Chapter 5

Empirical Results: Application of different Value at Risk Models and Backtesting of the Model

5.1. Background of the study

The study devolves round the comparative analysis of VaR measures with criticality as to choice of statistical distribution and subsequent backtesting. It is in fact a critical issue as to estimation of the Monte Carlo simulation methods as the researcher as to choose the statistical distribution to use for the market factors. This flexibility may allow the researcher to make an injudicious choice, in the sense that the chosen distribution might not adequately approximate the actual distribution of the market factors. Here lies the challenge of implementation of the same. As such, a detailed empirical examination is made hereunder.

5.2. Analytical framework of the study

The objective of the study is to find our value at risk with suitable model and backtest the same. Banks with a considerable trading activity have been required to hold capital against substantial portfolio losses since 1998 by regulatory guidelines (Basel Committee,
These guidelines allow banks to use internal models for risk management, but the quality and accuracy of these models is evaluated and validated through backtesting. If the risk model produces too low quality and inaccurate estimates, the required capital guarantee is increased by the regulatory body. The regulatory objective is to minimize the probability of bankruptcies and financial crises, which can be achieved by setting strict enough guidelines for risk reporting and by formulating a conservative enough minimum requirement framework. Basel Committee (1996b) obliges the market risk measurement model in a bank to fulfill the following requirements at the minimum:

- Value-at-Risk must be computed on a daily basis. No particular approach must be used, as long as it captures all the relevant risks.

- Confidence level of 99 % and a forecast horizon of 10 trading days9 are to be used in VaR calculations.

- The historical observation period is constrained to one year (i.e. the period must be at least 250 trading days effectively). The bank must update its data sets at least quarterly or whenever market prices are subject to substantial changes.

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Banks have discretion to recognize empirical correlations within market risk categories (i.e. interest rates, exchange rates, equity prices and commodity prices). Bank’s model must also capture accurately the unique risks associated with options within each of the market risk categories. The minimum forecast horizon for repo-style transactions is set to 5 days.

5.3. Data sources

We have used S&P CNX nifty returns series as available from National Stock Exchange website for the period from 1st April 2000 to 31st March 2007. We have used the index as it is a proxy to portfolio returns, which is professionally managed to as to churning of the same. We have considerable data points i.e. 1755 for fitting.

5.4. Methodology and Derivation of Empirical Results

We have applied different methods of Value at Risk depending on the nature of the distribution and subsequently applied backtesting. In this section we have discussed the methodologies along with empirical result in a step by step manner.

Confidence Level

VaR measures the expected maximum loss in portfolio value and is not the worst possible outcome. The confidence level is \( p = (1-\alpha) \), which defines the probability of the expected maximum loss. The market risk surface can be analyzed by varying the level of confidence. The most common confidence levels are between 95 % and 99 %, although they can vary between 90 % and 99.9 %
The Basel Committee requires the use of 99% confidence level in official reporting (Basel Committee, 2005), as it must be high enough for capital requirement calculations, but a lower level of confidence (e.g. 95%) can be used for internal reporting. In our study, we have selected 95% level of confidence in order to find out VaR in a little lower level.

*Forecast Horizon*

The length of the period, for which the expected maximum loss is forecasted, is an important factor that affects VaR. This period, denoted by \( k \) earlier, is known as forecast horizon or holding period. Large deviations in the portfolio value are more probable over a long period than a short one, and VaR is usually greater for a holding period of one month than for a day, for instance. The portfolio composition is assumed to stay unchanged for VaR over the holding period. The adequate length of the holding period depends on the asset class (e.g. equity versus bonds), on the industry (e.g. banking versus insurance), on the internal position (e.g. trading unit versus financial control), and whether the risk is measured from a private or a regulatory perspective (Christoffersen et al., 1998). Trading activity and the liquidity of the assets (i.e. the time and ability to convert a position to cash) have also an impact on the adequate length of the holding period (Khindanova

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and Rachev\textsuperscript{48}, 2000). In practice, the holding period can vary from one trading day to some years, but the Basel Committee requires the use of 10-day holding period for official reporting. They still permit the use of a shorter holding period and scaling of VaR to correspond 10-day holding period\textsuperscript{1} (Basel Committee\textsuperscript{49}, 2006). Khindanova and Rachev\textsuperscript{50} (2000) suggest that a 10-day holding period is inadequate for frequently traded assets and restrictive for illiquid assets. As such we have taken 5-days horizon for computing VaR.

