In this Chapter, which is divided into three sections, we have investigated electron scattering by atomic hydrogen. In Section A, we (29) have investigated the problem of elastic scattering of slow electrons by hydrogen atom, by the variational method of Hulthen. We have considered the polarisation effect through the process of virtual excitations to 2s and 2p₀ states but we have neglected the exchange effect. We (30) have extended this work in Section B, where we have made allowance for the exchange effect. The polarisation effect has been included as before through the process of virtual excitations to 2s and 2p₀ levels. In Section C, we (31) have computed excitation cross sections for 1s - 2s transition in atomic hydrogen, including the exchange effect and distortion of the incident plane wave by S wave elastic scattering only.
SECTION A

ELASTIC SCATTERING OF SLOW ELECTRONS
BY HYDROGEN ATOM

Introduction.

The scattering of electrons by hydrogen atom is theoretically the simplest of all electron-atom collision problems, hence it has been the subject of detailed theoretical investigation. This has many important applications, in Astrophysics, controlled thermonuclear devices and other processes.

A number of experiments has been carried out on slow electron scattering by atomic hydrogen by Neynaber (93), Brackmann, Fite et al. (94). Recently Schulz (96) has reported an experimental evidence for a resonance in the elastic scattering of electrons by atomic hydrogen, below the onset of excitation of the electronic states of hydrogen atom.

The scattering at low energy involves the effect of exchange because of the indistinguishability of the incident electron from atomic electron. There is a great probability of the incident electron being captured and the atomic electron being ejected in the process of slow collision. Moreover, due to the influence of the incoming electron, there is a polarization effect arising out of the distortion of the spherically symmetrical charge distribution of 1s-electron cloud in the neutral atomic hydrogen.
Massey and Moiseiwitsch (24) have used variational method to calculate the S-wave phase shift due to scattering of electrons by atomic hydrogen, taking into account the exchange effect and the partial distortion of the atomic cloud. They have treated this distortion by introducing explicitly in the trial wave function a term which depends on the electron-electron correlation distance $V_{e2}$. In some of the recent works (cf. Burke et. al. (9,10)) with the use of modern computing machines, sets of coupled equations resulting from the retention of only a few selected excited states of the hydrogen atom in the wave function, have been solved numerically. Burke and Schey (9) have utilised a close-coupling approximation in which the total wave function has been expanded in hydrogen eigen states and only terms corresponding to the 1s, 2s and 2p states are retained. In their investigations, they have found that except for the very low energy region below .68 ev the short-range distortion effect dominates over the long-range one and the 2s-state gives a larger contribution to the phase shift than 2p state. Moreover, once all states corresponding to second quantum level have been incorporated in the formulation of the problem, all other remaining states have negligible effects. Recently, Temkin and Pohle (32) have calculated the singlet S-wave phase shifts in electron hydrogen atom collisions below the inelastic threshold; they have obtained two resonances, the first one centred at 9.4 ev is broader than the second one occurring at 10.1 ev.
In the present work, we have calculated the S-wave phase shift values in electron-hydrogen atom collisions. Since the proton is very massive compared to the electron the wave function for the electron-hydrogen system will depend only upon the coordinates of the bound and free electrons. Here we assume that the distortion of the initial 1s-state is in the form of a superposition of higher excited states induced temporarily when the incident electron is close to the target; but when the colliding electron is far away, the original 1s-state of the bound electron is restored. Our formulation is in conformity with the remark of Burke and Schey (9). For simplicity of calculations, we have considered the virtual excitations to 2s and 2p_0 states only, the polar axis being taken along the direction of the incident electron. We have used Hulthén's variational method to calculate the S-wave phase shifts as a function of energy and we have neglected the effect of exchange. By making use of Breit-Wigner formula we have obtained a resonance at an energy 9.74 ev which agrees favourably with recent experimental findings of Schulz (36).

Theory.

The wave function \( \Psi (r_1, r_2) \) of the system of two electrons moving in the field of a proton satisfies the wave equation

\[
(\mathcal{H} - E) \Psi (r_1, r_2) = 0 \quad \cdots \quad (2.1)
\]
with \[ H = \left[ -\frac{\nabla^2}{2} - \frac{\nabla^2}{2} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{i}{r_2} \right] \]

in atomic units (i.e. \( e = m_c = \hbar = c = 1 \)); here \( r_1 \) and \( r_2 \) are the co-ordinate of the atomic electron and impinging electron, respectively, relative to the proton and \( r_2 \) is the distance between the electrons.

Expanding the total wave function \( \Psi \) in terms of the eigen states of the target Hamiltonian, we have

\[ \Psi (r_1, r_2) = \left( \sum_{n} \int \right) \psi_n (r_1) F_n (r_2) \cdots \cdots (2.2) \]

where the summation and integration signs have their usual meanings. Here \( \psi_n (r_1) \) represents the wave function for the \( n \)-th state of the hydrogen atom and satisfies the eigen-value equation \((-\nabla^2 - \frac{2}{r_1} - 2E_n)\psi_n (r_1) = 0\) where \( E_n \) represents the eigen-energy of the \( n \)-th state of the atom. If the kinetic energy of the incident electron is \( \frac{K^2}{2} \), the functions must have the asymptotic forms

\[ F_1 \sim e^{\frac{iK_1 r_2}{r_2}} + e^{\frac{iK_2 r_2}{r_2}} \}

\[ F_n \sim e^{\frac{iK_n r_2}{r_2}} \}

where \( E = \frac{K^2}{2} + E_1 = E_1' + E_n = \frac{K^2}{2} + E_n \).

Here \( E_1 \) and \( E_n \) represent the energies of the ground state.
and the n-th state of the atom respectively and \( \kappa \) and \( \kappa_0 \) represent respectively the momenta of the incident electron and scattered electron after excitation of the n-th state of the atom.

