INTRODUCTION

A common problem plaguing the emerging economies is the shallowness of their financial sector. Financial sector play an important role in the process of economic growth and development by facilitating savings and channeling funds from savers to investors. While there have been numerous attempts to develop the financial sector, small island economies are facing the problem of high volatility in numerous fronts including volatility of financial sector. Volatility which has a dominant influence impairs the smooth functioning of the financial system and adversely affects economic performance. Similarly, stock market volatility also has a number of negative implications. One of the ways in which it affects the economy is through its effect on consumer spending (Campbell, 1996; Starr-McCluer, 1998; Ludvigson and Steindel 1999 and Poterba 2000). The impact of stock market volatility on consumer spending is related via the wealth effect. Increased wealth will drive up consumer spending. However, a fall in stock market will weaken consumer confidence and thus drive down consumer spending. Stock market volatility may also affect business investment (Zuliu, 1995) and economic growth directly (Levine and Zervos, 1996 and Arestis et al 2001).

A rise in stock market volatility can be interpreted as a rise in risk of equity investment and thus a shift of funds to less risky assets. This move could lead to a rise in cost of funds to firms and thus new firms might bear this effect as investors will turn to purchase of blue-chip or growth stocks. While there is a general consensus on what constitutes stock market volatility and, to a lesser extent, on how to measure it, there is far less agreement on the causes of stock market volatility. Some economists see the causes of volatility in the arrival of new, unanticipated information that alters expected returns on a stock (Engle and Ng, 1993). Thus, changes in market volatility would merely reflect changes in the local or global economic environment. Others claim that volatility is caused mainly by changes in trading volume, practices or patterns, which in turn are driven by factors such as modifications in macroeconomic policies, shifts in investor tolerance of risk and increased uncertainty. The degree of stock market volatility can help forecasters predict the path of an economy’s growth.
and the structure of volatility can imply that “investors now need to hold more stocks in their portfolio to achieve diversification” (Krainer, J, 2002:1).

Indian stock market has seen many microstructure changes such as global capital flow in the form of FII, private equity during last one decade or so. This has helped the market to grow and attract substantial foreign investment. Last decade has seen a few market debacles when an illegal trade practices manipulated the market to earn abnormal return. However, few settlement problems have occurred. The market has crashed few times, specifically on May 17, 2004, but settlement has passed off without any hitch. This has been possible due to sound and alert risk management practices (systemic and non systemic) followed by the leading exchanges in the country.

Wide price fluctuations are a daily occurrence on the world's stock markets as investors react to economic, business, and political events. Of late, the markets have been showing extremely erratic movements, which are in no way in tandem with the information that is fed to the markets. Thus chaos prevails in the markets with investor optimism at unexpected levels. Irrational exuberance has substituted financial prudence. Has the stock market volatility increased? Has the Indian market developed into a speculative bubble due to the emergence of "New Economy" stocks? Why is this volatility so pronounced?

Volatility analysis is important to investigate the behaviour of stock market because issues of volatility and risk have become increasingly important in recent times to financial practitioners, market participants, regulators and researchers. Amongst the main concerns, which are currently expressed include: - has the world’s financial system become more volatile in recent times? Has financial deregulation and innovation lead to an increase in financial volatility or has it successfully permitted its redistribution away from risk averse operators to more risk neutral market participants? Is the current wave of financial innovation leading to a complete set of financial markets, which will efficiently distribute risk? Has global financial integration led to faster transmission of volatility and risk across national frontiers? Can financial managers most efficiently manage risk under current circumstances? What role the regulators ought to play in the process?
As a concept, volatility is simple and intuitive. It measures variability or dispersion about a central tendency. To be more meaningful, it is a measure of how far the current price of an asset deviates from its average past prices. Greater this deviation, greater is the volatility. At a more fundamental level, volatility can indicate the strength or conviction behind a price move. Despite the clear mental image of it, and the quasi-standardised status it holds in the field of finance, there are some subtleties that make volatility challenging to analyse. Since volatility is a standard measure of financial vulnerability, it plays a key role in assessing the risk/return tradeoffs and forms an important input in asset allocation decisions. In segmented capital markets, a country's volatility is a critical input in the cost of capital (Bekaert and Harvey 1995). Peters (1994) noted that stock prices and returns are cyclical, imperfectly predictable in the short run, and unpredictable in the long run and that they exhibit nonlinear, and possibly chaotic, behavior related to time-varying positive feedback.

