Chapter 4

Sierpinski Carpet Fractal Patterned Dielectric Resonator Antenna (SCFDRA)

4.1 Objective
This chapter aims at investigating the behaviour of Sierpinski Carpet fractal dielectric resonator antenna. In the first half of this chapter, a wideband Sierpinski carpet patterned cylindrical Dielectric Resonator Antenna (DRA) operating in the X-Band is presented. This DRA is realized from low cost Teflon. Antenna design methodology is discussed along with its resonance and radiation characteristics.

In the second half, a wideband Sierpinski Carpet fractal patterned rectangular Dielectric Resonator Antenna (DRA) operating in the X-Band which is characterised with Dielectric Waveguide Model (DWM) method is presented. For validation of simulated results, a prototype was realized. A comparison between conventional rectangular DRA and fractal shaped rectangular DRA of first iteration and second iteration and radiation characteristics of these structures are discussed.

Section A

4.2 Sierpinski Carpet Patterned Fractal Cylindrical Dielectric Resonator Antenna

4.2.1 Introduction
Dielectric resonator antennas (DRAs) have been subjected to many investigations since their introduction in 1983 [1]. The DRA is useful for high frequency applications where Ohmic losses become predominant for conventional metallic antennas. In addition, they offer higher bandwidths and gain when compared with microstrip patch antennas. Over the past few years researchers have tried to improve the impedance bandwidth of these DRAs to increase its functionality. Wideband DRA has been demonstrated for cylindrical DRA (CDRA) by Chair et al. [2] and for rectangular DRA by Li and Leung [3]. Systematic analysis of improving bandwidth using this mode merging technique has been reported by Young and Long [4]. In [4] it was revealed that the actual reason for wideband operation of CDRA was due to unequal rate
of variation in resonant frequency of the TM110 and TM111 modes with change in radius to height ratio as shown in Figure 4.1 illustrating an edge fed CDRA.

Guha and Antar [5] reported a new design by using a set of four CDRAs symmetrically arranged around a centrally located coaxial probe covered by a dielectric rod to achieve wideband performance by merging closely spaced resonances. In [6] Walsh et al. investigated a set of three different configurations of CDRA like the stacked, core plug embedded and embedded stacked respectively to achieve wideband resonance characteristics. It was reported in [6] that for core plug embedded CDRA maximum impedance bandwidth could be achieved by simultaneously making the embedded plug region dielectric constant as one and ensuring the maximum occupancy of the plug region. Keefe and Kingsley [7] reported RF range liquid DRA that uses water as dielectric which adds a new range of dielectric antenna research. Lai et al. [8] revisited the radiation efficiency of DRA and using Wheeler cap technique and confirmed that radiation efficiency of DRA is much higher than microstrip antenna millimetre wave frequencies. Recently integration of DRA with other circuits for on chip application using silicon substrate was reported in [9].

Chen et al. has proposed a miniature dual band dielectric resonator antenna with a parasitic c-slot fed by microstrip line [10]. The proposed antenna achieved bandwidth of 3.3% & 4.3 dBi at 2450 MHz and 5.7% & 3.8 dBi at 5640 MHz.

The enhanced radiation characteristics of a cylindrical DRA are presented by A. Singh and Satish K. Sharma [11] by employing the dual coaxial probes in the differential feed arrangement. The DRA offers an impedance bandwidth of 68% and broadside radiation patterns with good gain and low cross polarization levels. In comparison to this, a similar single probe fed DRA provided 82% bandwidth but with the mix of the omni-directional and directional radiation patterns with high cross polarization levels.

Liboli Z. et al. presented a broadband dielectric resonator antenna, formed by carving out notches from cylindrical geometry to form the bowtie shape and fed by coaxial probe on one of the notched sides [12]. The proposed bowtie DRA combines shape deformation with low permittivity resonator achieved impedance bandwidth of 49.4% covering frequency range of 4.194 - 6.944 GHz. The simulated and measured radiation patterns are consistent throughout the operational bandwidth.