To solve equation (2.1) under the prescribed boundary conditions in (2.3), we shall apply the variational method of Hulthén (25). We make a choice of the following trial wave function

\[
\Psi (Y_1, Y_2) = \chi (Y_1, Y_2) \cdot F_1 (Y_2)
\]

where

\[
\chi (Y_1, Y_2) = \chi_{s0} (Y_1) \left( 1 - \frac{\alpha^2 + \beta^2}{2} \cdot e^{-2Y_2} \right)
\]

\[
+ \alpha \chi_{s2s} (Y_1) E^{Y_2} + \beta \chi_{s2p_0} (Y_1, Y_2) e^{-Y_2}
\]

\[
F_1 (Y_2) = \left[ e^{iY_1} a + (\alpha e^{Y_2} + b e^{-Y_2}) (1 - e^{-Y_2}) \cos \frac{\kappa_0 Y_2}{\kappa Y_2} \right]
\]

here we have taken only the S wave part of the scattered wave and \( \chi (Y_1, Y_2) \) satisfies the normalisation condition \( \int \chi^* \chi \, dY_1 = 1 \) correct to terms of the order of \( \alpha^2 \) and \( \beta^2 \). The wave function \( F_1 (Y_2) \) having adjustable parameters \( \alpha \) and \( \beta \) has the asymptotic form \( \left\{ e^{iY_2}, Y_2 + \alpha \cos \frac{\kappa_0 Y_2}{\kappa Y_2} \right\} \) and is finite at the origin. The S-wave phase shift \( \eta_0 \) is given by \( \eta_0 = \left| a_n \right| a \).

We substitute the trial wave function \( \Psi (Y_1, Y_2) \) in the variational integral

\[
I = \int \int \Psi (Y_1, Y_2) \left[ H - E \right] \Psi^* \, dY_1 \, dY_2 \ldots \ldots (2.4)
\]
and evaluate it. The value of the phase-parameter 'a' is obtained from the following set of simultaneous equations

\[ I(\alpha, \beta, b; a) = 0 \]
\[ \frac{\partial I}{\partial a} = 0 \]
\[ \frac{\partial I}{\partial \alpha} = 0 \]
\[ \frac{\partial I}{\partial \beta} = 0 \]
\[ \frac{\partial I}{\partial b} = 0 \]

It is to be noted that cubes and higher powers of \( \alpha \) and \( \beta \) in \( I \) have been neglected.

The elastic S wave cross section \( Q_0 \) is \( \frac{4\pi}{K^2} \sin^2 \eta_0 \).

In discussing the resonance effect, we decompose the total phase shift into two parts as \( \eta_{\text{total}} = \eta_{\text{potential}} + \eta_{\text{resonance}} \), where \( \eta_{\text{potential}} \) is the slowly varying potential part of the phase shift and \( \eta_{\text{resonance}} \) is the phase shift due to the resonance effect. Following Burke and Schey (9), we write

\[ Q_{\text{total}} = \frac{n}{K^2} \left| A_{\text{res}} + A_{\text{pot}} \right|^2 \]

where

\[ A_{\text{res}} = e^{2i\eta_{\text{res}}} \left[ e^{2i\eta_{\text{res}}} - 1 \right] \]

and

\[ A_{\text{pot}} = \left[ e^{2i\eta_{\text{pot}}} - 1 \right] \]

The S wave resonant part of the cross section is written as

\[ Q_{\text{res}} = \frac{n}{K^2} \left| A_{\text{res}} \right|^2 = \frac{4n}{K^2} \sin^2 \eta_{\text{res}} \]

Hence we get \( Q_{\text{res}} \) as a function of \( K^2 \).
Calculation of $I$.

The variation integral $I$ in (2.4) can be written in the form:

$$I = I_1 + I_2$$

where

$$I_1 = \int A^* P \, d\gamma^2 + \int B^* Q \, d\gamma^2$$

and

$$I_2 = \int \left( \frac{\partial}{\partial \gamma^2} - \frac{2}{g} \right) \left[ A^* \psi_{ls}^* (\gamma_1) + B^* \psi_{ls}^* (\gamma_1) \right]$$

and

$$+ \left[ A \psi_{ls} (\gamma_1) + B \psi_{ls} (\gamma_1) + C \psi_{ls} (\gamma_1) \right].$$

in which

$$A = (1 - \frac{d^2 + B^2}{z} \cdot e^{-2t_2}) \tilde{F}_1 (t_2)$$

$$B = \alpha^* \cdot e^{r_2} \cdot \tilde{F}_1 (t_2)$$

$$C = \beta^* \cdot e^{r_2} \cdot \tilde{F}_1 (t_2)$$

$$Q = \alpha \cdot \nabla^2 \left\{ e^{r_2} \cdot \tilde{F}_1 (t_2) \right\} + \frac{A^2 + B^2}{z} \cdot \nabla^2 \left\{ e^{-2r_2} \cdot \tilde{F}_1 (t_2) \right\}$$

$$R = \frac{1}{C \cos \theta_2} \cdot \nabla^2 \left\{ e^{r_2} \cdot \tilde{F}_1 (t_2) \right\}$$

$A^*$, $B^*$, and $C^*$ are the complex conjugates of $A$, $B$, and $C$ respectively, and $\epsilon_1$ and $\epsilon_2$ are the binding energies of the ground state (1s) and 2s (also 2p) states of the atom respectively.
On carrying out the integrations, over $d\Omega$ and over the angular co-ordinates $\theta_2, \phi_2$ of $d\Omega$-space, we get

\[
I_1 = \frac{2\pi}{K_1^2} \int_0^\infty \left[ \frac{p}{(1 + \cos 2K_1r_2 + 8 \sin 2K_1r_2)} \right] d\gamma_2
\]

\[
- \frac{q\pi}{K_1} \int_0^\infty \left[ \frac{q}{(1 - \cos 2K_1r_2 + 8 \sin 2K_1r_2)} \right] d\gamma_2
\]

\[
- \frac{\pi d_1^2}{2K_1^2} \int_0^\infty \left[ \frac{7p^2}{(1 + \cos 2K_1r_2 + 8 \sin 2K_1r_2)} \right] e^{2\gamma_2} d\gamma_2
\]

\[
- \frac{\pi d_2^2}{2K_1^2} \int_0^\infty \left[ \frac{7\gamma_2^2}{(1 + \cos 2K_1r_2 + 8 \sin 2K_1r_2)} \right] e^{2\gamma_2} d\gamma_2
\]

\[
- \frac{\pi d_3^2}{K_1^2} \int_0^\infty \left[ \frac{9 \gamma_2^2}{(1 + 2\gamma_2^2 + 8 \sin 2K_1r_2)} \right] e^{2\gamma_2} d\gamma_2
\]

\[
- \frac{\pi d_4^2}{K_1^2} \int_0^\infty \left[ \frac{9 \gamma_2^2}{(1 + 2\gamma_2^2 + 8 \sin 2K_1r_2)} \right] e^{2\gamma_2} d\gamma_2
\]

where

\[
p = (a + b \ e^{-\gamma_2}) \ (1 - e^{-\gamma_2})
\]

\[
q = a \ e^{-\gamma_2} + b \ (2 \ e^{2\gamma_2} - e^{-\gamma_2})
\]

\[
s = -a \ e^{-\gamma_2} + b \ (e^{-\gamma_2} - 4e^{2\gamma_2})
\]