Asset-return variability can be summarised by statistical distributions. Typically, the normal distribution is used to characterise a series of returns. The distribution is centered at the mean and its width is determined by the standard deviation (volatility). Return series may not be normally distributed and often tend to exhibit excess kurtosis, so that extreme values are more likely than the normal distribution would suggest. Such fat-tailed distributions are common with financial parameters. Skewness is also common, especially with equity returns, where big down moves are typically more likely than comparable, big up-moves. Time-variation in market volatility can often be explained by macroeconomic and micro structural factors (Schwert 1989a, b). Volatility in national markets is determined by world factors and part determined by local market effects, assuming that the national markets are globally linked. It is also consistent that world factors could have an increased influence on volatility with increased market integration. Bekaert and Harvey (1995) showed this using time-varying market integration parameter.
i. PROBLEM STATEMENT:

Investors and other stakeholders of the stock market rely on market estimates of volatility as a barometer of the vulnerability of the financial markets. The existence of excessive volatility or “noise” also undermines the usefulness of stock prices as a “signal” about the true intrinsic value of a firm, a concept that is core to the paradigm of international efficiency of the markets.

There are several reasons for this study there are differing views about the dispersions of Indian stock prices. Comparison of time-series volatility of Indian equity market, with other emerging and developed markets, distributional characteristics of the variance process and evidence if any, of asymmetries in volatility under different market conditions may shed interesting light on the evolving characteristics of Indian equity market. The level of the investor, frequent and wide stock market variations cause uncertainty about the value of an asset and affect the confidence of the investor. Risk averse and risk neutral investors may shy away from the market with frequent and sharp price movements. An understanding of the market volatility is thus important from the viewpoint of developing a strong capital market and also regulatory policy perspective.

When the total volatility of individual stock is decomposed into systematic volatility and idiosyncratic volatility, it is clearly evident that idiosyncratic volatility has trended up. The study attempts to measure idiosyncratic volatility of market, industry and the firm. While idiosyncratic volatility can be eliminated in a well-diversified portfolio, individual investors may still care about the specific risk of the securities they hold. Because of wealth constraints or by choice, many investors do not hold diversified portfolios. Those investors might feel that the risk of their portfolios has increased when idiosyncratic volatility is rising. Moreover, high idiosyncratic volatility could increase potential total transactions costs if investors with relatively limited means choose to achieve adequate diversification. This is so because an increase in idiosyncratic volatility will have an important effect on increasing the number of securities one must hold to achieve reasonably “full” diversification. Idiosyncratic volatility is also important to arbitrageurs and option traders, whose profits depend on total volatility instead of market volatility.
The studies on the volatility of Indian Stock Market have focused only on forecasting aggregate market volatility. But the volatility of on an individual stock depends on the volatility of industry-specific and firm-specific shocks as much as the volatility of aggregate market returns. There have not been studies on disaggregating volatility which needs to be investigated for several reasons:

- Many investors hold large number of individual stocks i.e., they do not diversify their portfolios in the manner recommended by Financial theory. This is especially true for investors in Indian stock markets. These investors are affected by the shifts in industry-level and idiosyncratic volatility, just as by shifts in market volatility.

- It is important to understand the volatility of Indian Stock Market because risk premium may be directly related to the volatility of stock returns in the market. Higher volatility implies higher capital cost. Higher volatility may also increase the value of the “option to wait” hence delay in investment. Hence extensive study about market, industry and firm level volatility in the Indian Stock Market would be beneficial to the investor and regulators for determining the “Cost of Capital” for evaluating direct investment and “asset allocation” decisions.

- It is also necessary to study these issues for an Indian stock market since the importance of emerging market have been increasing as more and more investors include the stocks of these emerging market into their portfolios.

- Arbitrageurs who trade to exploit mispricing of individual stock face risks that are related to idiosyncratic return volatility, not aggregate market volatility.

- The price of an option on an individual stock depends on industry-level and idiosyncratic volatility as well as market volatility.
ii. IMPORTANCE OF THE STUDY:

Despite the importance of these disaggregated volatility measures, there is very little empirical research on volatility at the level of industry and firm. There is need for extensive study of Indian Stock market which is an emerging stock market with a potential in the global market. This research in a way tries to fill this gap by characterizing the historical movement in aggregate as well as disaggregate volatility of Indian Stock Market.