In this work we have presented a rule based method to achieve wideband performance by drilling out Sierpinski carpet fractal patterned regions from a CDRA. Section 4.2.2 deals with antenna design methodology and parametric study. This is followed by resonance and radiation characteristics in Section 4.2.3.
4.2.2 Antenna Design and Parametric Study

The group at University of Houston led by Liang C Shen reported their preliminary studies on cylindrical dielectric resonators as radiators in 1983 [1]. An illustration of the antenna geometry and feeding technique is displayed in Fig 4.2. Resonant frequency based on magnetic wall boundary condition for $TM_{110}$ mode was put forward as given by

$$f_{TM_{110}} = \frac{1}{2\pi a} \sqrt{\frac{\mu}{\epsilon}} \sqrt{X_{11} + \left(\frac{\pi a}{2d}\right)^2}$$  \hspace{1cm} (4.1)

Figure 4.1. Geometry of a typical cylindrical dielectric resonator antenna. The main design dimensions are the radius “$a$” and height “$h$” as indicated.

Figure 4.2 Cylindrical DRA with probe feed. [1] © IEEE 1983.
In (4.1) $X'_{11} = 1.841$, radius of the cylindrical resonator antenna is denoted as ‘$a$’ where as height of the cylindrical DRA is given by ‘$d$’. The semi-analytical results on radiation pattern match well with experimental ones. Investigations on impedance characteristics revealed that the input impedance depends on the resonance of the cylindrical resonator as well as on the probe feed. The feed probe length affects the magnitude of the impedance at resonance and the dimension of the cylinder influence the resonant frequency.

The proposed antenna is analyzed and parametrically studied using CST Microwave Studio™. As discussed in the previous section that a wide bandwidth can be achieved by proper merging of closely spaced modes, similarly the first design step was to parametrically vary radius to height ratio for a CDRA with $\varepsilon_r = 2.1$ (Teflon). The radius of the Teflon rod based CDRA is taken as 19 mm. For a height of 24 mm the closely spaced resonances merged resulting in a wide impedance bandwidth as shown in Figure 4.3.

![Figure 4.3](image)

**Figure 4.3.** Simulated S11 (dB) with different DRA height (h).

This is followed by drilling out from the centre a cylindrical region of radius $a'$ which is 1/3 the radius “$a$” of the original cylinder. This CDRA is a first iteration Sierpinski carpet fractal patterned CDRA with edge feed which is named as SCFCDRAEFI1. Similarly the second iteration is created by further drilling out eight cylinders of radius $a''$ where $a''$ is one third of $a'$. This is named as SCFCDRAEFI2 and its return loss plotted against frequency is shown in Figure 4.4. There is a slight improvement in impedance bandwidth as compared to the first iteration and the original CDRA.
Figure 4.4. Comparison of S11(dB) plot of a typical CDRA with first and second iteration Sierpinski carpet fractal CDRA with SMA probe feed at the circumference.

Figure 4.5. Variation in S11 curves for different pin lengths in circumference fed Sierpinski carpet fractal CDRA of second iteration.

The excitation is a Z-directed coaxial probe with the length “l” mm and the radius $r = 0.635$ mm. The probe is located at the edge circumference of Cylindrical DRA and connected to a SMA connector. The probe pin length “l” is parametrically varied to observe its effect on resonance frequency. It is seen from Figure 4.5 that the resonance frequency decreases as probe pin length increases. A probe pin length of 6 mm was chosen for which 10 GHz centre frequency could be achieved.
The depth of drill “$d_{\text{drill}}$” is also parametrically varied to observe its effect on impedance bandwidth as shown in Figure 4.6. It is observed that increasing the depth of drill improves impedance bandwidth. When the depth of drill is equal to height of DRA the impedance bandwidth is maximum.

![Graph showing variation in S11 (dB) curves](image)

**Figure 4.6.** Variation in S11 (dB) curves after varying the depth of the drilled out region in Sierpinski carpet fractal CDRA of second iteration.

