CHAPTER 3

PAPR REDUCTION USING ALL PASS FILTERING

3.1 PRELUDE

PAPR reduction has been a subject of intense research in the past decade. Among them, Partial Transmit Sequence (PTS) and Selected Mapping (SLM) are two promising techniques because they are simple to implement, no distortion is introduced in the transmitted signal and they can significantly improve the performance of the OFDM signal. PTS along with an algorithm is an efficient technique for improving the statistics of a multicarrier signal. In that, the stochastic optimization algorithm is very effective. This chapter considers the use of the PTS technique in reducing the PAPR of an OFDM signal [34]. The O-PTS technique is highly successful in PAPR reduction for OFDM signals, but the considerable computational complexity in search of the optimum phase value is a potential problem for practical implementation. To reduce the search complexity and to improve the, signal performances stochastic optimization techniques is proposed along with All Pass filter, in this work.

3.2 THE PARTIAL TRANSMIT SEQUENCE (PTS) TECHNIQUE

In the PTS technique as shown in Fig. 3.1, an input data block of N symbols is partitioned into disjoint sub-blocks.

The sub-carriers in each sub-block are weighted by a phase factor for each sub-block. The phase factors are selected such that the PAPR of the combined signal is minimized. In the O-PTS technique, input data block X is partitioned into V disjoint sub-blocks $X_i = [X_{i0}, X_{i1}, \ldots, X_{i,N-1}]^T$, $i = 1, 2, \ldots, V$, such that $\sum_{i=1}^{V} X_i = X$ and the sub-blocks are combined to minimize the PAPR in the time domain.
The L-times oversampled time domain signal of $X_i$, $i = 1, 2, \ldots, V$, is denoted $x_i = [x_{i,0}, x_{i,1}, \ldots, x_{i,NL-1}]^T$. $x_i$, $i = 1, 2, \ldots, V$, is obtained by taking an IFFT of length $NL$ on $X_i$ concatenated with $(L - 1)N$ zeros. These are called the Partial Transmit Sequences (PTS) [1]. Complex phase factors, $b_i = e^{j\phi_i}$, $i = 1, 2, \ldots, V$, are introduced to combine the PTS. The set of phase factors is denoted as a vector $b = [b_1, b_2, \ldots, b_V]^T$.

The time domain signal after combining is given by

$$x'(b) = \sum_{i=1}^{V} b_i x_i$$  \hspace{1cm} (3.1)

where, $x'(b) = [x'_0(b), x'_1(b), \ldots, x'_{NL-1}(b)]^T$. The objective is to find the set of phase factors that minimizes the PAPR. Minimization of PAPR is related to the minimization of

$$\max_{0 \leq n \leq NL-1} |x'_k(b)|$$ \hspace{1cm} (3.2)

In general, the selection of the phase factors is limited to a set with a finite number of elements to reduce the search complexity. The set of allowed phase factors is written as $P = \{e^{j2\pi/kW}, k = 0, 1, \ldots, W - 1\}$, where, $W$ is the number of allowed phase factors. So, exhaustive search for $(V - 1)$ phase factors should be
performed. Hence, $W^{V-1}$ sets of phase factors are searched to find the optimum set of phase factors. The search complexity increases exponentially with the number of sub-blocks $V$.

There are three kinds of sub-block partitioning schemes: adjacent, interleaved and pseudo-random partitioning. Among them, pseudo-random partitioning has been found to be the best choice. The PTS technique works with an arbitrary number of sub-carriers and any modulation scheme, the O-PTS technique has exponentially increasing search complexity [46-50].

3.3 OBJECTIVE OF THE MODULE

➢ To obtain optimal phase factors for minimum PAPR in PTS using All Pass filters and using Phase Optimization Algorithm.

3.4 PROPOSED METHOD

The proposed schemes are PTS based to find the optimal combination of phase factors with less number of iterations for keeping good PAPR reduction.

Partial Transmit Sequence (PTS) technique has been proposed by Muller and Hubber in 1997 [19]. This proposed method is based on the phase shifting of sub-blocks of data and multiplication of data structure by random vectors. This method is flexible and effective for OFDM system. The main purpose behind this method is that the input data frame is divided into non-overlapping sub blocks and each sub block is phase shifted by a constant factor to reduce PAPR.