The contribution of this research would be three fold:

- There is little research on Indian Stock market relative to other emerging markets. Volatility in Indian stock market has not been examined at disaggregated level so far. Researcher’s goal is to fill this gap and extract small stock evidence on these issues in the context of emerging stock market.
- This study will also provide broad and rigorous analysis of the Indian stock market with respect of stock returns. It is important to understand the volatility of stock market because risk premiums may be directly related to volatility of stock returns in the market. Higher volatility implies higher capital cost. Higher volatility may also increase the value of the ‘option to wait’ hence delaying investment. An extensive study about market, industry and firm level volatility would be beneficial to investors and regulators for determining the cost of capital for evaluating direct investment and asset allocation decisions.
- It is necessary to evaluate the disaggregated volatility measures because more and more investors include the stocks of emerging stock market into their portfolio.

iii. RESEARCH OBJECTIVES:

1. To assess volatility of aggregate market indices using traditional method.
2. To evaluate the disaggregated volatility applying attribution procedure.
3. To evaluate volatility in relation to business cycle
4. To identify relationship between market, industry and firm level volatility and macroeconomic variables
5. To identify cyclical behavior of volatility measure in Individual industries.
iv. RESEARCH METHODOLOGY:

This is an exploratory study of stock market volatility based on secondary data and application of quantitative techniques to analyze stock market volatility. Some of the models used for the study are explained.

**Estimation of Aggregate Volatility**

To determine whether Indian Stock Market is characterized by high volatility French, Schwert and Stambaugh (1987) and Schwert (1989) model, is used to calculate monthly standard deviation of stock return as a measure of volatility. Monthly standard deviation of stock returns using the daily returns. The estimator of the variance of monthly return is the sum of the squared daily returns after subtracting the average daily returns in the month:

\[
\sigma^2_t = \frac{1}{N_t-1} \sum_{i=1}^{N_t-1} r_{it}^2.
\]

Where there are \(N_t\) daily returns \(r_{it}\) in month \(t\). Using non-overlapping samples of daily data to estimate the monthly variance creates estimation error that is uncorrelated through time.

**Estimation of Volatility Components**

Campbell et al. (2001) studied the historical movement of the market, industry and firm level volatility using U.S data over the period 1962 – 1977 by characterizing the behavior at disaggregated level.

Following Campbell and Lettau (2001) the return on a stock is decomposed into three components: the market-wide return, an industry-specific residual and a firm-specific residual. Based on the return decomposition, time-series of volatility measures of the three components for a “typical” firm is constructed.

Industries are denoted by \(i\) subscript while individual firms are indexed by \(f\). The excess return of firm \(f\) that belongs to industry \(i\) in period \(t\) is denoted as \(R_{ift}\). The excess return of industry \(i\) in period \(t\) is given by \(R_{it} = W_{ift} \cdot R_{ift}\) where \(W_{ift}\) is the weight of firm \(f\) in industry \(i\).
The value-weight is based on market capitalization. The industries are aggregated correspondingly. The weight of industry $i$ in the total market is denoted by $W_{it} = \sum W_{it}$ and the excess market return is $R_{mt} = W_{it} R_{it}$ All the excess return is measured as an excess return over the Treasury bill rate.

In the next step the firm and industry returns is decomposed into the three components. Decomposition is based on the CAPM implies that can set intercepts to zero in the following equations:

$$R_{it} = \beta_{mi} R_{mt} + \bar{e}_{it} \quad (1) \text{ For industry returns and}$$

$$R_{ift} = \beta_{mf} R_{mt} + \beta_{if} \bar{e}_{it} + \bar{n}_{ift} \quad (2) \text{ For individual firm returns.}$$

In (1) $\beta_{mi}$ denotes the beta for industry $i$ with respect to the market return, $\bar{e}_{it}$ is the industry-specific residual. In (2) $\beta_{mf}$ is the beta of firm $f$ with respect to the market, $\beta_{if}$ is the beta of firm $f$ in industry $i$ with respect to its industry shock $\bar{n}_{ift}$ is the firm-specific residual. The weighted sum of the different betas equals unity:

$$\sum W_{it} \beta_{mi} = 1, \sum W_{ift} \beta_{mf} = 1, \sum W_{ift} \beta_{mf} = 1 \quad (3)$$

The CAPM decomposition guarantees that the different components of a firm’s return are orthogonal to one another. Thus it permits a simple variance decomposition in which all covariance terms are zero.

$$\text{Var } (R_{it}) = \beta_{mi}^2 \text{Var } (R_{mt}) + \text{Var } (\bar{e}_{it}) \quad (4)$$

$$\text{Var } (R_{ift}) = \beta_{mf}^2 \text{Var } (R_{mt}) + \beta_{if}^2 \text{Var } (\bar{e}_{it}) + \text{Var } (\bar{n}_{ift}) \quad (5)$$

This decomposition requires knowledge of firm-specific betas, which are difficult to estimate. Therefore, simplified industry returns decomposition which drops the industry beta coefficient $\beta_{mi}$ (1) is used.

$$(R_{it}) = R_{mt} + e_{it} \quad (6)$$
Campbell, Lo and MacKinlay (1997) refer this as “Market-adjusted-return model” in contrast to the market model. Comparing these two equations i.e., (1) and (6),

\[ e_{it} = \tilde{e}_{it} + (\beta_{mi} - 1)R_{mt} \quad (7) \]

