CHAPTER 5

INTERMITTENT INSTABILITY IN POWER ELECTRONIC CONVERTERS

5.1 INTRODUCTION

Intermittency is a phenomenon which may arise frequently in periodically driven nonlinear systems, where the frequency of the coupled signal may not be consistent with the system’s driving frequency. Such intermittency has been usually observed in switching power supplies which are not protected against the intrusion of spurious signals or when parasitic inductances and capacitances are present. The intrusion can also take the form of coupling via conducted or radiated paths present in the same circuit board or its close proximity. As this intermittency causes unwanted oscillations of the essential control signals which apparently affect the transfer efficiency and the switching stress of the converters, a better analysis of their nonlinear dynamics provide essential design-oriented information that allows the system parameters to be selected in a manner to avoid the occurrence of such undesirable bifurcations. In this chapter, the effect of different types of periodic interference signals like sinusoidal, triangular and saw-tooth waveforms is considered and the influence of these signals in the input, and control voltages of a voltage-mode controlled buck converter is investigated. Further, the effect of simultaneous presence of interference signals in more than one place is considered. The dynamics of this converter system is mathematically described and analysed with a simple discrete map. The analysis is used to investigate the mechanism and conditions for the
emergence of intermittency and remerging chaotic band attractors (or Feigenbaum sequences) in a voltage-mode controlled buck converter. The ordered and the chaotic dynamics of this system are investigated with suitable analytical, numerical and experimental setup.

5.2 VOLTAGE-MODE CONTROLLED BUCK CONVERTER WITH PERIODIC INTERFERENCE SIGNALS

The structure of DC-DC converters changes with the status of the switch in the converter, characterising them as a class of continuous-time switched systems. The schematic diagram of the voltage-mode controlled buck converter with an inductor $L$, a capacitor $C$, a switch $S$, a diode $D_1$ and a resistive load $R$ is shown in Figure 5.1(a). For simplicity, CCM operation is assumed with the system toggling between only two linear circuit topologies. The voltage-mode control operation of the buck converter is briefly described as follows:

The output voltage error with respect to the reference voltage is amplified to give a control voltage $v_{con}$ as:

$$v_{con} = A(v_o - V_{ref}),$$  \hspace{1cm} (5.1)

where $v_o$ is the output voltage, $A$ is the gain of the error amplifier $A_1$ and $V_{ref}$ is the reference voltage. The control voltage $v_{con}$ is compared with an externally generated saw-tooth voltage $v_{ramp}$ of time period $T_s$ and lower and upper threshold voltages $V_L$ and $V_U$ respectively, to determine the switching instants. The saw-tooth voltage $v_{ramp}$ is given by:

$$v_{ramp} = V_L + (V_U - V_L)\left(\frac{t}{T_s} \mod 1\right),$$  \hspace{1cm} (5.2)
The output $u$ of the op-amp comparator $A_2$ gives the pulses necessary for driving the switch and is described by:

$$u = \begin{cases} 
0, & \text{if } v_{con} > V_{ramp} \\
1, & \text{if } v_{con} < V_{ramp}.
\end{cases} \quad (5.3)$$

when $u = 0$, switch $S$ is turned OFF and when $u = 1$, switch $S$ is turned ON as illustrated in Figure 5.1(b). The inductor current ramps up during ON time and falls during OFF time.

![Diagram](image)

**Figure 5.1** Voltage-mode controlled buck converter with interference signals $v_{SE}$, $v_{SR}$, $v_{SC}$ included in input, reference and control voltages respectively. (a) Schematic diagram; (b) key operation waveforms

The state equations can be written as:

$$x = A_{on}x + b_{on}E \quad \text{switch } S \text{ is ON},$$

$$x = A_{off}x - b_{off}E \quad \text{switch } S \text{ is OFF}, \quad (5.4)$$
where $E$ is the input voltage, $\mathbf{x}=\begin{bmatrix} v_x & i_L \end{bmatrix}^T$ denotes the state vector of the system, $\mathbf{A}_{\text{on}}, \mathbf{A}_{\text{off}}$ and $\mathbf{b}_{\text{on}}, \mathbf{b}_{\text{off}}$ are the system matrices and vectors respectively and are given below:

$$
\mathbf{A}_{\text{on}} = \mathbf{A}_{\text{off}} = \begin{bmatrix}
-1 & 1 \\
\frac{RC}{L} & \frac{1}{C} \\
-1 & 0
\end{bmatrix},
$$

(5.5)

$$
\mathbf{b}_{\text{on}} = \begin{bmatrix} 0 \\ 1 \\ \frac{1}{L} \end{bmatrix}, \quad \mathbf{b}_{\text{off}} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
$$

(5.6)

The circuit parameters used are as follows: $L = 20 \text{ mH}$, $C = 47 \mu\text{F}$, $R = 22 \Omega$, $T_s = 400 \mu\text{s}$, $V_{\text{ref}} = 11.3 \text{ V}$, $A = 8.4$, $E = 22 \text{ V}$, nominal duty ratio $d = 0.5$, $V_L = 3.8 \text{ V}$ and $V_U = 8.2 \text{ V}$. The overall effect of the interference signal is lumped into one spurious source $v_s$ and this type of coupling is modelled as an additive process which superposes $v_s$ directly on some crucial parameter $V$ such as the reference voltage, input voltage etc [Zhou et al, 2008]. The resulting perturbed voltage $V^*$ is then given by:

$$
V^* = V + v_s.
$$

(5.7)

Without the interference ($v_s = 0$), the buck converter will experience a typical period-doubling bifurcation cascade with the input voltage $E$ varying from 22 V to 34 V as shown in Figure 5.2. The first bifurcation occurs when $E \approx 23.8 \text{ V}$ and the buck converter enters into a chaotic regime when $E \approx 31.4 \text{ V}$. 


Figure 5.2 Bifurcation diagram of a voltage-mode controlled buck converter with $E$ as the bifurcation parameter

5.3 INFLUENCE OF SINUSOIDAL INTERFERENCE SIGNAL IN INPUT POWER SUPPLY, $E$

The considered voltage-mode controlled buck converter has been simulated using Equations (5.1) – (5.3) in MATLAB/ Simulink platform by considering the switching logic mentioned in Section 5.2. With the circuit parameters, $L = 20$ mH, $C = 47$ μF, $R = 22$ Ω, $T_s = 400$ μs, $V_{ref} = 11.3$ V, $A = 8.4$ and input voltage $E$ fixed at 22 V, the unperturbed buck converter operates in stable period-1 regime. Considering the interference $v_{E}$ as a periodic sinusoidal signal, the perturbed input voltage $E^*$ is given by:

$$E^* = E + \bar{v}_E \sin(2\pi f t)$$
$$= E\left(1 + \alpha \sin(2\pi f t)\right)$$

(5.8)
where \(f_o\) is the frequency of interference, \(f_s\) is the switching frequency, \(\hat{v}_{SE}\) is the amplitude of the effective interference source and \(\alpha = \hat{v}_{SE} / E\), the strength of the spurious source. Based on the frequency ratio \(\omega_f = f_o / f_s\), the system is found to exhibit varied behaviours such as intermittent operation, quasi-periodic operation, sub-harmonic operation etc.