PTS is a probabilistic method for reducing the PAPR problem. It can be said that PTS method is a modified method of SLM. PTS method works better than SLM method [54]. The main advantage of this scheme is that there is no need of more side information to the receiver of the system, when differential modulation is applied in all sub-blocks. The modified form O-PTS with All Pass filters is shown in Fig.3.2.
The alternative OFDM sequences and produced by rotating the symbol phase using multiple All Pass filters in the proposed PTS, whereas the phase rotation of O-PTS schemes is performed with multiple complex multiplication modules with IFFT modules. Also many IFFTs are replaced as single N point IFFT. By this, the computational complexity can be reduced [33].

Assuming that $W$ is the number of allowed phase factors, the optimum set of phase factors required to perform an exhaustive search over $C=W^{V-1}$ combinations. The computational complexity is $\text{LNVC} + \text{LNC}$ complex multiplications and $2\text{LNC} (V-1) + \text{LNC}-1$ real additions. The amount of PAPR reduction depends on number of sub-blocks $V$ and the number of allowed phase factors $W$. In [57], the authors propose a new PTS scheme where a cost function $Q_n$ is generated by summing the power of the time-domain samples at time ‘n’ in each sub-block. From equation (3.3) and by applying the Cauchy-Schwartz inequality

$$\left|\sum_{\nu=1}^{V} b_\nu x_\nu \right|^2 \leq \sum_{\nu=1}^{V}|b_\nu |^2 \times \left|\sum_{\nu=1}^{V} x_\nu \right|^2 = VQ_n$$  (3.3)
\[ VQ_n \geq \max_{0 \leq t \leq N_L - 1} |x_n'(b)|^2 \geq \Phi_N \]  \hspace{1cm} (3.4)

where, phase factor \( \Phi_N = z \times Pav. \)

If \( \Phi_N \) is the minimum possible peak power among different time-domain symbols in an OFDM system with \( N \) sub-carrier, then only those samples with \( Q_n \) greater than, or equal to, a preset threshold \( \alpha = \Phi_N / V \) are used for peak power calculation during the process of selecting a candidate signal with the lowest PAPR [23]. This minimum peak power is applied to All Pass filters.

### 3.4.1 All Pass Filters

An All Pass filter is a signal processing filter that passes all frequencies equally, but changes the phase relationship between various frequencies [122]. It can be done by varying its propagation delay with frequency. Generally, the filter is described by the frequency at which the phase shift crosses 90° (i.e., when the input and output signals go into quadrature when there is a quarter wavelength of delay between them).

An All Pass filters passes all input frequencies with the same gain, but the phase of the signals will be modified. All Pass filter has a gain of one and such filters are used for group delay equalization, notch filtering design, Hilbert transform implementation and musical instruments synthesis. All Pass filters delay the OFDM sequence by providing delay and reduced interference. In a PTS scheme, different frequency domain OFDM sequences are generated and then transformed into time domain sequences by using multiple IFFT modules. In the proposed scheme, different OFDM sequences are directly generated in the time domain, hence the need for IFFT modules can be eliminated. Generation of the alternative time domain OFDM sequences can be performed using multiple All Pass filters [8]. The general function of an All Pass filter is given by

\[ H(Z) = \prod_{k=1}^{K} \frac{z^{-1} - c_k^*}{1 - c_k z^{-1}} \]  \hspace{1cm} (3.5)
where, $c_k$, $c_k^*$ and $K$ are the $k^{th}$ complex pole, its complex conjugate and the number of the complex poles, respectively. The magnitude response $|H(e^{j\omega})|$ and the phase response are given as angle of $H(e^{j\omega})$ are given as

$$|H(e^{j\omega})| = 1$$  \hspace{1cm} (3.6)

Thus the existing scheme has a drawback that it has lesser number of choices for selecting the optimum phase sequence in O-PTS methods. Therefore the proposed scheme has a prime advantage that it has more PAPR reduction capability as compared to existing and O-PTS methods. PAPR reduction in OFDM using PTS with optimization algorithm gives better performance than other PAPR reduction techniques.