The market-adjusted-return residual \( e_{it} \) equals the CAPM residual of (4) only if the industry beta

\[ \beta_{mi} = 1 \text{ or } R_{mt} = 0. \]

\( R_{mt} \) and \( e_{it} \) are not orthogonal in this decomposition, so covariance between them cannot be ignored. Computing the variance of the industry yields

\[ Var (R_{it}) = Var(R_{mt}) + Var(e_{it}) + 2 \text{Cov} (R_{mt} - e_{it}) \quad (8) \]

Taking the covariance term again introduces the industry beta into the variance decomposition. To eliminate the individual covariance’s weighted average of variance across industries are taken:

\[ \sum W_{it} Var(R_{it}) = Var(R_{mt}) + \sum W_{it} Var(e_{it}) \quad (9) \]

\[ = \sigma_{mt}^2 + \sigma_{et}^2. \]

The terms involving betas aggregate out from (3) because \( \sum W_{it} \beta_{mi} = 1. \) Therefore the residual \( e_{it} \) in (6) can be used to construct a measure of average industry-level volatility which does not require any estimation of betas.

Individual firm returns can be decomposed in the same fashion, consider a firm return decomposition that drops betas from (2)

\[ R_{it} = R_{mt} + e_{it} + n_{ift} \quad (10) \]

Where \( e_{it} \) is defined in (7) and

\[ n_{ift} = \tilde{n}_{ift} + (\beta_{mf} - 1)R_{mt} + (\beta_{if} - 1) \tilde{n}_{it} \quad (11) \]
As with industry residuals, \( n_{ift} = \bar{n}_{ift} \) only if the firm betas equal to one or market and industry shocks are zero. The variance of the firm return is

\[
\text{Var}(R_{it}) = \text{Var}(R_{mt}) + \text{Var}(e_{it}) + \text{Var}(\eta_{ift}) + 2\text{Cov}(R_{mt}, e_{it}) + 2\text{Cov}(e_{it}, \eta_{ift}) + 2\text{Cov}(R_{mt}, \eta_{ift})
\]

The weighted average of firm variances in industry I after expressing the covariance’s in terms of betas and volatility become

\[
\sum W_{ift} \text{Var}(R_{ift}) = \text{Var}(R_{mt}) + \text{Var}(e_{it}) + \sigma_{\eta_{ft}}^2 + 2(\beta_{mi} - 1)\text{Var}(R_{mt})
\]

Computing the weighted average across industries yields again variance decomposition without betas since the industry betas sum to one:

\[
\sum W_{lt} \sum W_{ift} \text{Var}(R_{ift}) = \text{Var}(R_{mt}) + \sum W_{lt} \text{Var}(e_{it}) + \sum W_{lt} \sigma_{\eta_{ft}}^2
\]

\[
= \sigma_{mt}^2 + \sigma_{et}^2 + \sigma_{nt}^2
\]

The simplified decomposition of firm returns (10) yields a measure of average firm-level volatility that does not require estimation of betas.

**Data & Estimation:**

- Firm-level return data set to estimate volatility components in (12). Is based on the return composition (6) and (10). Aggregate individual firms are clubbed according to the industry classification of SIC.
- Sample runs from 1st January 2000 to 31st December 2009.

Following procedure based on the methodology presented above is used to estimate the three volatility components in (12). The sample volatility of the market return in period \( t \) is computed as

\[
MRK_t = \sigma_{mt}^2 = \sum_{det} (R_{md} - \mu_m)^2
\]
Where $\mu_m$ is defined as the mean of the $R_{md}$ over the sample. $d$ refers to daily return and $t$ refers to months. Market capitalization is used for the weights. For weight in period $t$ the market capitalization of a firm in period $t-1$ is used and the weights as constant within period $t$.

For volatility in industry $I$ the sum of the squares of the industry-specific residual in (6) within period $t$:

$$\hat{\sigma}^2_{eit} = \sum_{det} e^2_{id}$$  \hspace{1cm} (14)

Average over industry is considered to ensure that the covariance’s of individual industries cancel out. The average industry volatility is computed as

$$IND_t = \sum_t W_{it} \hat{\sigma}^2_{eit}$$  \hspace{1cm} (15)

For the firm-specific volatility, first sum the squares of the firm-specific residual in (10) for each firm in the sample

$$\hat{\sigma}^2_{nit} = \sum_{det} n^2_{ifd}$$  \hspace{1cm} (16)

To compute the weighted average of firm-specific volatilities within an industry;

$$\hat{\sigma}^2_{nit} = \sum_{fet} W_{ift} \hat{\sigma}^2_{nift}$$  \hspace{1cm} (17)

And lastly average over industries is used to obtain a measure of average firm-level volatility as

$$FIRM_t = \sum_{det} W_{it} \hat{\sigma}^2_{nit}$$  \hspace{1cm} (18)

As for industry volatility this procedure ensure that the firm-specific covariance’s cancel out.

