Chapter 4

Research Methodology

4.1 Introduction

In this chapter we have briefly discussed various risk assessment and measurement methodologies we have applied in our empirical work. As mentioned on the earlier chapter on summary of existing literature, the study undertakes Market Risk, Credit Risk and Extreme Value Theory methods to apply the Indian context. Under the market risk, the study applies historical simulation Value at Risk and bootstrapped historical simulation VaR for non-linearity and conduct backtesting. Under Credit Risk, the study applies validation of credit rating model by Cumulative Accuracy profile and p values. Under Peaks Over Threshold method, the study applies graphical data exploratory techniques and Peaks Over Threshold method of Extreme Value Theory for ascertaining Value at Risk, taking cognizance of movement in the tails.

4.2 Market Risk - Application of different Value at Risk Models and Backtesting of the Model

Value at Risk (VaR) can be defined as the maximum expected loss for a defined horizon at a determined level of confidence. In other
words, VaR measurement offers a single number that summarizes sufficient information about portfolio risk exposure. The VaR figure can inform senior managers of the probability of losing for example (x) dollars is one in a hundred over a specified holding interval. VaR is becoming widely popular because it is a simple measure of risk; easy to interpret; but is best used by good senior managers specifically, value at risk is a measure of losses due to ‘normal’ market movements. Losses greater than the value at risk are suffered only with a specified small probability. Subject to the simplifying assumptions used in its calculation, value at risk aggregates all of the risks in a portfolio into a single number suitable for use in the boardroom, reporting to regulators, or disclosure in an annual report. Once one crosses the hurdle of using statistical measure, the concept of value at risk is straightforward to understand. It is simply a way to describe the magnitude of the likely losses on the portfolio.” Therefore, VaR is the negative of the appropriate percentile (normally in the lower tail) of the portfolio return distribution over a specific horizon that is under normal market conditions.

VaR is a simple method of quantifying the probable loss (amount of money that a firm is likely to lose) in terms of currency; it thus provides for better risk management, since it summarizes the likely loss of the portfolio in a single monetary term. Consequently, it enables senior staff to better manage a firm’s wide risk exposure. VaR figures are used for several needs including: determining risk target; risk performance and evaluation; and setting position limits for traders.
In addition, senior managers can be aware of the risks undertaken by different business units including trading and investment activities; risks of the various investment opportunities can then be assessed, even after the event and appropriate capital allocation can be established (Jorion, 2001). Generally, riskier investments entail higher VaR measures and consequently higher capital requirements. VaR is also used in the comparison of risky activities in different markets; positions can be revealed that contribute most to the portfolio risk because the procedure can be reported and the total risk can be split into incremental VaR. The VaR figure, and thus the financial risks can then be clearly and easily transmitted to shareholders (Dowd, 1998). The benefits of VaR as a measure of market risk promote social benefits by preventing fraud and presenting accurate control systems. Regulators expect that Value-at-Risk based risk management will decrease the potential of recent severe financial losses.

Risk managers can decide whether they are satisfied with their risk exposure or not, or make efforts to reduce risk, by examining VaR figures; in addition they can be used to determine position limits for traders and evaluate the returns on a portfolio based on risk adjustment.

Broadly speaking, Value-at-Risk is rapidly becoming the most commonly used measure to calculate and control market risk. There is

much emphasis on the need to manage risk and enhance the transparency and soundness of financial markets, which has led to the adoption of VaR as a uniform defensive tool against financial risks. The Securities and Exchange Commission (1997) requires that all financial institutions report their market risk exposure using VaR, for compliance and conformity. In addition, the Basle Committee on Banking Supervision authorized the use of Value-at-Risk models; as such, regulators, the private sector and banks are increasingly using VaR. In fact, financial institutions such as banks and other investment firms use the VaR approach for the determination of their requisite capital ratios and are obliged to meet their capital requirements based on VaR calculations. The Securities and Exchange Commission (1995) required publicly traded U.S. firms to reveal their derivatives activities using a VaR measure.

As indicated, VaR is the probability of loss expected on a portfolio over a specified period of time and for a certain confidence level. Mathematically, VaR is the one side percentile of the probability distribution function for the confidence level.

\[ \Pr(r_{t+1} + VaR_{t+1}) = \alpha \]

Where \( r_{t+1} \) denotes the change in the value of the portfolio for the holding period, and \( \alpha = (1 - \text{the confidence level}) \). The negative sign appears since we assume that returns and extreme tail revenues are losses.
Simple VaR methods commonly assume that returns follow a joint normal distribution; this makes the calculation of VaR easier and is given by:

$$VaR_{t+1} = \delta(\alpha)|\sigma_{t+1}$$

Where $\sigma_{t+1}$ is the standard deviation of the portfolio return conditional on time, t, information set, and $\delta(\alpha)$ is the $\alpha$ - quantile of the standard normal distribution. Here the mean return is assumed to be zero and this is a common assumption when modeling the volatility of short horizon asset returns. At the 95% confidence level the value of $\alpha$ will be $-1.65$, for 90% confidence level the value would be $-1.282$, and for a 99% level, $\alpha$ would be 2.326.