5.3.1 Irrational Frequency Ratio

The effect of the sinusoidal interference \(v_{SE}\) in input voltage \(E\) as in Figure 5.1(a) is studied for different irrational frequency ratios between \(f_o\) and \(f_s\). For all irrational frequency ratios, since \(f_o\) and \(f_s\) are not commensurate, a low oscillating frequency modulating the other frequency is expected and the steady state is quasi-periodic. Figures 5.3 and 5.4 show three such examples with \(\omega_f = \sqrt{3}/2\), \(\omega_f = (1+\sqrt{5})/2\) (Golden ratio) and \(\omega_f = (1+\sqrt{2})\) (Silver Ratio).

Figures 5.3(a) - (d) show the sampled data responses and Poincaré sections for \(\omega_f = \sqrt{3}/2\) and Figure 5.4 shows the Poincaré sections of responses induced by different levels of interference for the golden and silver frequency ratios. In all three cases, it is observed that when the strength of the interference signal is very weak, the effect of interference is not significant and the converter exhibits a quasi-periodic behaviour whereas for sufficiently high interference signal strength, the converter experiences chaotic behaviour.
Figure 5.3 Sampled inductor current waveforms and Poincaré sections with sinusoidal interference signal ($v_{SE}$) in input $E$ for $\alpha_f = 3/2$. (a) Sampled quasi-periodic waveform with $\alpha_v = 0.001$; (b) Poincaré section of (a); (c) sampled chaotic waveform with $\alpha_v = 0.5$; (d) Poincaré section of (c)
Figure 5.4 Poincaré sections of responses induced by different levels of sinusoidal interference ($v_{SE}$) in input $E$. (a) $a_v = 0.001$, $a_f = \text{Golden Ratio } (1+\sqrt{5})/2$; (b) $a_v = 0.9$, $a_f = \text{Golden Ratio}$; (c) $a_v = 0.001$, $a_f = \text{Silver Ratio } (1+\sqrt{2})$; (d) $a_v = 0.5$, $a_f = \text{Silver Ratio}$

5.3.2 Rational Frequency Ratio

For rational frequency ratio $\alpha_f = \frac{f_o}{f_s} = \frac{N_{num}}{N_{den}}$, where $N_{num}$ and $N_{den}$ are positive integers, the buck converter operates periodically with periodicity equal to $N_{den}$. For example, Figure 5.5(a) shows the Poincaré section for $\omega_f = 2, 3$ which corresponds to period-1 operation and is characterised by one intersection on the Poincaré section. In this case, the buck converter operates in a period-1 orbit always. Figure 5.5(b) shows three examples for $\alpha_f = 1, 1/2,
1/3 which corresponds to period-1, period-2 and period-3 operation respectively and are characterised by 1, 2 and 3 intersections on the Poincaré section. In this case, the buck converter operates in a period-$N_{den}$ sub-harmonic orbit.

![Figure 5.5 Poincaré sections of responses induced by sinusoidal interference ($v_{SE}$) in input $E$ for different rational frequency ratios. (a) $\alpha_v = 0.01$, $\alpha_f = 2$ or 3; (b) $\alpha_v = 0.01$, $\alpha_f = 1$ or 1/2 or 1/3](image)

### 5.3.3 Interference Frequency Approaching Switching Frequency or its Rational Multiples

As the interference signal is coupled unintentionally, it is also possible that the interference frequency $f_o$ approaches the switching frequency $f_s$ or its rational multiples (i.e.) $f_o = pf_s + \hat{f}$ where $\hat{f}$ is a small number compared to $f_s$ and $p$ is a rational number. This kind of interference in buck converter results in intermittent operation. In this case, the perturbed input voltage as represented in Equation (5.8) can be rewritten as:

$$E^* = E\left[1 + \alpha_s \sin(2\pi(pf_s + \hat{f})t)\right]. \quad (5.9)$$
Taking \( \hat{f} = 1 \) and \( p = 1/2, 1, 2 \), the interference frequencies considered are 1251Hz, 2501Hz and 5001Hz respectively. Numerical simulations are carried out in MATLAB and a specific form of intermittent operation exhibiting period-doubling bifurcation in two symmetrical directions over the time domain is observed as depicted in Figures 5.6 – 5.8.

In order to reveal the periodicity of operation and facilitate investigation of the intermittent behaviour, the inductor current is sampled at the switching frequency and plotted. When the sampled waveform is plotted as a function of time, it is possible to observe how the operation changes from time to time. To distinguish it from the usual bifurcation in parameter space, such plot is referred to as time bifurcation diagram as it provides information about the change of the qualitative behaviour of the system as time elapses [Zhou et al, 2003].

For very weak signal strength \( (\alpha_v = 0.0001) \), the effect of interference is not significant and the converter exhibits steady state period-\( N_{den} \) operation superimposed on a quasi-periodic orbit as shown in Figures 5.6(a), 5.7(a) and 5.8(a). For \( \alpha_v = 0.2 \), period-\( 2N_{den} \) bubbles are observed as shown in Figures 5.6(b), 5.7(b) and 5.8(b) and further increase in \( \alpha_v \), results in period-\( 2^2N_{den} \) bubbles as shown in Figures 5.6(c), 5.7(c) and 5.8(c). Similarly at a higher value of \( \alpha_v \), period-\( 2^3N_{den} \) bubbles as shown in Figures 5.6(d), 5.7(d) and 5.8(d) are observed. As \( \alpha_v \) is increased further, more bubbles are created until an infinitely branched tree appears as depicted in Figures 5.6(e), 5.7(e) and 5.8(e). Finally, it is observed that the two bands of the chaotic attractor merge into a single band resulting in a fully developed chaotic regime as depicted in Figures 5.6(f), 5.7(f) and 5.8(f). The bifurcation diagrams plotted for all three cases by sampling at the intermittent bubbling periods, with amplitude of the interference signal as parameter are shown in Figure 5.9.
Figure 5.6 Time bifurcation diagrams with sinusoidal interference signal ($v_{iE}$) of frequency $f_0 = 5001$Hz in input $E$. (a) period-1 cycle for $\alpha_v = 0.0001$; (b) the primary bubble for $\alpha_v = 0.2$; (c) period-4 bubble for $\alpha_v = 0.34$; (d) period-8 bubble for $\alpha_v = 0.415$; (e) chaos for $\alpha_v = 0.45$; (f) fully developed chaos for $\alpha_v = 0.5$
Figure 5.7  Time bifurcation diagrams with sinusoidal interference signal ($v_{iE}$) of frequency $f_o = 2501$Hz in input $E$. (a) period-1 cycle for $\alpha_v = 0.0001$; (b) the primary bubble for $\alpha_v = 0.2$; (c) period-4 bubble for $\alpha_v = 0.34$; (d) period-8 bubble for $\alpha_v = 0.35$; (e) chaos for $\alpha_v = 0.3595$; (f) fully developed chaos for $\alpha_v = 0.5$
Figure 5.8 Time bifurcation diagrams with sinusoidal interference signal ($v_{iE}$) of frequency $f_o = 1251Hz$ in input $E$. (a) period-2 cycle for $\alpha_v = 0.0001$; (b) period-4 bubble for $\alpha_v = 0.2$; (c) period-8 bubble for $\alpha_v = 0.25$; (d) period-16 bubble for $\alpha_v = 0.255$; (e) chaos for $\alpha_v = 0.261$; (f) fully developed chaos for $\alpha_v = 0.34$
Figure 5.9 Bifurcation diagrams with amplitude of the interference signal $a_v$ as the bifurcation parameter. (a) $f_o = 5001\text{Hz};$ (b) $f_o = 2501\text{Hz};$ (c) $f_o = 1251\text{Hz}$

An experimental setup with the same design parameter values used in the simulation has been built to compliment the numerical results. Figure 5.10 shows the basic schematic of the hardware implementation of the buck converter controlled using DS1104, where the dotted box $M$ shows the control logic realised using the DSP. The output capacitor voltage $v_o$ is sensed and fed back to the DSP controller through the ADC channel. The data is acquired in a Compact Flash (CF) card of TEKTRONIX TPS 2024, which is then sampled at the switching period and captured using DAC.
The experimentally observed time-domain waveforms for \( f_o = 5001 \) Hz and 1251 Hz are shown in Figures 5.11 and 5.12. Further, for comparison purposes, the corresponding numerical and experimental phase portraits and Poincaré sections for \( f_o = 2501 \) Hz and 1251 Hz respectively are depicted in Figures 5.13 and 5.14 at the intermittent bubbling periods.