**Correlation**

The most commonly used measure for linear relationship between two variables is the Pearson Correlation coefficient. The Two variables must be measured
by interval or ratio scale. The value of the coefficient ranges from -1 to +1. If there is no linear relationship between two variables, the value of the coefficient is 0. If there is a perfect positive relationship, the value is +1. If there is a perfect negative relationship, the value is -1. The Pearson correlation coefficient \( r \) is calculated by the following formula:

\[
    r = \frac{N \sum xy - (\sum x)(\sum y)}{\sqrt{[N \sum x^2 - (\sum x)^2][N \sum y^2 - (\sum y)^2]}} \tag{19}
\]

Where \( x \) and \( y \) are two variables and \( N \) is the number of observations.

Many studies have indicated that the higher the degree of correlation between returns in stock markets, the greater the integration of those stock markets. Within that context, Pearson product moment correlation analysis will be performed to assess the level of integration between the two stock markets returns (Alsuhaibani, 2004).

**Unit Root Test**

The purpose of a unit root test is to determine if a series is “stationary”. Many of the procedures used in conducting financial studies require that series to be stationary. Therefore, a unit root test must be carried out to verify that the data series are not violating this key assumption.

Economic series data often show structural changes that affect both the unit root and stationarity test. In the first case, Perron (1989) demonstrated that the estimator of the autoregressive parameter goes asymptotically to values close to 1 when the series exhibits stationary fluctuations around a level or a trend with a shift.

Unit root tests are important in examining the stationary of a time series. Stationary is an important issue in three contexts. First, a crucial question in the autoregressive integrated moving average (ARIMA) modeling of a single time series is the number of time series that needs to be first differenced before an autoregressive moving average (ARIMA) model is fitted. Each unit root requires a differencing operation. Second, stationary of regressors is assumed in the derivation of standard inference procedures for regression models. Non-stationary regressors invalidate many standard results and require special treatment. Lastly, in co-integration analysis,
an important question is whether the disturbance term in the co-integrating vector has a unit root (Perron, 1989).

Time series data reflect a process that also involves trend, cycle, and seasonality. By removing these deterministic patterns, the remaining data become stationary. Regressions with high R-squares but near-zero Durbin-Watson statistics frequently occur in time series analysis. Such regressions typically are associated with the analysis of non-stationary data. The unit root tests determine the stationarity characteristics of the time series data. When stationarity problems surface in the time series data, the original data are differenced and retested. Through this process, the order of the integrated process for each data series is established. Once each data series has completed this process, the series are regressed together and tested for a co-integration relationship.

If a variable is stationary, i.e., it does not have unit roots, it is said to be integrated of order zero of $I(0)$. If a variable is not stationary in its level but stationary in its first-differentiated from, it is said to be integrated of order one or $I(1)$. More generally, the series $x_t$ will be integrated of order $d$, that is, $x_t \sim I(d)$, if it is stationary after differentiate $d$ times, so $x_t$ contains $d$ unit roots (Dickey and Fuller, 1981).

To test for stationarity in the study, the application of the Augmented Dickey-Fuller (ADF) test in an ARIMA framework will be applied to the appropriate data sets. The ADF test is a unit-root test of stationarity that involves running regression analysis. In fact, the Augmented Dickey-Fuller tests are variants of the Dickey-Fuller test. The equation for the standard Dickey –Fuller test regression analysis is as follows:

\[ y_t = p \ y_{t-1} + u_1 \]  

(20)

Where $y_t$ is the value of stochastic variable, $y$ at time $t$, $y_{t-1}$ value of $y$ at time $t - 1$, $u_1$ is the stochastic error term. Parameter of $y_{t-1}$, $p$ determines the presence of unit root. If $p = 1$, then $y$ has a unit root.
The problem associated with the ADF test is that it requires that \( u_1 \) be white noise process. Therefore the standard Dickey-Fuller regression equation is adjusted to improve the accuracy and reliability of the analysis of longitudinal data. The Augmented Dickey-Fuller test adjusts the model and permits the researcher to test for stationarity in difference with intercept only or with intercept and trend. It allows differentiation of a unit root data series from a deterministic trend data series. The ADF test is conducted within the context of three distinct models of generating process of a series \( y \) as follows:

Model (1): without any constant and trend

\[
\Delta y_t = p y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t \tag{21}
\]

Model (2): with constant but no trend

\[
\Delta y_t = \alpha + p y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t \tag{22}
\]

Model (3): with constant and trend

\[
\Delta y_t = \alpha + \beta_t + p y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + u_t \tag{23}
\]

Where \( \Delta y_t \) is the first difference of the series, \( \alpha \) is an intercept, \( \beta_t \) is a time trend, \( p \) is the number of lagged difference terms, which is determined empirically when error term, \( u_t \) becomes white noise process. A widely used method is to choose the value of \( p \) that minimizes Akaike information criterion (AIC). Parameter of \( y_{t-1}, p \) determines the presence of unit root. The null hypothesis is:

\[
H_0 = p = 0, \text{ A unit root exists in } y \text{ (i.e., } y \text{ is non-stationary)}
\]

And the alternative hypothesis is:

\[
H_1 = p \neq 0, \text{ A unit root doesn’t exists in } y \text{ (i.e., } y \text{ is stationary)}
\]
The research design for the study relies on the use of unit root analysis to establish stationarity of the data sets for the stock markets included in the study. The stationarity determinations for each of the exchanges will involve the performance of a unit root test. The ADF then will be run separately for each exchange index as stated in previous equations, where \( y_t \) is the country index (in log), and the order of \( p \) is determined by AIC. The rejection of the null hypothesis \( H_0 = p = 0 \) at 5 percent implies the absence of a unit root, which in turn implies stationarity.

**Co-integration Test**

Co-integration examines long-run relationship between a set of variables, in this case the long-run relationship between Market index and Sector Indices. Percent study applied the Johansen Maximum Likelihood co-integration test in order to find any long-term stochastic relationships.

We first examine the stationarity of all the variables using the augmented Dickey-Fuller unit root test to insure that the regression results obtained are robust. If all the variables are not stationary in the form of a unit root, the first order difference should be used in the modeling procedure. We then check for Co-integration in terms of stock prices variables. If co-integration exists among all variables, an error correction term should be added to the estimation procedure (Engle and Granger, 1987).

The Johansen Co-integration procedure firstly specifies the unrestricted \( n \)-variable VAR:

\[
x_t = \mu + \sum_{i=1}^{k} \Pi_i x_{t-1} + u_t
\]  
(24)

Where \( x_t \) is an \( n \times 1 \) vector of \( I (1) \) or stochastic variables integrated in the same order, \( \mu \) is a vector of intercepts and \( u_t \) is a vector of error terms. This equation, however, can be reparameterized in order to obtain long-term response matrix (Johansen, 1988; Johansen and Juselius, 1990):

\[
\Delta x_t = \mu + \sum_{i=1}^{k-1} \Gamma_i \Delta x_{t-1} + \Pi x_{t-k} + u_t
\]  
(25)
Equation 5.7 is now a VAR re-parameterised in error correction form, where \( \Pi = -(\Pi_1 - \cdots - \Pi_k) \) and represents the long run response matrix. Rewriting this matrix as \( \Pi = \alpha \beta \) then the linear combinations of \( \Pi = \alpha \beta_{t-k} \) will be \( I(0) \) in the existing Co-integration, where the adjustment of coefficients and the matrix is \( \Pi \) is of reduced rank. The Johansen Co-integration approach is useful to determine the rank \( r \) of the matrix, if \( r = 0 \) then all the variables are \( I(1) \) and there are no Co-integration vectors, if \( 0 < r < n \) then there are \( r \) Co-integration vectors, and finally if \( r = n \) then all the variables are \( I(0) \), there are \( n \) Co-integration vectors given that any linear combinations are stationary.

Some special features in determining a long-run equilibrium are the dynamics that influence the long run patterns. These dynamics are tested by applying the Vector Error Correction Model (ECM) which searches the temporal direction and causality of the short run dynamics. The VECM is of the form:

\[
\Delta z_t = \Gamma_1 \sum \Delta z_{t-1} + \cdots + \Gamma_{k-1} + \Delta z_{t-k+1} + \Pi z_{t-k} + \mu_t
\]  

(26)

Where \( \Delta \) denotes first differences, \( \Gamma_i \) and \( \Pi \). The long and short-run adjustments are specified by \( \Gamma_i \) and \( \Pi \). If we denote \( \Pi = \alpha \beta \) then \( \alpha \) is the speed of adjustment to disequilibrium and \( \beta \) is a matrix of long-run coefficients that represents up to \( n-1 \) Co-integration relationship and provides that \( z_{t-s} \) converges to its long-run steady state.