Similarly, we get

\[
I_2 = \frac{q\pi}{K_1^2} \int_0^\infty \left[ \frac{(1 + \frac{4}{\gamma_2^2})}{e^{2\gamma_2}} \right] e^{2\gamma_2} d\gamma_2
\]

\[
+ 8\pi \int_0^\infty \left[ \frac{\gamma_2^2}{e^{2\gamma_2}} \right] d\gamma_2
\]

\[
+ \frac{\pi d_1^2}{2K_1^2} \int_0^\infty \left[ \frac{(\gamma_2^2 + 2\gamma_2^4 + 6 + \frac{9}{\gamma_2^2})}{e^{3\gamma_2}} \right] d\gamma_2
\]

\[
- 8(1 + \frac{1}{\gamma_2^2}) \ e^{-\gamma_2} d\gamma_2
\]
+ \pi \alpha^2 \int_{0}^{\infty} \left[ (\frac{a^4}{\nu^2} + 2 \frac{a^3}{\nu^4} + 6 \frac{a^2}{\nu^6} + 8 \frac{a}{\nu^8} ) e^{-3 \gamma \nu} - 8 (\nu^2 + \tau_2) e^{-4 \nu} \right] d\nu \\
+ \pi \beta^2 \int_{0}^{\infty} \left[ \left( -\frac{96}{\nu^2} \right) e^{-2 \nu} + (\nu^2 + 6 \tau_2 + 22 + 5 \frac{6}{\nu^2} + \frac{96}{\nu^2} + \frac{96}{\nu^4} \right) e^{-3 \gamma \nu} \\
- 8 (1 + \frac{1}{\nu^2}) e^{-4 \nu} \right] d\nu \\
+ \pi \nu \beta \int_{0}^{\infty} \left[ \left( \frac{48}{\nu^2} \right) e^{-2 \nu} + (2 \nu^2 + 8 \tau_2 + 24 + \frac{48}{\nu^2} + \frac{96}{\nu^4} \right) e^{-3 \gamma \nu} \right] d\nu \\
+ \pi \nu \beta \int_{0}^{\infty} \left[ \left( \frac{48}{\nu^2} \right) e^{-2 \nu} + (2 \nu^4 + 8 \nu^2 + 24 \nu^2 + 4 \nu^2 + 48) e^{-3 \gamma \nu} \right] d\nu \\
- \frac{32}{27} \pi \nu \frac{\gamma}{\nu^2} \int_{0}^{\infty} \left[ (3 \gamma^2 + 2) e^{-\gamma \nu} \right] d\nu \\
- \frac{32}{27} \pi \alpha \int_{0}^{\infty} \left[ (3 \gamma^2 + 2 \gamma \nu^2) e^{-\gamma \nu} \right] d\nu \\
+ \frac{32}{27} \pi \nu \frac{\beta}{\nu^2} \int_{0}^{\infty} \left[ (2 \gamma^2 + \gamma \nu^2 + \frac{96}{\nu^2} + \frac{64}{\nu^2}) e^{-\gamma \nu} \\
- \frac{64}{\nu^2} e^{-\gamma \nu} \right] d\nu \\
+ \frac{32}{27} \pi \beta \int_{0}^{\infty} \left[ (2 \gamma^2 + \gamma \nu^2 + \frac{96}{\nu^2} + \frac{64}{\nu^2}) e^{-\gamma \nu} \\
- \frac{64}{\nu^2} e^{-\gamma \nu} \right] d\nu \\
- 64 e^{-\gamma \nu} \right] d\nu \\

where \( t = \frac{\beta^2}{(1 + \cos 2 \alpha \nu^2) + 2 \pi \sin 2 \alpha \nu} \)

There is no further difficulty in carrying out the integrations appearing in \( I_1 \) and \( I_2 \) although the calculations become very tedious due to the occurrence of a large number of terms. For simplicity in calculations, we have taken \( b = 0 \).
In the low energy region we first find the solutions of the quadratic equation $I = 0$ obtained by putting $\delta = \beta = 0$. One of the solutions agrees with the results of Massey and Moiseiwitsch (24). Now to find the required solution of the sixth degree equation at low energy we search the root in the neighbourhood of the particular solution mentioned above; a root is obtained differing only slightly from the particular solution chosen of the quadratic equation. Once the root has been fixed for a particular low energy, the phase shifts for higher energies have been obtained by solving the sixth degree equation and using the continuity property of the phase shift.

Results and Discussions.

We have evaluated the $S$-wave phase shift values for the case $\beta = 0$ in the trial function $F(r)$ for energies ranging from 3.4 ev to 9.85 ev by using only the coupling of $1s$, and the virtually excited $2s$ and $2p_0$ states.

We give a plot of $S$-wave phase shift values $\eta_0$ versus $k_0^2$ in Fig. II-1. In the present calculation the $2s$ state gives a large correction to the $S$-wave phase shift than the $2p_0$ state at the energy range under consideration. This fact has been corroborated by the results obtained by Burke and Schey (9)

The most notable characteristic part of our result is the sharp increase in phase shift values above $k_0^2 = 70$. The curve
The s-wave phase shift is plotted as a function of $k^2$ in the neighbourhood of resonance.
has a pronounced resonating behaviour, with a definite flattening out before the threshold is reached.

In the Fig. II-2, we have plotted \( \Theta_{\text{res}} \) as a function of \( \kappa^2 \) using Breit-Wigner cross-section formula

\[
\Theta_{\text{res}} = \frac{\Gamma/4}{(E - E_{\text{res}})^2 + \Gamma^2/4}
\]

we obtain \( E_{\text{res}} = (4.7158 \text{a.u.}) 9.735 \text{ ev} \), which agrees very favourably with experimental findings by Schulz (26). The value of our resonance cross section comes out to be \( 5.203 \text{ mo} \) and the calculated value of the width \( \Gamma \) of the resonance state is found to be about \( 0.055 \text{ ev} \).