**Figure 5.10** (a) Schematic diagram depicts the hardware implementation of a voltage-mode controlled buck converter; (b) snapshot of the experimental setup
Figure 5.11 Experimentally observed time bifurcation diagrams with sinusoidal interference signal ($v_{E}$) of frequency $f_{o} = 5001$Hz in input $E$. (a) $a_{r} = 0.2$; (b) $a_{r} = 0.34$; (c) $a_{r} = 0.415$; (d) $a_{r} = 0.45$; (e) $a_{r} = 0.5$
Figure 5.12  Experimentally observed time bifurcation diagrams with sinusoidal interference signal ($v_{sE}$) of frequency $f_o = 1251\text{Hz}$ in input $E$. (a) $\alpha_v = 0.01$; (b) $\alpha_v = 0.2$; (c) $\alpha_v = 0.25$; (d) $\alpha_v = 0.261$; (e) $\alpha_v = 0.34$.
Figure 5.13 (a) Simulated phase portraits with sinusoidal interference signal $(V_{SE})$ in input $E$ of frequency $f_0 = 2501$Hz. (i) $\alpha_v = 0.01$; (ii) $\alpha_v = 0.2$; (iii) $\alpha_v = 0.5$; (iv) Poincaré section of (iii). (b) Experimental phase portraits (i) $\alpha_v = 0.01$; (ii) $\alpha_v = 0.2$; (iii) $\alpha_v = 0.5$; (iv) Poincaré section of (iii)
Figure 5.14 (a) Simulated phase portraits with sinusoidal interference signal ($v_{SE}$) in input $E$ of frequency $f_o = 1251\text{Hz}$. (i) $\alpha_v = 0.01$; (ii) $\alpha_v = 0.2$; (iii) $\alpha_v = 0.34$; (iv) Poincaré section of (iii). (b) Experimental phase portraits (i) $\alpha_v = 0.01$; (ii) $\alpha_v = 0.2$; (iii) $\alpha_v = 0.34$; (iv) Poincaré section of (iii)
5.3.4 Analysis of Intermittent Bifurcation with Sinusoidal Interference Signal

As there is no steady-state fixed point which can be used to do the bifurcation analysis in time-domain bifurcation, a transformation to change the time-bifurcation to parameter-bifurcation of another new variable is applied. The new variable is considered as a conceptual ‘phase shift’, which is used to represent the equivalent drift in the interfering frequency from the switching frequency and the perturbed input voltage as represented in Equation (5.9) is rewritten as:

\[ E' = E(l + \alpha_v \sin(2\pi f_N t + \hat{\phi})) \]  \hspace{1cm} (5.10)

where \( \hat{\phi} = 2\pi \hat{f} t \) is conceptually a phase shift. This kind of interference in buck converter results in intermittency in few different forms (such as intermittent subharmonics, intermittent chaos) and the intermittent period \( T_i \) is found to be the reciprocal of \( N_{den} \hat{f} \). Taking \( \hat{f} = 1 \) and \( p = 1/2, 1, 2 \), the interference frequencies considered for analysis are 1251Hz, 2501Hz and 5001Hz respectively. The analysis is carried out separately for two cases i.e. \( N_{den} = 1 \) (for \( p = 1, 2 \)) and \( N_{den} = 2 \) (for \( p = 1/2 \)). At first, the iterative discrete-time model is derived with the perturbed input voltage and then the stability of the converter is analyzed by examining the loci of the characteristic multipliers of the Jacobian.

5.3.4.1 Case – 1: \( (p = 1, 2 \text{ where } N_{den} = 1) \)

Here, with the steady state operation being a period-1 orbit, the converter first loses its stability and bifurcates from period-1 to period-2 sub-harmonic operation as \( \hat{\phi} \) increases. The discrete-time map takes the form \( x_{n+1} = f(x_n, \alpha_n) \), where \( x_n \) is the state vector at a clock instant \( t_n \) and \( x_{n+1} \) is
the state vector at the next clock instant $t_{n+1}$. For a closed loop system, the feedback equation that relates $\bar{d}_n = (1 - d_n)$ to $x_n$ is also found. From the steady state waveforms shown in Figure 5.15, the operation over the period $(t_n - t_{n+1})$ is divided into two phases:

1. $t_n < t \leq t_m$, switch $S$ OFF.
2. $t_m < t \leq t_{n+1}$, switch $S$ ON.

The state vectors of the converter at the end of phase 1, $x_m$ and phase 2, $x_{n+1}$ are expressed as:

\[
x_m = N_{off} (\bar{d}_n) x_n + M_{off} (\bar{d}_n) E^r, \tag{5.11}
\]

\[
x_{n+1} = N_{on} (1 - \bar{d}_n) x_m + M_{on} (1 - \bar{d}_n) E^r. \tag{5.12}
\]

Figure 5.15 The ON-OFF driving signal $u$ of switch $S$ and the inductor current waveform of Figure 5.1(a) with $pf_s = 2500$ Hz
Combining Equations (5.11) and (5.12), the discrete-time iterative map over one switching period is written as:

\[ x_{n+1} = N_{on}(1 - d_{n})N_{off}(d_{n})x_{n} + \left[ N_{on}(1 - d_{n})M_{off}(d_{n}) + M_{on}(1 - d_{n}) \right]E. \]

\[ = f(x_{n}, d_{n}) \]  

(5.13)

where

\[ N_{on}(1 - d_{n}) = e^{\lambda_{on}t}d_{n}, \]

\[ M_{on}(1 - d_{n}) = A_{on}^{-1} N_{on}(1 - d_{n}) d_{n}. \]

\[ N_{off}(d_{n}) = e^{\lambda_{off}t}d_{n}, \]

\[ M_{off}(d_{n}) = A_{off}^{-1} N_{off}(d_{n}) - 1. \]

The switching function \( s(x_{n}, d_{n}) \) is defined as:

\[ s(x_{n}, d_{n}) = v_{con}(d_{n}T_{s}) - v_{ramp}(d_{n}T_{s}), \]

\[ = \begin{bmatrix} v_{r}(d_{n}T_{s}) - V_{ref} \\ i_{L}(d_{n}T_{s}) \end{bmatrix} - V_{L} - (V_{U} - V_{L})d_{n}T_{s}, \]

\[ = \begin{bmatrix} N_{off}(d_{n})x_{n} + M_{off}(d_{n})E \end{bmatrix} - AV_{ref} - V_{L} - (V_{U} - V_{L})d_{n}T_{s}, \]

\[ = \begin{bmatrix} N_{off}(d_{n})x_{n} + M_{off}(d_{n}) \left[ E(1 + \alpha_{r} \sin(2\pi T_{n} + \phi)) \right] - AV_{ref} - V_{L} - (V_{U} - V_{L})d_{n}T_{s}. \]  

(5.14)
To find the defining equation for the duty cycle, it is noted that the switch \( S \) is turned OFF when \( v_{con} = v_{ramp} \) i.e., the switch \( S \) is turned OFF when

\[
s(x_n, d_n) = 0. \tag{5.15}
\]

Solving Equation (5.15) for \( d_n \) and combining with Equation (5.13), the discrete-time iterative map for the closed loop system is obtained. Using the discrete-time iterative map derived, the bifurcation diagrams are plotted for \( p = 1 \) and \( 2 \) as in Figures 5.16 and 5.17.