The theory of co-integration, which was developed by Engle and Granger (1987), has powerful statistical and economic implications. From an economic point of view, it is known that some pairs of data tend to move closely and systematically over time (for example, inflation and nominal interest rates), even though these series are individually non-stationary. Economic theory provides explanation of this equilibrium. Co-integration supports economic theory by representing a statistical characterization of such equilibrium. Therefore, co-integration is the statistical implications of the existence of a long-run relationship between economic variables.
Moreover co-integration allows us to capture the equilibrium relationship between non-stationary series within a stationary model. It is therefore a method of preempting both the spurious and inconsistent regression problems that otherwise occur with the regression of non-stationary data series.

The definition of co-integration states that if $y_t \sim I(d)$ and $x_t \sim I(c)$ but if $z_t = (y_t - \alpha + \beta x_t) \sim I(d - c)$ then $x_t$ and $y_t$ are said to be co-integrated. In other words, if $y$ and $x$ are $I(1)$ then the residuals from the regression of those series would be also $I(c)$, unless they are co-integrated. Thus if the residuals are $I(0)$, we reject the null hypothesis of no co-integrated, while if the residuals are $I(1)$, then we don’t reject the null hypothesis, which implies that $y$ and $x$ are not co-integrated.

The Engle-Granger (1987) co-integration method involves the application of ordinary least squares (OLS) regression to the data, and then applying the Augmented Dickey-Fuller test to the residuals. If the null hypothesis of no co-integration is valid, the residuals are $I(1)$ and should approximate zero. Under the alternative hypothesis of stationarity, the value should be negative to a statistically significant extent.

The Engle-Granger methodology of testing for co-integration involves two steps: in the first step, a researcher should proceed with co-integration if both variables turn out to be $I(1)$ (integrated of order one). We estimated the long-run equilibrium relationship in the following form:

$$y_t = \beta_0 + \beta_1 x_{t-1} + e_1$$  \hspace{1cm} (27)

In the second step, the estimated residuals ($e_t$) will be tested for stationarity. We had performed ADF test on the residuals to determine their order of integration as follows:

$$\Delta e_t = \alpha + \beta \Delta e_{t-1} + \sum_{i=1}^{P} \delta_i \Delta e_{t-i} + u_t$$  \hspace{1cm} (28)
The null hypothesis is $H_0: \beta = 0$ if it is not rejected; one concludes that the $z_t$ contains a unit root. Thus, $y_t$ and $x_t$ are not co-integrated. A determination of co-integration is made (Enders, 1995).

There are several problems with the Engle-Granger procedure. One of them is that it is two-step procedure, so any error in estimating the error term leads to misleading results. Moreover, the results of one regression may indicate that the variables are co-integrated, while the other one suggests no co-integration. The Johansen (1988) approach avoids the use of two-step procedure and estimates for the presence of multiple co-integrating vectors based on the relationship between the rank of a matrix and its characteristic root or Eigen values (Gilmore and McManus, 2002).

The methodology developed by Johansen (1991, 1995) involves multivariate test that is based on the specification of an initial VAR and the establishment of a corresponding vector error correction model (VECM). For any set of $n$ variables, each of which is $I(1)$, there can be up to $n - 1$ separate independent relationships among these variables. The approach in the present study is to consider the VAR model of the form:

$$y_t = u + A_1 y_{t-1} + \ldots + A_p y_{t-p} + \epsilon_t$$  \hspace{1cm} (29)

Where the $y_t$ is $n \times 1$ vector of $n$ time series variables, each of which is integrated of order one, $u$ is $n \times 1$ vector of intercept, $\epsilon_t$ is $n \times 1$ vector of error terms at time $t$, and $p$ is order of VAR. this VAR model can be represented in VECM as follows:

$$\Delta y_t = u + \Pi A_1 y_{t-1} + \sum_{j=1+1}^{p-1} \Gamma_j \Delta y_{t-j} + \epsilon_t$$  \hspace{1cm} (30)

Where

$$\Pi = \sum_{i=1}^{p} A_i - 1 \quad \text{and} \quad \Gamma_i = - \sum_{j=i+1}^{p} A_j$$  \hspace{1cm} (31)
The number of Co-integration vectors can be obtained by determining the significance of the characteristic roots $\Pi$, which can be identified by trace and maximum Eigen value test as follows:

$$
\lambda_{\text{trace}}(r) = -T \sum_{r=r+1}^{k} \ln (1 - \lambda)
$$

$$
\lambda_{\text{max}}(r, r + 1) = -T \ln (1 - \lambda_{r+1})
$$

Where $\lambda_{i}$ equal the estimated values of the characteristic root (Eigen values) obtained from the estimated $\Pi$ matrix, $r$ is number of Co-integration relations ($r = 0, 1, k - 1, k$) is number of variables in the system, and $t$ is number of observations. The trace test evaluates the null hypothesis that the number of Co-integration vectors is less than or equal to $r$ against the alternative hypothesis (Johansen, 1995).