This illustrates the simplicity of a VaR calculation. All we need is to estimate $\sigma$ then insert the figure into the VaR formula. Obviously, this indicates that VaR calculation is closely related to the estimation of the tail distribution of the portfolio return.

In the calculation of VaR, there are two significant factors that need to be realized and combined. These are the sensitivity of market value of portfolios according to the change of market rates and the joint probability distribution of the change in market rates during a given time period.

Basically, when calculating VaR we begin by ‘marking to market the portfolio’, which is assumed to remain constant for the whole horizon, the return on the portfolio is then calculated at the end of the period. Based on the probability distribution of these different
returns, VaR can be calculated. This means that Value at Risk is only valid for liquid portfolios and is not applicable to illiquid assets such as real estate.

Since we consider returns to follow a certain probability distribution, then we can define the expected loss with a certain level of confidence. This means that there are two basic parameters that first need to be defined for VaR estimation: the length of the holding period and the appropriate choice of confidence level. As mentioned earlier, the Basle Committee requires a 99% confidence level and 10 days holding period for VaR calculation. The percentile is frequently in the lower tail of the distribution, often as high as the 99\textsuperscript{th} percentile or at least the 95\textsuperscript{th} percentile. In other words, \( \alpha \) is equal to 1\% or 5\%.

\textit{Holding period}

The choice of holding period differs depending on the use of VaR calculation by management and is necessary for regulators to perceive any problems and adopt appropriate solutions. A one day holding period was stated in the revised document of the internal model 1994 (www.bis.org [2006]) that banks normally use for trading purposes, because when the choice of holding period is long, the price volatility is higher and the measured risk is greater. However this is only considered to be realistic under normal market conditions. Even with normal approximation, when portfolio returns include options a shorter horizon period is more frequently used, but, the holding period accounts for non-linearities in price options. The choice of holding period should be realistic and the Committee agreed that
for market risk measurement, banks must use a holding period of two weeks or ten business days in their VaR calculation. This can accommodate changes in the portfolio and allows for frequent adjustments to risk exposures. Banks can choose a different holding period, but have to scale up their value at risk figure to a ten day holding period; for example, by multiplying by the square root of time which is the ten holding days (Basel Committee, 2006). An important factor that affects the choice of holding period is the length of time required to liquidate positions in the markets where any financial institution is trading. For shorter liquidation, a shorter horizon is utilized for VaR measurement; and if a position requires a longer time to liquidate, a longer horizon interval is used. Another factor that requires a short holding period is the validation procedure in the VaR calculation since this requires a large data set in order to obtain a reliable validation process.

The confidence level

The second parameter is the choice of confidence level preference; this depends on the reason for VaR measurement and the behavior of the distribution: normal or otherwise. The choice of confidence level is extremely important if the VaR figure is to be used for capital cushion and to ensure safe and sound risk management, in case the firm exceeds the VaR measure. If firms are averse to risk, they use a higher confidence level, in order to ensure adequate amounts of capital. Banks typically use a confidence level ranging from 90% to 99%. If VaR is used for comparison among different
corporations, the importance of confidence level is lessened and it becomes entirely arbitrary. The Basel Committee on banking supervision advocates the use of a more conservative 99% confidence level; subsequently Bankers Trust uses 99%. Albeit JP Morgan uses a 95% confidence level in their riskmetric, others like Citibank prefer to report their VaR with a 95.4% level. Therefore, confidence levels need to be altered to be the same if comparisons are to be made; this shows that there is no preference to one confidence level over another; the choice is subject to the task at hand.

*Different approaches to measuring VaR*

There are a variety of methods that have been implemented to calculate VaR, all of which have the potential to allow for immensely different estimates. According to Jorion\(^40\) (2001), there are two types of VaR approaches: the local valuation when the distribution is anticipated using a Taylor series approximation such as the delta-normal method; and the full valuation approach (when distributions are estimated using different generated scenarios) which is implemented in either the historical-simulation method; the stress testing method, or the structured Monte Carlo method.