![Bifurcation Diagrams](Figure 5.16 Parameter bifurcation diagrams with sinusoidal interference signal \((v_{sE})\) in input \( E \) for \( pf_s = 5000\)Hz. (a) \( \alpha_s = 0.0001 \); (b) \( \alpha_s = 0.2 \); (c) \( \alpha_s = 0.34 \); (d) \( \alpha_s = 0.5 \))
Figure 5.17 Parameter bifurcation diagrams with sinusoidal interference signal ($\nu_{\text{is}}$) in input $E$ for $pf_i = 2500\text{Hz}$.

(a) $\nu_{\text{is}} = 0.0001$; (b) $\nu_{\text{is}} = 0.2$; (c) $\nu_{\text{is}} = 0.34$; (d) $\nu_{\text{is}} = 0.5$

The discrete-time iterative map derived above is used to study the stability of the system. Specifically, by computing the characteristic multipliers about the equilibrium point, the way the system loses stability as the amplitude of the interference signal is varied is analysed. For use of this conventional method, the Jacobian of the discrete-time map is evaluated by exploiting the implicit function derivation theorem as follows:

\[
J(\mathbf{x}_n, \mathbf{d}_n) = \frac{\partial f}{\partial \mathbf{x}_n} - \frac{\partial f}{\partial \mathbf{d}_n} \left( \frac{\partial s}{\partial \mathbf{d}_n} \right)^{-1} \frac{\partial s}{\partial \mathbf{x}_n},
\]  

(5.16)
where

\[ \frac{\partial f}{\partial x_n} = N_{on}(1 - \overline{d}_n)N_{off}(\overline{d}_n), \]

\[ \frac{\partial f}{\partial d_n} = \left[ -A_{on} T_s N_{on}(1 - \overline{d}_n)N_{off}(\overline{d}_n) \right] x_n - \left[ -A_{on} T_s N_{on}(1 - \overline{d}_n)M_{off}(\overline{d}_n) + N_{on}(1 - \overline{d}_n)N_{off}(\overline{d}_n) b_{off} T_s - b_{on} T_s \right] E \]

\[ - \left[ N_{on}(1 - \overline{d}_n)M_{off}(\overline{d}_n) + M_{on}(1 - \overline{d}_n) \right] \frac{\pi}{2} \varepsilon_{\alpha_s} \rho \cos(2\pi p \overline{d}_n + \theta). \]

\[ \frac{\partial s}{\partial x_n} = \begin{bmatrix} A & 0 \end{bmatrix} N_{off}(\overline{d}_n), \]

\[ \frac{\partial s}{\partial d_n} = \begin{bmatrix} A & 0 \end{bmatrix} N_{off}(\overline{d}_n) x_n - b_{off} E y_s - (V_U - V_L) y_s \]

\[ + \begin{bmatrix} A & 0 \end{bmatrix} M_{off}(\overline{d}_n) \frac{\pi}{2} \varepsilon_{\alpha_s} \rho \cos(2\pi p \overline{d}_n + \theta). \]

The following polynomial equation in \( \lambda \) is solved using a numerical algorithm written in MATLAB and the characteristic multipliers are found.

\[ \det \left[ \lambda I - J(x_{\lambda}, \overline{d}_n) \right] = 0. \]  \hspace{1cm} (5.17)

where \( I \) is the identity matrix, \( x_{\lambda} \) and \( \overline{d}_n \) are the equilibrium values. The movement of the characteristic multipliers, as \( \theta \) varies is studied for any crossing from the interior of the unit circle to the exterior indicating a bifurcation. It is found that the movements of the loci of characteristic multipliers as shown in Figures 5.18 and 5.19 agree perfectly well with the parameter-bifurcation diagrams shown in Figures 5.16 and 5.17. The following observations are made:
For a very weak interference signal \( (\alpha_v = 0.0001) \), the loci of the characteristic multipliers as phase shift \( \theta \) varies are given in Figures 5.18(a) and 5.19(a). From these figures, it is observed that as the phase shift \( \theta \) increases from 0, two characteristic multipliers move towards each other along a circle of radius 0.824 for \( p = 1 \) and two characteristic multipliers move apart along a circle of radius 0.824 for \( p = 2 \). For further increase in \( \theta \), they move back along the original path implying stable operation always.

As the strength of the interference signal increases, finite period-doubling sequences “merging” as it were with inversely advancing ones arise to form a finite number of “bubbles” on some cross-sections of the full parameter space. An important consequence of such remerging period-doubling sequences (or Feigenbaum trees) is that low-order periodic orbits become stable again. For the case of \( p = 2 \), remerging Feigenbaum trees exist in the ranges \( \alpha_v = (0.05, 0.5) \) and \( \theta = (0, 6.28) \) in the parametric space (Figure 5.16). For the case of \( p = 1 \), remerging Feigenbaum trees exist in the ranges \( \alpha_v = (0.07, 0.5) \) and \( \theta = (0, 6.28) \) in the parametric space (Figure 5.17).

For \( p = 1 \) and \( \alpha_v = 0.2 \), as \( \theta \) increases to 2.68, one characteristic multiplier crosses the unit circle along the real axis from inside and the converter experiences a period-doubling bifurcation. It remains in this unstable state till the characteristic multiplier enters the unit circle again which happens at \( \theta = 6.04 \) as in Figure 5.18(b). For \( p = 2 \), as \( \theta \) increases to 2.79, one characteristic multiplier enters the unit circle along the real axis and the converter becomes stable. It
remains in this stable state till the characteristic multiplier leaves the unit circle again which happens at $\beta = 6.28$ as in Figure 5.19(b). The loci of the characteristic multipliers for even higher values of $\alpha_v$ are shown in Figures 5.18(c), 5.19(c) and 5.18(d), 5.19(d).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.18}
\caption{Loci of the characteristic multipliers of Equation (5.17) for $p_f = 5000$ Hz. (a) $\alpha_v = 0.0001$; (b) $\alpha_v = 0.2$; (c) $\alpha_v = 0.34$; (d) $\alpha_v = 0.5$}
\end{figure}
Figure 5.19  Loci of the characteristic multipliers of Equation (5.17) for $p f_s = 2500$ Hz. (a) $\alpha_v = 0.0001$; (b) $\alpha_v = 0.2$; (c) $\alpha_v = 0.34$; (d) $\alpha_v = 0.5$
5.3.4.2 Case – 2: \((p = 1/2 \text{ where } N_{\text{den}} = 2)\)

In this case, with the steady state operation being a period-2 orbit, the converter first loses its stability and bifurcates from period-2 to period-4 sub-harmonic operation as \(\beta\) increases. A discrete-time map in the form of \(x_{n+2} = f(x_n, d_{in}, d_{2n})\) is constructed for stability analysis. From the steady state waveforms shown in Figure 5.20, the operation over the period \((t_n - t_{n+2})\) is divided into four phases:

1. \(t_n < t < t_{m1}\), switch \(S\) OFF.
2. \(t_{m1} < t < t_{n+1}\), switch \(S\) ON.
3. \(t_{n+1} < t < t_{m2}\), switch \(S\) OFF.
4. \(t_{m2} < t < t_{n+2}\), switch \(S\) ON.