In experiments by Schulz (26), it has been observed that the width of resonance in atomic hydrogen is approximately double that of helium, which could not be resolved due to limited energy resolution (\( \sim 0.3 \text{ ev} \)) and the drop in the elastic scattering cross section, indicative of resonance is centred in the vicinity of \( 9.7 \pm 0.15 \text{ ev} \). Hence the position of hydrogen resonance in our case is in very good agreement with the experimental findings by Schulz (26). Also our partial S-wave cross section \( \Theta_0 \) for the energy \( 3.4 \text{ ev} \) comes out to be equal to 12.32 in units of \( \text{mo}^2 \) whereas the experimental value of the total cross section from the curve of Neynaber (93) is about \( 11.18 \text{ mo}^2 \). Hence our result for total S-wave cross section without exchange in the low energy region agrees
reasonably well with experiment.

Recently, several theoretical papers have appeared, showing the existence of resonance level below the threshold for excitation of the second quantum level. Thus, Burke and Schey (9) have obtained a resonance at 9.61 ev with a width of 0.109 ev in 'S state, the corresponding resonance cross section being 5.66 \( \mu^2 \).
ELASTIC RESONANCE IN ELECTRON - HYDROGEN SCATTERING

Introduction.

Several experiments have been carried out on slow electron scattering by atomic hydrogen. Schulz (86), Kleinpoppen et al. (89) and McGowan et al. (88) have found experimentally elastic resonances below the threshold for excitation of electronic states of atomic hydrogen.

A number of theoretical investigations has been made on e - H collision problem which has been discussed in our previous work, where we have dealt with the same problem by Hulthén's variational method considering the polarisation effect through the virtual excitations to 2s and 2p₀ levels but neglecting the exchange effect. Here in the present work, we have used the same variational method and have considered the exchange effect by explicitly antisymmetrising the wave function with respect to the atomic and incoming electrons, the polarisation effect has been taken into account in the same way as in our previous work so as to include virtual excitations to 2s and 2p levels. Recently, Geltman (28) has applied variational method in e - H scattering with a particular choice of trial function so as to take into consideration the virtual excitations to higher excited states and
has obtained very narrow resonances at electron energies below the first inelastic threshold. Burke and Taylor (34) also have carried out close-coupling calculations including correlation effects in their trial function in their investigations on the resonances in e - H scattering.

Our calculations for singlet S wave phase shifts indicate a resonance at an energy of 9.55 ev, which agrees satisfactorily with the experimental findings of Schulz (36), Kleinpoppen et. al. (37), McGowan et. al. (38) and also with the results of other theoretical calculations.

Theory.

The wave function $\Psi (\gamma_1, \gamma_2)$ of the system of two electrons moving in the field of a proton satisfies the wave equation $(H - E) \Psi (\gamma_1, \gamma_2) = 0 \ldots \ldots \ldots (2.5)$

with $H = -\frac{1}{2} \nabla_1^2 - \frac{1}{2} \nabla_2^2 - \frac{1}{\gamma_1} - \frac{1}{\gamma_2} + \frac{1}{\gamma_1 \gamma_2}$ in atomic units (i.e. $\varepsilon = m = \hbar = 1$ = $\alpha$), here $\gamma_1$ and $\gamma_2$ are the co-ordinates of the atomic and incoming electrons, relative to the proton and $\gamma_2$ is their mutual distance.

Since the total wave function of the system of two electrons must be antisymmetric for exchange of their space and spin co-ordinate, we therefore choose for $\Psi$ the forms
\[
\psi_{\pm} (\gamma_1, \gamma_2) = \frac{1}{\sqrt{2}} \sum_n \{ \psi_n (\gamma_1) f_n (\gamma_2) \pm \psi_n (\gamma_2) f_n (\gamma_1) \}
\]

where + and − signs correspond to singlet and triplet states respectively and \( \psi_n \) represent the wave function for the \( n \)-th state of the hydrogen atom and satisfies the eigen value equation

\[
(\nabla^2 + \frac{2}{\rho} + 2 F_n ) \psi_n (\gamma) = 0.
\]

\( F_n \) represents the eigen energy of the \( n \)-th state of the atom.

If such functions can be had in which \( f_n^\pm \) have the asymptotic forms

\[
\frac{f_n^\pm}{\rho} \sim e^{\frac{i K_i \gamma}{\rho}} \delta n + \frac{e^{\frac{i K_m \rho}{\rho}}}{\rho} \int_n^\pm (\theta, \phi)
\]

then the respective differential cross sections for excitations of the \( n \)-th state for antiparallel and parallel alignment of electrons are

\[
\frac{K_n}{K_i} \left| \int_n^\pm (\theta, \phi) \right|^2
\]

Here total energy \( E = \frac{K_i^2}{2} + E_i = E + E_n + \frac{K_n^2}{2} + E_n \) and \( E_i \) represent the eigen-energy of the ground state of the atom, \( K_i, K_n \) represent respectively the momenta of the incident electron and scattered electron after excitation of the \( n \)-th state of the atom.

We shall consider only the symmetric wave function corresponding to the singlet case and neglect all states for \( n \neq i \).
Therefore, 
\[ \psi^\pm(y_1, y_2) = \frac{1}{\sqrt{2}} \left[ \psi(y_1) f_1(y_2) + \psi(y_2) f_1(y_1) \right] \]

with 
\[ f_1(y_1) \sim e^{ik_1r_1} + e^{ik_2r_1} \int (\theta, \phi) ... \]  

(2.6)

To solve eqn. (2.5) under the boundary condition in (2.7), we shall use Hulthen's variational method choosing our trial wave function \( \Psi^\pm(y_1, y_2) \) as

\[ \Psi^\pm(y_1, y_2) = \chi(y_1, y_2) f_1(y_1) + \chi(y_2, y_1) f_1(y_1) \]

where, \( \chi(y_1, y_2) = \psi(y_1) \left\{ -\frac{\beta^2}{2} e^{2y_2} \right\}

+ \alpha \psi(y_2) e^{-y_2} + \beta \psi_0(y_1, y_2) e_{y_2} \]

the polar axis being along \( \gamma_2 \),

and \( f_1(y_2) = \left\{ \frac{\sin k_1 y_2}{k_1 y_2} + (a + b e^{-y_2})(1 - e^{-y_2}) \right\} \frac{\cos k_1 y_2}{k_1 y_2} \}

It may be mentioned that we have considered only S-wave scattering and as in our previous work \( \chi \) satisfies the normalisation condition \( \int \chi^* \chi d\gamma_1 = 1 \) correct to terms of the order of \( \alpha^2 \) and \( \beta^2 \).