### 3.4.2 Optimization Techniques in PTS

There are different types of optimization techniques to reduce PAPR in OFDM. The conventional stochastic optimization techniques such as the Simulated Annealing (SA) algorithm, Cross-Entropy (CE) method, Particle Swarm Optimization (PSO) and iterative flipping algorithm have recently been proposed to search for a phase factor that reduces both the PAPR statistics and the computational load [41]. In the proposed one, the PTS scheme is modified using All Pass filter with new optimization algorithm such as Electromagnetic-like (EM) algorithm.

### 3.4.3 Proposed Optimization Algorithm

The Electromagnetism-like Method (EM) is a population based stochastic global optimization method inspired by the Coulomb’s Law of the electromagnetism theory. The EM method starts with an initial solution set (particles) and an attraction-repulsion mechanism is then used iteratively to move those particles towards optimality. The general scheme for the EM method consists of four main procedures:

(i) Initialization
(ii) Local search

(iii) Calculation of the total force

(iv) Movement of the particles

These procedures are interpreted as follows:

**Initialization**

The initialization is used to sample $M$ points (particles) randomly from the feasible region. As a first step, the particle population is initialized at $k = 0$. Like most of the stochastic algorithms, the EM method starts with generating $M$ random sample particles from the feasible region

$$\left\{ \{ \theta^k_{m,v} \}_v \right\}_m^M$$  \hspace{1cm} (3.7)

where, $V$ is the dimension of the problem (i.e., the number of sub-blocks) and $\theta^k_{m,v}$ denotes the $v^{th}$ coordinate of the particle $m$ of the population at iteration $k$ analogous to Electromagnetism, each particle of

$$\Theta^k_m = \{ \{ \theta^k_{m,v} \}_v \}_v$$  \hspace{1cm} (3.8)

The eq (3.8) is regarded as a virtually charged particle that is released in the space. It should be noted that in a multi-dimensional solution space where each particle represents a solution, a charge is associated with each particle. As such, each coordinate of a particle, denoted as $\theta^k_{m,v}$ is computed by

$$\theta^k_{m,v} = lv + \lambda(u_v - lv)$$  \hspace{1cm} (3.9)

where, $lv$ is the lower bound of the $v^{th}$ dimension and $\lambda$ is a uniform random number generator within $[0$ and $1]$. The values of phase factors in the range of $0$ to $2\pi$ the upper bound and lower bound are set to $0$ to $2\pi$ respectively.
Therefore, the range of phase factor will be bounded at [0 to 2 \pi].

Meanwhile, since \( \lambda \) is a uniform random number generator within [0 and 1], the distribution of phase factor is uniform distribution with [0 to 2 \pi]. After a particle is sampled from the space, the objective function value for the particle is calculated. Given a particle (i.e., phase factor vector), the fitness function, defined as the amount of PAPR reduction, can be expressed as

\[
f(\Theta^k_m) = 10 \log_{10} \frac{\max|x(\Theta^k_m)|^2}{E|[x(\Theta^k_m)]^2|}
\]  \hspace{1cm} (3.10)

When the \( M \) particles are all identified, the particle with the best objective function value is stored into

\[
\Theta^k_{best} = \{\theta^i_{best,v}\}_{v=1}^V
\]  \hspace{1cm} (3.11)

- **Local Search**

The Local search is used to gather the neighborhood information for a sampled particle, which can be applied to one particle or to all particles in the population for local refinement at each iteration. Theoretically, the local search is expected to find a better solution especially when it is applied to all particles. However, the local search is usually time consuming. Therefore, in this study, the EM algorithm is implemented with local search on the current better particle.

- **Calculation of Maximum Feasible Random Step Length**

The following steps are used to calculate the maximum feasible random step length:

1. The length is calculated by the maximum difference of each dimension’s upper and lower bound. Since the upper and lower bound of each dimension is 2\pi and 0, respectively, the maximum difference of each dimension’s upper and lower bound is 2\pi. Second, it makes use of the parameter \( \delta \in [0,1] \) to have a feasible
random length. Therefore, the maximum feasible step length can be computed using the following equation:

\[ S_{\text{max}} = \max_{1 \leq v \leq V} (u_v - l_v) \]  

(iii) A new particle is generated from the current best particle

(iv) Decide whether to update the current best particle \( \Theta^k_{\text{best}} \).

(v) If the new particle \( \Theta_{\text{new}} \) observes a better particle, the sample particle \( \Theta^k_{\text{best}} \) is replaced by this new particle.