To further improve the bandwidth it was desirous that different resonances must be properly excited within the SCFCDRAEFI2. This could be achieved by shifting the position of feed from the circumference to an inner point. This new design is named as inset fed second iteration Sierpinski carpet patterned CDRA which in short form is written as SCFCDRAEFI2. To insert a probe in a simple CDRA holes need to be drilled. Here, as a cylindrical region of radius $a'$ is already removed from the original CDRA for designing SCFDRAEFI2 so it is easy for inserting the coaxial probe pin in the inner circumference as shown in Figure 4.7.

![Diagram of proposed antenna](image)

**Figure 4.7.** Illustration of the proposed antenna (a) side view and (b) top view.
To further study the effect of the drilled out regions the holes were filled with dielectric materials and the effect on resonance behaviour was observed. It is seen from Figure 4.8 that the maximum bandwidth and proper frequency tuning is achieved using air i.e. by simply removing the drilled out material.

4.2.3 Resonance and radiation characteristics

The proposed antenna is analyzed using CST Microwave Studio™. From the S11 (dB) plot of the initial designs of the antenna as shown in Figure 4.4 it is seen that there is an improvement of approximately 6% in impedance bandwidth after first iteration and improvement of approximately 14% after second iteration is achieved. The impedance bandwidths of the individual designs are also tabulated in Table 4.1.

The simulated S11 (dB) of the inset feed Sierpinski carpet fractal CDRA, as illustrated in Figure 4.7, is given in Figure 4.9. The impedance bandwidth is calculated to be 50%.

Table 4.1. Impedance bandwidth of various designs of fractal based CDRA SCFCDRA:
Sierpinski carpet fractal cylindrical dielectric resonator antenna.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Impedance Bandwidth (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDRA</td>
<td>4.1967</td>
</tr>
<tr>
<td>SCFCDRAEF1</td>
<td>4.4262</td>
</tr>
<tr>
<td>SCFCDRAEF2</td>
<td>4.7541</td>
</tr>
<tr>
<td>SCFCDRAIF2</td>
<td>5.0</td>
</tr>
</tbody>
</table>

EF: Edge feed; IF: Inset feed.
The E-field distribution inside the fractal DRA (FDRA) is illustrated in Figure 4.10. for three frequencies at 9 GHz, 10 GHz and 11 GHz. At lower frequency of 9 GHz it is observed that the field is confined within the DRA confirming that it is the fundamental mode. With a definite pattern of drilled off region which in this case is a fractal geometry, the resonance frequencies could be fine tuned resulting in a wide impedance bandwidth.
The radiation pattern in E-theta for Phi = 0° and Phi = 90° are shown in Figure 4.11. A similar radiation characteristic is observed within the band in both cut planes. The simulated antenna peak gain against frequency of inset fed Sierpinski carpet fractal CDRA of second iteration is shown in Figure 4.12. The simulated peak gain varies from 5.25 dBi to 6.25 dBi within the band. At the resonant frequency 10 GHz the peak gain is 5.511 dBi. The computed efficiency is found to be above 90%.

Figure 4.11. Simulated radiation patterns: E-theta pattern for Phi = 0 deg at (a) 9 GHz; (b) 10 GHz and (c) 11 GHz. E-theta for Phi = 90 deg at (d) 9 GHz; (e) 10 GHz and (f) 11 GHz respectively.

Figure 4.12. Simulated antenna peak gain of inset fed Sierpinski carpet fractal CDRA of second iteration against frequency.
Section B

4.3 Sierpinski Carpet Patterned Fractal Rectangular Dielectric Resonator Antenna

4.3.1 Introduction

Present and future wireless communication systems need antennas capable of supporting higher data rates and increased user densities. It has renewed interest in development of wideband and multiband antennas with spatial ability. Newer services require greater channel capacities and therefore they are continuing to move into higher frequency bands. Due to their small size, planar microstrip antennas are common choice as radiators in mobile communication devices. Ohmic and surface wave losses increase with frequency in case of the microstrip antenna. Besides, they are inherently narrowband. On the other hand, DRA proposed in 1983 [1], has several advantages like low Ohmic loss (due to the absence of conductors), low cost, small size and wider bandwidth when compared to microstrip antennas [13]. Its resonant frequency is a function of size, shape and material permittivity. Traditional mono-modal DRAs were not suitable for wideband application due to their bandwidth limits. The major problem of DRA in comparison to microstrip antenna is its lower gain. Therefore, engineered configurations had been proposed to enhance their gain [14] and bandwidth [15 -18].