Substituting \( \Psi(y_1, y_2) \) in the variational integral \( I = \int \psi^* (H - E) \psi^* d\gamma_1 d\gamma_2 \) and using Hulthen's variational method, the phase shift is found in terms of the phase parameter 'a', i.e., \( \eta = \tan^{-1} a \).

It is to be noted that cubes and higher powers of \( \alpha \) and \( \beta \) in \( I \) have been neglected.
Calculation of $I$.

The variational integral $I$ can be written as

$$I = \frac{2}{\hbar} \int \left( E + \mathcal{H}_0 \right) d\rho d\sigma$$

where

$$I_1 = \left\{ \chi^* (\mathbf{r}_1, \mathbf{r}_2) \cdot F^*_i (\mathbf{r}_2) \right\} \cdot (H - E)$$

and

$$I_2 = \left\{ \chi^* (\mathbf{r}_1, \mathbf{r}_2) \cdot F^*_i (\mathbf{r}_1) \right\} \cdot (H - E).$$

After simplification, we get

$$I_1 = -\frac{\hbar^2}{2} \left[ \left( \mathcal{P}_1 + \mathcal{P}_3 \right) \cdot \chi_2^* (\mathbf{r}_1) \cdot \chi_1 (\mathbf{r}_1) \right] e^{-2 \mathcal{P}_2} \cdot F_i^* (\mathbf{r}_2)$$

$$- \frac{\hbar^2}{2} \left[ \left( \mathcal{P}_1 + \mathcal{P}_3 \right) \cdot \chi_2^* (\mathbf{r}_1) \cdot \chi_2 (\mathbf{r}_1) \right] e^{2 \mathcal{P}_2} \cdot F_i (\mathbf{r}_2)$$

$$+ \alpha \left[ \mathcal{P}_2 \cdot \psi_{2s} (\mathbf{r}_1) \cdot \psi_{2p_0} (\mathbf{r}_1, \mathbf{r}_2) \right] e^{2 \mathcal{P}_2} \cdot F_i (\mathbf{r}_2)$$

$$+ \alpha \left[ (\mathcal{P}_2 + \mathcal{P}_3) \cdot \chi_2 (\mathbf{r}_1) \cdot \chi_{2s} (\mathbf{r}_1) \right] e^{-\mathcal{P}_2} \cdot F_i (\mathbf{r}_2)$$
\[ + \beta \left[ \frac{1}{2} \left( p_{2} + p_{1} \right) - \frac{2}{\gamma_{2}} \right] F_{1} (\gamma_{2}) \int \psi_{1}s (\gamma_{1}) \cdot \psi_{2}p_{0} (\gamma_{1}, \gamma_{2}) \cdot e^{-\gamma_{2}} \cdot F_{1} (\gamma_{2}) \]

\[ + \left[ \rho_{3} \psi_{1}s (\gamma_{1}) \psi_{1}s (\gamma_{1}) \right] F_{1} (\gamma_{2}) \]

and

\[ I_{2} = -\frac{\alpha^{2}}{2} \left[ \left( \omega_{1} e^{-2\gamma_{1}} + \omega_{2} e^{-2\gamma_{2}} \right) \cdot \psi_{1}s (\gamma_{1}) \psi_{1}s (\gamma_{2}) \right. \]

\[ - 2 \omega_{2} e^{(\gamma_{1} + \gamma_{2})} \cdot \psi_{2}s (\gamma_{1}) \psi_{2}s (\gamma_{2}) \cdot \left. \right] F_{1} (\gamma_{2}) \]

\[ - \frac{\beta^{2}}{2} \left[ \left( \omega_{1} e^{-2\gamma_{1}} + \omega_{2} e^{-2\gamma_{2}} \right) \cdot \psi_{1}s (\gamma_{1}) \psi_{1}s (\gamma_{2}) \right. \]

\[ + \frac{\rho_{3}}{2} F_{1} (\gamma_{1}) - 2 \omega_{1} \int e^{(\gamma_{1} + \gamma_{2})} \psi_{2}p_{0} (\gamma_{2}, \gamma_{1}) \cdot \psi_{2}p_{0} (\gamma_{1}, \gamma_{2}) \cdot e^{-\gamma_{1}} \cdot F_{1} (\gamma_{1}) \]

\[ + \beta \left[ \omega_{2} e^{-(\gamma_{1} + \gamma_{2})} \right] \psi_{2}s (\gamma_{1}) \psi_{2}s (\gamma_{2}) \]

\[ + \frac{\rho_{3}}{2} \psi_{2}s (\gamma_{1}) \psi_{2}s (\gamma_{1}) \cdot \psi_{2}p_{0} (\gamma_{1}, \gamma_{2}) \cdot \psi_{2}p_{0} (\gamma_{2}, \gamma_{1}) \cdot F_{1} (\gamma_{1}) \]

\[ + \alpha \left[ \omega_{2} e^{-\gamma_{1}} \psi_{1}s (\gamma_{1}) \psi_{2}s (\gamma_{2}) \right. \]

\[ + \omega_{3} e^{-\gamma_{2}} \psi_{1}s (\gamma_{2}) \psi_{2}s (\gamma_{1}) \right] F_{1} (\gamma_{2}) \]

\[ + \beta \left[ \omega_{3} e^{-\gamma_{2}} \psi_{1}s (\gamma_{2}) \psi_{2}p_{0} (\gamma_{1}, \gamma_{2}) \right. \]

\[ + \frac{\rho_{3}}{2} \psi_{1}s (\gamma_{1}) \psi_{1}s (\gamma_{1}) \cdot \psi_{2}p_{0} (\gamma_{1}, \gamma_{2}) \cdot \psi_{2}p_{0} (\gamma_{2}, \gamma_{1}) \cdot F_{1} (\gamma_{1}) \]

\[ + \left[ \omega_{3} \psi_{1}s (\gamma_{1}) \psi_{1}s (\gamma_{1}) \right] F_{1} (\gamma_{1}) \]
where

\[
P_1 = \begin{align*}
    & \sum q + 2 \left( \frac{1}{\gamma_2} - \frac{1}{\gamma_{1\omega}} \right) \int_1 (T_2) \\
    & + \frac{1}{k_1 \gamma_2} \sum (S_2 - q \gamma_2 - q \gamma_1) \cos k_1 \gamma_2 \\
    & + (q \gamma_1 p_2 - 2 k_1 \gamma_2) \sin k_1 \gamma_2 \\
    & \int_1 (S_2 - 2 \gamma_2 - 2 k_1) \cos k_1 \gamma_2 + (2 k_1 p_2 - 2 k_1 \gamma_2) \sin k_1 \gamma_2 \\
\end{align*}
\]

