CHAPTER-3

BLOOD FLOW THROUGH STENOSED INCLINED TUBES WITH PERIODIC BODY ACCELERATION IN THE PRESENCE OF MAGNETIC FIELD

3.1. INTRODUCTION

In this paper, using finite Hankel and Laplace transforms, analytical expressions for velocity profile, volumetric flow rate and wall shear stress have been obtained and their natures are portrayed graphically for different parameters such as Hartmann number, phase angle, time etc. in an inclined tube under stenoses.

3.2. MATHEMATICAL FORMULATION

Let us consider the axially symmetric and fully developed pulsatile flow of blood through a stenosed porous circular artery with body acceleration under the influence of uniform transverse magnetic field. Blood is assumed to be Newtonian and incompressible fluid. Also for mathematical model, we take the artery to be a long cylindrical tube with the axis along z-axis. The pressure gradient and body acceleration are respectively given by the expressions (2.2.1) and (2.2.2).

The governing equation of motion for flow in cylindrical polar coordinates is given by

$$\rho \frac{\partial u}{\partial t} = -\frac{\partial P}{\partial z} + \rho G + \mu \nabla^2 u - \sigma B_0 u + \rho g \sin \theta$$  \hspace{1cm} (3.2.1)

where \(u\) is the axial velocity of blood; \(P\), blood pressure; \(\frac{\partial P}{\partial z}\), pressure gradient; \(\rho\), density of blood; \(\mu\), the viscosity of blood; \(B_0\), the external magnetic field along the radial direction and \(\sigma\) is the conductivity of blood.

The geometry of stenosis is shown in figure-1.

$$R(z) = \begin{cases} \ a - \delta(1 + \cos \frac{\pi z}{2z_0}), & -2z_0 \leq z \leq 2z_0 \\ a, & \text{otherwise} \end{cases}$$  \hspace{1cm} (3.2.2)

where \(R(z)\) is the radius of the stenosed artery, \(a\) is the radius of artery, \(4z_0\) is the length of stenosis and \(2\delta\) is the maximum protuberance of the stenotic form of the artery wall.
\[ \xi = \frac{r}{R(z)} \text{ where } R(z) \text{ depends on } \delta. \]

The equation (3.2.1) becomes

\[ \rho \frac{\partial u}{\partial t} = A_0 + A_i \cos(\omega_p t) + \rho a_0 \cos(\omega_q t + \phi) + \frac{\mu}{R^2} \left[ \frac{\partial^2 u}{\partial \xi^2} + \frac{1}{\xi} \frac{\partial u}{\partial \xi} \right] - \frac{\mu C^2 u}{\xi} + \rho g \sin \theta \quad (3.2.3) \]

where

\[ C = \sqrt{\frac{M^2}{R^2}} \quad M = \sqrt{\frac{\sigma}{\mu} RB_0 (Hartmann number)} \]

We assumed that \( t < 0 \) only the pumping action of the heart is present and at \( t = 0 \), the flow in the artery corresponds to the instantaneous pressure gradient i.e.,

\[ -\frac{\partial p}{\partial z} = A_0 + A_i \]

As a result, the flow velocity at \( t = 0 \) is given by

\[ u(\xi, 0) = \frac{A_0 + A_i}{\mu C^2} \left[ 1 - \frac{I_0(CR\xi)}{I_0(CR)} \right] \quad (3.2.4) \]

where \( I_0 \) is modified Bessel function of first kind of order zero.
The initial and boundary conditions to the problem are

\[ u(\xi,0) = \frac{A_0 + A_1}{\mu c^2} \left[ 1 - \frac{I_0(CR\xi)}{I_0(CR)} \right] \]

\[ u = 0 \quad \text{at} \quad \xi = 1 \quad \text{(3.2.5)} \]

\[ u \quad \text{is finite} \quad \text{at} \quad \xi = 0 \]

3.3. SOLUTIONS

Applying Laplace transform to equation (3.2.3) and first boundary condition of (3.2.5), we get

\[ \rho s \bar{u} - \left( \frac{\rho (A_0 + A_1)}{\mu c^2} \right) \left[ 1 - \frac{I_0(CR\xi)}{I_0(CR)} \right] = \frac{A_0}{s} + \frac{A_1}{(s^2 + \omega_0^2)} + \frac{\rho \omega_0 (s \cos \phi - \omega_0 \sin \phi)}{(s^2 + \omega_0^2)} + \mu \frac{\partial^2 \bar{u}}{\partial \xi^2} + \frac{\delta \bar{u}}{\partial \xi} - \frac{\mu c^2 \bar{u} + \rho g \sin \theta}{s} \]

\[ (3.3.1) \]

where \( \bar{u}(\xi,s) = \int_0^\infty e^{-st}u(\xi,t)dt (s > 0) \)