![Figure 5.20](image)

**Figure 5.20** The ON-OFF driving signal \(u\) of switch \(S\) and the inductor current waveform of Figure 5.1(a) with \(p_f = 1250\) Hz
The state vectors at the end of phase 1 - $x_{m1}$, phase 2 - $x_{m+1}$, phase 3 - $x_{m2}$ and phase 4 - $x_{m+2}$ are expressed as:

$$x_{m1} = N_{off}(d_{1a})x_n + M_{off}(d_{1a})E^*,$$  \hfill (5.18)

$$x_{m+1} = N_{on}(1 - d_{1a})x_{m1} + M_{on}(1 - d_{1a})E^*,$$  \hfill (5.19)

$$x_{m2} = N_{off}(d_{2a})x_{m1} + M_{off}(d_{2a})E^*,$$  \hfill (5.20)

$$x_{m+2} = N_{on}(1 - d_{2a})x_{m2} + M_{on}(1 - d_{2a})E^*.$$  \hfill (5.21)

Following similar procedure as in case 1, the discrete-time iterative map over two switching periods is written as:

$$x_{m+2} = N_{on}(1 - d_{2a})N_{off}(d_{1a})N_{on}(1 - d_{1a})N_{off}(d_{1a})x_n$$

$$+ N_{on}(1 - d_{2a})N_{off}(d_{2a})N_{on}(1 - d_{1a})M_{off}(d_{1a})E(1 + c + \sin(2\pi d_{1a} + \theta))$$

$$+ N_{on}(1 - d_{2a})N_{off}(d_{2a})M_{on}(1 - d_{1a})E(1 + c + \sin(2\pi d_{1a} + \theta))$$

$$+ [N_{on}(1 - d_{2a})M_{off}(d_{2a}) + M_{on}(1 - d_{2a})]E(1 + c + \sin(2\pi d_{2a} + \theta)],$$

$$= f(x_n, d_{1a}, d_{2a}).$$  \hfill (5.22)

Similar to the previous case, the switching functions used to define the duty cycles are written as:

$$s_1(x_n, d_{1a}) = [A \ 0 \ N_{off}(d_{1a})x_n + M_{off}(d_{1a})E(1 + c + \sin(2\pi d_{1a} + \theta))]$$

$$- AV_{ref} - (V_U - V_L)d_{1a}T_s,$$  \hfill (5.23)
Solving Equations (5.23) and (5.24) for $d_{1n}$ and $d_{2n}$ and combining with Equation (5.22), the discrete-time iterative map for the closed loop system is obtained. Using the discrete-time iterative map derived, the bifurcation diagram is plotted for $p = 1/2$ as in Figure 5.21.

Figure 5.21 Parameter bifurcation diagrams with sinusoidal interference signal ($v_{sE}$) in input $E$ for $pf_s = 1250$Hz.
(a) $\alpha_r = 0.0001$; (b) $\alpha_r = 0.2$; (c) $\alpha_r = 0.25$; (d) $\alpha_r = 0.34$
The Jacobian of the discrete-time map evaluated using the implicit function derivation theorem is given as follows:

\[
\mathbf{J}(\mathbf{x}_n, \mathbf{d}_{1n}, \mathbf{d}_{2n}) = \left( \frac{\partial \mathbf{f}}{\partial \mathbf{x}_n} - \frac{\partial \mathbf{f}}{\partial \mathbf{d}_{2n}} \right) \frac{\partial \mathbf{s}}{\partial \mathbf{d}_{1n}} \left( \frac{\partial \mathbf{s}_1}{\partial \mathbf{x}_n} \right)^{-1} \frac{\partial \mathbf{s}_1}{\partial \mathbf{x}_n} \\
- \frac{\partial \mathbf{f}}{\partial \mathbf{d}_{2n}} \left( \frac{\partial \mathbf{s}_2}{\partial \mathbf{d}_{2n}} \right)^{-1} \left[ \frac{\partial \mathbf{s}_2}{\partial \mathbf{x}_n} \frac{\partial \mathbf{s}_2}{\partial \mathbf{d}_{1n}} \left( \frac{\partial \mathbf{s}_1}{\partial \mathbf{d}_{1n}} \right)^{-1} \frac{\partial \mathbf{s}_1}{\partial \mathbf{x}_n} \right]
\]

(5.25)

where

\[
\frac{\partial \mathbf{f}}{\partial \mathbf{x}_n} = \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{1n} \right)
\]

\[
\frac{\partial \mathbf{f}}{\partial \mathbf{d}_{1n}} = \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) \left[ -\mathbf{A}_{\text{on}} \mathbf{T}_s \mathbf{N}_{\text{off}} \left( \mathbf{d}_{1n} \right) + \mathbf{N}_{\text{off}} \left( \mathbf{d}_{1n} \right) \mathbf{A}_{\text{off}} \mathbf{T}_s \right] \mathbf{x}_n
\]

\[
+ \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) \left[ -\mathbf{A}_{\text{on}} \mathbf{T}_s \mathbf{M}_{\text{off}} \left( \mathbf{d}_{1n} \right) \mathbf{b}_{\text{off}} \mathbf{T}_s \right] \mathbf{E}
\]

\[
+ \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{1n} \right) \mathbf{b}_{\text{off}} \mathbf{T}_s \mathbf{E}
\]

\[
+ \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{M}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) 2\pi \mathbf{E}_{\mathbf{c}_{\gamma}} \mathbf{p} \cos(2\pi \mathbf{p} \mathbf{d}_{1n} + \mathbf{\theta})
\]

\[
+ \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{2n} \right) \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{M}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) 2\pi \mathbf{E}_{\mathbf{c}_{\gamma}} \mathbf{p} \cos(2\pi \mathbf{p} \mathbf{d}_{1n} + \mathbf{\theta})
\]