\[
P_2 = \begin{align*}
    & \frac{1}{2} \sum q + 2 \left( \frac{1}{\gamma_2} - \frac{1}{\gamma_{1\omega}} \right) \int_1 (T_2) + \frac{1}{k_1 \gamma_2} \\
    & \int_1 (S_2 - 2 \gamma_2 - 2 k_1) \cos k_1 \gamma_2 + (2 k_1 p_2 - 2 k_1 \gamma_2) \sin k_1 \gamma_2 \\
\end{align*}
\]

\[
P_3 = 2 \left( \frac{1}{\gamma_2} - \frac{1}{\gamma_{1\omega}} \right) \int_1 (T_2) \\
    + \frac{1}{k_1 \gamma_2} \int_1 (S_2 \cos k_1 \gamma_2 - 2 k_1 \gamma_2 \sin k_1 \gamma_2) \\
\]

and \( P_2 = (a + b e^{\gamma_2}) (1 - e^{-\gamma_2}) \)

\[
\begin{align*}
    q_2 &= a e^{\gamma_2} + b (2 e^{2\gamma_2} - e^{\gamma_2}) \\
    S_2 &= -a e^{\gamma_2} + b (e^{-\gamma_2} - 4 e^{-2\gamma_2})
\end{align*}
\]

Now, \( \omega_1, \omega_2, \omega_3 \) and \( p_1, q_1, s_1 \) can be obtained from \( P_1, P_2, P_3 \) and \( P_2, q_2, s_2 \) respectively by replacing \( \gamma_2 \) by \( \gamma_1 \). The integrals occurring in \( I \) can now easily be worked out.

Results and Discussions.

We have evaluated only the singlet S-wave phase shift values (\( \eta_o \)) for the case \( b = 0 \) in the trial function \( T_1(r) \) for energies below the threshold for excitation of second quantum levels by using only the coupling of 1s, 2s and 2p\( _o \) states.
We have plotted the singlet S-wave phase shifts $\gamma_0$ against $\kappa^2$ in Fig. II-3, where we notice the peculiar resonating behaviour i.e. there is a sharp increase in phase shift values above $\kappa^2 = .70$.

We have shown $Q_{\text{res}}$ as a function of $\kappa^2$ in Fig. II-4. By making use of Breit-Wigner cross section formula

$$Q_{\text{res}} = \frac{\frac{\Gamma^2}{4}}{\left(\frac{E - E_{\text{res}}}{\Gamma/4}\right)^2 + \Gamma/4},$$

we get $E_{\text{res}} = 9.55$ ev, the width $\Gamma$ being $\sim .04$ ev and the resonance cross section turns out to be $5.56 \, \pi a_0^2$.

The first experimental report on a resonance in the scattering of electrons by atomic hydrogen has been made by Schulz (36) at $(9.7 \pm .15)$ ev. Such a resonance has been further confirmed by the observations of Kleinpoppen and Raible (37) who found it centred around $(9.73 \pm .12)$ ev. Recently, McGowan et. al. (33) have reported two resonances, one centred near $9.45$ ev and the other near $9.68$ ev.

Burke and Schey (9) have obtained a resonance at $9.62$-ev with a width of .109 ev in the $^1S$ state and the corresponding resonance cross section being $5.66 \, \pi a_0^2$. Nearly similar results were obtained by O'Malley and Geltman (74), Temkin and Pohle (32) and others. Most recently Burke and Taylor (34)
FIG. 2-3. THE SINGLET S WAVE PHASE SHIFT IS PLOTTED AS A FUNCTION OF $k^2$ IN THE NEIGHBOURHOOD OF RESONANCE.
FIG. II: The resonant part of S-wave cross section is plotted as a function of $k_1^2$. 
have obtained two resonances, one at 9.560 ev and the other
at 10.178 ev with widths of .0475 ev and .00279 ev respectively.

In conclusion, we may say that our present formulation
yields satisfactory results for the position of elastic
resonance in e - H scattering.
SECTION C

EXCITATION OF 1s-2s PROCESS IN ATOMIC HYDROGEN BY ELECTRON IMPACT

Introduction:

Several theoretical attempts have been made (cf. Burke and Smith (1); Mott and Massey (2)) towards the calculations of the excitation cross sections in hydrogen atom by electron impact. Recent experiments have been carried out by Hillo et. al. (3) for the remeasurement of the excitation cross section in 1s-2s process in atomic hydrogen by electron impact.

Here in the present work we have investigated the same problem, taking exchange into consideration in Born-Oppenheimer approximation. We have also taken into account the distortion due to S wave elastic scattering. For the sake of simplicity, we have considered the distortion due to S wave elastic scattering only whereas the distortion due to all other remaining partial waves has been neglected in the scattering amplitude.

The singlet and triplet S wave phase shifts (\(\eta_0^\pm\)) corresponding to the phase-parameters \(\alpha^\pm\), determined from the variational trial wave function of Massey and Moiseiwitsch (24) have been utilised for considering the distortion due to elastic scattering.
Here we have computed the excitation cross sections in the energy range of 13.6 ev to 54.4 ev. The results have been compared with several other theoretical calculations and recent experimental findings.

**Theory.**

The wave function \( \psi(\vec{r}_1, \vec{r}_2) \) of the system of two electrons moving in the field of a proton satisfies the wave equation (in atomic units)

\[
\left( \nabla^2 + \frac{2}{r_1} + \frac{2}{r_2} - \frac{2}{r_1} + 2E \right) \psi^\pm(\vec{r}_1, \vec{r}_2) = 0.
\]

where \( E \), the total energy of the system is \( E = E_1 + \frac{k_n^2}{2} \), \( \vec{r}_1, \vec{r}_2 \) are the position vectors of electrons 1 and 2 relative to the proton and \( \gamma_{12} = |\vec{r}_1 - \vec{r}_2| \), \( \pm \) signs refer to singlet and triplet states respectively.