Then applying the finite Hankel transform to equation (3.3.1), we obtain

\[ \bar{u}'(\lambda, s) = \frac{J_0(\lambda R)}{\lambda [\rho s^2 + \mu (C^2 R^2 + \lambda^2)]} \left( \frac{A_0}{s} + \frac{A_1}{(s^2 + \omega_0^2)} + \frac{\rho \omega_0 (s \cos \phi - \omega_0 \sin \phi)}{(s^2 + \omega_0^2)} + \frac{\rho (A_0 + A_1) R^2}{\mu (C^2 R^2 + \lambda^2)} + \frac{\rho g \sin \theta}{s} \right) \]

\[ (3.3.2) \]

where \( \bar{u}'(\lambda, s) = \int_0^1 r u(r,s)J_0(r \lambda)dr \) and \( \lambda \) are zeros of \( J_0 \), Bessel function of first kind and \( \nu = \frac{\mu}{\rho} \)

The Laplace and Hankel inversions of equation (3.3.2) give the final solution for blood velocity as

\[ u(\xi, t) = e^{\frac{-\lambda_0^2 g \sin \theta}{\mu (\lambda_0^2 + C^2 R^2)}} \left\{ \frac{A_0 + \rho g \sin \theta}{\mu (\lambda_0^2 + C^2 R^2)} \right\} + \frac{A_1}{(s^2 + \omega_0^2)} + \frac{\rho \omega_0 (s \cos \phi - \omega_0 \sin \phi)}{(s^2 + \omega_0^2)} + \frac{\rho (A_0 + A_1) R^2}{\mu (C^2 R^2 + \lambda_0^2)} + \frac{\rho g \sin \theta}{s} \]

\[ (3.3.3) \]
which can be written in the form

\[
\begin{align*}
\frac{2A_0R^2}{\mu} & \sum_{n=1}^{\infty} J_n(\lambda_n R) \left\{ A_n + g \sin \theta \right\} + \frac{\epsilon (\lambda_n^2 + C^2 R^2) \cos \omega t + \alpha^2 \sin \omega t}{(\lambda_n^2 + C^2 R^2) + \alpha^2} + \frac{\rho a_b}{A_0} \left\{ \frac{(\lambda_n^2 + C^2 R^2) \cos(\omega t + \phi) + \beta^2 \sin(\omega t + \phi)}{(\lambda_n^2 + C^2 R^2) + \beta^2} \right\} \\
& - e^{-t \lambda_n^2 C^2 R^2} \frac{u_R}{\lambda_n^2 + C^2 R^2} \left( (\lambda_n^2 + C^2 R^2)(\alpha^2 + (\lambda_n^2 + C^2 R^2))^2 \right) + \frac{\rho a_b}{A_0} \left\{ \frac{(\lambda_n^2 + C^2 R^2) \cos(\omega t + \phi) + \beta^2 \sin(\omega t + \phi)}{(\lambda_n^2 + C^2 R^2) + \beta^2} \right\} \\
& + g \sin \theta \left( \frac{u_R}{R} \right) \frac{\mu}{(\lambda_n^2 + C^2 R^2)}
\end{align*}
\]

(3.3.4)

where \( \alpha^2 = \frac{\omega_p R^2}{v} = \text{Re}_p \), \( \beta^2 = \frac{\omega_b R^2}{v} = \text{Re}_b \), \( \varepsilon = \frac{A_1}{A_0} \)

The analytical expression of \( u \) consists of four parts. The first and second parts correspond to steady and oscillatory parts of pressure gradient, the third term indicates body acceleration and the last term is the transient term. As \( t \to \infty \), the transient term approaches to zero. Then from equation (3.3.4), we get

