CHAPTER - 1
INTRODUCTION

1.1. Introduction:

The study of algorithms is at the very core of Computer Science. For algorithm practitioners, graphs bear a special connotation. Graphs represent most of the real life problems in a framework easy to represent and manipulate by a computer. Algorithms on graphs are worth studying not because they only solve some complex problems of graph theory but because they also give a general approach for the solution of a wide spectrum of computer algorithms. For example, so called intractable problems of computer science arise very often with algorithms on graphs. For these graph algorithms which are intractable in the sense of requiring exponential time to solve, it is felt that discussions on their design and analysis will be of much help for both pure graph theorists and computer scientists.

Intractable graph algorithms are generally classified into two categories. The first category contains those problems which require generation of an exponential number of subgraphs. Obviously no polynomial bounded algorithm can be found in this case. These problems include generation of all spanning trees, all circuits, all cutsets, all paths, all cliques etc. The present thesis discusses three of the above problems. These are: generation of all spanning trees, circuits and cliques. The graphs considered are always nondirected, simple, and connected.
The second category of problems include classical problems like travelling salesperson, determining isomorphism, clique decision problem, chromatic number problem, Hamiltonian circuit problem etc. All problems of this category have no polynomial bounded algorithms for their solution and unfortunately nonexistence of such algorithms is yet to be proved. One such problem namely chromatic number problem or node coloring problem is chosen for discussion in the thesis.

It is worth mentioning here that regardless of the speed of the computer, the brute force technique can hardly solve any graph theoretic problem of practical interest. The purpose of the thesis is to suggest improvement of existing algorithms and design of new algorithms on graphs which can handle larger graphs within the constraints of the power of the existing computer. Next, we give the chapterwise break-up of the thesis.

1.2. Contents of the Thesis:

The thesis contains altogether 6 chapters including this introductory chapter. Chapter 2 contains input output graph representation and generation of them by computer. Eventually these graphs will serve as input graphs in the rest of the chapters. Chapter 2 also contains discussions on the design approach of the graph algorithms and their analysis techniques.

The remaining four chapters 3, 4, 5 & 6 contain discussions on four distinct graph algorithms. Chapters 3, 4, 5 contain discussions on three graph theoretic problems for which it is simply not possible to have polynomial
bounded algorithm. These algorithms require generation of exponential number of subgraphs. The problems discussed in these chapters are:

(i) **Chapter-3**: Generation of all spanning trees of a non-oriented graph: Since the number of trees of a $n$ vertex graph may be as high as $n^{n-2}$ the algorithms for this problem requires at least $n^{n-2}$ units of time.

(ii) **Chapter-4**: Finding all circuits of a non-oriented graph: A graph of $F$ fundamental circuits consists of $2^F$ circuits and hence its time complexity is of the order of $2^F$ time units.

(iii) **Chapter-5**: Finding all cliques of a non-oriented graph: Number of cliques of a graph is sometimes as high as $3^n$ and hence any algorithm will require time proportional to $3^n$ to generate all of them.

The pattern of discussions in these three chapters can be grossly narrated in the following steps:

(a) Classification of existing algorithms if possible.

(b) Discussions on some existing algorithms which lead us to the design of more promising algorithms.

(c) Discussion on the complexities of the algorithms.

(d) Test results with discussions.

Chapter 6 contains discussions on node coloring algorithms. The algorithms are approximate and in this chapter attempt is made to give better heuristics depending upon the degrees of nodes and structure of a graph to color its nodes with minimum number of colors. This chapter is singled out because the problem belongs to classical NP complete problems.
1.3. Conclusion:

The strategy and philosophy of the algorithms discussed in the thesis have special bearing to the algorithm design in general. The studies, it is hoped, will give us foresight to design graph algorithms of some special class which are of interest in the algorithms research of the present time. The algorithms have remarkable application areas like scheduling problems, information storage and retrieval problems, switching theory, computer aided network analysis and synthesis etc.