These approaches follow different calculation techniques and carry various underlying assumptions for each, which is why a normally generated VaR figure differs according to the method

adopted. For this reason, it is important to comprehend the underlying assumption for each approach, the quantitative methods and mathematical paradigms utilized, so that the best model is chosen to meets the needs and objectives. This is clearly illustrated: “The actual benefits to be derived from the VaR estimates depend crucially on the quality and accuracy of the models on which the estimates are based. To the extent that these models are inaccurate and misstate the banks’ true risk exposures, then the quality of the information derived from any public disclosure will be degraded. More important, inaccurate VaR models or models that do not produce consistent estimates over time will undercut the main benefit of a models-based capital requirement: the closer tie between capital requirements and true risk exposures. Thus, assessment of the accuracy of these models is a key concern and challenge for supervisors.” (Hendricks 41, 1996)

Broadly speaking, VaR is a simple method to calculate and a simple risk measurement to interpret. Hitherto, there is a broad argument among financial firms and supervisory bodies on how it should be calculated; its calculation underlies some challenging statistical techniques. Nonetheless, all different approaches to VaR share a common general structure including: marking to market, estimating the portfolio distribution; then calculating the portfolio value at risk. However, it is worth mentioning that generally in VaR

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calculation, it is assumed that returns follow a conditional normal distribution where the portfolio is a linear function of normal risks. In reality this is not usually the case and will ultimately mislead VaR estimates.

**The Variance Covariance Approach**

This is a simple method derived from the assumption that: risks are normal; portfolio is a linear function of normal risks; and standard deviation is a linear function of volatilities and covariances. Thus, VaR can be easily estimated from the portfolio standard deviation using variance covariance matrix, which is then multiplied by the confidence level parameter. Linear function is usually referred to as: parametric, variance-covariance, closed-form, or delta-normal VaR measures. The appeal of normality assumption is that VaR numbers then inform the different possible sets of parameters: the holding period and the confidence level. Therefore, the VaR number based on 95% confidence level can be deduced from another figure based on 99% confidence level. The same applies to the holding period if normality is assumed to be subject to scaling by the ratio of the square root of the different holding periods. However, when the returns are non-linear functions, as is often the case, the first order linear approximation (Delta normal) or the second order approximation (Delta gamma) is used to handle the non-linearity or non-normality. Banks often use the variance covariance method based on risk factor and correlation volatilities to measure their market risk exposure.
Historical Simulation Method

As it sounds, historical simulation techniques use past price movements; the actual historical distribution of the returns over some period to estimate a hypothetical distribution of returns to calculate the VaR on the current portfolio. This is a full valuation technique, which means that the non-linearities and non-normalities are accounted for. To apply this method, weights are allocated to the past returns assuming that past distribution is a good proxy for the next holding period, where returns are revalued based on today’s scenario. Thus, VaR is found from the total distribution of hypothetical returns, in other words: set equal to the percentile of the returns at a certain confidence level.

The historical simulation is a non-parametric method, thus there are no parameters to assume, nor models, and there is no need to assume returns are normally distributed given that the data determines the distribution of returns. Consequently, there is no need for approximation. The use of non-parametric models is appropriate to determine if returns are best characterized by a normal or non-normal distribution. Another advantage of the Historical Simulation Method is that the measures of skewness, kurtosis, and expected tail losses are also captured, which gives a more accurate estimation of the riskiness of a portfolio. Indeed it can be applied to different types of market risks including gamma and volatility risks. HS is a rather simple technique to implement when data is readily available and is well suited to
smaller portfolios with options, which do not need a particular distribution.

However, this technique has shortfalls, mainly reliance on historical data that assumes that the past represents the future; that future risks are similar to past risks. Clearly this is true in certain markets but in reality, this does not hold, especially at sudden unusual regime events like stock market crashes and exchange rate risks where the VaR calculation is affected. Furthermore, difficulties in obtaining sufficient historical data for each instrument in the portfolio can be encountered. Another problem is that it assumes that changes in distribution of market factors are roughly stable for long periods; moreover, the choice of estimation period length would influence VaR calculation.

The need for a reliable figure requires a longer observation period in order to obtain reliable inferences about the tail, particularly when a high confidence level is used. Recent observation may be required, particularly when correlations and volatilities change over time and are reflected in the VaR forecasts, however, if the sample size is short, the VaR forecasts will probably be subject to estimation error or sampling variation. Boudoukh, Richardson and Whitelaw (1997), proposed Historical Simulation with exponentially weighted past returns in an attempt to overcome such weaknesses.