If the atom is initially in the ground state, the asymptotic form of the wave function is given by,

\[
\psi^\pm(\vec{r}_1, \vec{r}_2) \sim e^{-\frac{i k_n r}{\sqrt{2}}} \psi(\vec{r}_1) + \sum_n \sqrt{n} (\Theta_n) e^{\frac{i k_n r}{\sqrt{2}}} \psi_n(\vec{r}_1) \pm \frac{\sum_n \sqrt{n} (\Theta_n) e^{\frac{i k_n r}{\sqrt{2}}} \psi_n(\vec{r}_1)}{\sqrt{2}} \psi(\vec{r}_2) \ldots \ldots \ldots \ldots \ldots (2.9)
\]

where \( k_n^2 = 2(E - E_n) \) and \( E_n \) is the eigen energy of the \( n \)-th state of the hydrogen atom, the other symbols having their usual meanings. Writing \( \psi^\pm = \xi \pm \eta \) we get (cf. Drukarev (3))
\[
\sigma_{V}^{\pm} = -\frac{\sqrt{2}}{2\pi} \int \int e^{-i k_0 \cdot \mathbf{r}_2} \Psi_{25}(\mathbf{r}_1) \left[ \frac{1}{\mathbf{r}_{12}} - \frac{1}{\mathbf{r}_{12}} \right] \cdot 
\Psi(\mathbf{r}_1, \mathbf{r}_2) \, d\mathbf{r}_1 \, d\mathbf{r}_2
\]

where

\[
\Psi(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{\sqrt{2}} \left[ \Psi_{16}(\mathbf{r}_1) \, F_{1}^{\pm}(\mathbf{r}_2) \pm \Psi_{16}(\mathbf{r}_2) \, F_{1}^{\mp}(\mathbf{r}_1) \right]
\]

and

\[
F_{1}^{\pm}(\mathbf{r}) = \left[ e^{i k_1 \cdot \mathbf{r}} + \frac{e}{\sqrt{1 + \alpha^2}} \left\{ \frac{\cos k_1 r}{k_1 r} J_0(k_1 r) - J_0(k_1 r) \right\} \right]^{\pm}
\]

with the symbols having their usual meanings.

The total excitation cross section is then given by

\[
\sigma_{n} = 2\pi \cdot k_0 \frac{k_{n}}{k_{1}} \sum \frac{1}{q_{1}^2} \left| q_{1} \right|^2 + \frac{3}{q_{2}} \left| q_{2} \right|^2 \sin \theta \, d\theta \quad \cdots \cdots (2.10)
\]

We have computed \( \sigma_{n} \) in the energy range concerned by numerical integration of (2.10) by Simpson's rule.
Calculation of $q^\dagger$.

The expression for $q^\dagger$ in (2.9) can now be written as

$$q^\dagger = -\frac{1}{2\pi} \left[ \pm A \pm B + C \right]$$

where

$$A = \iint e^{ik_0\cdot\nu \cdot} \Psi_{25}(\eta) \left[ -\frac{4}{\nu_2} \right] \Psi_{15}(\nu_2) \cdot F_{1}(\nu_2) d\eta_1 d\nu_2$$

$$B = \iint e^{-ik_0\cdot\nu \cdot} \Psi_{25}(\eta) \left[ \frac{4}{\nu_2} \right] \Psi_{15}(\nu_2) \cdot F_{1}(\nu_2) d\eta_1 d\nu_2$$

$$C = \iint e^{-ik_0\cdot\nu \cdot} \Psi_{25}(\eta) \left[ \frac{1}{\nu_2} \right] \Psi_{15}(\nu_2) \cdot F_{1}(\nu_2) d\eta_1 d\nu_2$$

Now,

$$A = \frac{1}{\eta_2 \sqrt{2}} \left[ A_1 + \left( \frac{e^{i\eta_{0}\cdot}}{\sqrt{1+\nu^2}} - 1 \right) A_2 + \frac{e^{i\eta_{0}\cdot}}{\sqrt{1+\nu^2}} A_3 \right]$$

with

$$A_1 = \frac{16\eta_0^2}{\eta_{0} k_{1} k_{2}} [ S_{2,1}^{n} ( S_{4,\nu_{2}} - 2 S_{3,\nu_{2}} )]$$

$$A_2 = \left( \frac{e^{i\eta_{0}\cdot}}{\sqrt{1+\nu^2}} - 1 \right) \frac{16\eta_0^2}{\eta_{0} k_{1} k_{2}} [ S_{2,1}^{n} ( S_{4,\nu_{2}} - 2 S_{3,\nu_{2}} )]$$

$$A_3 = \alpha^* \frac{e^{i\eta_{0}\cdot}}{\sqrt{1+\nu^2}} \frac{16\eta_0^2}{\eta_{0} k_{1} k_{2}} \left[ S_{2,1}^{n} \left( R_{4,\nu_{2}} - R_{4,\nu_{2}} - 2 ( R_{3,\nu_{2}} - R_{3,\nu_{2}} ) \right) \right]$$
$$B = \frac{1}{4\pi^2} \left[ B_1 + \left( \frac{i\eta^\pm}{\sqrt{1+\alpha^2}} - 1 \right) B_2 + \frac{i\eta^\pm}{\sqrt{1+\alpha^2}} \cdot \Theta_3 \right]$$

with

$$B_1 = \int e^{-i\gamma_n \cdot \gamma_3} \frac{1}{\Pi_2} \cdot \gamma_1 (\gamma_2) \gamma_2 (\gamma_1) \cdot e^{i\gamma_n \cdot \gamma_2} \cdot d\gamma_1 \cdot d\gamma_2$$

(this has been explicitly evaluated by Corinaldesi and Trainor (96))

$$B_2 = \frac{8\pi^2}{k_n \lambda_n} \left[ \frac{\varphi_1}{(k_n^2 + \mu_0^2)^2} \cdot \left( R_{2,3/2}^{n} - R_{2,3/2}^{n+} \right) \right. + \frac{\beta_1}{(k_n^2 + \mu_0^2)^2} \left( S_{2,3/2}^{n+} + S_{2,3/2}^{n-} \right) \left. \right]$$

$$+ \frac{16\pi^2}{k_n \lambda_n} \left[ \frac{\varphi_2}{(k_n^2 + \mu_0^2)^3} \left( R_{2,3/2}^{n-} - R_{2,3/2}^{n+} \right) \right.$$

$$+ \frac{\beta_2}{(k_n^2 + \mu_0^2)^3} \left( S_{2,3/2}^{n+} + S_{2,3/2}^{n-} \right)$$

$$- \frac{\varphi_1}{(k_n^2 + \mu_0^2)^2} \left( R_{2,3/2}^{n+} - R_{2,3/2}^{n-} \right)$$

$$- \frac{\beta_1}{(k_n^2 + \mu_0^2)^2} \left( S_{2,3/2}^{n+} + S_{2,3/2}^{n-} \right) \right]$$
\[ B_3 = a^\pm \frac{3 \pi^2}{k_1 k_n} (G_{\mu} - G_{\mu+1}) \]
\[ \quad + a^\pm \frac{32 \pi^2}{k_1 k_n} (H_{\mu} - H_{\mu+1}) \cdot S_{2,1}^{\ast} \]