\[
\frac{\partial \mathbf{s}_2}{\partial \mathbf{d}_{1n}} = \left[ \begin{array}{cc} \mathbf{A} & 0 \\ \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) \mathbf{A}_{\text{on}} \mathbf{M}_{\text{off}} \left( \mathbf{d}_{1n} \right) \mathbf{E} & \left( 1 + \mathbf{\alpha}_\mathbf{c} \sin(2\pi \mathbf{d}_{2n} + \mathbf{\theta}) \right) \mathbf{T}_s \\ -\left[ \begin{array}{c} \mathbf{A} & 0 \\ \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) \mathbf{A}_{\text{on}} \mathbf{M}_{\text{off}} \left( \mathbf{d}_{1n} \right) \mathbf{E} \left( 1 + \mathbf{\alpha}_\mathbf{c} \sin(2\pi \mathbf{d}_{2n} + \mathbf{\theta}) \right) \mathbf{T}_s \\ -\left[ \begin{array}{c} \mathbf{A} & 0 \\ \mathbf{N}_{\text{off}} \left( \mathbf{d}_{2n} \right) \mathbf{N}_{\text{on}} \left( 1 - \mathbf{d}_{1n} \right) \mathbf{M}_{\text{off}} \left( \mathbf{d}_{1n} \right) \mathbf{b}_{\text{off}} - \mathbf{b}_{\text{on}} \mathbf{E} \left( 1 + \mathbf{\alpha}_\mathbf{c} \sin(2\pi \mathbf{d}_{2n} + \mathbf{\theta}) \right) \mathbf{T}_s 
\end{array} \right]
\right.
\]
\[
\frac{\hat{c}_1}{\hat{d}_2} = N_{\text{on}} (1 - \hat{d}_{2n}) T_s N_{\text{off}} (\hat{d}_{2n}) + N_{\text{off}} (\hat{d}_{2n}) A_{\text{on}} T_s N_{\text{on}} (1 - \hat{d}_{1n}) N_{\text{off}} (\hat{d}_{1n}) \hat{s}_n
\]

\[
- N_{\text{on}} (1 - \hat{d}_{2n}) A_{\text{on}} T_s N_{\text{off}} (\hat{d}_{2n}) N_{\text{on}} (1 - \hat{d}_{1n}) M_{\text{off}} (\hat{d}_{1n}) E (1 + \alpha, \sin (2 \pi \hat{d}_{1n} + \theta))
\]

\[
+ N_{\text{on}} (1 - \hat{d}_{2n}) N_{\text{off}} (\hat{d}_{2n}) A_{\text{off}} T_s N_{\text{on}} (1 - \hat{d}_{1n}) M_{\text{off}} (\hat{d}_{1n}) E (1 + \alpha, \sin (2 \pi \hat{d}_{1n} + \theta))
\]

\[
+ N_{\text{on}} (1 - \hat{d}_{2n}) [-A_{\text{on}} T_s M_{\text{off}} (\hat{d}_{2n}) + N_{\text{off}} (\hat{d}_{2n}) b_{\text{off}} T_s - b_{\text{on}} T_s] E
\]

\[
+ [N_{\text{on}} (1 - \hat{d}_{2n}) M_{\text{off}} (\hat{d}_{2n}) + M_{\text{on}} (1 - \hat{d}_{2n})] 2 \pi \alpha, \cos (2 \pi \hat{d}_{2n} + \theta)
\]

\[
\frac{\hat{c}_1}{\hat{d}_1} = [A \ 0] N_{\text{off}} (\hat{d}_{1n})
\]

\[
\frac{\hat{c}_1}{\hat{d}_1} = [A \ 0] N_{\text{off}} (\hat{d}_{1n}) [A_{\text{on}} \hat{x}_n + b_{\text{off}} E] T_s - (V_U - V_L) \hat{y}_s
\]

\[
+ [A \ 0] M_{\text{off}} (\hat{d}_{1n}) 2 \pi \alpha, \cos (2 \pi \hat{d}_{1n} + \theta),
\]

\[
\frac{\hat{c}_2}{\hat{d}_2} = [A \ 0] N_{\text{off}} (\hat{d}_{2n}) N_{\text{on}} (1 - \hat{d}_{1n}) N_{\text{off}} (\hat{d}_{1n})
\]

\[
\frac{\hat{c}_2}{\hat{d}_2} = [A \ 0] N_{\text{off}} (\hat{d}_{2n}) A_{\text{off}} N_{\text{on}} (1 - \hat{d}_{1n}) [N_{\text{off}} (\hat{d}_{1n}) \hat{s}_n + M_{\text{off}} (\hat{d}_{1n}) E] T_s
\]

\[
+ [A \ 0] N_{\text{off}} (\hat{d}_{2n}) [A_{\text{off}} M_{\text{on}} (1 - \hat{d}_{1n}) + b_{\text{off}}] E T_s - (V_U - V_L) \hat{y}_s
\]

\[
+ [A \ 0] [N_{\text{off}} (\hat{d}_{2n}) N_{\text{on}} (1 - \hat{d}_{1n}) M_{\text{off}} (\hat{d}_{1n})] 2 \pi \alpha, \cos (2 \pi \hat{d}_{2n} + \theta)
\]

\[
+ [A \ 0] N_{\text{off}} (\hat{d}_{2n}) M_{\text{on}} (1 - \hat{d}_{1n}) 2 \pi \alpha, \cos (2 \pi \hat{d}_{2n} + \theta)
\]

\[
+ [A \ 0] M_{\text{off}} (\hat{d}_{2n}) 2 \pi \alpha, \cos (2 \pi \hat{d}_{2n} + \theta)
\]
Then, the characteristic multipliers $\lambda$ are found by solving

$$\det\left[\lambda I - J(\mathbf{X}_Q, \overline{d}_{1Q}, \overline{d}_{2Q})\right] = 0,$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{X}_Q$, $\overline{d}_{1Q}$ and $\overline{d}_{2Q}$ are the equilibrium values.

As can be seen in Figure 5.21, the parameter bifurcation diagrams over the interval $[0, \pi]$ are exactly equivalent to the interval $[\pi, 2\pi]$ which makes it sufficient to calculate the characteristic multipliers only for the interval $[0, \pi]$. It is found that the movements of the loci of characteristic multipliers as shown in Figure 5.22 agree perfectly well with the parameter-bifurcation diagrams shown in Figure 5.21. The observations are summarised as follows:

- For a weak interference signal ($\alpha_v = 0.0001$), the loci of the characteristic multipliers as phase shift $\theta$ varies are given in Figure 5.22(a). From the figure, it is observed that as the phase shift $\theta$ increases from 0, two characteristic multipliers move towards each other along a circle of radius 0.68. For further increase in $\theta$, they move back along the original path implying period-2 operation always.

- As the strength of the interference signal increases ($\alpha_v = 0.2$), period-bubbling bifurcation is observed as shown in Figure 5.21(b). As $\theta$ increases to 0.21, one characteristic multiplier crosses the unit circle along the real axis from inside and the converter period doubles upto period-4. It remains in this unstable state till the characteristic multiplier enters the unit circle again which happens at $\theta = 0.99$ (Figure 5.22(b)). For even higher values of $\alpha_v$, the intermittent period-4 sub-harmonic state doubles upto period-8 sub-harmonic state with increase in $\theta$ and for very high values of $\alpha_v$, the intermittent state results in chaotic regime. The loci of the characteristic multipliers for higher values of $\alpha_v$ are shown in Figures 5.22(c) and 5.22(d).
Figure 5.22 Loci of the characteristic multipliers of Equation (5.26) for $p_f = 1250$ Hz. (a) $\alpha_v = 0.0001$; (b) $\alpha_v = 0.2$; (c) $\alpha_v = 0.25$; (d) $\alpha_v = 0.34$
5.4 INFLUENCE OF PERIODIC INTERFERENCE SIGNALS IN CONTROL VOLTAGE

There are several potential sources of spurious signals which can coexist in a signal loop in the power electronic converter circuit such as phase locked loop, sample clock, digital interface, power supplies, oscillators, PWM generators etc., and each have distinct characteristics. As an exemplar of these cases, three types of periodic spurious signals like sinusoidal, triangular and saw-tooth waveforms are considered. As this study is focused on the modulation of various parameters that leads to intermittency, the effect of these three types of spurious signals in control voltage is studied in this section.