Assuming that stock price are determined to be $I(1)$, the Johansen (1998) co-integration test will be applied to measure bi-variate and multivariate co-integration among stock markets included in the study. The vector auto-regression (VAR) element of the co-integration analysis will be lagged by finding the optimum number of lags empirically. The null hypothesis holds that no co-integration exists. The criterion for the reject of the null hypothesis in all instances will be a determination of statistical significance at 0.05.

Co-integration will be tested for each pair of markets among the eight selected stock market. When the stationarity tests permit the rejection of the null hypothesis (non stationarity) for both markets in a pair, it will be proceed to co-integration testing, which will be performed through the application for vector auto-regression (VAR) analysis testing the relationship between the two stock markets. The vectors tested for co-integration will be $(z(t, 1))$ for market one in a pair and $(z(t, 2))$ for market two in a pair. If the null hypothesis holds, then no Co-integration exists between the vectors, while if the alternative hypothesis holds, then Co-integration does exists. Co-integrated variables, if disturbed, will not drift apart from each other and thus possess a long-run equilibrium relationship. A finding of co-integrated of
stock markets implies that the values of the markets will not vary greatly over the long term.

**Granger Causality Test**

Granger (1969) proposes a method of describing the relationship between some variables in order to observe the direction of causality. Consider the variables: $X_t$ and $Y_t$, the Granger-causality test can be applied as follows:

$$Y_t = \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{j=1}^{p} \beta_j X_{t-j} + \mu_t \quad (34)$$

Where the restricted model is:

$$Y_t = \sum_{i=1}^{p} \alpha_i Y_{t-i} + \nu_t \quad (35)$$

Where $\mu_t$ and $\nu_t$ are white noise, $p$ is the order of lag $Y$, and $q$ is the order of lag $X$. The null hypothesis for equation (5-18) is:

$$H_0 = \sum_{i=1}^{q} \beta_i = 0 \quad (36)$$

Suggesting that the lag terms $X_t - j$ does not Granger cause $Y_t$ in the regression. The hypotheses are tested using an F-test.

The rejection of the null hypothesis will establish the co-integration of the vectors of the daily market-value weighted values of the two exchanges in the pair tested. Establishing the presence of the co-integration, however, does not establish a causal relationship between the two indices. Determining the presence or absence of a causal relationship between movements in the two indices in a pair will rely on the application of the Granger causality test. The Granger test for causality involves using F-tests to test whether lagged information on a variable $y$ provides any statistically
significant information about a variable \( x \) in the presence of lagged \( x \). If not, then “\( y \)
does not Granger-cause \( x \).”

To test whether \( x \) cause \( y \), we thus proceed as follows. First, we test the null hypothesis “\( x \) does not Granger-cause \( y \)” by running two regressions:

Unrestricted regression:

\[
Y = \sum_{i=1}^{m} \alpha_1 Y_{t-i} + \sum_{i=1}^{m} \beta_i X_{t-i} + \epsilon_t
\]  

(37)

Restricted regression:

\[
Y = \sum_{i=1}^{m} \alpha_1 Y_{t-i} + \epsilon_t
\]  

(38)

Then, we use the sum of squared residuals to calculate an \( F \) statistics as follows:

\[
Y = (n - k) \frac{(RSS_R - RSS_{UR})}{q(RSS_{UR})}
\]  

(39)

Where \( RSS_x \) and \( RSS_{UR} \) are the sum of squared residuals in the restricted and unrestricted models respectively; \( n \) is number of observations, \( k \) is the number of estimated parameters in the unrestricted regression, and \( q \) is the number of parameter restrictions. If the calculated \( F \) is larger than critical \( F \) with \( k - 1 \) and \( n - k \) degrees of freedom, we can reject the null hypothesis that “\( X \) is does not cause \( Y \)” (Pindyck and Rubinfeld, 1991).

**Data Sources:**

The research is based on secondary data available with on-line sources and financial database like Capital Line, Capital charts, Industry Analysis by CMIE, journals, magazines, books, RBI bulletin etc.,
v. SCOPE OF THE STUDY:

- The study focuses on selected indices of Indian Stock market with particular reference to Bombay Stock Exchange.
- Firms Listed on BSE are considered for the study.
- Sensex is taken as proxy for market for computation of market return
- Treasury bill rate would be considered as risk-free return for that particular year.
- The study is for the period of January 2000 to December 2009.