Drilling off a tunnel in a rectangular DRA reduces the Q-factor and hence improves the impedance bandwidth [19]. Coulibaly et al. achieved broadband using microstrip-fed dielectric resonator antenna for X-band [20]. This antenna suffers from periodic mismatch, particularly at high frequency end. However, the air-gap introduced due to the tunnel adversely affects the resonant frequency and impedance characteristics [21-24], when the volume of the dielectric is low in X-Band frequencies and above. Also, air gaps usually produce undesired effects on the antenna characteristic impedance [25]. Use of liquids has been suggested to avoid air-gap losses [26].

Broadband antennas have also been obtained exploiting the self-similarity property of fractal geometry [27]. Authors have reported preliminary findings of using fractal geometry in DRA [28-29] for broad-banding. Simulated study on modifying the boundary of rectangular DRA using fractal concept has also been reported for Wi-MAX application [30].

In this section 4.3 we use the concept of fractal geometry in rectangular DRA to achieve broadband while maintaining gain. To achieve this we drill off holes in Sierpinski carpet pattern from rectangular DRA. For design and optimization, we have done a parametric study using commercial code CST Microwave Studio. One of the simulated structures has been
realized as a prototype for experimental validation. Section 4.3.2 describes the proposed structure along with simulation, parametric studies and experimental results in details. This is followed by results and discussion in Section 4.3.3.

4.3.2 Antenna Design and Parametric Study

A solid rectangular DRA is designed to resonate at X-band frequency commonly used in RFID. A probe fed rectangular DRA with dimensions $a$, $b$ and $d$ is illustrated in Figure 4.13.

![Figure 4.13. A probe fed rectangular DRA.](image)

Using DWM model, the $TE_{y_{mnl}}$ mode resonant frequency $f_0$ of the DRA can be given in (4.2).

$$f_0 = \frac{c}{2\pi\sqrt{\varepsilon_r}} \sqrt{k_x^2 + k_y^2 + k_z^2} \quad \text{------------------- (4.2)}$$

where

$$k_z = l\pi / d \quad \text{------------------- (4.3)}$$

$$k_x = m\pi / a \quad \text{------------------- (4.4)}$$

$$k_y \tan \left( \frac{k_y b}{2} \right) = (\varepsilon_r - 1)k_0^2 - k_y^2 \quad \text{------------------- (4.5)}$$

In (4.5) $k_0$ is the free space wave number, $c$ is the speed of light in vacuum, and $k_x$, $k_y$, $k_z$ are wave number inside the DR in the three directions with $k_x^2 + k_y^2 + k_z = \varepsilon_r k_0^2$. The subscripts $m$, $n$, $l$ of the $TE_{y_{mnl}}$ denote the number of extremes in the $x$, $y$, and $z$ directions respectively. The most fundamental mode is the $TE_{y_{111}}$. 
Figure 4.14(a) shows the antenna configuration. In terms of Sierpinski carpet, this is the so-called zeroth order iteration. The DRA has a rectangular cross section, with the parameters given by length, \( a = 13.50 \) mm, width, \( b = 25.00 \) mm, dielectric thickness, \( d = 22.00 \) mm using dielectric waveguide model (DWM) method [1] and dielectric constant of \( \varepsilon_{\text{DRA}} = 2.1. \) Excitation to DRA is done by a SMA probe pin of length, \( l = 6 \) mm which is placed at the centre of the broad wall (Fig. 4.14). To generate the Sierpinski carpet fractal patterned DRA rectangular regions in the shape of Sierpinski carpet were removed from the solid rectangular DRA. In this section SCFDRAI1 & SCFDRAI2 represent respectively the SCFDRA of iteration 1 and SCFDRA of iteration 2 shown in Figures 4.14(b) and 4.14(c). The SCFDRAI1 is generated by removing a rectangular region from the centre of the dimension \( a' = a/3 = 4.50 \) mm and \( b' = b/3 = 8.33 \) mm. The SCFDRAI2 is created by removing out eight more rectangular holes of dimensions \( a'' = a'/3 = 1.50 \) mm and \( b'' = b'/3 = 2.78 \) mm in the shape of Sierpinski carpet (Fig 4.14).