\textit{Historical Observation Period}

The length of the data sample in VaR calculation is known as the historical observation period or the window length of sample data. This observation period connects VaR to the history of the market risk factors, as the volatility of the risk factors is determined based on the length of the historical observation period. In practice the observation period may vary from a month to several years in practice,\textsuperscript{1} A one-period VaR can be scaled to a long horizon VaR by multiplying by


the square root of the length of the horizon. For instance, a one-day VaR may be scaled to ten-day VaR by multiplying it by 10. However, this is permitted only if short horizon returns are i.i.d., which is not always the case (Christoffersen et al., 1998). The regulatory requirement of 250 trading days produces rather accurate VaR forecasts when used with the most common volatility models and Historical Simulation VaR (Hendricks, 1996). Longer historical observation periods provide the most accurate forecasts (Khindanova and Rachev, 2000). Hendricks (1996) reports the superiority of 1,250-day historical observation period on the basis of an analysis of several VaR models with 95% and 99% levels of confidence. He finds the stability of unconditional distribution of changes in portfolio value to support the use of long periods. Hendricks’s (1996) results highlight the Basel Committee requirement for a minimum historical observation period of 250 days, as he finds shorter periods to produce inaccurate VaR measures. We have taken considerable long period from 1st April 2000 to 31st March 2007 having 1755 data points.

Valuation

VaR approaches can be categorized by several arguments.

The most common classifications are based on the underlying forecast

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55 Ibid.
model and on the valuation method. The forecast model classifies VaR approaches to parametric and non-parametric, which are referred to as model and data approaches, respectively. Parametric approaches employ the probability distribution function (PDF) of some statistical distribution as a forecast model of return distribution function for VaR. Non-parametric approaches implicitly derive the PDF from sample data set and calculate VaR for one or several quantiles. Non-parametric approaches make no assumption about the PDF, because the empirical distribution is used. The quality and the accuracy of the entire empirical PDF must be evaluated. Parametric approaches include a well-defined PDF, which can be used more efficiently in statistical power tests than the non-parametric PDF, if the distribution shape accurately matches the actual distribution. The valuation method in the VaR model classifies the approach to local or full valuation. Local valuation approaches model the portfolio risk based on one-time full valuation at the starting position, and afterwards the risk from the market movements is modeled with local derivatives. Full valuation approaches model risk through direct revaluation of the portfolio. Full valuation approaches are often more computationally intensive, but they are not necessarily more accurate in risk modeling than local valuation. Local valuation can be accurate provided that the derivatives used in risk modeling capture the portfolio risk adequately. Jorion\textsuperscript{56} (2001) suggests full valuation to be the only adequate approach to capture the market risk correctly in special linear risk

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cases and with portfolios of nonlinear instruments. In our case, we have used full valuation method.

We have generated the profit and loss from the index returns, which replicate that of a portfolio. The profit and loss generated by an asset (or portfolio) over the period t, \( P/L_t \), can be defined as the value of the asset (or portfolio) at the end of t minus the asset value at the end of t-1:

\[
P/L_t = P_t - P_{t-1}
\]

The positive value indicates profit and negative value indicate loss. For preliminary analysis of the data, we first plotted our reference data, the profit and loss distribution, in histogram as below.

**Figure 5.4.1: Histogram of absolute returns**

![Histogram of absolute returns](image)

In the next step, distribution fitting is attempted where as fit 1 is
normal and fit 2 is non-normal. The objective of the fitting is to find the nature of the distribution as methods of value at risk computation that we shall approach in the next step is dependent on distributional assumptions. Therefore, the identifying the underlying distribution is one of the most important inputs to any value at risk model.

5.4.2: fitting of Profit and Loss distribution

It could well be viewed that the distribution in our study is a good candidate for non-normal one. Therefore, it is not a good candidate for general parametric value at risk measure, where underlying assumption is returns are normally distributed.

In the next step, we attempted historical simulation. In historical simulation approaches, the distribution of the future shifts in the risk factors of a portfolio is a treated as the same way as the prior period
distribution of shits to simulate the value at risk. The most advantage
is that it is non-parametric and as such does not assume any
distributional assumption as to normality. We computed Value at Risk
(called as VaR) at risk and subsequent expected shortfall (called ES)
as per historical simulation. Expected Shortfall is a coherent risk
measure which considers risk beyond VaR level.