\[
\begin{align*}
\frac{2A_0R^2}{\mu} & \sum_{n=1}^{\infty} J_n(\lambda_n R) \left\{ A_n + g \sin \theta \right\} + \frac{\epsilon (\lambda_n^2 + C^2 R^2) \cos \omega t + \alpha^2 \sin \omega t}{(\lambda_n^2 + C^2 R^2) + \alpha^2} + \frac{\rho a_b}{A_0} \left\{ \frac{(\lambda_n^2 + C^2 R^2) \cos(\omega t + \phi) + \beta^2 \sin(\omega t + \phi)}{(\lambda_n^2 + C^2 R^2) + \beta^2} \right\} \\
& - e^{-t \lambda_n^2 C^2 R^2} \frac{u_R}{\lambda_n^2 + C^2 R^2} \left( (\lambda_n^2 + C^2 R^2)(\alpha^2 + (\lambda_n^2 + C^2 R^2))^2 \right) + \frac{\rho a_b}{A_0} \left\{ \frac{(\lambda_n^2 + C^2 R^2) \cos(\omega t + \phi) + \beta^2 \sin(\omega t + \phi)}{(\lambda_n^2 + C^2 R^2) + \beta^2} \right\} \\
& + g \sin \theta \left( \frac{u_R}{R} \right) \frac{\mu}{(\lambda_n^2 + C^2 R^2)}
\end{align*}
\]

(3.3.5)

The volumetric flow rate \( Q \) is given by

\[
Q(\xi, t) = 2\pi \int_0^R rq \, dr
\]

(3.3.6)

The fluid acceleration \( F \) is given by

\[
F(\xi, t) = \frac{\partial^2 u}{\partial t}
\]

(3.3.7)
The expression for the wall shear stress \( \tau_w \) can be obtained from

\[
\tau_w = \mu \left( \frac{\partial u}{\partial r} \right)_{r=R}
\]

\[
\tau_w(\xi, t) = -2AR\sum_{s=1}^{\infty} \left[ \frac{A_0 + g \sin \theta}{A_0 (\lambda_n^2 + C^2 R^2)} \left( \frac{(\lambda_n^2 + C^2 R^2) \cos \omega_s t + \alpha^2 \sin \omega_s t}{(\lambda_n^2 + C^2 R^2)^2 + \alpha^2} \right)^t + \frac{\rho a_0 \left( \frac{(\lambda_n^2 + C^2 R^2) \cos (\omega_s t + \phi) + \beta^2 \sin (\omega_s t + \phi)}{(\lambda_n^2 + C^2 R^2)^2 + \beta^2} \right)}{A_0} \right]
\]

(3.3.8)

3.4. RESULTS

Fig. 3.2. Variation of velocity profiles for aorta artery against \( \xi \) with \( \phi=45^0, \theta=30^0, M=2.0 \)
Fig. 3.3. Variation of velocity profiles for aorta artery against $t$ with $\phi=45^0, \theta=30^0, M=2.0$
Fig. 3.4. Variation of flow rate for aorta artery against $\phi$ for $\theta = 30^0, g = 9.8, M = 2.0$
Fig. 3.5. Variation of flow rate for aorta artery against $t$ for $\phi = 30^\circ$, $g = 9.8$, $M = 2.0$, $\theta = 45^\circ$
Fig. 3.6. Variation of fluid acceleration for aorta artery against $\xi = 0.2, M = 2.0, \theta = 30^0, g = 9.8$
Fig. 3.7. Variation fluid acceleration for aorta artery against $\zeta$ for $M=2.0, \theta = 60^0, g = 9.8, \phi=45^0$. 
In fig(3.2), if we plot $u$ verses $\xi$ for fixed values of $\phi = 45^\circ$, $\theta = 30^\circ$, $M=2.0$, $u$ decreases with increasing $\xi$ for different $t$.

In fig(3.3), for fixed values of $\phi = 45^\circ$, $\theta = 30^\circ$, $M=2.0$, if we plot $u$ verses $t$, we get the oscillatory nature of curves for different $\xi$.

For $\theta = 30^\circ$, $g=9.8$, $M=2.0$, if we plot $Q$ verses $\phi$ for different $t$, we get the curves as shown in figure(3.4).

In fig(3.5), for fixed $\theta = 30^\circ$, $g=9.8$, $M=2.0$, $\phi = 45^\circ$, if we plot $Q$ verses $t$ for different $\theta$, as $\theta$ increases $Q$ is also increases.
In fig(3.6), for fixed $\xi=0.2, M=2.0, \theta = 30^\circ, g=9.8$, if we plot $F$ verses for increasing $\phi$, we get oscillatory nature of curves.

For fixed $M=2.0, \theta = 60^\circ, g=9.8, \phi = 45^\circ$, if we plot $F$ verses $\xi$ for different $t$, we get the curves as shown in fig(3.7).

In fig(3.8), for fixed $\phi = 45^\circ, M=2.0, g=9.8, \alpha = \beta = 1$, if we plot $\tau$ verses $t$ for $\theta$, as $\theta$ increases, $\tau$ decreases for increasing $t$. 