(vi) Repeat Step (i) to Step (v) until the maximum number of local search iteration is met.

- **Calculation of Total Force**

  In this procedure, an artificial electromagnetism field is built to propel the particles to new positions via the Coulomb’s law of the electromagnetism theory. The artificial charge \( q^k_m \) at particle \( f(\Theta^k_m) \) is determined by the fitness function value and is calculated using the following equation

\[ q^k_m = \exp \left\{ -V \frac{f(\Theta^k_m) - f(\Theta^k_{\text{best}})}{\sum_{m=1}^{M} [f(\Theta^k_m) - f(\Theta^k_{\text{best}})]} \right\} \]  

- **Movement of the Particles**

  The particle \( m \) is updated in \( v^{th} \) coordinate of the force by a random step length. Repeat local search and movement of particle steps for \( k=k+1 \) until the maximum number of iteration is met. The searched phase factors using optimized algorithms are multiplied with time domain values of partitioned sub-blocks. The multiplied values are then applied to All Pass filters. The All Pass filters are working
like equalizers and limit the high peaks and maintain magnitude as 1 and phase as 180°. Then the optimal combination of phase factors with lowest PAPR is selected.

3.5 SIMULATION RESULTS

The performance of the proposed scheme is realized by the simulation carried out in the MATLAB (version 7.9.0 R2009a). The simulation parameters are listed in Table 3.1.

Table 3.1 Simulation Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Software Tool</td>
<td>MATLAB 7.9</td>
</tr>
<tr>
<td>Sub-carriers</td>
<td>1024</td>
</tr>
<tr>
<td>Maximum Symbols</td>
<td>10000</td>
</tr>
<tr>
<td>No. of Sub-blocks</td>
<td>4</td>
</tr>
<tr>
<td>Modulation</td>
<td>QPSK</td>
</tr>
<tr>
<td>Channel Used</td>
<td>Rayleigh Channel</td>
</tr>
</tbody>
</table>

Complementary Cumulative Distribution Function (CCDF) versus calculated PAPR are evaluated for O-PTS and proposed optimized PTS with All Pass filters and compared their performances that are obtained using QPSK modulation at Rayleigh channel. BER versus SNR is calculated and plotted for proposed methods. The following section provides the results and discussion of the proposed system.

3.5.1 PAPR Performance Comparisons

The PAPR is calculated and compared for different methods. It is well known that if peak power is minimized during high peak signal, then the PAPR can
be controlled. The simulated results in Fig. 3.3 show the comparison of PAPR reduction capabilities of the proposed PTS and O-PTS. From Fig. 3.3 it is clearer that the PAPR performance is improved by applying the PTS with All Pass filter method. It can be observed that the performance of the proposed PTS scheme in terms of PAPR reduction is better than that of the O-PTS scheme for all configurations. This is because of the ability of the proposed PTS scheme.

**Fig. 3.3 Comparison of PAPR reduction with and without All Pass filter**

An All Pass filter is a signal processing filter that passes all frequencies equally, but changes the phase relationship between various frequencies. It can be done by varying its propagation delay with respect to frequency for more candidates having the same parameters. From Fig. 3.3 it can be observed that the PTS with All Pass filter gives better performance of 10.4 dB for $10^{-4}$ CCDF than compared to O-PTS where it needs 12 dB for $10^{-4}$ CCDF.

3.5.2 **Optimum Phase Factor Search**
In this subsection, Electromagnetism like algorithm is applied to find optimum phase value to reduce the PAPR of OFDM symbols. In this algorithm, the sub-carrier phases are systematically changed so that the PAPR is reduced. The objective of the algorithm is to find the minimum power needed in order to transmit the OFDM Symbol. The results show that the performances of the proposed PTS scheme can be improved by finding optimum phase value by using phase optimization algorithm. Using the proposed algorithm the optimum phase value is found as 260°.

Fig. 3.4 Searching of optimum phase value

Fig. 3.4 shows the searching process of optimum phase which gives minimum PAPR. From the graph, it is inferred that, the minimum PAPR of 9.8 dB is achieved at 260° phase. This 260° phase is implemented for the sub-carrier to obtain minimum PAPR. This algorithm is performed for entire 360° to find minimum power to transmit the symbol. This process is repeated until the PAPR is not decreasing further more.
For fairness of comparison of the PAPR performance, the data point Vs transmitted data phase representation is presented in the Fig.3.5. While the sub-carrier amplitudes stay the same during the transmission, the PAPR is reduced by adapting the sub-carrier phases. The choice of phase value for transmission has a great influence on the performance of the algorithm.