Historical Simulation (HS) approach generates the P/L
distribution for VaR estimation from historical samples and does not
rely on any statistical distribution or random process. Implementation
of Historical Simulation is straightforward but requires the aggregation
of daily price data. The approach uses full valuation and generates the
whole P/L distribution. Historical Simulation is nothing but the modified
bootstrapping method as it uses real historical sample of price
changes without replacement.
The empirical result for historical simulation VaR is 49.935 and Expected Shortfall is 92.5742. The major advantage of this method is that it neither assume returns are normally distributed nor it assumes returns are identically distributed over time. As a result, historical simulation model can well accommodate the fat tail for VaR computation unlike other simple approaches. This model does not bear much model risk for incorrect estimation of parameters as there is no necessity to estimate any parameter like volatilities, correlations or others.

Further, an attempt has been made to identify the behaviour of the tail as it is an important tool for risk measurement. We have constructed Exploratory tool like QQ plot to get a view of the
heaviness of the tail.

Figure 5.4.4: Application of Graphical Exploratory Tool: QQ Plot

As the QQ plot has steeper slopes at the tails and the tails have the slope different from the central mass, are suggestive of the empirical distribution have heavier, or thinner, tails than the reference distribution. This QQ plot is a good tool for identifying outliers (e.g. observations contaminated by large errors).

The improvement over the conventional Historical Simulation Approach is Bootstrap Approach. In bootstrap method the samples are drawn from same historical data with replacement. The benefit with it that it implicitly takes the volatilities and correlations present in the historical data. The major advantage of this bootstrap method is that we can draw any amount of large data which is essential for
model validation that may be not be case in historical simulation with less historical data. Then we computed bootstrapped VaR as below:

Figure 5.4.5: Bootstrapped Historical Simulation

The result of the bootstrapped historical VaR is 51.2312 with 10000 resample from the historical data set.

Then, we applied Cornish-Fisher approximation. As we have already seen that non-normality rules the our reference data set and we also tried normal analytical approximation called Cornish fisher expansion in order to find out percentiles of the distributions and then tried these percentiles to estimate VaR. The aim of it is to

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57 Peter Zangari, "A VaR methodology for portfolios that include options." RiskMetrics Monitor, First quarter, 1996, pp. 4-12
compare with historical Simulation VaR. Firstly we estimate the parameters required for Cornish fisher approximation i.e. mean, standard deviation, skewness and kurtosis.

In the Cornish Fisher Approximation, we compute adjusted critical values as a function of normal critical values.

\[ cv = z_\alpha + \frac{1}{6} (z_\alpha^2 - 1)^* \rho_3 + \frac{1}{24} (z_\alpha^3 - 3z_\alpha)\rho_4 - \frac{1}{36} (2z_\alpha^3 - 5z_\alpha) \rho_3^2 \]

Where \( cv \) is the critical value, \( \rho_3 = \text{skewness} \), \( \rho_4 = \text{kurtosis} \).

Then we computed Cornish-fisher VaR which is arrived at 58.8576.

**Backtest**

In the next step we applied the various backtest methods to validate the models as above. The motivation to backtesting is to improve risk management efficiency and accuracy of the risk measures. Basel Committee (1996a) specifies that “the essence of all backtesting efforts is the comparison of actual trading results with model-generated risk measures”. The benefits of backtesting from an organizational perspective are numerous. A major benefit of a qualified risk management is to disengage capital to operations. The worse the risk measure captures the probability of losses the higher the capital requirement for a financial organization is set by the regulatory authority, provided the risk model does not perform well in the first place. Too conservative risk measures elevate the capital requirement to an excessive level. Another incentive for backtesting in banks and other financial institutions is to uncover situations, where risk models
do not capture trading volatility accurately and do not decrease the capital requirement accordingly. Backtesting can lead to increased accuracy in risk management and to lower capital requirement. Efficient risk management also decreases the probability of a bankruptcy, because the institution can anticipate the occurrence of large losses.

We have applied both becktests of Christoffersen\textsuperscript{58}(1998) to our VaR estimation. Initially we have find out goodness for an out of sample interval forecast for given time series i.e. in our case the profit and loss series.

The properties are defined and interpreted, some other key factors in backtests are considered and types of backtesting error are defined. The properties tested by the backtesting methods are thus presented here along with other relevant issues in backtesting. A successful backtesting process essentially defines the properties that are tested and what exactly the backtests attempt to detect. The necessity of short-term and long-term VaR can also be considered through backtesting.

As a procedure, backtesting is not so straightforward if all the properties are tested. Methods primarily test one property, but also joint tests exist, which cover several properties simultaneously. Despite the broad coverage of these tests, they are not necessarily

considered more efficient to detect failures than tests that concentrate on a single property (Campbell\textsuperscript{59}, 2005). It is suggestive that strongly indicative backtesting results should always be reconfirmed by another test, which ideally takes a different approach.

