

It is inferred that for lower values of $\zeta$ or load, the Hopf bifurcation point is obtained at higher values of $k_0$.

![Figure 3.4](image_url)  
**Figure 3.4** Margin of stability curve of a hysteresis current-controlled cascaded-boost converter.
3.4 LOCAL TRAJECTORIES FROM DESCRIBING EQUATIONS

In this section, the behaviour of the system as it goes from a stable region to an unstable region is further investigated by plotting the local trajectories. The trajectory of the system near the equilibrium point is derived from the corresponding eigenvalues and eigenvectors. For the eigenvalues and corresponding eigenvectors

\[ \lambda_r, \sigma \pm j\omega \quad \text{and} \quad \vec{v}_r, \vec{v}_1 \pm j\vec{v}_2 \text{ respectively,} \]

the general solution is given by:

\[ x(t) = c_r e^{\lambda_r t} \vec{v}_r + 2c_c e^{\sigma t} \left[ \cos(\omega t + \phi_c) \vec{v}_1 - \sin(\omega t + \phi_c) \vec{v}_2 \right]. \tag{3.22} \]

where \( c_r, c_c \) and \( \phi_c \) are determined by initial conditions.

3.4.1 Local Trajectory before the Bifurcation Point

The local trajectory corresponding to a stable equilibrium point with \( \bar{c} = 0.00053, k_1 = 6.25 \) and \( k_0 = 10 \) is examined. The Jacobian matrix evaluated at the equilibrium point is

\[ J(X) = \begin{bmatrix} -0.0031 & 1.7621 & -0.6561 \\ -0.0019 & -0.0001 & -0.0050 \\ 0.0008 & 0.0001 & -0.0015 \end{bmatrix}. \tag{3.23} \]

The eigenvalues \( \bar{\lambda} \) and the corresponding eigenvectors \( \vec{v} \) are found as:

\[ \bar{\lambda} = -0.0032, -0.0007 \pm j0.0626; \]
Using MATLAB programming, the stable local trajectory is drawn as in Figure 3.5(a). Since the real eigenvalue is negative, the trajectory moves initially in the direction of $\mathbf{v}_r$. It moves in a helical motion converging towards $\mathbf{v}_r$, as the sign of the real part of the complex eigenvalue pair is also negative. After it reaches the $\mathbf{v}_1 - \mathbf{v}_2$ plane, the trajectory keeps spiraling towards the equilibrium point.

### 3.4.2 Local Trajectory after the Bifurcation Point

The local trajectory corresponding to an unstable equilibrium point with $\bar{\xi} = 0.00053$, $k_1 = 6.25$ and $k_0 = 75$ is investigated. The Jacobian matrix evaluated at the equilibrium point is

$$
\mathbf{J}(\bar{\mathbf{x}}) = \begin{bmatrix}
-0.0018 & 1.0227 & -0.1676 \\
-0.0014 & 0.0014 & -0.0031 \\
0.0004 & 0.0009 & -0.0016
\end{bmatrix}.
$$

The eigenvalues $\lambda$ and the corresponding eigenvectors $\mathbf{v}$ are found as

$$
\lambda = -0.0021, 0.0001 \pm j0.0391;
$$
Using MATLAB programming, the unstable local trajectory is drawn as in Figure 3.5(b). Since the real eigenvalue is negative, the trajectory moves initially in the direction of \( \mathbf{v}_r \). It moves in a helical motion diverging away from \( \mathbf{v}_r \), as the sign of the real part of the complex eigenvalue pair is positive. Upon reaching the \( \mathbf{v}_1 - \mathbf{v}_2 \) plane, the trajectory keeps spiraling away from the equilibrium point.

From the two trajectories, it is observed that the system loses stability via Hopf bifurcation as a stable spiral develops into an unstable spiral in the locality of the equilibrium point [Banerjee and Verghese, 2001].

\[
\mathbf{v} = \begin{pmatrix}
-0.8688 \\
0.0803 \\
0.4885
\end{pmatrix}, \quad \begin{pmatrix}
-0.9993 \\
-0.002 \pm j0.0366 \\
-0.0013 \pm j0.0096
\end{pmatrix}
\]

(3.26)

**Figure 3.5** Local trajectory of a hysteresis current-controlled cascaded-boost converter showing (a) stable operation for \( \bar{\xi} = 0.00053 \), \( k_1 = 6.25 \) and \( k_0 = 10 \); (b) unstable operation for \( \bar{\xi} = 0.00053 \), \( k_1 = 6.25 \) and \( k_0 = 75 \)
3.5 STABILITY STUDIES BY COMPUTER SIMULATION

Since the use of an averaged model for predicting nonlinear phenomena becomes inadequate when stability is lost, further investigation beyond the bifurcation point is done with computer simulation which employs an exact piecewise-switched model. MATLAB/Simulink with differential equation solver ode45, maximum step size of 1 μs and relative tolerance of 1 ms was used for this simulation. For all other simulation configuration parameters, default settings were used.