5.4.1 Sinusoidal Interference Signal

As evinced in Figure 5.23, when the frequency ratio is irrational, the converter exhibits quasi-periodic behaviour since \( f_o \) and \( f_s \) are not commensurate. The time bifurcation diagrams with sinusoidal interference of different strengths and frequencies taken as 1251Hz, 2501Hz and 5001Hz are explained by Zhou et al (2008). The converter performs similar to the sinusoidal interference in input voltage but for relatively lesser values of \( a_s \). The thresholds of the coupling signal strength for intermittent bubbling and intermittent chaos with periodic sinusoidal interference signal of frequency 2501Hz in the control voltage are summarised in Tables 5.1 and 5.2 respectively. Graphical presentations are also shown in Figure 5.24, where the upper surface gives the thresholds for intermittent chaos and the lower surface gives the thresholds for intermittent bubbling. For example, when input voltage \( E = 22V \), gain \( A = 8.4 \) and coupling signal strength \( a_s = 0.0099 \), the converter exhibits an intermittent bubble similar to Figure 5.7(b). When input
voltage $E = 22V$, gain $A = 8.4$ and coupling signal strength $\alpha_v = 0.0576$, the converter exhibits an intermittent chaos similar to Figure 5.7(e). These data though obtained for a particular converter operating with a specific set of circuit parameters, are useful for inspecting the general trend rather than providing absolute design data. Nonetheless, from these data, it is studied that for a relatively higher feedback gain or input voltage, the converter is more vulnerable to attack by spurious signal coupling.

Figure 5.23 Poincaré sections of responses induced by different levels of sinusoidal interference ($v_{SC}$) in control voltage ($v_{con}$) with (a) $\alpha_v = 0.001$, $\alpha_f = \sqrt{3}/2$; (b) $\alpha_v = 0.1$, $\alpha_f = \sqrt{3}/2$; (c) $\alpha_v = 0.001$, $\alpha_f =$ Golden Ratio; (d) $\alpha_v = 0.1$, $\alpha_f =$ Golden Ratio
Table 5.1  Threshold values of $\alpha_v$ at which intermittent bubbling occur for sinusoidal interference ($v_{SC}$) at control voltage ($v_{con}$)

<table>
<thead>
<tr>
<th>Gain $A$</th>
<th>$E=20$ V</th>
<th>$E=21$ V</th>
<th>$E=22$ V</th>
<th>$E=23$ V</th>
<th>$E=24$ V</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.8</td>
<td>0.0367</td>
<td>0.0346</td>
<td>0.0309</td>
<td>0.0267</td>
<td>0.0238</td>
</tr>
<tr>
<td>7.2</td>
<td>0.0319</td>
<td>0.0292</td>
<td>0.0249</td>
<td>0.0209</td>
<td>0.0178</td>
</tr>
<tr>
<td>7.6</td>
<td>0.0274</td>
<td>0.0242</td>
<td>0.0205</td>
<td>0.0156</td>
<td>0.0118</td>
</tr>
<tr>
<td>8.0</td>
<td>0.0224</td>
<td>0.0191</td>
<td>0.0152</td>
<td>0.0100</td>
<td>0.0057</td>
</tr>
<tr>
<td>8.4</td>
<td>0.0176</td>
<td>0.0141</td>
<td>0.0099</td>
<td>0.0044</td>
<td>0.0011</td>
</tr>
</tbody>
</table>

Table 5.2  Threshold values of $\alpha_v$ at which intermittent chaos occur for sinusoidal interference ($v_{SC}$) at control voltage ($v_{con}$)

<table>
<thead>
<tr>
<th>Gain $A$</th>
<th>$E=20$ V</th>
<th>$E=21$ V</th>
<th>$E=22$ V</th>
<th>$E=23$ V</th>
<th>$E=24$ V</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.8</td>
<td>0.0773</td>
<td>0.0759</td>
<td>0.0751</td>
<td>0.0732</td>
<td>0.0703</td>
</tr>
<tr>
<td>7.2</td>
<td>0.0735</td>
<td>0.0724</td>
<td>0.0709</td>
<td>0.0686</td>
<td>0.0655</td>
</tr>
<tr>
<td>7.6</td>
<td>0.0697</td>
<td>0.0686</td>
<td>0.0666</td>
<td>0.0640</td>
<td>0.0604</td>
</tr>
<tr>
<td>8.0</td>
<td>0.0657</td>
<td>0.0643</td>
<td>0.0622</td>
<td>0.0590</td>
<td>0.0555</td>
</tr>
<tr>
<td>8.4</td>
<td>0.0621</td>
<td>0.0600</td>
<td>0.0576</td>
<td>0.0544</td>
<td>0.0504</td>
</tr>
</tbody>
</table>

Figure 5.24 Graphical representation of the thresholds of coupling signal strength for intermittent chaos (upper surface) and intermittent bubbling (lower surface)
5.4.2 Triangular Interference Signal

A periodic triangular wave is a piecewise linear continuous function and is quite often viewed as piecewise linear approximation of sine wave. Then as expected, the triangular interference in control voltage intrudes the same behaviour in the converter as sinusoidal interference in control voltage. Figure 5.25 shows the Poincaré section for irrational frequency ratios and Figures 5.26 – 5.28 show the time bifurcation diagrams for frequencies 1251Hz, 2501Hz and 5001Hz respectively.

Figure 5.25 Poincaré sections of responses induced by different levels of triangular interference ($v_{SC}$) in control voltage ($v_{con}$). (a) $a_v = 0.001$, $a_f = \text{Golden Ratio}$; (b) $a_v = 0.05$, $a_f = \text{Golden Ratio}$; (c) $a_v = 0.001$, $a_f = \text{Silver Ratio}$; (d) $a_v = 0.1$, $a_f = \text{Silver Ratio}$
Figure 5.26 Time bifurcation diagrams with triangular interference signal ($v_{SC}$) in control voltage ($v_{con}$) for $f_o = 5001$Hz. (a) $\alpha_v = 0.005$; (b) $\alpha_v = 0.01$; (c) $\alpha_v = 0.05$; (d) $\alpha_v = 0.06$
Figure 5.27 Time bifurcation diagrams with triangular interference signal \((v_{SC})\) in control voltage \((v_{con})\) for \(f_o = 2501\text{Hz}\). (a) \(\alpha_v = 0.01\); (b) \(\alpha_v = 0.02\); (c) \(\alpha_v = 0.056\); (d) \(\alpha_v = 0.065\)
Figure 5.28 Time bifurcation diagrams with triangular interference signal ($v_{SC}$) in control voltage ($v_{con}$) for $f_o = 1251$Hz.
(a) $\alpha_r = 0.001$; (b) $\alpha_r = 0.066$; (c) $\alpha_r = 0.083$; (d) $\alpha_r = 0.093$