**Test of Unconditional Coverage**

The property of unconditional coverage defines the coverage rate $\alpha$ to be on average equal to the proportion of VaR violations in a sample of returns.

Under the hypothesis/prediction of correct unconditional coverage, the test statistic:

$$LR_{uc} = -2 \ln[(1-P)^{x-n} P^n] + 2 \ln[(1-x/n)^{x-n} x/n^x]$$

Is distributed as a $\chi^2(1)$ where $x$ is the number of exceedences and $n$ is the total number of observations.

We have conducted Christoffersen\textsuperscript{60}(1998) backtest for test of unconditional coverage. The result comes at 0.0916. The test statistic is distributed as $\chi^2(1)$ and the probability of a test value of 0.0916 or more under a $\chi^2(1)$ is 0.7621, which indicates that the null hypothesis is acceptable. We can not reject the hypothesis that the model generates the correct frequency of tail losses.

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Test of Independence

The property of independence holds when all the autocorrelations of the lags and leads of the indicator function are not systematically different from zero. Independence requires the autocorrelations of the indicator function to be statistically equal to zero. In other words the VaR violations are not clustered and the occurrence of a violation must not contain any information of future VaR violations. When the elements of the violation sequence are uncorrelated with each other the VaR violations can be assumed to be independently distributed (Christoffersen and Pelletier, 2004).

Independence is another fundamental property of an adequately specified VaR model, which must hold together with unconditional coverage. Independence restricts any dependence in the indicator function over time and it is a necessary condition for the adequacy of a VaR model. Clustering of violations in the VaR violation sequence indicates a violation of the independence property. The violation of independence can cause VaR to react too slowly to changes in the market and it can imply a negligence of the chosen coverage rate (Campbell, 2005). If for example two VaR violations continually occur consecutively then the second violation takes place after the first one with a probability of 100 %. Now, the probability of a VaR violation is not consistently $\alpha \%$, which contradicts the property

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of independence. Independence must be carefully analyzed, because some VaR models, for example Historical Simulation VaR, are correctly specified for unconditional coverage by definition.

Then we applied Christoffersen\textsuperscript{62\textsuperscript{}}(1998)’s backtest for test of independence.

The result comes at 1. The test statistic is distributed as $\chi^2 (1)$ and the probability of a test value of 0.0916 or more under a $\chi^2 (1)$ is 1 and probability value associated with indicate that we can not reject the null hypothesis and so we can not reject the model.

Independence and unconditional coverage are fundamental properties of an adequate VaR model, and they can be formalized into a joint property. The combined property of independence and unconditional coverage is known as conditional coverage, where the coverage condition must hold in condition of the independence in the violation sequence. It is a stronger condition than the previous ones and it defines an adequately specified VaR model. Conditional coverage is also known as an efficient forecast (Christoffersen\textsuperscript{63}, 1998). The property of conditional coverage is a sufficient condition for the adequacy of a VaR model and it can be readily tested to validate a given VaR model. A VaR model should be classified invalid only if it does not satisfy either the condition of unconditional coverage or the


\textsuperscript{63} Ibid.
condition of independence (Christoffersen\textsuperscript{64}, 1998). Conditional coverage is a stronger assumption of the accuracy of the VaR model than independence or unconditional coverage, because it includes them both.

5.5. Summary

Our study on historical bootstrapped VaR is estimated at 51.2312, which is a little higher than historical VaR i.e. 49.935. The main purpose of using bootstrapped historical VaR is that it takes care of the necessity of large data for model validation even the sample size is not adequate. As we have shown above that historical simulation is used for non-linearity present in the data set as historical simulation method takes care of volatilities and correlations present in the referenced historical data. As such, in present context, the use of bootstrapped historical VaR method is a better choice. It also be noted that the main motivation for this comparative study is that a well-defined optimization process of VaR accuracy would be a valuable asset to risk managers, though analytical derivation of such optimization process can be difficult, as the portfolio composition is not often static and the market risk factors change randomly. Accordingly, the statistical properties of VaR can vary. The relevance for backtesting is to improve the accuracy of VaR, through backtesting, which is continual statistical testing of the accuracy of VaR estimates. Backtesting is also required by the regulatory bodies, as the Basel Committee has set up standards for the quality of VaR data in the

\textsuperscript{64} Ibid.
backtesting framework (Basel Committee, 1996a). A VaR model with correctly specified statistical qualities and well-defined distributional properties, which are confirmed by several tests, is likely to be a good model for market risk management.