3.5.1 Hopf Bifurcation for Variation in $K$

Since the actual circuit is simulated, the parameters used are $\mu$ and $K$ instead of the dimensionless ones used for analysis. The cascaded-boost converter dynamics is studied by varying $K$, keeping $\mu$ as constant at 0.125. The system parameters used were the same as listed in Tables 3.1 and 3.2. The following observations are made:

- For low values of $K$, the system exhibits stable periodic operation as shown in Figures 3.6(a) – 3.6(d). Figures 3.6(a) and 3.6(b) show the time domain waveforms of inductor current $i_L$ and output capacitor voltage $v_c$ with the close up view of $v_c$. The phase portrait which is very useful in uncovering the subtle periodicity is plotted in Figure 3.6(c) which clearly indicates the period-1 operation of the system. Figure 3.6(d) shows an equilibrium point in the Poincaré section corresponding to Figure 3.6(c).
Figure 3.6 Simulated waveforms of a hysteresis current-controlled cascaded-boost converter exhibiting stable periodic operation with $K = 3$ and $\mu = 0.125$. (a) $v_{C2}$ and $i_{L2}$; (b) close-up view of $v_{C2}$; (c) phase portrait showing stable period-1 operation; (d) Poincaré section of (c).

For a larger value of $K$, the period-1 orbit becomes unstable and slow-scale oscillation is observed as shown in Figures 3.7(a) and 3.7(b). The phase portrait and Poincaré section shown in Figures 3.7(c) and 3.7(d) reveal a limit cycle.
Figure 3.7 Simulated waveforms of a hysteresis current-controlled cascaded-boost converter exhibiting slow-scale oscillation with $K = 30$ and $\mu = 0.125$. (a) $v_{c2}$ and $i_{L2}$; (b) close-up view of $v_{c2}$; (c) phase portrait showing limit cycle; (d) Poincaré section of (c)

For yet a larger $K$, a stable closed curve is observed in the Poincaré section as illustrated in Figures 3.8(a) and 3.8(b) signifying quasi-periodicity.
Figure 3.8 Simulated waveforms of a hysteresis current-controlled cascaded-boost converter exhibiting slow-scale oscillation with $K = 40$ and $\mu = 0.125$. (a) Phase portrait showing quasiperiodic orbit; (b) Poincaré section of (a)

3.5.2 Hopf Bifurcation for Variation in $E$

Since the voltage of a solar cell decreases with rise in temperature of the cell, which results in the module voltage drop, the attention is also focussed on effects of varying the input voltage of the converter on the dynamical behaviour of the system. The following observations are made:

- For designed operating conditions, the reduction in output voltage of PV module is less and the system exhibits normal period-1 stable operation as in Figures 3.9(a) and 3.9(b).

- For larger changes in temperature, the reduction in output voltage is very high and the system is found to exhibit supercritical Hopf bifurcation as shown in Figures 3.9(c) and 3.9(d).

- For higher input voltages of the converter, the bifurcated limit cycle becomes unstable and the system is found to exhibit subcritical Hopf bifurcation as can be seen in Figures 3.9(e) and 3.9(f).
Figure 3.9 Simulated waveforms of a hysteresis current-controlled cascaded-boost converter with $K = 3$ and $\mu = 0.125$. (a) Stable periodic operation with $E = 5$; (b) Poincaré section of (a); (c) supercritical Hopf bifurcation with $E = 3$; (d) Poincaré section of (c); (e) subcritical Hopf bifurcation with $E = 20$; (f) Poincaré section of (e)
3.6 EXPERIMENTAL VERIFICATION

An experimental setup with the same design parameter values as in the simulation has been built to validate the numerical results. Figures 3.10(a) and 3.10(b) show the basic schematic diagram and photograph of the experimental setup controlled using dSPACE 1104, built for validating the simulation studies. The output voltage and the two inductor currents are sensed and given to the ADC channels of a DSP controller, where the hysteresis control algorithm is implemented.

Figure 3.10 (a) Schematic diagram depicts the hardware implementation of a hysteresis current-controlled cascaded-boost converter; (b) snapshot of the experimental setup

Figures 3.11(a) and 3.11(b) (correspond to simulated waveforms shown in Figures 3.6(a) and 3.6(d) respectively) depict the stable operating regime of the system for $K = 3$, keeping $\mu$ constant at 0.125. As $K$ is changed to 40, slow-scale oscillations are observed in the system as can be seen in Figures 3.11(c) and 3.11(d) (correspond to simulated waveforms shown in
Figure 3.8). To measure the phase portraits of the system as shown in Figures 3.11(b) and 3.11(d), the AC component of the measured signal alone is used. The experimentally observed phase portraits are found to agree well with the simulation results.

Figure 3.11 Experimental waveforms of a hysteresis current-controlled cascaded-boost converter with $\mu = 0.125$ showing (a) stable periodic operation with $K = 3$; (b) Poincaré section of (a); (c) slow-scale oscillation with $K = 40$; (d) Poincaré section of (c)

3.7 CONCLUSION

In this chapter, detailed analytical, numerical and experimental investigations have been carried out to show the mechanism of loss of stability of the periodic orbit in a hysteresis current-controlled cascaded-boost
converter which is a very popular choice in recent years for solar energy systems. Due to its robustness and simplicity, hysteresis current-mode control is commonly employed in cascaded-boost converter applications. Analysis of the describing autonomous equations has revealed that the system loses stability via Hopf bifurcation. Numerical analysis has been done by examining the movement of the complex eigenvalues of Jacobian matrix at the equilibrium point, as some chosen parameters are changed. Computer simulations using MATLAB/Simulink have been performed to delineate the trajectories and Poincaré sections. The same simulated dynamics also has been verified using a suitable experimental setup based on DSP. Such analysis provide useful guidelines for the appropriate design of the converter and its control, thereby keeping the bifurcations far enough from the operating conditions in parameter space.