5.4.3 Saw-tooth Interference Signal

Due to the asymmetrical nature and sudden dip in amplitude of saw-tooth signal, even lesser amplitude interference results in intermittent chaos. As the strength of the interference signal increases, the converter poses more rigorous intermittent chaos. Figures 5.29 and 5.30 show the Poincaré sections of responses induced by different levels of interference.
Figure 5.29  Poincaré sections of responses induced by different levels of saw-tooth interference (v_{SC}) in control voltage (v_{con}). (a) \( \alpha_s = 0.001, \alpha_f = \text{Golden Ratio} \); (b) \( \alpha_s = 0.05, \alpha_f = \text{Golden Ratio} \); (c) \( \alpha_s = 0.001, \alpha_f = \text{Silver Ratio} \); (d) \( \alpha_s = 0.05, \alpha_f = \text{Silver Ratio} \).
Figure 5.30 Time bifurcation diagrams with saw-tooth interference signal \( v_{SC} \) in control voltage \( v_{con} \). (a) \( f_o = 1251\text{Hz} \), \( a_o = 0.001 \); (b) \( f_o = 1251\text{Hz} \), \( a_o = 0.075 \); (c) \( f_o = 2501\text{Hz} \), \( a_o = 0.001 \); (d) \( f_o = 2501\text{Hz} \), \( a_o = 0.075 \); (e) \( f_o = 5001\text{Hz} \), \( a_o = 0.001 \); (f) \( f_o = 5001\text{Hz} \), \( a_o = 0.03 \)
5.5 INFLUENCE OF SIMULTANEOUS PRESENCE OF INTERFERENCE SIGNALS

In Sections 5.3 and 5.4, the effect of interference signal present in any one stage of the converter is studied. But in practical applications, the spurious signals may intrude in more than one stage and the combined effect can be perceived in crucial parameters. The changes in the system’s performance in the presence of a periodic sinusoidal interference signal simultaneously in input and reference voltage is considered here for both irrational and rational frequency ratios.

5.5.1 Irrational Frequency Ratios

When the interference signals in the input and reference voltages have irrational frequency ratios with that of the switching frequency, the following observations are made.

- When the amplitude of the interference signal in both the input and reference voltages are less, the converter which otherwise with one interference would have perceived quasi-periodicity as shown in Figures 5.4(a) and 5.4(c), now exhibits a 3-frequency quasi-periodicity as there are three incommensurate frequencies in the system (i.e.) switching frequency, interference frequency of input and reference. Figures 5.31(a) – 5.31(i) show six examples of different combinations of input and reference frequency ratios.
Figure 5.31  Poincaré sections of responses induced with two interference signals (\(v_{SE}\)) and (\(v_{SR}\)) in input \(E\) and reference \(V_{ref}\) respectively. Input \(a_v = 0.001\), \(a_f = \sqrt{3}/2\) and Reference \(a_v = 0.001\), (a) \(a_f = \sqrt{3}/2\); (b) \(a_f = \text{Golden ratio}\); (c) \(a_f = \text{Silver ratio}\). Input \(a_v = 0.001\), \(a_f = \text{Golden ratio}\) and Reference \(a_v = 0.001\) (d) \(a_f = \sqrt{3}/2\); (e) \(a_f = \text{Golden ratio}\); (f) \(a_f = \text{Silver ratio}\). Input \(a_v = 0.001\), \(a_f = \text{Silver ratio}\) and Reference \(a_v = 0.001\) (g) \(a_f = \sqrt{3}/2\); (h) \(a_f = \text{Golden ratio}\); (i) \(a_f = \text{Silver ratio}\). (j) Input \(a_v = 0.001\), \(a_f = \sqrt{3}/2\) and Reference \(a_v = 0.01\), \(a_f = \sqrt{3}/2\); (k) Input \(a_v = 0.01\), \(a_f = \text{Golden ratio}\) and Reference \(a_v = 0.01\), \(a_f = \text{Golden ratio}\)
If the amplitude of one interference signal is high, however less the other may be, the converter still exhibits chaotic behaviour as shown in Figures 5.31(j) and 5.31(k). It is also noticed that the converter enters into the chaotic regime for lesser values of \( \alpha \), in comparison with the presence of interference signal at one stage. As an example, for silver input frequency ratio of strength 0.001 and golden reference frequency ratio of strength 0.01, the converter exhibits chaotic behaviour whereas with one interference signal, the converter would have entered chaotic regime only at silver input frequency ratio of strength 0.5.

5.5.2 Closer to Rational Frequency Ratios

When the interference signals in the input and reference voltage have frequencies closer to the switching frequency or its rational multiples, the following observations are made.

- When both the interference signals are of different frequencies, the converter follows the greatest \( N_{den} \) periodicity at steady state but with oscillation. As an example, with the input and reference interference signal of frequency 1251Hz and 5001Hz respectively, the steady state operation of the converter is period-2 as depicted in Figure 5.32(a).

- When the strength of one interference signal is high enough to induce intermittent chaos even when present alone, irrespective of the other interference signal’s strength, the converter endangers intermittent chaos. For instance,
Figures 5.32(b) and 5.32(c) show the case when input and reference interference signals are of strength 0.3 and 0.001 respectively but in Figure 5.32(b), both the frequencies are 1251Hz whereas in Figure 5.32(c), the frequencies are 1251Hz and 2501Hz respectively.

Figure 5.32 Time bifurcation diagrams with the presence of two interference signals ($v_{SE}$) and ($v_{SR}$) in input $E$ and reference $V_{ref}$ respectively. (a) Input $a_e = 0.001$, $a_f = 1251$Hz and Reference $a_e = 0.0001$, $a_f = 5001$Hz; (b) Input $a_e = 0.3$, $a_f = 1251$Hz and Reference $a_e = 0.001$, $a_f = 1251$Hz; (c) Input $a_e = 0.3$, $a_f = 1251$Hz and Reference $a_e = 0.001$, $a_f = 2501$Hz; (d) Input $a_e = 0.3$, $a_f = 1251$Hz and Reference $a_e = 0.5$, $a_f = 1251$Hz
When the strengths of both the interference signals are sufficiently high so that they induce intermittent chaos when present solely, the converter surprisingly exhibits the greatest $N_{den}$ periodicity with oscillation. To illustrate, the same condition as in Figure 5.32(b) is considered but the strengths are taken as 0.3 and 0.5 respectively. As in Figure 5.32(d), the time bifurcation diagram clearly confesses the steady state period-2 operation.

5.6 CONCLUSION

In this chapter, detailed analytical, numerical and experimental investigations have been carried out to show the mechanism of loss of stability of the periodic orbit in a classic voltage-mode controlled buck converter when it experiences period-bubbling and intermittency due to the presence of periodic intruding signal that can easily be coupled to the converters via unintended paths. A quasi-periodic behaviour is observed when the ratio of the interference frequency to the switching signal frequency is irrational for the cases of symmetrical periodic interference signals and period-$N_{den}$ operation when the ratio is rational. It is shown that intermittency occurs with the intended period-$N_{den}$ operation when the interference frequency approaches the switching frequency or its rational multiples. It is also inferred that due to the asymmetrical nature of saw-tooth signal even lesser amplitude of saw-tooth interference results in intermittent chaos as compared to sinusoidal interference. Also the possibilities of the simultaneous presence of two interference signals with commensurate and incommensurate frequency ratios have been considered and the occurrence of 3-frequency quasi-periodicity has been identified.
Mathematical analysis has been carried out by deriving the exact discrete-time map and examining the movements of the characteristic multipliers of the Jacobian as some chosen parameters are varied. Numerical simulation results are presented using suitable Poincaré sections, time-bifurcation diagrams and parameter-bifurcation diagrams. The dynamics also has been verified by developing a prototype and control based on DSP.