For simulation, generating rectangular holes in the DRA to obtain SCFDRAs is not difficult. But, for fabrication, it is difficult to realize rectangular holes using circular drill bits with the available facilities at our labs. However, realizing circular holes is not difficult. For this determination of equivalent radii of the holes is necessary. As discussed already, one of the advantages of a hole in the DRA is that it reduces the Q-factor and hence increases the bandwidth. This in effect, depends on the volume of the hole. So, we now assume that to maintain the same bandwidth, the volume of the circular hole shall be equal to the volume of the rectangular hole. Using this principle, the radii (\( r' \) or \( r'' \)) of circular holes in the two iterations are obtained. After comparing the simulated characteristics of the SCFDRA with rectangular and circular holes, the SCFDRA with circular
holes are fabricated for measurements. For the SCFDRAIs, the first iteration cylindrical hole radius is $r' = 4.5$ mm and the second iteration hole radius is $r'' = 1.5$ mm (Fig. 4.15).

![Figure 4.15. Top view of SCFDRAI2 with rectangular and circular holes with side feed.](image)

The first simulation is that of the DRA with optimized dimensions to merge $TE_{111}$ and $TE_{113}$ modes for broadband operation. It is followed by simulations of SCFDRAI1 and SCFDRAI2, which embed rectangular holes in fractal pattern. The final set of simulations is for SCFDRAI1 as well as SCFDRAI2 embedding circular holes in fractal pattern. These simulations also include parametric studies on the drill-depth, feed-pin length and feed placement.

**Figure 4.16** compares the $S_{11}$ variation for RDRA, SCFDRAI1 and SCFDRAI2 with rectangular holes, when they are fed with a SMA probe on the outer circumference. Impedance matching is better for SCFDRAI1 as well as SCFDRAI2 than the RDRA. It is also evident that the bandwidths of SCFDRAI1 and SCFDRAI2 are higher than that of the RDRA by at least 14%. Besides, the SCFDRAI2 shows marginal increment in bandwidth with better matching than SCFDRAI1.

![Figure 4.16. Comparison of $S_{11}$ (dB) plots of typical rectangular DRA with first and second iteration Sierpinski carpet fractal RDRA as shown in Fig. 4.13.](image)
The effect of the drill depth ‘$d_{DRILL}$’ on impedance bandwidth is shown in Figure 4.17. It is observed that impedance bandwidth increases with the drill depth. Also it is observed that with increased drill depth, mismatch is of incremental value. Impedance bandwidth is maximum when the drill depth of is equal to height of rectangular DRA with best matching. It probably confirms our assumption that introduction of vertical air-gaps can also increase the bandwidth. Also the figure seems to indicate that the bandwidth is approaching a saturation value with increasing gap volume.

![Figure 4.17](image)

Figure 4.17. Variation of $S_{11}$ (dB) with frequency for different drill depths of SCFDRAI2 having rectangular holes.

The effect of probe pin length ‘$l$’ on resonant frequency, matching and bandwidth is observed in Figure 4.18. It is seen that the resonance frequency decreases with increase in probe pin length. A probe pin length of 6 mm was chosen for achieving 10 GHz centre frequency. It is also seen that as the pin length increases, initially matching improves but then starts to degrade gradually. Furthermore, the impedance bandwidth decreases with increasing pin length. The reasons for such behaviour may be the inductive nature of the feed probe, which increases with the feed length.
The next simulation on SCFDRAI2 with rectangular holes is on the feed probe placement effects on impedance bandwidth. In this configuration, the feed probe placement is shifted from the outer circumference to innermost circumference (Fig. 4.19). It is termed as SCFRDRAIFI2.