Furthermore, regulatory bodies are concerned with accuracy and the conservatism of the different models to measuring risk;
conservative models produce high estimates and reduce the capital reserve that needs to be held.

Thus, it is crucial for estimates of risks to be extremely precise. Regulatory bodies evaluate the performance of the different approaches to VaR given that banks are now able to develop their own in-house models. Otherwise, this badly influences capital allocation and with that, the profitability and financial health of organizations. Therefore, financial organizations will carry out the most appropriate method to fit their needs in measuring market risk; they will have to appraise the relative performance of contesting models and consider all the possible approaches.

**Back testing**

Basel Committee 42 (1996a) has set up a backtesting framework to control the quality and accuracy of the risk models in banks and to monitor that the market risk model requirements are complied with. The framework is a traffic light approach, which categorizes the risk models into three zones. It computes the number of trading outcomes that are not captured by the VaR measure and classifies the risk model on the basis of these VaR violations. The framework involves the use of 99 % VaR with 1-day forecast horizon, and VaR violations during the past 250 trading days are used. The framework consists of three zones. The green zone corresponds to

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backtesting results that do no suggest any problems in the risk model. The yellow zone covers results that may suggest a problem in the risk model but are not definitive. And the red zone indicates almost certain quality and accuracy problems in the model. The backtesting framework entails formal testing and accounting of VaR violations on a quarterly basis using the most recent twelve months of data (Basel Committee\textsuperscript{43}, 1996a). Banks are urged to perform backtests with actual trading outcomes as well as with hypothetical trading outcomes, which exclude changes in portfolio positions (i.e. intra-day trading) and fee income. Intraday trading and fee income are to be excluded because they are not within the scope of what VaR is intended to capture and they might distort trading outcomes to be too large or too small (Basel Committee, 1996a). The three zones in the framework are chosen to eliminate statistical errors of classifying an accurate model as inaccurate and of not detecting an inaccurate model as inaccurate. The boundaries for the zones are chosen according to binomial probabilities of a certain number of exceptions from a risk-model (Basel Committee, 1996a). The green zone consists of results that indicate a small probability of erroneously validating an inaccurate model, and models with four or less VaR violations are classified into this category. The yellow zone contains models with five to nine VaR violations, and it contains results that do not definitively imply accuracy or inaccuracy. Although the assumption of

inaccuracy in this category increases with the number of exceptions, banks whose models are in this category are usually required to present a clarification of their risk model, backtesting procedures and main reasons for the violations. The supervisory body also retains the right to consider whether the capital requirement is increased on the basis of i) the basic integrity of the model, ii) the accuracy of the model, iii) bad luck or extreme market movements and iv) intra-day trading and fee income as described in (Basel Committee\textsuperscript{44}, 1996a). Red zone covers situations with ten or more violations, and outcomes in this zone imply by assumption a problem with the risk model. An accurate and high quality model could be classified into red zone only under the most extraordinary market circumstances. Basel Committee (1996a) recognizes that the backtests in their framework have a limited power to distinguish an accurate model from an inaccurate model. The reason for the use of the number of exceptions as a basis for classification in the backtesting process is the simplicity and straightforwardness of the method. This approach necessitates the assumption that each day’s outcome in backtests is independent of the outcome on any other day. We have applied different Value at Risk Models depending on the distributional assumption of the reference dataset and backtest the same.

4.3 Credit Risk - Validation of Credit Rating Model

Credit rating systems perform a critical role in credit risk assessment. Financial regulations such as Basel II and Solvency II emphasize the use of internal rating-based models (IRB) for credit risk management of borrowers. These internal ratings can be mapped to external grades in order to estimate critical risk components such as the probability of default (PD). Also, supervisory authorities consider external ratings for detecting solvency problems of financial institutions. Nevertheless, the usefulness of credit rating systems depends on the comprehensibility of their dynamic characteristics and the validation and calibration of risk models over time. Basel II introduces requirements on the validation of IRB systems as stability, integrity, objectivity and discriminatory power; the fulfillment of these requirements are particularly difficult for the external mapping approach because supervisors and risk managers must first confirm the accuracy of the PDs associated with the external rating scale, and then they must validate the accuracy of the bank’s mapping between internal and external grades.