Again

\[ C = C_1 + \left( \frac{e^{i \phi_0}}{\sqrt{1 + \alpha^2}} - 1 \right) C_2 + \frac{e^{i \phi_0}}{\sqrt{1 + \alpha^2}} C_3 \]

\[ C_1 = \frac{1024 \sqrt{2}}{(t^2 + 9)^3} \pi \quad \text{where} \quad t^2 = 4 |k_n - k_1|^2, \]

\[ C_2 = \frac{4 \pi \sqrt{2}}{9 k_1 k_n} \left[ (R_{3,3/2}^{n-} - R_{3,3/2}^{n+}) \right. \\
\[ \quad + \left. \frac{2}{3} (R_{2,3/2}^{n-} - R_{2,3/2}^{n+}) \right] \]
\[ C_3 = \frac{a^+ \eta \eta^2}{\Theta K^* K_n} \cdot \left( S_{\frac{n^+}{3}, \frac{3}{12}} + S_{\frac{n^-}{3}, \frac{3}{12}} - S_{\frac{n^+}{3}, \frac{3}{12}} \right) \]

\[ + \frac{2}{3} \left( S_{\frac{n^+}{2}, \frac{3}{2}} + S_{\frac{n^-}{2}, \frac{3}{2}} - S_{\frac{n^+}{2}, \frac{3}{2}} - S_{\frac{n^-}{2}, \frac{3}{2}} \right) \]

where

\[ \mu = \frac{1}{2}, \quad \varphi_1 = \mu^2 - k_1^2, \quad \varphi_2 = (\mu^2 - k_1^2) \mu - 2 \mu k_1, \]

\[ \beta_1 = 2 \mu k_1, \quad \beta_2 = k_1 (\mu^2 - k_1^2) + 2 \mu^2 k_1, \]

\[ R_{\alpha, \beta} = \int_0^\alpha r^{\alpha - 2} e^{-\beta r} \cos \kappa \pi r \, dr \]

\[ R_{\alpha, \beta}^{n+} = \int_0^\alpha r^{\alpha - 2} e^{-\beta r} \cos (k_{n+} k_1) r \, dr \]

\[ R_{\alpha, \beta}^{n-} = \int_0^\alpha r^{\alpha - 2} e^{-\beta r} \cos (k_{n-} k_1) r \, dr \]

and

\[ S_{\alpha, \beta} = \int_0^\alpha r^{\alpha - 2} e^{-\beta r} \sin \kappa \pi r \, dr \]

\[ S_{\alpha, \beta}^{n+} = \int_0^\alpha r^{\alpha - 2} e^{-\beta r} \sin (k_{n+} k_1) r \, dr \]

\[ S_{\alpha, \beta}^{n-} = \int_0^\alpha r^{\alpha - 2} e^{-\beta r} \sin (k_{n-} k_1) r \, dr \]
\[ S_{d, \beta}^{n^+} = \int_0^\infty r^2 e^{-\beta r} \sin(k_1 + k_2) r \, dr \]
\[ S_{d, \beta}^{n^-} = \int_0^\infty r^2 e^{-\beta r} \sin(k_1 - k_2) r \, dr \]

Again

\[ G_{\mu}^* = \frac{1}{(k_1^2 + k_2^2)^2} \left[ \left( \mu^2 - k_1^2 \right)^2 \left\{ S_{3, \mu^+}^{n^+} + S_{3, \mu^+}^{n^-} - 2 S_{2, \mu^+}^{n^+} - 2 S_{2, \mu^+}^{n^-} \right\} \\
+ 2/\mu k_1 \left\{ - R_{3, \mu^+}^{n^-} + R_{3, \mu^+}^{n^+} \right\} \\
+ 2 R_{2, \mu^+}^{n^-} - 2 R_{2, \mu^+}^{n^+} \right\} \right] \\
+ \frac{2}{(\mu^2 + k_1^2)^3} \left[ \left\{ \mu (\mu^2 - k_1^2) - 2/\mu k_1^2 \right\} S_{2, \mu^+}^{n^+} + S_{2, \mu^+}^{n^-} \right] \\
+ \left\{ k_1 (\mu^2 - k_1^2) + 2/\mu k_1^2 \right\} R_{2, \mu^+}^{n^+} - R_{2, \mu^+}^{n^-} \right\} \right] \]

and \[ H_{\mu} = \frac{\mu^2 - k_1^2}{(\mu^2 + k_1^2)^2} - \frac{\mu^3 - 3/\mu k_1^2}{(\mu^2 + k_1^2)^3} \]
FIG. II - 5. 1S - 2S EXCITATION CROSS SECTION ($\sigma_{1s}$) IS PLOTTED AGAINST ENERGY.

- --- CLOSE - COUPLING CALCULATION (10)
- --- --- CALCULATION BY CROITHERS AND Mc CARROLL (98)
- --- --- --- --- RUDGE (97)
- --- --- --- --- --- BORN APPROXIMATION.
- --- --- --- --- --- --- --- A - PRESENT CALCULATION.
- --- --- --- EXPERIMENTAL POINTS OF HILLO et al. (95)
Results and Discussions.

We have computed the excitation cross sections for $1s-2s$ process in atomic hydrogen for energies in the range of 13.6 ev to 54.4 ev and have compared them in Fig. II-5 with the experimental values of Hillo et. al. (95) and the results of other theoretical calculations viz. Rudge (97), Crothers et. al. (98), Burke et. al. (10). Our results compare favourably with experiment, particularly in the low energy region.

It may be mentioned here that we have neglected the effect of distortion of the incident plane wave due to other higher order partial waves, specially due to P wave and D wave elastic scattering. Moreover, no allowance has been made for the polarisation of the target atom under the influence of the incoming electron. Perhaps, the inclusion of all these effects could produce still better accord between theoretical and experimental findings.