In this structure, a bandwidth of 35 % is achieved as indicates in Figure 4.20. It also indicates that feed probe placement on outer circumference outweighs the placement on inner circumference in terms of impedance bandwidth.
Figure 4. 20. S11 (dB) of inset fed Sierpinski carpet fractal RDRA (SCFRDRAIFI2) of second iteration.

The next simulation is to verify the equivalence of SCFDRA with rectangular and circular holes. In this simulation, the feed probe is on the outer circumference of the DRA. The S11 parameters of SCFDRAI2 with rectangular and circular holes are compared in Figure 4.21. It is observed that the values follow each other very closely, indicating that our assumption of obtaining equal bandwidth with removal of equal volume of material from the RDRA.

Figure 4.21. Variation of S11 for rectangular shaped holes and circular shaped holes in circumference fed Sierpinski carpet fractal RDRA of second iteration.
4.3.3 Results and Discussion
A prototype of the SCFDRAI2 with circular holes was fabricated and tested. The inset in Fig. 4.22 shows the fabricated antenna. The DRA is made of 22.0 mm thick Teflon substrate of dielectric constant 2.1. The other parameters of the fabricated antenna are: $a = 13.50$ mm, $b = 25.00$ mm, $r' = 4.5$ mm and $r'' = 1.5$ mm. Simulated and measured $S_{11}$ parameters shown in Fig. 4.22. The measured results are obtained using an 8722C VNA. The plot shows a good agreement between the measured and simulated results. A bandwidth of about 48% ($S_{11} < 10$dB) is observed from the measured result.

![Figure 4.22. Measured and simulated $S_{11}$ plots of circumference fed Sierpinski carpet fractal RDRA of second iteration.](image)

Measured and simulated $E_0$ patterns at 9 GHz, 10GHz and 11GHz respectively are shown in fig. 4.23. It is observed that the patterns are more or less stable across the band. The ripples and back radiations in the measured patterns are due to diffraction at the edges of the square aluminium plate acting as a finite ground plane. The measured antenna gain remains on average 7.5 dBi over the entire band (Fig. 4.24).
Figure 4.23. Measured and simulated E-theta pattern for $\theta = 90$ deg at (a) 9 GHz, (b) 10 GHz and (c) 11 GHz respectively.

Figure 4.24. Variation of measured and computed antenna gain over the entire band.
4.4 Summary
In the first section a new broadband cylindrical dielectric resonator antenna is realized using drilling off Sierpinski carpet fractal shaped holes in the original cylindrical dielectric resonator. By shifting the feed position from the circumference to an inset position it is seen that matching over a wideband covering the entire X-band becomes uniform. In this design, impedance bandwidth of 50% is obtained covering the entire X-band with similar radiation pattern throughout the band. The average peak gain within the band is about 5.5 dBi. In addition, the antenna cost is very low as an attempt has been made to realize DRA using low dielectric permittivity material like Teflon. With these features, this design of fractal CDRA is suitable for broadband wireless communication systems like long range RFID operating in X-band.

In the next section a broadband rectangular dielectric resonator antenna realizable from Teflon is presented which is of a low cost and of small dielectric permittivity material. The proposed design obtains wide bandwidth by drilling holes into the solid rectangular DRA in a manner that the surface resembles a Sierpinski Carpet. Parametric studies have been carried out for optimizing the design. For fabrication simplification the square holes of Sierpinski carpet were replaced by circular holes such that volume of drilled out material is same. Simulation showed that the modified Sierpinski carpet DRA had similar performance as the one which had square shaped holes. A prototype has been fabricated and measured for validating the simulation. Measured and simulated results show bandwidth of 48% for $S_{11}$ less than 10 dB. Also, $E_{\theta}$ consistency is maintained over the band of measured & simulated frequency. A measured gain of 7.5 dBi over 9.0 - 11.5 GHz has been obtained in this band. The experimental and simulated results were in close agreement. These different features make the proposed antenna a suitable candidate for wireless communication in X-band.

References