If internal grades match the external rating scale, the mapping should reasonably remain stable over time. But if the bank and the external agency have different rating philosophies (point-in-time vs. through-the cycle) the mapping might change periodically. Thus, general risk perceptions on the macroeconomic and industrial environment may affect rating systems and be considered in validation and calibration processes.
The primary purpose of validation is to examine whether the internally constructed scoring model can fully explain the credit status of borrowers. As sampled data used to construct the model can mostly be explained by the model, it is necessary to see whether the model possesses sufficient explanatory power for different samples. Thus out-sample testing should be carried out to observe the tendency of over-learning, which will lower the predictability of model. In addition, improper sampling and omission of relevant information will lead to model bias. Thus external data should be employed to assess the validity of the rating model. Moreover, since the primary objective of the rating model is to make forecast, whether the model works normally under all circumstances, including significant changes of the macroeconomic environment, must also be validated. We have used different validation methodologies for validation of credit rating model.

4.4 Extreme Value Theory - Application of Peaks Over Threshold (POT) Method

The key insight for this approach is to quantify extreme losses with a determined level of confidence; it is a parametric robust approach that uses the statistical probability theory of extreme values (Dowd, 1998). In order to entail more accuracy, this approach uses the extreme values of stock prices for a specified interval.

This approach offers a methodological basis, which is used to estimate tail distribution in extreme conditions without enforcing any
assumption on the return distribution dependent only on sample data. This shows that the behavior of extreme values is the same regardless of the distribution that generates the data. Broadly, there are some major benefits to this approach because it offers more reliable results for VaR than the variance covariance and historical simulation methods. Extreme values include important information about the true movement of the observed prices; it is a simple method that can evade any unusual events that were observed in the historical simulation process as a result of historical data.

The basic premises on which Extreme Value Theory (EVT) is based is the extreme value theorem, a variant of the better-known central limit theorem, which tells us what the distribution of extreme values should look like in the limit, as our sample size increases. Suppose I have some return observations but do not know the density function from which they are drawn, and then subject to certain relatively innocuous conditions, this theorem tell us that the distribution of extreme returns converges asymptotically to:

\[
H_{\xi,\mu,\sigma}(x) = \begin{cases} 
\exp(-[1 + \xi(x - \mu) / \sigma]^{-1/\xi}) & \xi \neq 0 \\
\exp(-e^{-x/\sigma}) & \xi = 0 
\end{cases}
\]

The parameters \( \mu \) and \( \sigma \) correspond to the mean and standard deviation, and the third parameter, \( \xi \), gives an indication of the heaviness of the tails: the bigger \( \xi \), the heavier the tail. This parameter is known as the tail index, and the case of most interest in finance is where \( \xi > 0 \), which corresponds to the fat tails commonly
founded in financial return data. In this case, our asymptotic distribution takes the form of a Fréchet distribution.

This theorem tells us that the limiting distribution of extreme returns always has the same form — whatever the distribution of the parent returns from which our extreme returns are drawn. It is important because it allows us to estimate extreme probabilities and extreme quantiles, including VaRs, without having to make strong assumptions about an unknown parent distribution.

EVT provides a natural approach to VaR estimation, given that VaR is primarily concerned with the tails of our return distributions. To apply to VaR, the parameters of the distribution are estimated first, and there are a number of standard estimators available. Once these are computed, they can be plugged into a number of alternative formulas to obtain VaR estimates. To give a simple example, if it is to estimate a VaR that is out of (i.e., more extreme than) the sample range, I can project the tail out from an existing in-sample quantile $X_{k+1}$ — where $X_{k+1}$ is the k+1-th most extreme observation in our sample — and infer the (asymptotic) VaR from the projected tail using the formula:

$$\text{VaR} = \left[ CL / k \right] \hat{\xi} X_{k+1}$$

where CL is the confidence level on which the VaR is predicated. EVT also gives us expressions for the confidence intervals associated with our VaR estimates.

The EVT approach to VaR has certain advantages over
traditional parametric and non-parametric approaches to VaR. Parametric approaches estimate VaR by fitting some distribution to a set of observed returns. However, since most observations lie close to the center of any empirical distribution, traditional parametric approaches tend to fit curves that accommodate the mass of central observations, rather than accommodate the tail observations that are more important for VaR purposes. Traditional parametric approaches also suffer from the drawback that they impose distributions that make no sense for tail estimation and fly in the face of EVT theory. By comparison, the EVT approach is free of these problems and specifically designed for tail estimation.

Non-parametric or historical simulation approaches estimate VaR by reading off the VaR from an appropriate histogram of returns. However, they lead to less efficient VaR estimates than EVT approaches, because they make no use of the EVT theory that gives us some indication of what the tails should look like. More importantly, these approaches also have the very serious limitation that they can tell us nothing whatever about VaRs beyond our sample range. The methods of assessing market risk, credit risk and Peaks Over Threshold as explained above are applied in the following three chapters.