Fig. 3.5 Transmitted data with phase representations
Fig. 3.5 shows the phase representations of transmitted data for 64 data points. This is transmitted through Rayleigh channel. Fig 3.5 shows clearly the change in statistical characteristics of the OFDM output signals maintained for both the amplitude and power of the OFDM output signals resulting from the All Pass filter and by applying optimization algorithm.

Fig. 3.6 shows the OFDM signals for amplitude and time. From the impact of the proposed technique, it plays a vital role in not only reducing the transmitting power and to also maintain the magnitude responses throughout the transmission. Even though the transmission power is decreased the significant performances is still maintained by maintaining the magnitude response of the proposed system.
Fig. 3.7 Phase representations of received data

Fig. 3.7 shows the phase representations of received data for 64 data points after phase factor implementations by using the proposed technique using All Pass filter and phase optimization algorithm. From Fig. 3.7, it is observed that there is a minimum phase deviation between transmission and reception because of Rayleigh channel. But the significant performances can be achieved by using the proposed system at the receiver side. The proposed scheme can provide better performance that meets the requirements for various systems by adjusting the phase value with the help of the proposed phase optimization algorithm. Fig. 3.8 shows the comparison of PAPR Reduction with All Pass filter and phase optimization algorithm.
By comparing with the PTS with All Pass filters, the PTS with All Pass filter and phase optimization perform better in terms of PAPR reduction by 0.6 dB and also provide significant bit error performance.

Fig. 3.9 gives the detail about BER performance for proposed system which results that 12 dB SNR for $10^{-4}$ BER. Accordingly the proposed algorithm is used for reducing PAPR. The All Pass filter used to maintain the phase of the signal though out the transmission. It has been found that the proposed system performs better compared to existing system without affecting the receiver performances even though the transmission power is reduced.
The PAPR performances obtained by different techniques have been tabulated in Table 3.2. PAPR reductions in OFDM using PTS with optimization algorithm gives better performance than other PAPR reduction techniques.

**Table 3.2 Performance of Proposed scheme**

<table>
<thead>
<tr>
<th>Technique</th>
<th>PAPR at $10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original PTS (128 sub-carrier)</td>
<td>12 dB</td>
</tr>
<tr>
<td>PTS with All Pass filters (1024 sub-carrier)</td>
<td>10.4 dB</td>
</tr>
<tr>
<td><strong>PAPR when optimum phase $260^\circ$ at transmitter with 1024 sub-carrier (Proposed Technique)</strong></td>
<td><strong>9.8 dB</strong></td>
</tr>
<tr>
<td>BER Performance (1024 sub-carriers)</td>
<td>12 dB</td>
</tr>
</tbody>
</table>
3.6 CONSUMMATION

Partial Transmit Sequence (PTS) with All Pass filters is implemented to reduce the PAPR of OFDM signals. Using optimization technique, the phase factor which is giving minimum PAPR is found and it is multiplied with data vectors. The QPSK modulation technique is taken and transmitted through Rayleigh fading channel. By comparing the PAPR reduction performance for various schemes, the optimized PTS with All Pass filtering shows better performance with reduced complexity. It was found that with simple All Pass filters in the proposed scheme could significantly reduce the computational complexity, at the cost of slightly worse PAPR reduction performance over the O-PTS scheme without BER performance degradation.

It is obvious that the PAPR increase is due to the increase of sub-carrier. The results show that for the O-PTS, PTS with All Pass filters and PTS with fixed phase factors are having the simulated PAPR values of 12 dB, 10.4 dB and 9.8 dB at $10^{-4}$ CCDF respectively. So it is proved that the proposed optimized PTS with All Pass filtering method has better PAPR reduction performance than other techniques. But there is a limitation in the proposed system by finding the phase value within $360^\circ$. If the number of disjoint sub-blocks is going to be increased the complexity of the proposed system is going to be increased by finding the optimum phase value within $360^\circ$. So the technique called GPW and RPW is proposed in the next chapter along with PTS in order to find optimum phase and to reduce the complexity of the system in searching optimum phase value.